Physics of Materials

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Lecture No. # 03
Thermal Expansion

Hello welcome to this class on the physics of materials course, we have already gone through a couple of classes. We went through the range of topics we will do and the level of detail we will do it in. We also in the last class looked at a variety of properties as our first step towards the goals that we wish to accomplish in this course. We looked at the properties, we tried to understand, we tried to recognize that there were relationships between these properties or at least there were correlations that you could say in certain cases that materials with certain type of property would also imply that the same material would have certain range of values for another property.

So, in particular I linked the electronic conductivity to the thermal conductivity and as I mentioned the purpose of this one of the main themes in this course is that we wish to develop an understanding of where these properties originate from and how is it that you can link events or specific phenomena occurring at extremely small scales atomic level sub atomic level and so on to something that you measure at a macroscopic level in a lab.

So, this is the activity that we wish to pursue. So, in this process, as I mentioned our intention is to develop models for materials and I also mentioned that the model simply means that we will have a set of rules that we will expect the constituent of the material to obey. And as long as they obey those rules we will be able to extract from our calculations what is it that, what is the kind of behaviour that the material will therefore display at a macroscopic level.

So, this then forms the model and the implication of that model; to the extent that the model is correctly predicting those properties we can assume that our model is reasonable. So, we use a variety of inputs to figure out if a model is reasonable, including our knowledge of other aspects of that material and but, more specifically we want the final outcome of that model so match a variety of experimental data then we are satisfied that it is a good model.

What we will see as we progress through this course is that for specific properties we will be making fairly elaborate in depth models. We will evaluate those models and then to the extent that they succeed in meeting the properties or predicting the properties we will accept those models. If they do not, we will have to go through the exercise all over again and refine those models that we have developed and develop the subsequent models. So, this is the exercise we will do and from our very next class we will be getting in to one of these models in great detail. What I will like to forewarn you about is that some of the models will take few classes to reach their conclusion; it may retake three or four classes sometimes even five classes before we can develop all of the background that is required justify the background build the model and then test the model and see what is it at.

So, we are going to get into that kind of an elaborate exercise where at all stages I believe you will stay comfortable with the material that we are discussing. So, that is where we will go; as a prelude to it, today what we will do is we will look at one particular property of a material wherein we will, the kind of information we will use for the material is something that you are probable already familiar with ok. So, you may be familiar with it in different context, so we will pull that information together and say something about a particular property of a material such that and we will do this discussion such that we can sort of finish it in a single class. This will give you an idea of the kind of process that we go through although this is a much simpler process that we are going through today. So, but it is still give you an idea of you know the way in which we are correlating things and so on and then trying to extract some information out of it. So, we will do this today and it will be a sort of a prelude to all the subsequent classed that will come.

So, so this is the intent of today’s class. So, as I mentioned the properties that we highlighted last class were that any material you take could have mechanical properties, could have chemical properties, thermal properties, electronic properties, magnetic properties and optical properties. These are the major properties that we normally refer to normally evaluate materials for. So, this is the general range of properties that we are interested in.

Today we will look at a particular thermal property and see what we can understand of it.

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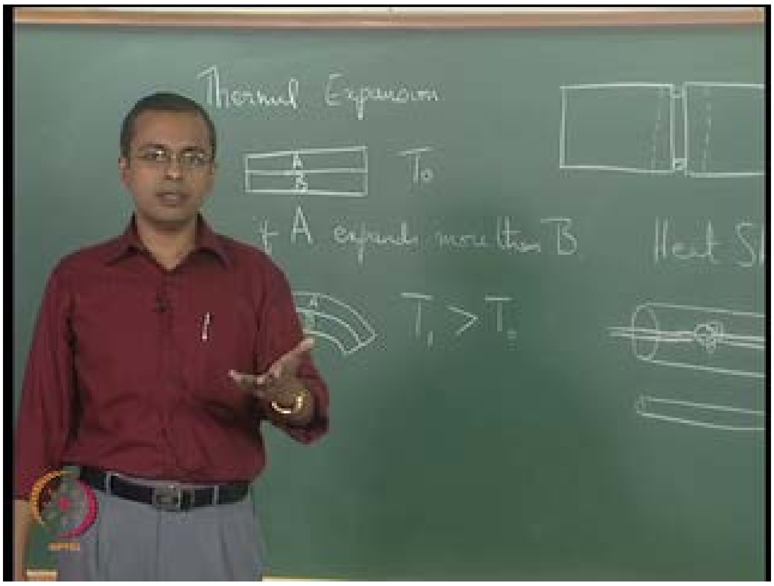

The thermal property that we are interested in today is simply the thermal expansion ok, so thermal expansion is what we are going to look at today. This is some this is the property that most of us are familiar with even from our high school days. We are thought of thought about thermal expansion and there are lot of day to day products which utilise the process of thermal expansion to carry out specific activities ok.

