Physics of Materials
Prof. Dr. Prathap Haridoss
Department of Metallurgical and Materials Engineering
Indian Institute of Technology, Madras

Lecture No. # 25
Fermi Energy, Fermi Surface, Fermi Temperature

Hello, so we have arrived now at the twenty fifth lecture in our physics of material's lecture series. So, in the last class, we specifically looked at more and more detail of the way in which electrons are distributed across energy levels. And particularly, we try to figure out if there is a way in which we can write an expression for the density of occupied states. So, we found out that there is such thing as density of allowed states, and then if you multiply that by the probability of occupancy, we get something called the density of occupied states.

So, taken together that is the picture now which conveys to us, what we have in the system as a function of energy. So, we originally had general idea that as the energy goes up, you fill in electrons, and then you will reach some, eventually you will reach an energy level, where you have run out of electrons, and and that then constitutes the system. And all these of course comes about, because we have incorporated Pauli's exclusion principle and to the picture, and so we say that first of all at any given energy level, there are a finite number of states, and since Pauli's exclusion principle applies, we cannot place all the electrons on just a finite number of states. If you have a very large number of electrons, which is typically the case in a solid that we are dealing with. So, 10 power 28 electrons per metre cube is the kind of situation we deal with.

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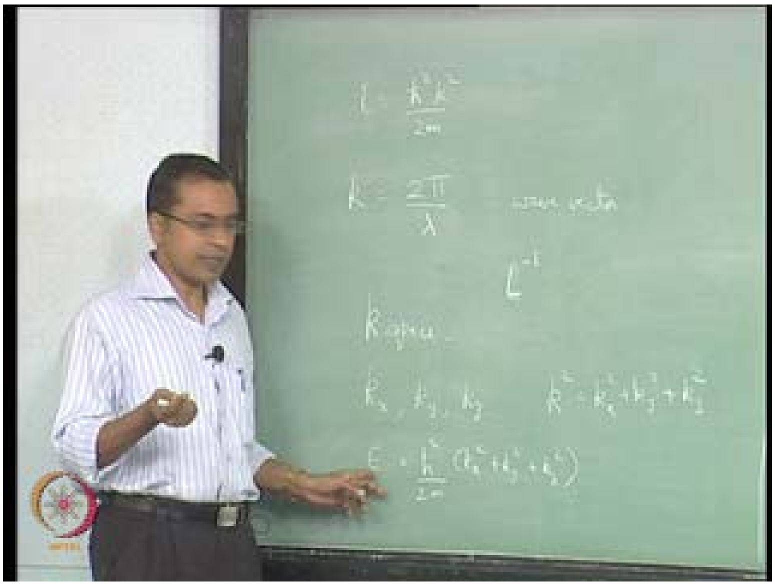

So, we wrote this equation for the energy; it simply h bar square k square by 2 m. Now, today what we will do is, we will actually explore few other ideas of associated with what we have done so far. And see what additional understanding we can get of the subject as well as better feel for the kinds of quantities that we are currently discussing. So, and this is important, because some of the quantities we have discussed mentioned so far or quantities that we have not really delt with perhaps in our high school physics or even early college physics. So, therefore these are all new quantities it is worth our time to actually try and understand may be how they relate to other aspects of what we know, and also what is the significance of some of these quantities, is it of any relevance, is it simply something that is an interesting term that that is around or is it something that is going to be significance for some usage little while little later down the road. So this is something that we want to understand.

So, we will we will at least initiate our discussion today on along those lines and depending on how far we proceed with that we will elaborate on specific aspects associated with the quantities. So, we write this equation down E is h bar square k square by 2 m. It simply the square of the momentum by two m. So that is what we have, k is the wave vector - we call it wave vector. So in principle we can, if it is written in pictorial form this is the modulus of it we did most of our work in one dimensional case. So that is why we have I mean... So, we have the results we have discussed so far have tended to have simply the x component of whatever it is that we were discussing and only where necessary we brought in the other components; we will continue with that in that same spirit in the discussion that we will do today.

So wave vector, you will see immediately the first of all it has the units of one by length - this is wave length, so this is in meters I mean meters in meaning it may be ten powers minus ten meters if it is an Armstrong, but still the dimension is in meters. So the dimension of this is length minus one. So that is the dimension of it L minus one metre minus one is the dimension of it. So L minus one so what we are looking at. And also mentioned in the last class that when you look at these quantities and they have certain relationships and we can plot information in any one of those variables - based on any one of those variables you have something on the left hand side something on the right hand side and so you can plot the expressions on and you have multiple relationships valid for, because you have different variables.

So, the same information can be presented in terms of the various variables that we have recognised and based on how they relate to each other. So depending on which variable you choose, the plot that you make may look different. So, that is the point I made last time I want to emphasis it again; based on the variable you will choose the plot that you make may look different, because it is a different variable - it simply a plot of different variables, but the information that you can always extract out of it will be the same. So, so that is the thing that we look at. So again some diagram energy may show up as a flat line, I mean you may just see if you write energy versus position it simply show up as a flat line for example, at for some free electron or some such thing where as if you, where to plot it in some other variable you will see a certain different surface where the surface may not be flat, but the surface corresponds to a constant energy. So that is the that is the point that I wish to make.

