

Reciprocal Space -2
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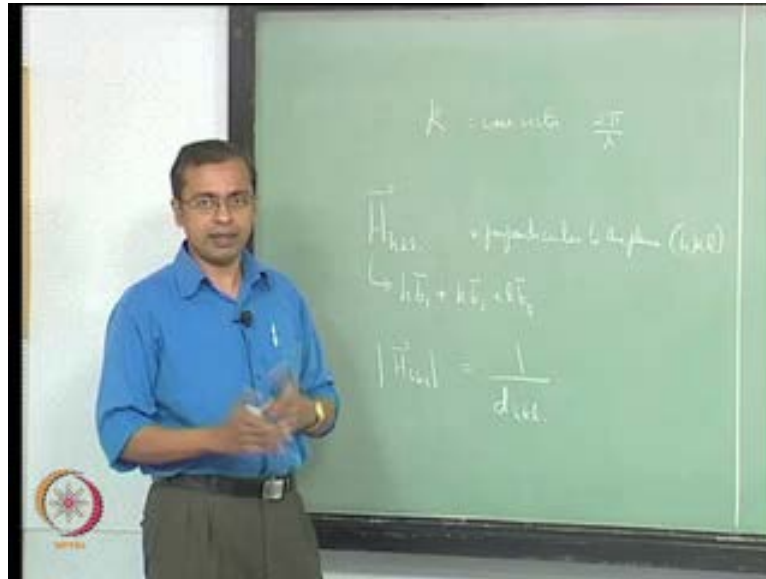
Lecture No. # 28
Condition for Diffraction

.Hello, we have now at twenty eighth class in this physics of materials course. So, in the last class we began to look at the reciprocal lattice notation, and we arrived at this topic for the specific reasons that we recognized that in the quantum mechanical scheme of things. When you described electrons in a solid, the electrons can of course show both particle like behavior as well as wave like behavior that is well established. So, within the solid they do appear to show wave like behavior or atleast some of the properties we can we are able to explain on the basis they are showing as wave like behavior. And we would also like to see. So, we have already gotten ourselves some descriptions of how the properties are for electronic and thermal properties are? We would like to define that description and in that contests we would like to see, how those waves interact those electron waves? How they interact with the periodic structure of the lattice?

And it is in these contests that we would like to understand the interaction between waves and the lattice in general. So, we are now trying to get feel for the interaction of waves, electromagnetic waves with a periodic crystal structure. Once we get that understanding clear we can extend to see the interaction between electron waves which are already within that solid with the same periodic structure. So, the frame work with in which we will do it is going to be the same. Because a in both cases we look at electromagnetic waves. We are starting off with a general discussion on just interactions of such waves with the periodic structure, then we will extended it to the specific case of these electron waves travelling within that structure.

So, that is the frame works with in which going to do it.

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We also notice that when we put in the description for the wave nature of those electrons especially with in the solid we were using the k vector, or the wave vector which was being which was which had this value 2π by λ , and its is λ is in the denominator this conveys to us that at least with respect to the wave vectors. We are looking at 1 by length dimensions.. So, we would actually as we do this process we want to get feel for the different ways in which we can represent the interaction between waves and the crystalline structure in that in the solid. And whichever works out most useful for us that is the description that we will use. We can actually we already have some descriptions in the real space so, to speak in the conventional $x y z$ space, there is description for how waves interact with the crystalline structure.

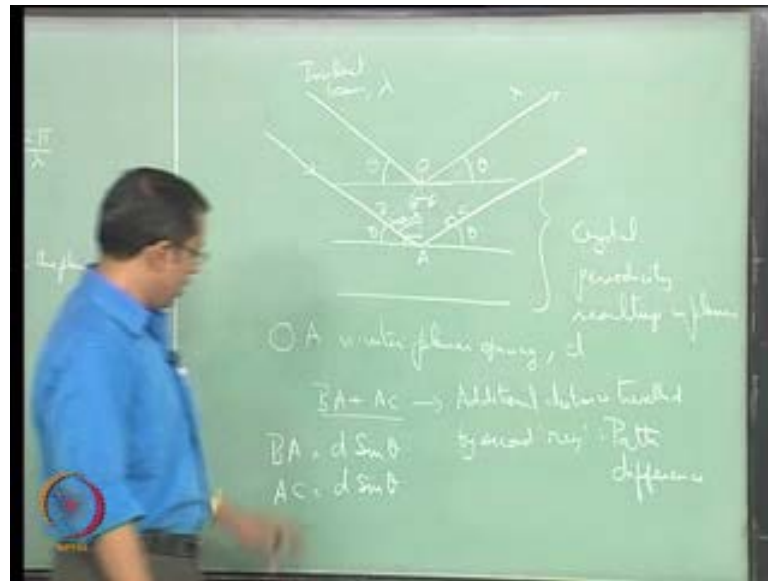
So, we will began there and we will look at that interaction because that is something that your already familiar with from high school or even from a early college classes. And then we will see how the same information can be represented in reciprocal lattice formation. So, we will do that of course, last class we looked at reciprocal lattice as an independent entity we just basically said that we can given real lattice which has unit vectors $a_1 a_2$ and a_3 . We can specifically create reciprocal lattice which corresponds to the real lattice with the definition of the reciprocal lattice vectors, unit vectors in the reciprocal space $b_1 b_2$ and b_3 having specifically relationships to $a_1 a_2 a_3$.