A simple device that does this is the bimetallic strip which you would normally see in many devices such as ion boxes possibly certain types of kettles and so on and may be to some degree in in a refrigerator in in a thermostat if if it is design based on a bimetallic strip. Where essentially you will have two different metals, so we will just say A and B; so we have two different metals and they are capable of expanding two different degrees for the same raise in temperature. So, if at a particular temperature both of them are such that their lens are equal and you see them in a horizontal disposition, so to speak. At some higher temperature, if A expands more than B then you will see the sample curve like this ok. So, we have a situation where the sample could be a straight sample or it could be a curved sample depending on the temperature of a sample fine. So, and if you so this is when at so this is at some temperature T naught, this is at some temperature T1 which is greater than T naught. So, that is the relationship between those temperatures that is what you would see for that sample fine. So, so what you see is physically the sample is changing its dimension changing its shape and therefore you could setup a switch based on it wherein, the circuit is complete or closed when this sample is straight and the circuit is open or switched off effectively when the sample is bent. So, this is how the thermostat of certain types work which use this bimetallic strip.

So, so there is there are lot of household appliances which are effectively making use of thermal expansion to satisfy the activity that they are designed for. There are also a number of places where you do not want thermal expansion so or or at least it is it is something that is not convenient to you. So, often this happens when you have a variety of dissimilar materials which are in contact with each other but, have to maintain their dimensions so that they do not put undue stress on a the neighbouring material or to for or for example, to to maintain some kind of a sealing. So, so for example, if you have two surfaces in contact and you have a gasket between them and there is a certain pressure applied on this two materials; so for example, you have a one object here and there is a gasket of some sort here and there is another object here and let say that this gasket is ensuring that seal between this these two pipe for let us say these two are pipes.

Now, if you have a situation where the due to temperature change both of these are shrinking significantly during regular use. So, let us say these pipes were intended for use at room temperature but, on that day for some reason the fluid flowing through it reaches minus hundred degree C, which is significantly lower than the temperature it was designed for. And if both these pipes contract then you will have a situation that this O ring or this ring that is or this gasket that is around this these pipes will no longer be able to ensure a seal between the two of them.

So quite simply you may have a situation, where one pipe simply becomes this long and the other simply becomes this long and suddenly this gasket is no longer able to although this is exaggerated, it it conveys the point that you know you may have a situation where the gasket no longer is in contact with either of the pipes and therefore you have a leak ok. So, there are a number of instances where thermal expansion is inconvenient for practical purposes ok. So, so like any property there are places where it is useful and there are places where it is actually hindrance. Also I wish to point out that most of us are familiar with materials that expand on heating ok. So, this is something we are commonly aware of and this is what we become exposed to early on. Just for your information sake I also want you to be aware of the fact, that there are specifically designed materials that shrink on heating ok. So, this is an interesting phenomenon, it is simply called that those kinds of materials are referred to as heat shrinks.

So, heat shrink material is what it is called, it has a very specific use and I mean you can use it for a variety of things but, one specific use is to help you seal, make good contact between wires and seal seal that wire such that there is no possibility of an electric short from it. These materials are typically polymer based materials, where what happens is you have a polymer in the form of a tube ok and it is supplied; there are various ways in which they accomplish this but one way is that this is supplied wherein the materials is still more in the monomer state, there is not much of the polymerization that has taken place have limited amount of polymerization has taken place. Normally when a significant amount of polymerization takes place that bonds becomes stronger and the material the molecules move closer as a result and often when we supply some heat the polymerization will take place. So, when you heat this cube because polymerization takes place all the atoms move closer to each other and over all the dimension of this cube decreases, so suddenly you have a same tube with the much smaller diameter.

So, this is the heat shrink material so what they would normally do is put a wire from one side and then make a knot between that wire and another wire so and then that whole two wire system will be sitting inside this and you have some joint here ok. So, some joint between those two wires and then you apply heat to this, normally the kind of device they use is something similar to hair dryer it is not exactly hair dryer they we have specific temperature that they are interested in. But in general it is something very similar to hair dryer it puts a fair amount of hot air on this region and be simply because of the hot air present there, this tube will shrink, it will form a very tight hold on the wire on this side on the wire on this side and completely cover this joint that you have made ok. So, it is a very effective way of sealing or joint between two wires, so this is one of the places where this material is in use. So, I just wanted to highlight the fact that there are materials that expand on heating and there are also materials that shrink on heating and there are specific uses for these kinds of materials.

Right, now we know of this property, we are at least aware of it in a macroscopic sense and the way we measure it is simply we write we have a certain temperature length L0 of the material at a temperature T0, we also have some other length Lt at a some other temperature Tt ok.

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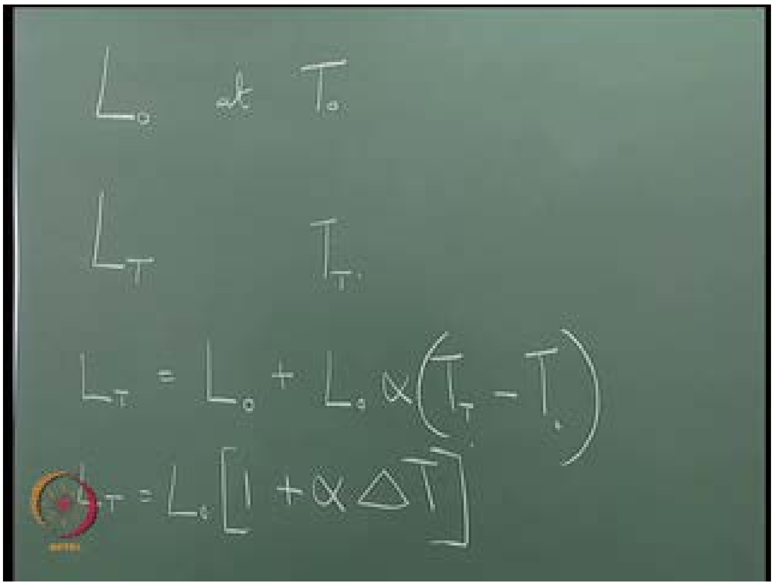