So based on the variables you choose and the kind of plot you make the topology of the plot may look different, but the information would be the same right. So in this case we have energy as a function of the wave vector. So, in fact, you can plot it in this is called - so this is simply k. So, we say that we can make a plot of we can make we can plot our information in k space. So we can plot our information in k space in which case we are actually looking at the components of this information when k x k y and k z right and then we make a plot of this of this information. So, simply again k square will be k x square plus k y square plus k z square.

So, if you now see, if you compare this equation here with the fact that you can actually write it down in terms of its components in k x k y k z. So we have. In fact, analogous to that is we had this discussion with respect to quantum number space in our previous class; it is essentially the same component that we have in a with respect to what we are seeing here. And that is simply that you have energy which is a constant or we can choose a particular value of energy, let me say put it that way. We will choose a particular value of energy and we can write this relationship saying h bar square by 2 m into k x square plus k y square plus k z square.

So, you can write it this way, now in this h bar is a constant, it simply the Planks constant by two pi m is a constant, because that is the mass of the electron. So these parameters here are all constant and let say I would like to do this calculation for a fixed value of energy - I will just say that this calculation I am doing for a fixed value of energy when I make that statement what I am essentially saying is that k x square plus k y square plus k z square has to now become a constant, because this is all ready a constant I am saying left hand side is a constant therefore, this term has become a constant, when you do that what you have is you have a spherical surface, you you are defining a sphere in k space k x square plus k y square plus k z square equals a constant is the equation of a sphere in k space, k space is simply a plot of all information in k x k y and k z coordinate system.

So access are k x k y and k z. If you make k x k y k z and then you plot whatever you wish to plot in terms of those three variables then then you are plotting the information in k space, and if you plot this information in k space subject to the constraint that you are talking of a fixed value of energy then you will get a sphere.

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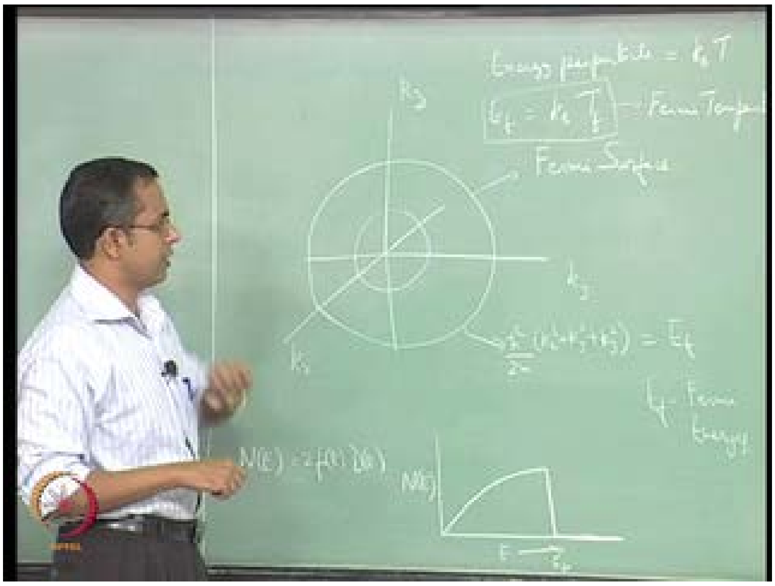

So, so that is basically what we have. So, now in other words if I make this plot here. So, we will say this is k x this is k y and this is k z right and then I arbitrarily define some energy - some energy value I would define. So what I will see is, I will see a sphere corresponding to that. Of course, in a two dimensional board that I am drawing it looks like a circle tube, but basically it is a sphere centred around the origin it is a sphere centred around the origin and these are wave vectors. So, they can actually be in any directions; it is a wave vector wave. So, that it so, it depends on the direction of the movement of the electrons so to speak.

So you have a sphere around the origin. Now we also mention that you know in our plot of the N of E or density of occupancy and also our plot of the Fermi dirac statistics itself. We said that in the ground state as the as we described in the physics in physics in the ground state in meaning the lowest energy state of the system and essentially in this case at zero Kelvin. So at zero Kelvin this is the manner in which the density of occupied states is behaves as a function of energy. And this energy where it drops to zero the density of occupied states drop to zero is the Fermi energy E f, E f is one energy where it drops to zero that is that is the in fact where the Fermi deriac statistics also drops from its probability of f of E equals 1 2 f of E equals zero.

So Fermi energy is defined as that energy at which it below which f of E is equal to one and above which f of E is equal to zero at zero Kelvin at zero Kelvin at any other Kelvin it will work out to f of E equals 0.5, but below at zero Kelvin it is it is exactly step function there it goes from 1 to 0. So so we have E of f, that is the Fermi energy E f equals Fermi energy right. So, this is something that we we recognised now that for the system that we have been discussing all along; simply by nature of how electrons distribute themselves across the various energy levels and the constraints we have placed on the system in terms of the number of energy levels that the fact that it has energy levels that there are that they are quantised that there is a fixed number of states and so on. Based on all those constraints and the fact that we have finite number of electrons and finite number of states.