So, we had a cross product $\mathbf{b}_1 \times \mathbf{b}_2$ by the volume of that unit cell and hence and similarly \mathbf{b}_2 and \mathbf{b}_3 were described. On the basis of those definitions we found, we already found that for example, in the reciprocal lattice the vectors are designated by we have standardly designating them using this $\mathbf{H} \mathbf{h} \mathbf{k} \mathbf{l}$ and the vector $\mathbf{H} \mathbf{h} \mathbf{k} \mathbf{l}$ as we found is perpendicular to the plane $h k l$. So, these are standard notations which your familiar with $h k l$ plane based on the intercepts of that plane with the crystal lattice. So, $h k l$ this is standard notations in material science is standard notation, and also in any other early physics that you have read this is standard notation.

So, this is of course, something in real space $h k l$ plane the way we describe it, and way we you know visualize it. And this is vector in reciprocal space and since the units vectors in these two spaces have been I mean defined are at least the reciprocal lattice vectors have been defined with respect to the real lattice factors in with in specific manner. That is why we and the finding this relationship wholes true. That the reciprocal lattice vectors $\mathbf{a} \mathbf{h} \mathbf{k} \mathbf{l}$ which would then be $h \mathbf{b}_1 + k \mathbf{b}_2 + l \mathbf{b}_3$. So, this is what the $h k l$ is. So, and this vector is going to be perpendicular to $h k l$ plane in real space. So, that is how it is, and the modules of $\mathbf{H} \mathbf{h} \mathbf{k} \mathbf{l}$ are simply $1/d_{h k l}$ the spacing between these planes that is the value of these modules of this.

So, therefore, we find that both this relationships are I mean clearly indicate that there I s some geometrical relationship between these two spaces that we have created. And that is being born out by this these two relationships. So, these were the two major relationships that we derived last class after having defined as reciprocal lattice. So, today we are going to look at the interaction of the waves with crystalline structure, periodic structure. And we will first do that in the real space, because that is something you are familiar with, and then we will do that in this reciprocal space. And we will see that essentially the results are comparable and that we can draw some conclusions from there.

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So, in real space we are conventionally is basically say that we know we have series of planes of atoms. As you are aware you know when we say planes of atoms that is just something for our convenience that we say. In principle atoms are occupying only periodic locations within the crystalline structure that is all there are. So, it is just a location its its an address for the atoms so to speak and it stays there within in the context of our discussion. The plane as such that we define is our convenience we just say that along a certain plane we are seeing several atoms. So, we defined that is plane in the crystal structure. So, that is how it is? So, when we pick up plane when we draw it like this it is our definitions that that is that this is **obtained**.

So, we will take an incident beam of us. So, this is a periodic structure. So, this crystal structure, the crystal periodicity permits us to define these planes. So, we have incident beam of wave length lambda. So, it is coming in with some wave length lambda then we look at diffracted beam also at wave length lambda. So, we are assuming that the interaction in such that the wave length is not being modified in any way. The incident beam is incident at some angle theta, and we will say this is also theta we will look see at that diffraction beam in that direction, and see what is happening? So, we will see for interference purposes when we say diffraction we are looking to see if there is a constructive interference from a collection of waves. So, it is not just the single wave we are interested, we will see also what happens with when you are sending a beam of waves.

We have said that this is beam. So, there is an adjacent wave also coming which will perhaps interact with this second layer and come out. So, we will look at similar wave which comes down here. So, this is again an incident wave and this is diffracted wave and again this angles are still theta. Now this spacing here between this point and this point we will just call this is O this is A. So, O A this inter planar spacing which we standardly designate as d , d is the inter planar spacing. So, this is what d O A is this distance between its play as spacing between these planes. So, if you see we would like we first find out what is path difference between this wave and this wave.

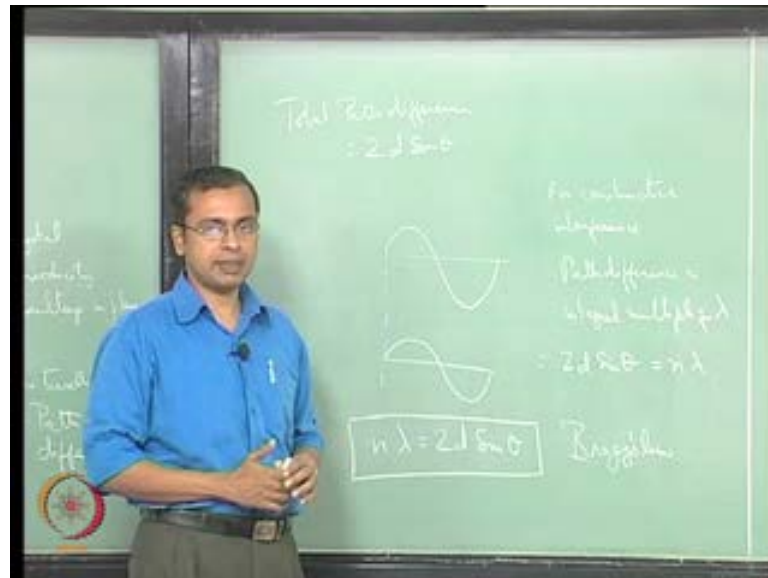
Since they are all coming in such that they are in phase both the waves are in phase. So, we will look at the path difference here. So, this is the additional distance being travel by the wave that is coming to the second plane, and this is the additional distance. So, this plus this. So, we will just say the we will call as B and this is C. So, B A plus A C is the additional distance being travelled by the waves that is a coming to the second plane relative to the first plane. So, this. So, up to here there are in a exactly in phase second wave travels this additional distance travels is additional distance. Then from here on there all travelling the same distance these two waves are travelling it seems. So, this is the additional distance being travel.