L_0 at T_0

L_T T_T

$$L_T = L_0 + L_0 \alpha (T_T - T_0)$$

$$L_T = L_0 [1 + \alpha \Delta T]$$

So, we have two different lengths in two different temperatures and so simply we have a situation where the length at temperature T is equal to the length at temperature T0 ok. So, length at L0 times this factor where L0 is still there but alpha times delta T where alpha is the linear coefficient of thermal expansion ok. So, if this is the linear coefficient of thermal expansion and you are familiar with it so, we would write it in another way write simply as ok. So, l equals L0 into one plus alpha delta T so that is your linear coefficient of thermal expansion right.

So, now we would like to understand why thermal expansion is occurring ok. So, for the purpose of our discussion we will no longer be concerned with materials such as heat shrinks, we will look at the more familiar examples that we are used to like the kinds of materials such as metallic system and so on or even we will in this case we will look at an example of an ionic crystal. But, in general we are looking at materials which we are which show the conventional behaviour which is that it expands on heating ok. So, that kind of a behaviour is what we will examine and we will try to understand why that expansion is occurring.

So, first thing we will do is let us just consider as a concept you should something expand on heating. The common idea that comes to mind is that when you heat people are often aware that the atoms present in a solid vibrate ok. So, this is something the people are aware of, if you go to 0 kelvin the atoms are stationary if you really accomplish 0 kelvin which is I mean it is only theoretically possible but, assuming you actually reach 0 kelvin the atoms become frozen they stay stationary. At any higher temperature the manner in which the solid reaches the temperature or attains the temperature is that the atoms within the solid are now vibrating in a manner that is commensurate with that temperature ok. So, that is one of the ways in which that material reaches the temperature ok, so this is the, this is what is happening.

So, if you rise the temperature the amplitude with which the vibrations occur in that solid go up right so at some level without realising it, often people think that since the vibration amplitude has gone up the material will expand ok. So, this is the general idea that people have that, this is the reason why materials expand. Now the problem with this issue with this situation is that, when a vibration occurs it is occurring both for a purpose of understanding let us say a forward direction and a reverse direction it is occurring both in the forward direction as well as in the reversed direction. When that happens, the mean position of that vibration is the same, so when the mean position of the vibration is the same on average for that solid there is no expansion that is going to occur. On average the atoms are still going to have the same position; they are simply going to get displaced more from their position in two different directions and they will come back to the mean position.

So, as long as the mean positions at of the atoms remains the same simply rising the amplitude of vibration is not going to result in expansion of the solid, this is the very fundamental concept ok. So, you need to understand that simply rising the amplitude of vibration and therefore, simply rising the temperature which causes this raise on amplitude of vibration cannot directly explain the expansion of a solid.

Therefore, we need to look deeper to understand why this expansion is occurring and understand clearly that you you cannot just explain it away by saying that amplitude has gone up of vibration ok, so that is the first concept that I would like to do become aware of. So therefore, we will now try to build some detail into how a material functions and see if you can understand thermal expansion to do this we will take the example of some ionic crystal and say say for example, k C1 or n a C,1 potassium chloride or the sodium chloride and we will see how it comes together. That is our first activity we will see how it comes together, what is the consequence of how it comes together and then based on how it has come together we will see what it implies with respect to thermal expansion; so, this is the two step process that we will go through today.

So, we will now look at this an ionic crystal either say potassium chloride or sodium chloride and we will see how this crystal is built up right. So, to do this what we need to consider is the fact that in the crystal we can think of it as containing k plus ions ok or n a plus ions and C1 minus ions.

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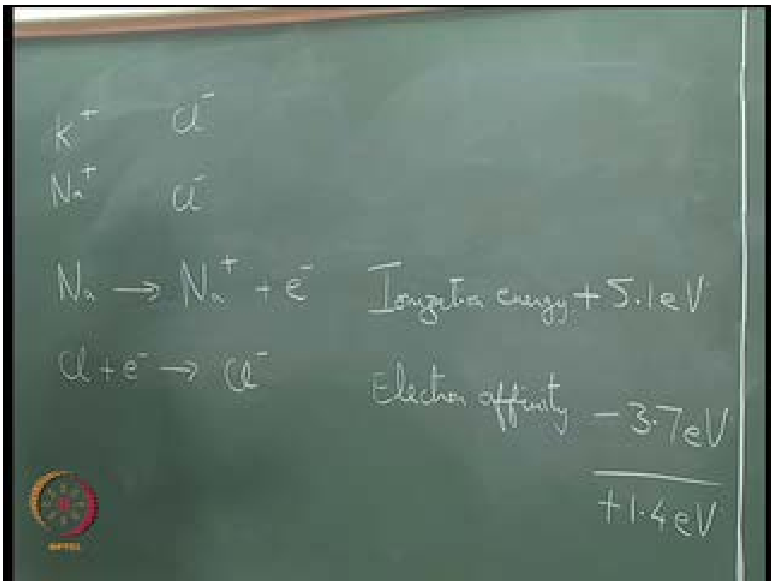

