Physics of Materials

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Lecture No # 24

Density of States

Hello, welcome to this is the twenty fourth classes in our series of lecture on physics of materials. So, in the last class, we have looked at what we can expect, when we have an electron that is bound in some manner in fact, in the form of potential well. And in and the fact that, you know in a material there are couple of different levels at which, the couple of different extend to which an electron can be bound. It can be bound within the potential well that is very narrow that and therefore, then it would be called bound electron or it could be nearly free electron in which case it is free to run through the extend of the solid.

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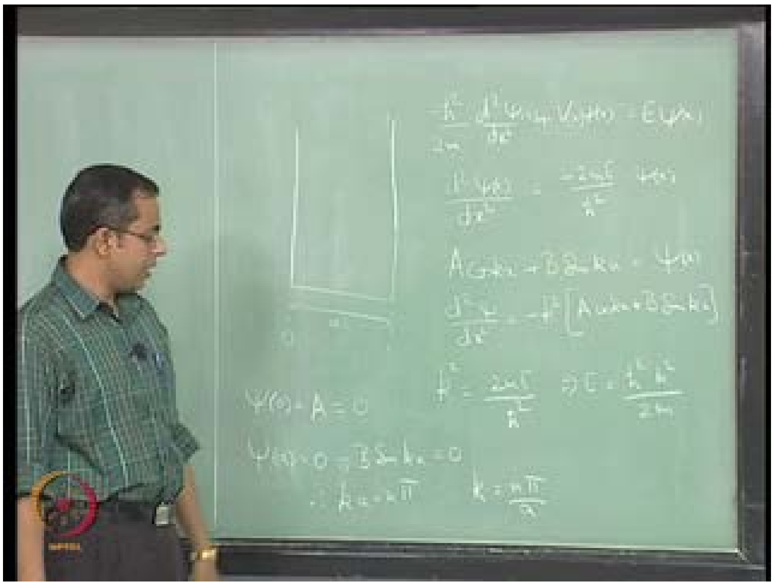

So, just to reiterate what we did? We had a potential well, we can do; we can make a minor modification, we can say that, you know the entire dimension is an even did from minus a to plus a relates this is 0 and this is a. So, we basically had when we solved the Schrodinger wave equation which was…

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We recognised that in this problem, we are saying that, the potential is 0 within this well and then, it is infinite on either side of the well therefore, the electron cannot come out. So, since, this is 0 this term reduces to 0. So, we simply had d square psi of x by d x square, d x square equals minus 2 m E by h bar square psi of x. And therefore, we recognised that the typical sin or cos function would satisfy this equation, because the second derivative of it would be the constant times the same function negative constant terms the same function. So, we wrote it as the possible solution as A cos k x plus B sin k x as the form of the psi of x or other psi of x is this and we do not know what A is and B is and what k is and so on.

So, we find that, if you differentiate this twice, you will find that d square psi by d x square equals minus k square of psi of x essentially A cos k x plus B sin k x. So, therefore, k equals this term here or k square equals that term, k square equals 2 m E by h bar square or we can also say m equals E implies h bar square k square by 2 m, simply taking this equation here and rearranging that terms we get that equation there. And, we also said that in particular problem, we used minus A to A, but we can also do this 0 to A. At 0 the wave function has to drop to 0, because the probability of the existence of the electron is; because this is an infinite potential pass this point, the probability of it is existence pass this point is 0.

And it has to sort of stake and we assume, that the functions are well behaved therefore, it has to drop to 0 at that point so that it can remain 0 pass that point, it should not being at some value are there appropriate drops to 0. We assume that of the function are well behaved therefore, it has to drop to 0 plus drop to 0 here, only then psi psi star d x the probability of finding the electron at location x 2 x plus d x. Which we said, you know was the bond interpretation of what the wave function was only then that, the probability of finding an electron will drop to 0 pass this point and pass that point and therefore, it has drop to 0 at that point.

Therefore, we were if you look at 0 when you put x is equal to 0 in this equation and you say that psi of x should become 0 or psi of 0. So, psi of 0 psi of x at x equal to 0 is simply psi of 0 cos of 0 is 1. So, it would become A and sin of 0 is 0 therefore, this term would reduce to 0. Therefore, we simply have psi of 0 is simply A and that has to be equal to 0 therefore, a is equal to 0 fine therefore, this term is no longer valid in this equation I mean not valid in the sense it reduces to 0. Then we also have psi of a it is also equals 0 what is psi of a? We already see that the first term is 0 so, we neglect this first term. So, we have B sin k a should be equal to 0, implies B sin k a should equal 0.