So, if you see here since this is theta and this is 90 degrees, thus this line has got to be perpendicular to them only then and this is the spacing. So, this is 90 degrees and this is 90 minus theta. So, this is theta. So, therefore, this alone 90 minus theta. Therefore, this is theta similarly this is also theta right. So, this angle is theta therefore, this angle in this triangle, this angle and this whole angle is 90 degrees therefore, the remaining angles is 90 minus theta. Therefore, the third angle here and again this is 90 degrees therefore, that is a theta. So, if you say this is d this is d which is O A is the inter planar spacing d then O B or I am sorry the A B is simply $d \sin \theta$ if that is it is straight forward trigonometric relationship.

So, B A equals $d \sin \theta$ that is theta and. So, this is d , $d \sin \theta$ would give us this distance, and similarly $d \sin \theta$ would also give us this distance because it is same relationship here this is d and sine theta of that would be would bring it here $d \sin \theta$ would bring it here. So, similarly A C also $d \sin \theta$. So, therefore, so this is the additional second ray if you want to call it that second ray of that beam of rays. So, the total additional distance or path difference this is also called path difference. So, path

difference is B A plus A C each is $d \sin \theta$. So, therefore, the total path difference is simply $2 d \sin \theta$.

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So, we can right that the so that is the total path difference between the two waves that have come to the adjacent planes and moved away from then. So, that is the total path difference between them. So, that is the total path difference. For constructive interference between those two waves that we are looking at this path difference should be an integral multiple of the wave length only then... So, for example, you have a wave like this. The wave from the adjacent plane could have traveled any amount of additional distance, but eventually when it comes to this point. It should once again come back the same way, then these two are exactly in phase there is constructive interference if it is not in phase then it is not going to be in constructive interference.

So, therefore whatever additional distance it has travel eventually when it comes back here it should adapt one integral multiple of that that extra path. That it has travel should be an integral multiple of wave length only then when it comes back to this location, they would again be exactly in phase. If it is off by any if it is 1 point if it is 7.1 wave lengths then it is not going to be in phase it has to be 7 wave lengths just for an example. So, this has therefore, got to be for constructive interference path difference should be equal to integral multiples of lambda. Therefore, we say $2 d \sin \theta$ which is the path

difference, total path difference should equal some integer n times λ , when this is true constructive interference occurs.

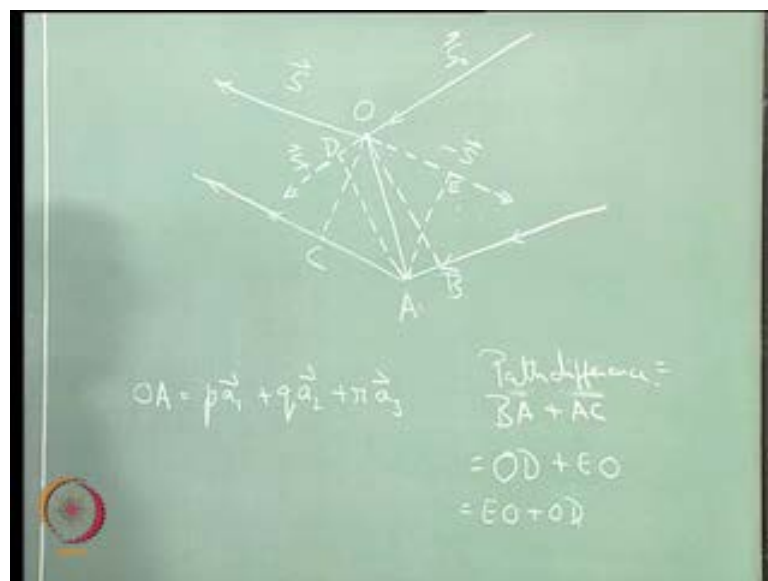
So, $n\lambda = 2d \sin \theta$ is just writing it other way $n\lambda$ equals $2d \sin \theta$. This is the Bragg law, Bragg's law right attributed to Bragg. So, $n\lambda$ is $2d \sin \theta$. So, this is the Bragg's law this is derived the concept behind it is derivation is valid is in fact we are going to use the essentially the same kind of approach. Even when we look at the reciprocal lattice representation of this interaction, but this is the way it is derived it is derived in real space. We are looking at real space planes and the interaction of waves as they are interactive with those real space planes, when they come off those planes, how is it that they are constructively interfering? How is it that they are interfering? And when they interfere? What is the rule that has to be true for the interference to be constructive? And so that is how we end up with this $n\lambda$ being here. ...