So, when you actually get the crystals, so sodium chloride is your common salt if you take it, it already has this ions in it right. So, for our understanding we will step back we will say that somehow we have separately got sodium and chlorine or chloride chlorine separately and then we would like to build the sodium chloride crystal.

So, to do that we will do we do it in stages so first we will assume that we have one atom of sodium which is available to us separately and one atom of chlorine which is also available to us separately and that these are infinitely separate from each other so at this moment they are not in a position to interact with each other ok.

So, what we will do is we will take sodium and from this we will remove an electron ok. So, I have now removed one electron from sodium and let us say I have removed the electron and taken it taken that electron infinitely away from that sodium, original sodium atom ok. So, in this process what I have done is I have ionized this sodium atom.

So, corresponding to this there is a certain amount of energy I must provide to take this electron out and that energy is referred to as ionization energy ok. So, I have to provide ionization energy to get the electron off of the sodium atom and that is of the order of about 5.1 electron volts ok. So, of the order of 5.1 electron volts is what I need to provide to get this electron off of the sodium atom right. So, this I have done independently for away, by the same and so just to also to highlight, this is energy that I have provided externally to that system.

So, if you wish to look at it from the perspective of the energy of the system or energy of the constituents in this case, the energy of that system has actually gone up ok. So, we will actually do calculation with respect to energy of the system in this case the energy of the system has gone up. So, by our convention we will simply say this is plus 5.1 electron volt, we will denote plus to mean that the energy has gone up minus to mean the energy has gone down; in general the nature nature prefers lower energy so we will move towards more and more negative energy to the extent that is possible ok, so and that is in a relative sense. So, we have plus 5.1 electron volts, independently we will take chlorine chlorine atom we will add on electron to it and we will create a C1 minus ion ok.

So, this we will do independently right now this even though I have written them next to each other they are not related to each other, I have done this these two processes independently ok. So, to do this when you do this the, there is a certain amount of energy that is released and that we refer to as electron affinity and in this case it is of the order of 3.7 electron volts per atom, in each of these cases I am talking of one pair atom this is 3.7 electron volts. In this case this process is releasing that energy because it prefers to be as a C l minus it is releasing that energy and therefore, we would call this we will that reduces the energy of the system and so we will call that as a minus 3.7 electron volt ok, so these two things have occurred independently right. So, simply generating a sodium plus ion and a chlorine minus ion requires a net energy of plus 5.1 electron volts and minus 3.7 electron volts, so if it total this this is about plus 1.4 electron volts ok. So, if those numbers based on those numbers we will proceed for our purpose because this is just a order of magnitude calculation that we are doing.

What we see is we have a plus 1.4 electron volts; in other wards simply creating a sodium plus ion and C1 minus ion and keeping them separate from each other requires us to provide a total of 1.4 electron volt. So therefore, you see this is not something that is energy energetically favourable energetically favourable means, the energy of the overall system should decrease right now it has gone up by 1.4 electron volts right. So, other processes must occur which help us to reduce the overall energy of the system so that at the end of it all we can have a crystal which has a sodium plus ions and C1 minus ions right. So, overall at that end we realise that in nature that already exist so that means there are other processes that are occurring over and above this step that we have just discussed which then enables you to reduce the energy of the system.

So, what happens is that now you have sodium plus ion and a chlorine minus ion, the process that is occurring that helps us reduce the energy of the system is that, these two attract each other and the closer you bring them to each other the energy of the system goes goes down ok. So, what we have is the plus 1.4 electron volts to create these two ions.

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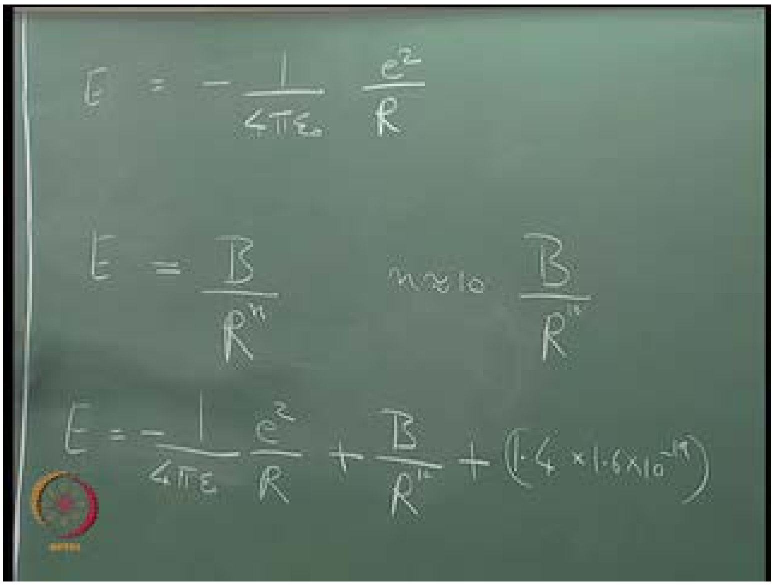

$$E = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R}$$

$$E = \frac{B}{R^n} \qquad n \approx 10 \qquad \frac{B}{R^{10}}$$

$$E = -\frac{1}{4\pi\varepsilon}\frac{e^2}{R} + \frac{B}{R^{10}} + (1.4 \times 1.6 \times 10^{-19})$$