We will always find there there is an energy value above which the occupancy drops to zero and that energy value we have defined as Fermi energy; and that is the highest energy level occupied by electrons at zero Kelvin. So, it is the highest energy level occupied by electrons at zero Kelvin. Now, so obviously for our system the Fermi energy given as particular system any any system that we take, you know you take silver as an example you take copper as an example that then forms the system that based on the normal standard way in which copper behaves or silver behaves we will have a certain number of and based on its density its actual density and so on. We will have a certain number of free electrons per unit volume. So that is very specific to that system. So that is very specific for that system. So that is very specific for the given system. So, we can defined that for the system that we are interested in and on the basis of that definition we can write figure out what will happen as you fill electrons at various energy levels.

So for every specific system that we discuss, at the particular value corresponding to that system which will be unique to that system at that particular energy value you will run out of electrons. So in other words based on the system that we choose we will have specific value of Fermi energy. So it is not it is not the that the Fermi energy is the same regardless of which metal you choose or regardless of which crystal structure you choose and so on, when you change the crystal structure when you do something else you are changing perhaps you are changing the number of free electrons per unit volume perhaps you are changing the density of the material and so on. So in all these things, there are lot of changes that are likely to occur; therefore from material to material the value of Fermi energy need not be the same if at all it is it is going to be quite accidental match; it is, in principle it need not be the same and even for the same material has to change if you put it on the different circumstances may be the face changes and so on.

If the density changes, if there is if there are additional changes that are occurring in the material that affect the number of free electrons per unit volume then you may have a change in and it and it if it also affects the way in which the energy levels are in the system it can affect the Fermi energy. So, the bottom line it is a fixed value for a given system under some circumstances. So so therefore, the Fermi energy is a constant in some ways for for that particular system under those circumstances.

So for a particular system you can uniquely identify of particular value of Fermi energy. So, for that system when you make this plot k x k y k z if you plot energy of the various electrons that are present within the system in this k space, then you can make several spheres each sphere will consists of all the electrons that possess the same amount of energy. And finally, you will reach a sphere which is at some size; you will reach a sphere here where k x square plus k y z k y square plus k z square times your value here h bar square by 2 m, h bar square by 2 m times k x square plus k y square plus k z square for this particular sphere, this largest sphere that I have drawn here, for this sphere h bar square by 2 m into k x square for this particular sphere will be equal to this value of E f. So for this particular sphere not for this spheres that are below it for the spheres that are below it they are also constant energy spheres, but they are for energy values less than E f less than the Fermi energy; for this particular sphere the value of k x square plus k y square plus k z square times h bar square by two m will work out to the E f.

This sphere in k space this sphere in k space is called the Fermi surface. So, now we have two concepts that I have just we have discussed so far. One is the existence of this energy called Fermi energy and its significance the fact that you when you run out of electrons you reach this that energy value where you running out of electrons becomes the Fermi energy and therefore, it corresponds to the electrons it corresponds to the energy of the highest energy electrons in the system. So, because you are putting them from the lowest energy upwards highest energy that that you run out of electrons is the Fermi energy that same energy you can plot, because of its relationship to the wave vector that is there in the system based based on its relationship to the wave vector based on its relationship to the wave vector of the electrons right; these are not some arbitrary wave vector these are the wave vectors of those electrons which have this energy. So based on this relationship.

You can identify certain surface in k space which consists of k x k y and k z axis right that surface will turn out to be a sphere simply, because of the nature of this equation and that sphere is referred to as the Fermi surface. So sort of a plot we can now see it has a certain shape and everything and that is the surface which is the Fermi surface and there is an energy corresponding to it called the Fermi energy. So these are two concepts there is an one other concept that we can look at, we will now see the significance of these in just a moment we are going to discuss the significance of all of these.

So we we right now, I am only defining this quantities I am defining them and trying to explain what it is and where they have come from, what is the use of the surface? What is the use of this energy? What is it? That we can say of about the system based on these energies that is the other piece of information which we will talk of just a moment. There is one other quantity which we will define in relationship to these information that we have put on here. In general in in systems where you have you know when you are discussing the thermal energy of the system and so on.

We we essentially associate energy we will put it on here we will keep it here; so that we can follow it. Energy on a per particle basis is given by k B times T. So, in fact, when we did our we would have come up with something similar the translational kinetic energy when we did Maxwell boltsman statistics and when we did the ideal gas behaviour came came out to three by two k B T half m v square is three by two k B T is what we came up with. So, it essentially relates to k B T that is the basic idea the energy relates to k B T; some constant is there it relates to k b t.

So, now, similarly we can write E f. So this is all on a per particle basics. So in our in our case the particles are electrons. So, we can write E f equals k B times T some T which in this case we will put subscript f T f to associated with the fact that is E f that are equating it with. So, E f is k b times T f. So that is just based on the general relationship between temperature and energy on a per particle basis for the kinds of system that we look in it. This T f is referred to as the Fermi Temperature. So T f is the Fermi Temperature.