So, this is Bragg's law it is derived with respect to real space. And In fact, lot of interaction between lots of the interaction between the waves and the periodic structure. And therefore, for lot of diffraction related experiments this is all that is really required it does satisfy most of the information; however, there are subtle features of the type action process which this law will not be able to clearly indicate. So, and for that, the reciprocal lattice rotation that we have that reciprocal lattice method of telling us, how and when diffraction occur is considered much more robust way, much more fundamental way of describing diffraction process. And therefore, is able to explain a lot more of the experiments that we encounter in diffraction lot more of the diffraction phenomenon.

So, but in real space this is the conditions for diffraction to occur in real space. So, this is the condition we will now look at same diffraction process as it is laid out in reciprocal space. And see whether or and see what is that we can understand from the process. As far as I will also say at this point that know as far as the actual diffraction process is concerned you have periodic lattice you have a wave, they just interact and you see diffraction. So, that is all there is to it in the real sense in the in terms of what is actually happening? Whether we describe it using real space notation or reciprocal lattice notation is simply a question of our convenience.

So, that is all there is to it. So, it is not that we are stating something different when we going to reciprocal lattice notation or we have change the interaction in some way. If that is not the case it is simply manner of describing interaction. So, therefore, fundamentally things cannot change from what we have just described. Fundamentally what is happening is the same thing that there is the path difference between waves and whether are not the path difference have stop to some integral multiple of wave length is all that we are looking at. So, that is something we have to keep in mind even though we would look at some other description for.

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So, we will do that. So, know let us take an atom at the origin O and some other atom at the location A. So, we have two atoms here which belong to this periodic crystalline structure. So, we have an atom at O and atom at E. Therefore, this vector E O A since this is real lattice vector. So, this is real lattice vector. So, O A is going to be we will say that we will just described it as p times a 1 plus q times a 2 plus r times a 3, were a 1 a 2 and a 3 are the unit vectors in the real space. In the real space a 1 a 2 and a 3 the unit vectors p q and r are integers and reason there are integers, it is because that is way we are defining the crystal structure. We define it so, that when you have atomic spacing you reach those atomic spacing are atleast lattice point you reach those lattice points based on integral multiples of a 1 a 2 and a 3, that is how a 1 a 2 and a 3 are defined mainly for our convenience.

So, in terms of the standard convention used for describing lattice vectors and such p , q and r would be integers. So, that we just need to keep that in mind. So, by definition we are saying this vector here is $p \hat{a}_1 + q \hat{a}_2 + r \hat{a}_3$ where p , q and r integers, because that would **live us** leads from one lattice point to another lattice point by based on the convention. So, now we have again similar situation, we will say that beam comes and strikes this atom and then goes off in some direction. So, we have this so parallel to this we have another beam striking this atom and it goes off in this direction, or at least we are looking at we are going to examine this interaction of the same beam with this other point when it comes off in this direction.

So, this is what we planning to do. So, now we will look at some vectorial notation. We will say that this is the incident beam direction, this is the diffracted beam direction. So, this is the incident beam direction this is the diffracted beam direction. So, this is the incident beam direction this is the diffracted beam direction, we will define unit vector in this direction, and the unit vector in this direction. So, it is only unit vector it is not please keep that in mind. So, its modulus is unity. So, we will say this is \hat{S} \hat{S} naught is the unit vector in this direction and \hat{S} is a unit vector in this direction. So, these are unit vectors fine. Now, I will extend this or we can we can extend that in just a moment, we will look at the path difference now.

So, say it this is going to be true even for these two waves, because it just unit vector. So, this is also going to be the same direction. So, it this is also going to be \hat{S} naught, this is also going to be \hat{S} . So far as the unit vector direction is concern is now what is the path difference. So, same thing we will do we will just draw line which is which is perpendicular to this wave and see where it comes and touch us this way. So, I will call this B . So, that is the additional distance wave as travel similarly here we have draw perpendicular, and that will be the additional distance travel by this wave. So, we will do that $A B C$. So, we have now so path difference is since we are looking at it vectorially we will make sure that we are using the correct directions this is $B A$ plus $A C$.