We also have the energy due to the attraction of these ions as they move closer and closer to each other and the fact these are oppositely charged ions ok. So, this is the energy that comes due to the attraction between the two ions that are oppositely charged as they move closer and closer to each other ok. So, the decrease in energy that is occurring energy as a function of the distance between them, so when they are infinitely apart this energy due to the interaction between them is 0 ok. So, as you bring them closer and closer and closer this gets going lower and lower and lower ok, so you get more and more negative energy proportional to 1 by R that is occurring as you bring them closer and closer and closer.

So, we now have a expression which tells us that the energy of the system is now going to depend on the distance between the ions ok. So, we have done a two step process so far we have taken, separately created the sodium and chlorine ions and then we have moved them together ok, so we have attempted to move them together. So, this is the second contribution to the energy in the system ok. The, there is one other contribution to the energy of the system which we will add to this picture, which is that as you bring the ions very very close then what happens is, the inner shell electrons of the two atoms or the two ions now begin to overlap; the electron clouds of the two atoms so to speak or two ions so to speak begin to overlap. When that happens the electrons in the inner shells are now forced to go to higher energy levels because to avoid this issue of having this more electrons in the same energy levels and so on.

So, they actually end up having to go to higher energy levels and so if you push the ions indefinitely close a repulsion force of repulsion sets in which tries to put the push these ions away and that repulsion occurs because of the inner shell electrons which now begin to overlap with each other or the electron clouds which begin to overlap with each other which begins to push these atoms.

So, we have a that energy corresponding to that given by an expression which is simply, where B is some constant R power n where n is of the order of 10 ok, so of the order of B by R power so this is B by R power 10 ok. So, this is the third contribution that we have. Now this is going to rise the energy of the system because it is going to push the electrons up to higher energy levels therefore, this is a positive contribution to the system. So, if you take all of these contributions together we now have the overall energy of the system being given as ok. So, I have just corrected this last term it is it is given to you in the form of electron volts we can just convert that to joules so therefore, I have just done that correction there using 1.6 into 10 to the power minus 19 but, it does not really that is not the critical thing because that is the constant that is all you need to remember. What you need to see is that this is the positive quantity ok, so it is not really favouring the formation of this process of this crystal, this is a positive quantity this also does not favour the formation of this sodium chloride crystal, this is the negative quantity and the negative quantity as I mentioned means the energy of the system is being lower therefore, this is favourable to the formation of the crystal ok.

So, we see that there is a competition between some quantities that are negative and some quantities that are positive the crystal eventually forms and it eventually has the certain dimension etcetera associated with the due to the interplay of this positive and negative quantities. So at the end of it what happens at equilibrium is when these things balance each other out ok, or you have the minimum with minimum energy of the system is attained, that is when the crystal becomes stable.

So, we will make a small diagram using these parameters and then we will see what the diagram tells us right. So, the diagram that we will make is simply energy as a function of the distance between the ions just to reiterate R is the distance between those ions ok which which initially starts out at infinite infinite distance between infinity the distance between ions and then you bring it closer d is the constant in in this term. In the attraction term between the ions the charge in this case happens to be just a unit positive charge and unit negative charge that is why we have simply used the electron charge here that is the small e, this epsilon naught is the permittivity of free space which which I am sure you are aware of from high school days and this this R again is the distance between the ions. So, that is this term that is this term and then there then you have a constant right.

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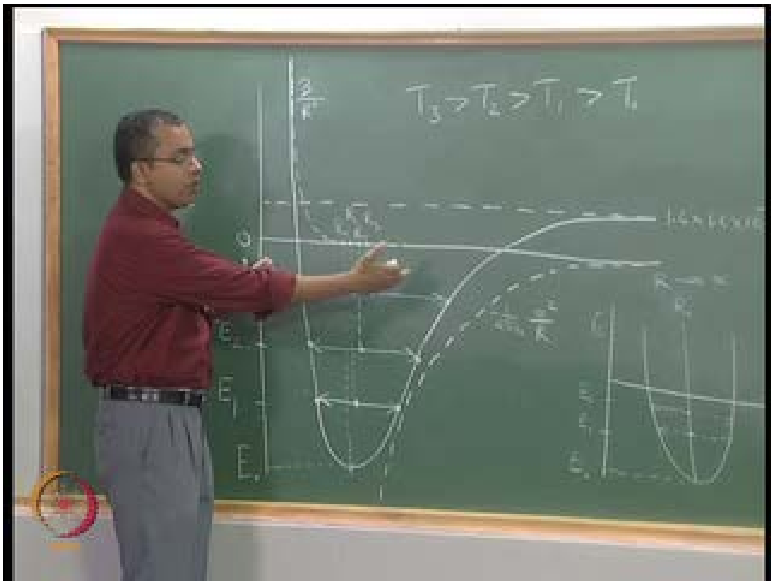