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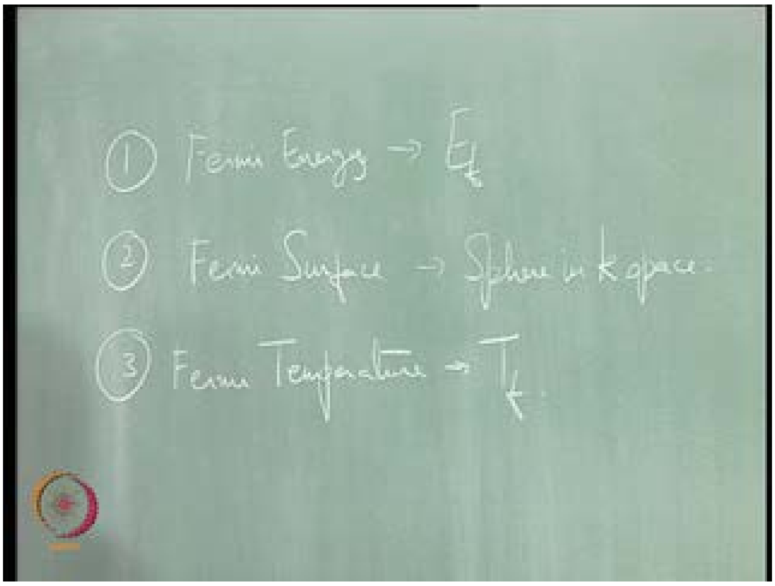

① Fermi Energy → E_f

② Fermi Surface → Sphere in k space.

③ Fermi Temperature → T_f.

So, so what we see are we have three quantities are quantities and say surfaces that we have; the first is Fermi energy which is E subscript f, the second is Fermi surface which is sphere in k space third is Fermi temperature, which is T subscript f right. We have these three things Fermi energy Fermi surface Fermi temperature, so these are all definitions that have come about as a result of the kind of system that we have discussed, the phenomenon in the system how they relate to each other and so on. And with respect to some other definitions that we have. So, on that basis we have come up with these three definitions.

So, now we will look at each of them one by one and try to understand what is the significance of these these three definitions these three terms and what we can we say based on these three terms I mean what if in a system you get these information about the system what is it that that you can sort of related to. So, and what is the kind of issue that we have with it what is the utility of it. So, where can we use this information, how it how does it help us understand something about material property, how does it help us make some predictions where in the in the process of making the predictions where is where is is the information going to appear. So this is a sort of this is a sort of layout for us this discussion will sort of layout for us where we can expect these quantities to show up in some of our related classes and perhaps to understand what is the manner in which it will show up and what is the level of significance of this quantity. So that is what we have going to look up.

So, we will start with Fermi energy, to discuss Fermi energy to tell you something more about the Fermi you already know that when you place electrons you know from the lowest energy level upwards the highest energy level that is occupied is the Fermi energy that is the definition of Fermi energy at zero Kelvin right. So, we keep that definition in mind, we will look at few other quantities that we are more familiar with and then we will come back to Fermi energy right.

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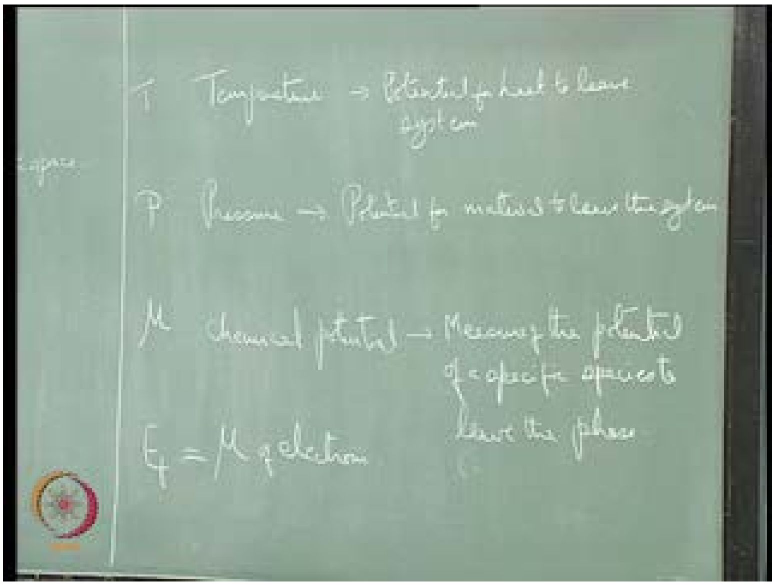

So, let us look at say temperature just temperature in general. Now temperature if you if you look at the thermo dynamic associated with the temperature when we say temperature what we are saying is, it is the potential it represents the potential for heat to leave a system; represents (No audio from 23:01 to 23:14) you can think of the temperature as being the potential of heat it is the measure of the heat in the system, but more importantly; it is measure of the potential for heat to leave the system.

In other words if you bring two objects of different temperatures next to each other and bring them and contact with each, whichever has a higher temperature has a greater tendency to get the heat out of the system both of them have the tendency to both of them have the certain potential to get the heat out of the system that temperature is the measure of that potential the higher the temperature more greater is the potential for heat to leave that particular object. So, you bring two objects together whichever has a higher temperature it will push heat into the whichever object has the lower temperature and therefore, potential can be described as the I mean sorry the temperature can be described as the potential for heat to leave the system and higher the temperature higher is the potential for heat to leave the system. So, this is something we talk of in thermodynamics and so... And therefore, we may or may not have thought of it in specifically these lines, but that is the way that is the reasonably good way of saying it.