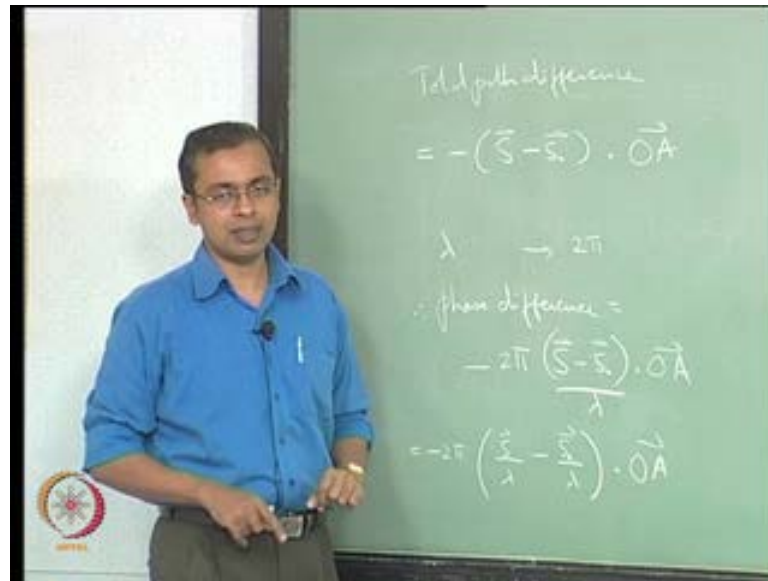
So, $B A$ plus $A C$ we can say. So, or we are only the modules at this stage. So, any way we would say $B A$ plus $A C$ that is additional distance that is being travel by this second wave. So, wave travels $B A$ plus $A C$ that is what as being travel. Now, what we will do is, we will actually for our convenience of calculation we will define both of them with respect to \hat{S} naught and \hat{S} . So, to do that more easily we will just extend this backwards,

if I extend this line backwards the opposite direction this is the origin right. So, this opposite direction here is minus s . So, this is vectorial notation. So, this is plus S direction this is origin, I am going opposite direction here. So, this is minus S fine, in if I extend this line forward this is in the same direction as the S naught. I have not changed the direction I am coming I am continuing in the same direction right with respect to the origin with respect to the origin I am continuing with the same direction and. so, this is still S naught.

So, now what I will do I simply look at this direction $A B$ along this. So, to do that I will I have just draw line parallel to this starting from here. So, that will just come out to something like this. So, I am just transferring this distance e on to this vector here. So, that I can designated with respect to that vector that is I am doing, it is **it is** still the same distance. So, $A B$ and C is a already taken here. So, with call this D and similarly this distance we will transfer on to here. So, that we can take it respect to S minus S . So, this is now this is $A B C D E$. So, this same path difference $B A$ plus $A C$, now $B A$ is simply $O D$ they are are both the same B to A is the same as O to D . So, this is equal to $O D$ and $A C$ is here A to C it is the same as E to O .

So, $B A$ is $O D$ and $A C$ is $E O$. So, therefore the path difference is simply $E O$ plus $O D$ the same thing. So, this is the path difference this vector a fine $O A$ if you look at the what $O D S$ it is simply the it is component of $O A$ along this direction. So, there is some angle θ here. So, $O A \cos \theta$ is what $O D$ is right $O A \cos \theta$ is $O D S$ some other θ is I mean does also θ here $O A \cos \theta$ would that θ here would give us this value $O E$. So, in other words it is a components of $O D$ is the component of this $O A$ along this direction along the S naught direction and $O E$ is the component of $O A$ vector along the minus S direction.

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So, path difference is in fact just by convention it is written we had S naught minus S . I will simply write it as minus of S minus S not simply by convention its written in this way naught $O A$. So, this is what we have now we basically say that you know path difference of λ its space difference of 2π . So, to get this path difference we have simply going to have therefore, the space difference is equal to minus 2π into minus S minus S naught simply. So, this quantity times this by this is the space difference it is. So, it just a proportional matter of proportion. So, path difference of λ gives us this space difference of 2π therefore, path difference of this gives us this space difference this path difference times 2π divided by λ . So, that is what we have this path difference times 2π divided by λ that is the space difference.

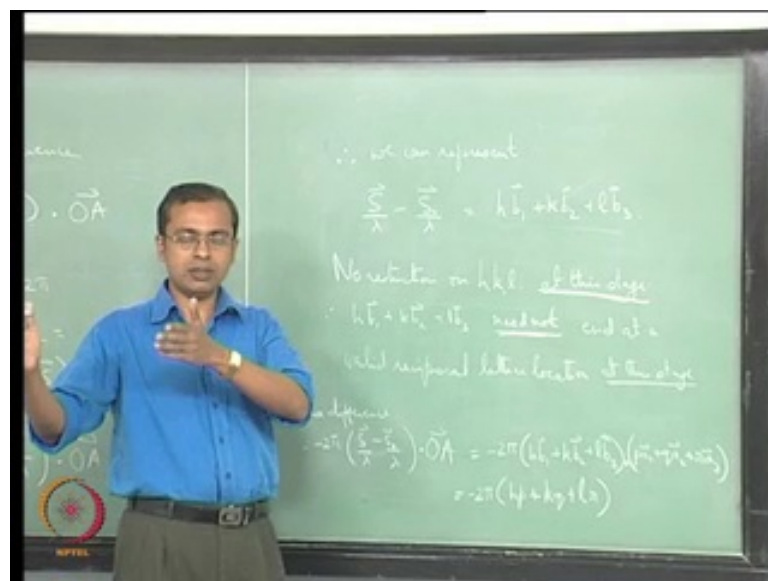
Now, we have already see this is unit vector in the direction of the incident wave this is \hat{I} am this is a unit vector in the direction in the incident wave this is unit vector in the direction of the diffracted wave therefore, divided by λ . So, we have already we have the 1 by λ 1 by length quantity beginning to appear here. So, what we will say is we can in fact write this more specifically as minus 2π \hat{S} bar by λ minus S naught by λ . So, what we will we need to keep in mind is that this S naught by λ . This S naught is the direction of the incident wave and 1 by λ is be is the magnitude of it in reciprocal space if you want put it in inverse space. We need not worry about that a in a specific way except that we need to understand this is a direction and the quantity here is 1 by λ . So, it is in reciprocal length quantity. So, in other

words this vector is a reciprocal lattice vector. In reciprocal lattice vector 1 by length quantities are being portrayed and directions are portrayed. We now have a direction plus magnitude of 1 by length.