so let us take a diagram now, this is energy on your y axis and we will have this is R that is R tending to infinity ok. So, this is energy on your y axis that is R tending to infinity the position or or the distance between the ions ok. So, this is what we have; so there is no, this is just arbitrary units on the y axis just it is some linear scale that we will make use of to our convenience. This is not really drawn to scale, this is to help us understand what is happening right. So, when you are at infinity this term here is 0 R is infinity therefore, this is 0 this term here is also 0 because R is infinity so the only term you have is this constant term which is what we originally calculated right. So, we will and we see that that is some kind of a positive constant, so we will just put some positive mark here ok. So, and we will just say that that is that 1.4 into 1.6 into 10 power minus 19 right. And if you just look at it term by term this last term is independent of R, there is no R value there therefore, it is independent of R therefore regardless of the position of these two ions the contribution of that term is the same ok.

So, I am going to plot here the contribution of each term independently and then we will sum up those terms which will give you this total term here or this total expression here right; so independently we will do it first and then we will total it up. So, if you take it independently that last term there is independent of position and therefore, at all values of R the last terms' contribution is the same ok. So, this dotted line here shows you the contribution of the last term which is now independent of position ok, so that is what this is. The first term if we take is a negative term but, at R equal to infinity it is actually 0 ok, so and as you reduce R it goes as a 1 by R behaviour it shows here 1 by R behaviour, some constant times 1 by R behaviour.

So, what we actually see is a behaviour which starts at infinity in a manner that is say if this is 0 energy, 0 on your y axis this is at infinity the energy contribution due to the first term is 0, so we start at the coinciding with y equal to 0 and then as you come closer and closer and closer it shows you 1 by R behaviour. So, if you just draw this and continue this at R equal to 0 you will see minus infinity ok, at R equal to 0 you will reach minus infinity so that is the contribution of this first term that you see here this first term here the contribution of it is that when R equal to 0 it is minus infinity and when R equals infinity it is 0, so this dotted line here is the contribution of first right. So, I will just write here the the same term here minus 1 by 4 pi epsilon naught E square by R right.

Now let us look at the middle term B by R power n which is in this case b by R power 10. The, it is actually a positive quantity at R equals infinity it is also 0, so it is also going to start coinciding with the y equal to 0 position in other wards effectively coinciding with the x axis but, because of the R power 10 behaviour at even as you decrease R it initially stays closer and closer to 0 ok. But, when you go very close to 0 it starts increasing in its its impact starts increasing and you see a behaviour that is a very steep increase in positive contribution by the middle term, which is B by R power n ok.

So, this is the behaviour and you can easily understand this, so for example if there is some value of R for both of these terms and you decrease that value of R by a factor of 2 so let us go to half the value of R then when you go to half the value of R here the contribution of this term doubles because it is divided by 1 by 2 so it goes up by a factor of 2 ok, so this goes up by a factor of 2 so from somewhere here it will double the value right. In in the same kind of a situation here when you go from a certain value of R to half the value of R because it is R power 10 so you are actually going by you are rising the quantity by 2 power 10.

So, of the order of a thousand, so thousand is the kind of thousand times increase more or a even more if what you are bringing about here ok. So, so this is what you see right; so thousand time increase increase up here but there is only a double doubling the value the negative of a direction. So, now these are the three contributions right, so now what is the total of them the sum of them which is that final term that we see there. So, when you are at R equals infinity the only term that is contributing is this constant so your actual value is starts off somewhere here as you move closer and closer to the origin initially only the first term is additionally adding to this last term here ok. So, only this curve is additionally adding to this straight line here because this this one is still remain remaining very close to 0 fine. So, what you see is a behaviour that looks like this it starts getting closer and closer to the second dotted line that we just drew right. So, we start seeing this behaviour but what happens is once you cross a certain threshold you start seeing this positive value showing up here which is going up at a rate much faster than this thousand times the rate at which that is going down ok. So therefore, at some value the contribution the positive contribution from this term is significantly greater than the negative contribution from the other term; so what you see is you see a behaviour that comes down like this and then turns around and goes up ok, so this is the curve that we see ok.

So, this straight line that I have drawn here or this continuous line I should say not straight line continuous line that I have drawn here is the sum of the contributions of all of those terms, each curve term has been independently identified here this continuous line is the contribution from all of those terms right. And this straight this continuous line now is the energy as a function of position of these two ions as you state a, as you take sodium and chlorine as two independent two as two independent atoms and then from there from that as a starting point you build this crystal ok.