Similarly if we look at pressure it represents potential for material to leave the system, and usually you are talking of a liquid or gas and some and some such thing and you and so, basically when you have two objects with differing pressures or you have two containers where species are in at two different pressures. So you put a pressure gauge at one pressure gauge on the other you see that they are two distinctly different pressures. So whichever container has the higher pressure there is the greater potential for the material within that container to leave that container.

So, as long as there is pressure both of the containers have material which can leave the container, but when you connect the two containers whichever has higher pressure has higher potential to for the material inside the container to leave it right. So, therefore, that is how you that is how pressure equilibrium arises. So when you connect those two containers more material will leave one container, the container which has higher pressure and move to the container which has lower pressure there by reducing the pressure of the container which was high increasing the pressure of the container which was low in pressure till the pressure in both the containers equalises; when pressure in both the container equalises the tendency for material to leave the first container is the same as the tendency for material to leave the second container at that point it will reach equilibrium right, that is how it happens.

Same thing is true for temperature when the tool the heat moves from one body to the other body the hotter body to the colder body when it does so the temperature of the hotter body will come down temperature of the colder body, will go up till such time that the two temperatures are yet to equilibrium when the temperatures equilibrate when you have the same temperature at both the objects then the potential for heat to leave the first object is the same as the potential for the heat to leave the second object, then heat transfers becomes equilibrated any heat going from one side to other is equated by the heat going from the other side to the first then you reach an equilibrium. So that this is what it is. The third quantity, so we write this as T we write this as P there is another quantity called chemical potential - chemical potential is typically designated by mu. Now chemical potential of it is usually associated with a particular species.

So in a in a more metallurgical material sense you may have you know let us say you have whatever you have iron which contains say carbon in it. So you have iron containing carbon in it so we will associate chemical we will we can designate certain chemical potential to the carbon present in the iron. We can also designated to the iron itself which is present there the iron atoms that are represented and also... So, it is assigned to the very specific species.

So in your system you may contain lot of different species you may have a solution where you have several different solutes present; for each solute you can which you are identifying as species you can designate a chemical potential. Chemical potential is a measure of (No audio from 27:48 to 28:20) it is a measure of the potential of the specific species to leave that phase, that phase for which you have in in which you have found out its chemical potential. In other words I can have let say two different samples two different samples one contain carbon of certain percentage another contains carbon of another percentage, when you typically bring the samples together, our conventional engineering experience is that carbon will diffuse from one sample to the other right. So, it it is as long as the percentage are not the same.

It will diffuse we normally simply talk in terms of percentages, but the more appropriate term there is in fact, chemical potential. So, and they need not necessarily be the same that is the important thing you have to remember based on the kind of solutions that are found those to need not necessarily be the same in many cases in an engineering sense it works out to be the roughly the same. So, we can use it that way that is why we use percentages in general, but they do not have to be the same. So percentage of the species does not directly equate to the chemical potential of the species, but it is a reasonable measure.

So anyway our macroscopic understanding of this phenomenon of diffusion, is that diffusion occurs when the concentrate of the species in two different locations is not the same. And as I said the appropriate term is the chemical potential of the two of the same species in two different locations is not the same. So chemical potential therefore, is a measure of the potential of a species to leave that particular face. So whichever that face is and go to another face where the chemical potential is lower. Equilibrium in thermodynamic sense occurs when the chemical potential of the species is exactly the same in all the faces that are in contact with each other, that are in the position to interact with each other that is when thermo dynamic equilibrium occurs with respect to that system. Now, with respect to what we have just discussed. So, I have said that we will relate Fermi energy to this.

So, Fermi energy E f represents the highest energy electrons present within the system right. So it represents at least at zero Kelvin represents the highest energy electrons in the system. So, what happens is as you are aware in general when you bring systems together where the energy levels of some components of the system is not the same; the systems try to rearrange themselves or the component raise to rearrange itself in such a way that energy is minimised. Therefore, what normally happens is that when you when you bring together materials where the Fermi energies of the two two samples is not the same, the two materials is not the same. The electrons will tend to move from the material that has a higher Fermi energy to the material that has lower Fermi energy such that the energy of the system decreases.

So, the Fermi energy of an electron is equal to the chemical potential of the electron; Fermi energy is the chemical potential of electrons at at certainly at this is this is an this is in a sense this is an approximation this is not exactly correct there is there is some collective factors to be thrown it, but for all lower temperature applications typically the kind of applications we are going to be, we typically do this is an acceptable statement (( )). So, Fermi energy is the same as the chemical potential of the electrons. So higher the Fermi energy of particular material, it is the same as saying that the chemical potential of electrons in that material is high.

Therefore, when you put two materials together the chemical potential the material with having higher Fermi energy and therefore, where the electrons have the higher chemical potential will tend to get the electrons to leave the system and they will go to the material which where the Fermi energy is lower. And therefore, the chemical potential of the electrons in that material is lower till the chemical potential is equate or some other phenomena occurs. So, so it is a measure of the chemical potential of electrons and therefore, in that context it is a very useful piece of information it is not simply a definition of just r k n interest. It is a very or of academic interest; it is a very useful measure, because it tells directly about something about what will happen when you bring materials together.