Therefore, this is a reciprocal lattice vector, or in other words it is ready to be represented in reciprocal lattice you can directly represented in the reciprocal lattice, because it has all those attribute it has got the 1 by length convention associated with it and it as direction. So, it is a reciprocal valid reciprocal will not say it can be represented in reciprocal space when you say it is valid reciprocal lattice vector, it has to end at a reciprocal lattice point. So, right now we have not yet placed that restriction on this vector. So, and when I get to the we in a couple steps I will re emphasize what I mean by that right.

Now, it is simply it as got all the attributes to represented in a reciprocal space, that is the first thing we will note here same thing is true here. This is also direction and this is 1 by lambda. So, in terms of the magnitude, in terms of dimensions it is correct for reciprocal space representation. Because it has got both direction as far as 1 by length designation. So, in reciprocal space so this is vector this is vector in reciprocal space, this is a vector in a reciprocal space fine. So, therefore, difference between these two vectors is also going to be vector in reciprocals space fine.

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So, right now therefore what we can say is... So, this can be represented in reciprocal space that is all we are saying. Because it got the attributes to fit into reciprocal space. Now it does very important point that we need to note here b_1 , b_2 and b_3 are reciprocal unit vectors in reciprocal lattice, in reciprocal space or unit vector in reciprocal space, corresponding to the a_1 , a_2 and a_3 which we use to define $O A$. The difference when we position 2 atom at 1 atom at O and 1 atom at A we use real lattice vector a_1 , a_2 and a_3 those are real lattice vectors define for that real lattice unit vector define for that particular lattice. So, they have specific magnitudes and directions using that, we were able to decide on the distance $O A$ as $p a_1$, plus $q a_2$, plus $r a_3$ where p , q and r were integers. Because the moved us from lattice point which was O to another valid lattice point which was A , those were two valid lattice point.

Therefore, $O A$ was a valid real lattice vector. So, therefore, that is the reason why p , q and r had be integers, because that way in which we define such lattice as we defined them so, that when we use integers moments of a_1 , a_2 and a_3 you will move from 1 lattice point to another lattice point, that is the way we defined that right. So, that is p , q and r were valid were integers. So, and a_1 , a_2 , a_3 are the unit vector in real space. Now corresponding those unit vectors we can define b_1 , b_2 and b_3 as the corresponding unit vectors in reciprocal space. So, that is what this b_1 , b_2 and b_3 . Now, having said that the same logic holdss in reciprocal space. In reciprocal space also, if started the origin if you move through integers multiples of b_1 , b_2 and b_3 you will arrive at valid reciprocal lattice points.

In reciprocal space as in real space there are specific point which are valid lattice points. So, that those are the points that define the lattice, the rest of the space in between does not really it is just empty space so to speak fine. So, if you do not move through integers values you will reach some open location where there is no atom available or there is no lattice point available fine. So, that is a point you have to understand. So, in other words when you simply have b_1 , b_2 and b_3 you can multiply them by integers or you can multiply them, by non integers also. So, you can multiply them by fraction if you multiply them by fraction you will arrive at some non valid lattice point you will be in some open location where there is no lattice point. If you multiply them by integers you will arrive at lattice point the same is true for a_1 , a_2 and a_3 as it is true for b_1 , b_2 and b_3 .

3. You have to multiply them by integers you arrive at a lattice point if you multiply them with non integers you will arrive at some open location, in general.

So, right now when we when we wrote $p a_1 + q a_2 + r a_3$ because we moved from one valid lattice point to another valid lattice point p , q and r are to be integers. At the moment the way we have defined this vector we have not said anything about we have not placed any requirement that this has to be at a valid lattice point. So, at the moment it is simply the difference between two vectors in reciprocal space and these two vectors we have not placed any restriction on them. These two vectors are based on a beam that we have sent from outside. We have taken a sample, a beam of x-ray is for example, or an electron, if you are working with let us say with electron microscope a beam of electron arrived at that sample and headed off in some direction for some interaction.