So, as you can imagine nature tries to attain the lowest energy that it can possibly attain and this is then the lowest energy that you can attain. So, at this point we have not taken in to account the temperature effect on the system, so assuming that this is at 0 kelvin this is the position this is the equilibrium state of this system or this is the lowest energy of the system and when I say lowest energy corresponding to that lowest energy there is a position R 0 ok. So, corresponding to the lowest energy of the system and we will say that we will we will keep our discussion at this point saying that we are starting this from 0 kelvin, at the lowest energy of the system there is an equilibrium position corresponding to that lowest energy which is R 0 in other wards if you did reach 0 kelvin you will find that the average spacing between all the sodium and chlorine ions present in that ionic crystal would be R 0, that is the lowest energy that that system is able to attain given the fact that there is an ionization energy present in the system, there is a electron affinity present in the system, there is columbine attraction between those ions and there is a repulsion between those electron right.

So, now this is the composite picture we have got the complete picture. So, and as you can see we have looked at this system from you know in this case this is an electronic contribution to the system because we are looking at an electron being removed and being added this is sort of an ionic contribution to the system because of course, it is due to the charges on this ions but, still that attraction is there at an as a two ion two ions attracting each other. So, it is slightly larger scale than what this occur, then you have a repulsion which is due to electron cloud overlap ok. So that is again at a sub atomic level.

So, you have a sub atomic contribution, you have an electronic contribution, you have a ionic contribution all of these are come together to see show you the picture of that solid of as of energy as the function of the position for that solid as the function of inter ionic position in that solid right. So, this is now the picture of what happens in the solid, so what we would now like to see is, given that we have this energy as a function of position what does it tell us about the if impact of raising the temperature of the system ok. So, that is what we would like to understand because our purpose as I said is to understand thermal expansion process so this is what we are interested in looking at so we would like to consider this. Now, we have R 0 corresponding to that we will also have an energy E 0 R 0 and E 0 are the two energy levels that we have that we are interested in fine.

So, so this is the system now let us say, we raise the temperature of the system so when you raise the temperature of the system what is happening is, the atoms are gaining that energy so when you provide energy to the system it is not an independent entity. It is that that system in in this case that solid let us say one you have a wire you have a strip made out of this particular material when it raises temperature when the temperature of it has gone up when you are able to measure a higher temperature on that system it means all the atoms present inside inside it have all also gain that energy in principle ok. That there will be a distribution and so on we are only talking of some average, so an average their energy has gone up right. So, originally at equilibrium at T equal to 0 kelvin the energy of those atoms was E 0 the lowest energy that the system could attain is there.

Now we have raised the energy of the system so it is now having a slightly higher energy level all the atoms on average are having an an higher energy level. So we will just say for example, that it has an energy level E1 at temperature T1 ok, so we start at T 0 we raise the temperature to T1 and so that is the energy level now of the atoms. What does is mean? What is it that we can understand of the system given that we have now developed this diagram and we are now imposing this additional condition on it that the temperature has gone up and therefore, the energy has gone up?

What it means is if you look at this diagram consistent with this energy there are two positions that are possible you could be here or you could be somewhere here ok. So, these are two positions I mean if you just draw the horizontal line if you draw the horizontal line correctly you will find that there there are two positions in this curve where the inter ionic separation between those ions is such that the energy they have a certain energy associated with them which is higher than the lowest energy that they could obtain is a higher energy level but, that energy level is consistent with that temperature so that temperature corresponds to some energy that energy corresponds to two values as spacing ok. So there is spacing here and there is inter ionic spacing here, those two values of spacing now are able to accommodate this level of energy.

So what happens is that the ions now are vibrating which is something that we measure write at the beginning but, the important additional detail that we are now able to say about the system is that they are vibrating between these two positions ok. So, the vibration is occurring like this between these two positions between the two positions that are consistent with this higher level of energy right. So, that is the thing that we which is now going to give us this insight into thermal expansion process right. So, this is the vibration that is occurring consistent with this new level of energy, you can raise the temperature further let us say E2 and we can again identify in this curve this position is consistent with this value of E2 this inter ionic spacing, similarly somewhere here is the next position that is also consistent with this E 2 ok.

So again we can draw this, so that arrow then indicates the vibration now that is possible so the amplitude is going up because you can see that initially initially at 0 Kelvin there was no amplitude, it is struck at the single position as you raise the temperature as you keep raising the temperature this is the amplitude this distance that is covered by this curve. So now this is the amplitude you can think of as the amplitude of this you can associate with these ions similarly, you can associate this amplitude and just to finish off we just to take one more temperature and with that we will we will see our next understanding of the system.

So, three different temperatures I have taken here T1 T2 and T3 for which the energy values are E1 E2 and E3, corresponding to them these are the amplitudes of vibration and they are all consistent with the all the rules that the solid has followed to form ok; all the all in other words the kind of to use our term, the model that we have put on the solid or we have believe that we believe that the solid is following ok, so we have reached this condition. The so so far so good, we have we understand that in in this sense it is it is consistent with our picture that the amplitude goes up as you raise the amplitude of vibration of the ions goes up as you raise the temperature of the system.