So, even when you know when you bring semi conducting materials together when you try to you make junctions of different materials; this is the very important phenomenon the chemical potential of the electrons across those various junctions is a very important phenomenon in the sense that there will be the attempt to equate those chemical potentials. So, what will happen as soon as you bring two materials together is largely determine by this value of Fermi energy.

Therefore, it is a very important piece of information; we understand also physically it is a it is a it is an information that something fundamental about the material, because it tells us something about the highest level of highest energy level of occupied by electrons, but it has a very practical applications is what I am trying to bring out. So the fact that in a lot of electronics that will read, what is happening to the Femi energy where is the Fermi energy what is happening to the Fermi energy of the system as you do various things to it. It is something that is of feather less of it is an important part of the discussion that, that will exist in most part of the electronics. So, therefore, this is a very important concept and this is the way it relates to some other things that you have perhaps been aware of... So, that is our first quantity Fermi energy and we now understand its significance that it is it is quite valuable piece of information to have about the system. (No audio from 33:57 to 34:19)

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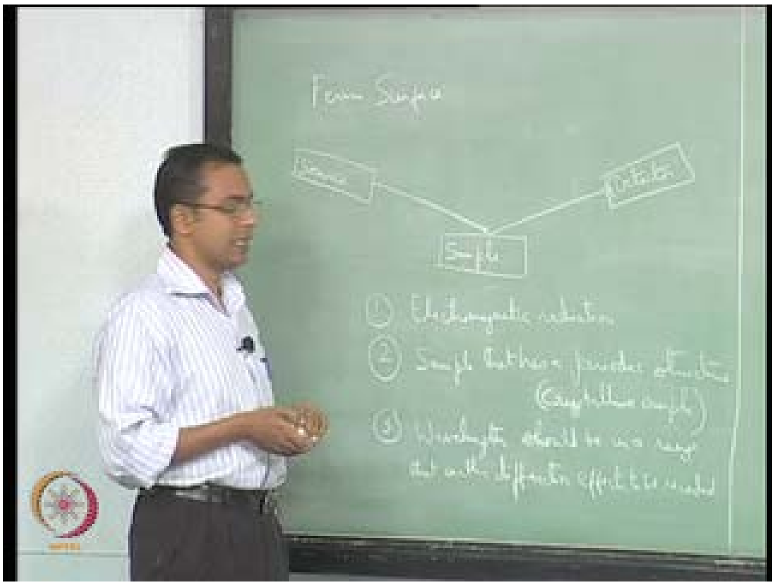

The next thing we will look at is the Fermi surface. Now the Fermi surface as we saw is a sphere. So, we will as we discuss here as a sphere in k space where you have plotted k x k y and k z and so we have a sphere Fermi surface.

Now, it is as I mentioned it basically says that, that is the surface in k space that corresponds to the Fermi energy. It is the surface in k space that corresponds to the Fermi energy we already just discuss that the Fermi energy itself is the very significant quantity, because it represents the boundaries, so to speak of that of the energy values that are occupied by the electrons and past which it is z of empty states at z zero Kelvin. So that is number one. So, therefore, on that basis itself energy you can you can anticipate that this surface may represent something important, because it is representing that energy values right.

So, therefore you can expect that this can represent something important; also in k space what have you plotted, you have plotted k x k y and k z these are wave vectors. So, and they therefore are of these of k is simply 2 pi by lamda right. What is this lamda? This lamda is a wavelength; so it is the k - it is the wave length that we are associating with an electron right. We have said that in the quantum mechanical view of the world view of the universe we can think of most I mean we can look at matter either as consisting of particles or consisting of waves. And so, this wave particle duality is a is an aspect that we have to accept and based on what experiment we are doing? What phenomena

phenomena we are observing? One or the other of these behaviours will tend to dominate or at least, we we will be able to better interpret something based one of these phenomena or more easily interprets something based on one of these phenomenon.

So, we understand that if you have electrons moving within a solid, they have a linear momentum associated with them, associated with that linear momentum is a based on the De Broglie relationship we have a wavelength associated with it. So, this wavelength corresponds to electrons the wavelength corresponding to the electrons moving within the material right. So, it is something that is very specific to the electrons. Now and and in this particular case, it is wavelength corresponding to this Fermi energy Fermi surface.

So, so the Fermi surface therefore, represents all the wave vectors corresponding to that the largest energy electrons that are present within the surface. Now, we will see later in one of our other classes I mean and your also anyway familiar with this basic idea that when you send in electromagnetic radiation electromagnetic radiation when it when it sent into a sample it interacts with a sample right. So, it is on that basics that we have x rays interacting with matter we have waves of electrons - electron beams so to speak interacting with matter. What is this interaction that we talk of in physics and material science and engineering? The interaction that is of significant interest to us is the diffraction process right.