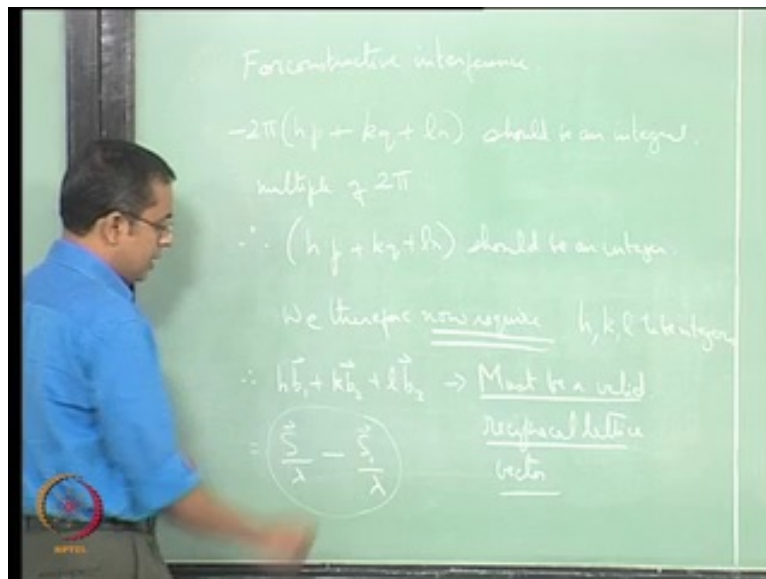
So, that beam is kind of independent of the sample that we have put right. So, the direction in which that beam can be, the wavelength with which that beam comes is completely at your discretion are the instrument's discretion. So, therefore, this S could have been anything this S could have been anything. Similarly, this λ could have been anything. So, you have full freedom to select all of these 3 quantities which is what this left hand side of this equation. Since you have full freedom to select anything which satisfies I mean for the left hand side of this equation it therefore, means that at the moment h , k and l can also be any arbitrary number, it could be an integer, it could be a fraction, it could be a one of them could be a fraction anything could anything could be true.

So, therefore, at the moment the way it is written even though we have used h , k and l no restriction is actually being placed on the values of h , k and l they could be in absolutely anything they could be integers, they could be fraction any number that you can think of you can place here. So, no restriction will be placed on h , k and l . So, at this stage of our definition no restriction has been placed on h , k , l therefore, $h a_1 + k a_2 + l a_3$ need not... So, at this stage in our derivation at this stage of in our derivation is what we are looking at we have placed any other restriction on this system. So, therefore, this difference even though it has all the dimensions of being reciprocal space. And therefore, can be written this way it need not end at if there is no if this h , k , l and need not be integers and therefore, this sum here which is known vector need not end at a valid

reciprocal lattice point. It could end of it some open location, where there is no valid reciprocal lattice point. So, that is the important now.

We are saying that the space difference is equal to minus 2 pi S minus S naught by lambda or S minus... So, this is what our space difference is? Therefore, this is equal to minus 2 pi this is the S by lambda minus S naught by lambda is what this quantities is. So, therefore we can write that as that vector there h b 1 plus k b 2 plus l b 3. And O A we have defined as p a 1 plus q a 2 plus r a 3 and this is a dot product. Dot product of h b 1 k b 2 and l b 3 dot product with p a 1 q a 2 r a 3 were p q and r have to be integers at the moment h k l could be anything they need not to be integers. So, if you take the dot product so this is equal to minus 2 pi h p plus k q plus l r this is what we get of which p q and r integers h k l and could be anything. Now for constructive interference so we have this space difference for constructive interference this has to be such that the path difference is space difference should be integral multiple of 2 pi .When we say integral multiple of wave length, wave length is 2 pi in terms of space. So, if this becomes integral; multiple of 2 pi we have constructive interference because; that means, the two waves are exactly in phase. So, one may have travelled several additional wave length, but are end of it all there are both exactly in phase. So, for constructive interference this complete quantity here should be integral multiple of 2 pi.

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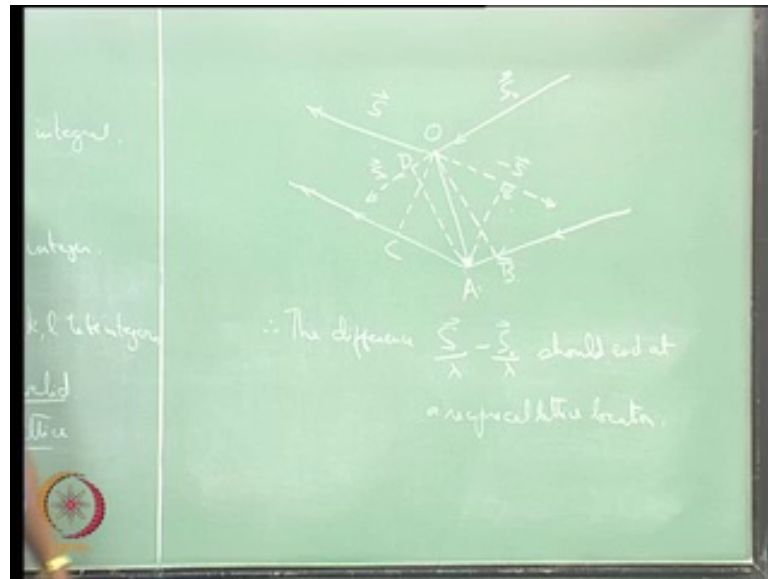
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So, it has to be integral multiple of 2π , this is phase difference is whole quantities is phase difference that as to integral multiple of 2π for constructive interference. We already have 2π here. Therefore it means therefore we are requiring that $h p$ plus $k q$ plus $l r$ should be an integer. So, while we when first defined the S by λ minus S naught by λ . We define that to be $h d_1$ plus $k b_2$ plus $l b_3$, but we place no restriction and values of $h k l$ at that stage; however, we find that we are forced to we are forced to deal with a situation for where in for constructive interference to occur a product sum of this term $h p$ plus $k q$ plus $l r$ should be an integer, in this $p q$ and r already integers and it could be any integer.