What is specifically interesting to us is the additional information that we now have which is the mean position of those atoms ok, which is not something that we spoke about when we just simply said that the amplitude has gone up. We in fact we said that you know if amplitude goes up and mean position is the same there will be no expansion of the solid right. So, if you look at this here we find that the mean position of this which is midpoint of this line here is somewhere here ok, so somewhere here is the midpoint of this line if you take this line somewhere here is the midpoint of this line and if you take this line somewhere here is the midpoint of this line ok. So, this is a hand drawn schematic if you draw it more to scale you can see it more clearly. Now if you see now look at the positions of these points the those midpoints corresponds to specific values of R right.

So, if you just take this point here, so this is then R 1 this is R 2 and this is R 3 so we have R 0 at E 0 which is at 0 kelvin at temperature T1 energy is E1 and the position mean position between the ions is R 1 at temperature T2 energy is E2 mean position between the ions is R 2, so those are all marked here and a temperature T3 which is all where you know T3 is greater than T2 is greater than T1 is greater thanT0 which is your 0 kelvin. So finally at temperature T3 energy is E3 and the corresponding mean position between the ions is R 3 all right. So, now we have a situation where we are able to see that as you raise the temperature of the system, the mean not only is the vibration amplitude amplitude of vibration of the ions going up we actually find that the mean position at the end as a result of that vibration the mean position between the ions is actually going up it is going up from the value that was R 0 at at T equal to 0 kelvin.

Therefore, the solid is now able to is now showing an expansion because the mean position between all the ions is gone up from R 0 to R 3 in this case as you go up fromT0 to T3 since the average spacing has gone up the overall solid has expanded ok. So, that is the important insight that we get that form the solid to expand the average spacing between the ions to go up only then the solid will expand all right.

So, in fact if you look back at this picture we see diagrammatically how this has happened but more importantly if you wish to correlate with all those other information that we have put together in those equations, what we find is that the form of those three terms ok; the form which is what these dotted lines here are this dotted line that is the form of this term here, this dotted line here which is the form of this middle term that B by R power 10 and the form of this which is the straight line the last term that is there which is just a constant. It is because of the form of those terms the the way in which those terms actually display their influence in the form of these curves, it is because of that that we end up with a resultant curve which has a shape that we see here. It turns out, that because of the way those terms are there, this resultant curve is not symmetric ok. So that is the important thing that we wish to identify highlight.

If you see this curve upfront I have drawn it without specifically bringing it out earlier but, you will find that the curve on this side is not symmetric with respect to the curve on this side if it take any kind of a midpoint if it take this lowest energy line if you take for example, this R 0 line this dotted line here you see more region this side than on this side of this of that line. So, this curve is definitely upfront for visual inspection it is asymmetric all right.

So, the expansion process is occurring as a direct result of the fact that this curve is asymmetric, that is a very important piece of information that we are able to now recognise as a result of the understanding that we have built into the system alright. More specifically if this curve happens to be symmetric in some hypothetical system in some hypothetical system if for example, this curve were to be exactly like this exactly u shaped curve if we had in some hypothetical system and this is at 0 kelvin. This is only a hypothetical system in some hypothetical system if this were exactly symmetric this part of the curve if it were exactly symmetric, then what will happen is even as you raise the temperature to E1 or E2, yes the amplitude of vibration will go up but the mean position of the vibration will remain the same ok. So, that is the very important piece of information to know the amplitude of vibration goes up but, the mean position remains exactly the same as a result this hypothetical material will not show you thermal expansion. Whereas, this real material where this curve is asymmetric shows you thermal expansion because of this asymmetric present alright.

So, in fact additionally we we can also extend this to see that depending on the exact material that it choose the exact ions and atoms that you choose the impact the shape of this curve can change from material to material. Materials where this is a very deep turf a very very deep turf very narrow and deep turf, will show you a very small coefficient of thermal expansion because you will have to raise the temperature quite significantly before you see a change in the at reasonably large change in the value of R. In other materials where this is a very shallow but, very widely spaced curve even for small changes in temperature you will see a very large change in the value of R the mean position. So that is how you explain the difference in coefficient of thermal expansion between materials ok, based on the kind of bonding that is present in the material the strength of the bond and so on. So, more strongly bonded materials you will find a very deep turf narrow turf, so that will show you less coefficient of thermal expansion, material which are less strongly bonded will show you a much much more shallow curve more widely positioned curve and that will show you much greater coefficient of thermal expansion.

So, this is the general idea that I wanted to cover today’s class. We have seen one full one property which is very commonly present in many engineering materials which is used in a lot of engineering situations, which is a thermal expansion coefficient; thermal expansion process. We have understood what thermal expansion is we have looked at it from a fundamental perspective, we have a built a model for the material and we have looked at it at a sub atomic level at an atomic level and at a macroscopic level and we put all these pictures together and now developed an understanding of why or how in a more fundamental sense thermal expansion occurs ok.

So, with this we will conclude today’s class in the next as I said this is the class specifically to show you, in a single class how this process could occur how this process, the kinds of detail that we go through to evolve this kind of a information. In subsequent classes when we look at other properties we may need to develop much more fundamental information to help us reach kinds of models which would give us information or which would give us greater insight into the materials and so that may take few classes for every property that we are discussing and so even though it is spread out if you look back at this class you will get an idea of how it might all fall together. So, with that we will conclude this class thank you .

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