So, you send x rays. Of course, one of the recruitment is then therefore, is that the wavelength of the beam is of the order of the kinds of spacing that exists within the system only then you tend to see much more of the interaction, but still we will see the at least it should be at that order it can be less. So, you can have wavelengths much smaller for you have to see the interaction, but at least you should be at that order. So, you we see that we are aware that you know that there is diffraction process is that occur right.

So when a diffraction process in a convictional sense when you use diffraction what we have is a sample that is separate. So, you have a sample that is existing separately some sample. Some source of radiation some radiation from a source is in pinched on a sample and then we have some detector. So, based on the experiment you are doing the exact position of the source the position of the sample the position of the detector will change they do not have to be in the configuration that I have just shown you, you can have the source on the top you can have the sample in the middle you can have the detector at the bottom. So, you can have all these possibilities

So there is several possibilities then how they can be arrange, but the basic idea is that you have the source of radiation it sends an electromagnetic radiation which is then which then in pinches of the sample and then what comes of with the samples is captured in the detector right. And then based on what we what we difficulty find is when diffraction occurs you will find that as the detector is moved or as a function of position you will find maxima or minima you will find that you know there is constructive interference in some directions, there is destructive interference in lot of the other directions and based on the periodicity of the sample some places you have constructive interference and some places you have destructive interference.

So very strong signals in some particular locations of that detector and very and essentially zero signal at other location of the detector right. So this is the diffraction process. So the diffraction process essentially request a few requirements I mean few points that have to be satisfy for diffraction to occur first of all you have to have electromagnetic radiation for in the in the sense that we are discussing it at least.

In the general sense this - these are the once we are looking at two you need to have a sample that has periodic structure.(No audio from 40:38 to 40:51) The sample that has a periodic structure and therefore, we are basically talking of a some crystal in sample. We we want a crystal in sample right, so some kind of periodic structure has to be there some crystal sample has to be there and and the wavelength should be wavelength should be of should be in in a range that enables identified to be recorded. So, it is simply has to be in a range that permit as to record the diffraction effects and at least it which typically means it has to be at least in the wave length should be at least in the order of magnitude of this of this spacing in the crystal in sample if not less. So, that is that is what we are looking at.

So, these are the these are the general requirements and the typical way in which we are used seeing diffraction in labs is that you have a separate sample separate source separate detectors that is how we see diffraction right. But nowhere in this set of requirements have we stated that; we have never really stated that the source should be somewhere else the sample should be somewhere the detector should be somewhere else, that is just an experimental convenience that we have ended up doing it that way, but we have not there is no real fundamental requirement that the source has to be somewhere else the sample has to be somewhere else and the detector has to be somewhere else right. It is only an experimental convenience.

So, therefore, if you already have wave within the sample. If you already have some kind of of behaviour of particles within the sample which is equivalent or if not exactly equal to electromagnetic wave radiation passing through the sample then the basic idea that diffraction can occur will still happen all right. So to restated if you have particles within the sample which are showing you all the characteristics of electromagnetic radiation in based on their behaviour right, then those particles and the electromagnetic radiations behaviour kind of thing that they are demonstrating can also interact with the periodic structure of the sample within the sample to show you diffractions effects, this is something that can be expected, there is nothing here that says that cannot hold to.

So what in our particular discussion, what do we have? We have a sample that has electrons inside it, those electrons have energy and based on the energy and momentum that they have they have a wave vector associated with that all right. So that is the wave vector that we have designated as this two pi by lamda or k. So, therefore, we find that within the sample you have electrons that are moving there is a wavelength associated with them. And so so we now have the equivalent of electromagnetic radiation within the sample if you want the call it that; you have an equivalent of an electroma electron beam within the sample.

So, you need not specifically have an electron beam from outside, when we do electron diffraction in an electron microscope you have an source of electrons from outside it sends an electron beam it goes through the sample and it shows you diffraction right. Whereas, here the source of electrons is the sample sample itself; the electrons are within the sample they are moving around, but they have all the other characteristics they do have the wavelength associated with that and based on the wavelength you may or may not have diffraction based on the spacing of the appliance here.

Therefore, when we look at the Fermi surface information that we have discussed; the Fermi surface is of used to us, because it represents the wave vectors of the electrons; it represents all the possible wave vectors of the electrons from the lowest energy electron to the highest energy electron right. So, and this is reciprocal length space, in this same space we can also plot the periodicity of that crystal structure that we are talking about. So, when I take the particular sample there are two pieces of information that are unique to that sample or specific to that sample one is its crystalline crystal structure which means what are the axis what are the dimension and so on of those axis unit vectors and everything. So, that is an information that is length scale information and we can plot it in a in a manner that we will fit into the k space we will discuss that plotting little later, but that information.

So, the periodicity of the lattice can be plotted on this picture and it represents the periodicity of the lattice, the wave vectors permitted by the system based on the number of free electrons available in the system can also be plotted in the same plot and when you plot the two of them together in other words when you plot the Fermi surface as well as the periodicity of the lattice together on the same picture what we will see is - we can we will be able to identify conditions under which the electron which are present within the material will undergo diffraction, because of the crystalline structure of the same material. So this is a very important phenomenon; we will we will explore this in one of our upcoming classes by putting numerical values against it, but I am making diagrams; these same diagrams that I have just described will be made in those classes and we will look at the numerical values associated with it, but as a concept.