So, therefore for this complete sum to be on integers regardless of the value of the $p q$ and r we now require. We therefore, we now require h comma k comma l to be integers. So, we therefore, now require $h k$ and l to be integers in are derivation we find that we have reached the stage were originally we were at stage were $h k l$ could have been anything. But the conditions we are placing on those interference process is the conditions that the interference places on the set of relationship, create a situation where the $h k$ and l arrive at this particular sum where there multiplied by $p q$ and r respectively where $p q r$ are and integers and that sum should be an integers for this to be true regardless of the values of $p q$ and r we can now guarantee this to be true only if $h k$ and l are also integers.

So, therefore we now arrive at a situation were $h k l$, $h k$ and l which could have been anything earlier on now have to be integers for constructive interference to occur. Therefore, having reached this stage we are able to say therefore, $h b_1$ plus $k b_2$ plus $l b_3$ must be of **valid vector**, valid reciprocal lattice vector. So, therefore $h b_1$ plus $k b_2$ plus $l b_3$ must be of **valid vector**, valid reciprocal lattice vector. So, therefore, we find and this is simply S by λ minus S naught by λ .

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So, this is what it is. So, if you look at this quantities the way it will work out is we need this difference two end up at are reciprocal lattice location. So therefore...

((No audio from 43:44 to 44:10))

So, this should arrive at a reciprocal lattice location for constructive interference (()). So, what we see is, if you look at r equation here we fine that we have two different quantities here, we have quantity on the on this side of the equation which basically gives us all the properties of the waves arriving at the lattice arriving or leaving the lattice. This is the direction in which the incident wave is arriving S naught and 1 by λ is the magnitude of that wave when reciprocal quantities magnitude of the wave of reciprocal quantities of the wave length in reciprocal quantities. So, that is that this gives as the properties of the incident beam. This is the properties of the diffracted beam this is the direction of the diffracted beam S and 1 by λ is the wave length of that diffracted beam in the reciprocal lattice in the reciprocal space dimensions.

So, this is this gives as all the properties of the incident beam as well as the diffracted beam in the reciprocal space notation fine. So, all of the wave information is here regardless of the where the wave came from of what is the origin of the wave, that is a irrelevant to us all the wave information is here. What is up hear, on the other side of the equation the other side of the equation as valid reciprocal lattice vector. So, it is line that is connecting two valid reciprocal lattice points. So, this is got the crystal structure

information in it. This is got all the crystal structure information in it denoted in the reciprocal space notation. So, we find this equation this equation here $h^2 + k^2 + l^2 = \frac{S}{\lambda^2}$ that equation gives us the condition for diffraction. as indicated in the reciprocal lattice in the reciprocal space notation.

We have the reciprocal lattice vector representing information about the reciprocal lattice on one side of the equation. We have the reciprocal of the wave lengths with the directions indicating the wave information on the other side. And the equation of these the fact that they are equal to each other with h , k and l being integers and therefore, being valid reciprocal lattice point give us the complete picture of how we would represent the conditions for diffraction in reciprocal space. So, this is a... So, this is the important derivation here fine.

So, it has all the information that we require, in term of the both the reciprocal lattice crystals. So, in term of the crystal structure on one side in term of the wave the information on the other side and taken together, it is the diffraction information diffraction conditions. So, we have derived today the diffraction conditions both in real space as well as us in reciprocal space in both cases we have the structure information on one side and the a diffraction condition on the other side. So, the diffraction conditions we have derived in both format the real space and the reciprocal space. So, what we need to do, of course, in this case what we have done so, for is that we have treated this waves we have not really focused on the origin of this waves we have basically treated them as independent of we do not care what the origins is. They could be from some source that you have in a lab separate from your sample. So, this is information about your sample.

Your sample meaning whatever metal sample that you have or ceramics sample here this is information of your sample. This is information about the waves that are coming from any source, you could have x ray source, or you can have a electron beam source in a electron micro scope the waves coming from there is this information. And we have looked at this in interaction independent of all other information, you looked at this interaction come of with this condition of diffraction. So, this is as an independent picture. Were we will head from here is the fact that we have already seen that electrons within the sample itself have wave like behavior. So, to the extent that they have wave like behavior this a equation is going to apply to them also.

So, for this equation to hold the wave does not have to arrive from external to the sample we have not place any restriction saying that the wave must arrive external to the sample you must have separate source from which waves are arriving, that is not a restriction that we have placed. Any which place the waves arrive from as long as they satisfy this conditions diffraction is going to occur. If they do not satisfy the conditions diffraction will not occur that is the basic rule. So, we will now as we proceed forward in the next few classes we will look at what is the implication of this information as represented in reciprocal space the implication of this relationship as represented in reciprocal space, with respect to the electron waves that are already present within the sample itself.

So, and in that context we will see what properties of those electrons are then impacted due to this relationship as it happens within the same sample. So, that is the direction in which we will head off. Today, we have seen these two diffraction representations in real space and reciprocal space and we will look at our future discussion with respect to this and the discussion. Thank You.