What this represent is that there are certain energy values that the electrons can have corresponding to specific k values then they can have, at which point they are interactions with the crystal within which they are - crystal structure within which they are results in diffraction. It is this diffraction process that now results in energy values that are permitted or forbidden for those electrons that aspect we will elaborate on in one of our later classes, but as the concept I wanted you to at least the aware of it at the sage and therefore, the Fermi surface which now represents the highest energy electrons becomes very important parameter. So we are we now have an understanding that Fermi energy is an important parameter, we also have an understanding of what the Fermi surface is and the fact that the Fermi surface is also very important parameter in our ability to understand material properties.

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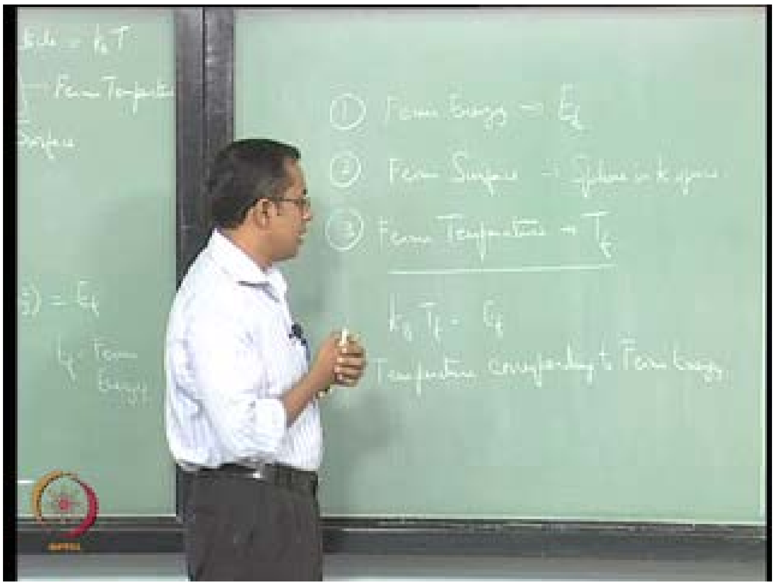

The last thing I will mention talk about is the Fermi temperature which are just very briefly touch upon today, because at least we only we will we will just say what it is and when it becomes when we use it in our in our next class we will actually use it at that point you will get a better sense of what it is.

As I said it is simply if you write k B T f equals E f, you will get a value of temperature; so, it is a temperature corresponding to (No audio from 48:01 to 48:16) so it is a temperature corresponding to Fermi energy, the point I want to highlight here is that if you actually did this calculation at zero Kelvin. Let say the material is setting at zero Kelvin the Fermi energy will be of the order of few electron volts - several electron volts it will be. So, if you take that and you just put in the value of k B and do this calculation you will find at the temperature what is being designated as the Fermi temperature will work out to be a very large temperature actually it will be several thousands of Kelvin. So, five thousand Kelvin ten thousand Kelvin something layer it is a very large temperature, but we are doing this...

So, in a way it is initially very confusing to look at, because we are saying that the material is at zero Kelvin. So, are is it are it is at it is ground state ground state is a lowest energy state. You will find that the Fermi temperature will work out to be very large number very large number the significance of the Fermi temperature we will see in our next class, but the basic point here is that what we are essentially saying is that when you arrange the electrons in various energy levels, if you assume that the given energy can only be present in the form of thermal energy then we can sort of equated to a temperature that is what this temperature works out to for the highest energy electrons in the system.

So that is what this temperature works out to and how do we rationalise it? How do I say that you know something is sitting at either zero Kelvin or very near zero Kelvin, but the temperature of electrons there is several Kelvin, how do we rationalise it where basically the idea is that when you talk of temperature we often do not talk of - we typically do not talk of temperature of a single particle in this case we do not talk of single electron, we are looking at the average temperature of the entire system whenever you do that there is the distribution of energies and therefore, the distribution of temperatures. So, it is always like that and you end up taking the average.

So the average is sitting at zero Kelvin at very tiny quantity of electrons which constitutes constitutes a very tiny quantity of the material present in the system is sitting at a very high temperature or it can be thought of at least in that in that of context. And that is how you can rationalise this idea that you have something sitting at very low temperature, but it has a Fermi energy which has the temperature corresponding to that Fermi energy which is a very high temperature. So, we have in this class we have looked at these three important quantities what they are how they have come about in relation to everything that we have discussed so far. And what they represent what is the importance of them and what is the physical significance of them and we find that there are all very relevant quantities.

So, we will halt here in this class, in the next class we will look at a calculation that as I mention we need to look at specific heat some more. So, we will start our discussion in the next class using specific heat where we will suddenly see that this Fermi temperature becomes becomes of relevance. And there in subsequent classes as we develop better even better models for our for our materials, we will actually use this Fermi energy and Fermi surface and the interaction of the Fermi the electrons in k space in in the plot that I was discussing with the periodic nature of that material. And on the basis of that interaction we will draw much better conclusions much more what will I say informative conclusions of how the material behaves why did why behaves that manner. So with that we halt for today. Thank you.