For this to happen regardless of the value of B, we are saying that, trivial solution is B is 0, but not trivial solution is that, k a should equal n pi whenever it is n pi here, sin of n pi is 0 where n is an integer therefore, or in other words k is only allowed the values n pi by a. So, this is the result we got yesterday, we got it couple of different ways, we looked at the string, we also looked at the Schrodinger wave equation again this is still the Schrodinger wave equation approach I use 0 to a.

And I also said that you know the extent of this. What shall I say, the extend of the well for the material, we have; we found there are two different levels of confinement, one is the length of the material and other is the ionic core itself, the dimension of the ionic core. So, in one case the a, the electrons that are bound by the ionic core this a works out to be the in the levels of Armstrong’s and for the electrons that are nearly free this a works out to be the value of the length of the material. So, this is this k can have significantly different meaning and we saw that, when you plot that this function here, for the free electrons all values of k are allowed therefore, all values of E are allowed, for the bound electrons in any form only specific values of k are allowed and therefore, specific values of E are allowed.

Now, the closer the space the k values are the closer space the E values are. And the larger the value of the denominator the more close the space those k values are, because they are the tiny values, the next value would be the very marginal difference from this value. So, therefore, for the nearly free electrons.

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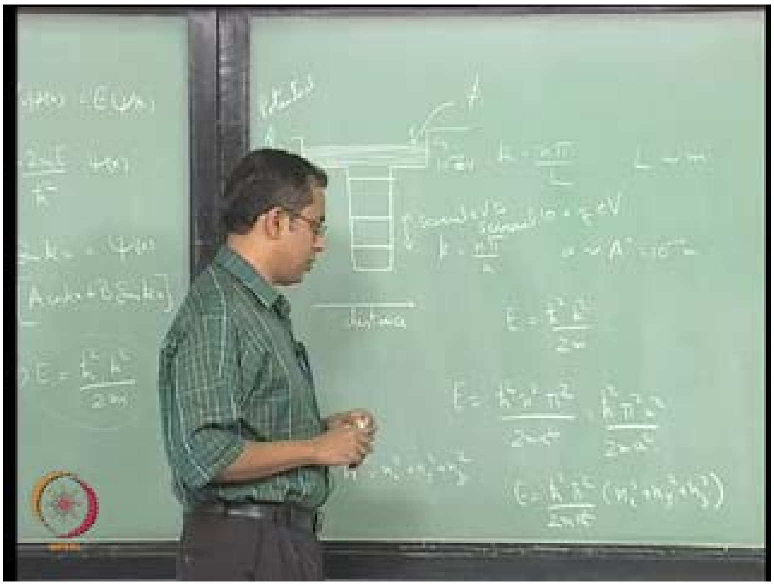

So, when we write this, when we draw this, I just draw one potential well here, just to illustrate the point, for the nearly free electrons which fill this space here, we also indicated this E pi there are sorry, E f Fermi energy and this is the work function phi, this is the potential and direction and this is distance or location distance. So, here, even though for all so, for all visual purpose I have done it like this, there is the word of difference in the dimension of this, versus the dimension of this, this is of the order of Armstrong, this is the order of metres. Therefore, here the k f values are n pi by L and here the k values are n pi by a, a is the order of Armstrong, L of the order of meters, 10 power minus 10 metres.

So, what we find is, because this is the very small value here, it dominates this term and therefore, for every value of n this spacing are different. The allowed energy level which are E equals h bar square k square by 2 m this values are very widely spaced for electrons that are bound within these narrow region. So, they are very widely space, you will see one energy level here allowed energy level, next allowed energy level will be somewhere there, somewhere there and so on. So, these are the exact location, you have to do the calculation to see, but the point is they are very widely spaced relatively speaking in energy and this spacing could be of the order of several terms of electron volts.

So, we will I put those number on, several terms of electron volts or even hundred electron volts between this values of energy levels, that are present in the bound electron. Here on the other hand, they are still discrete it is not a continuous spectrum, because only the specific value of k are allowed, but not all values of k are allowed, but they are much much much closer. (No Audio From: 09:40 to 09:49) So, they are very very very very close. In fact, drawn to the scale you would it able to distinguish between them, they are that close that is the reason by I mean of course, later we relate this by bands and all that. Just for very loose connection, if you want to make at this point, when we look at the band, when you say conduction band when it bands, at first glance it appears that that entire band consists of allowed energy values.

Actually, even within them, there is very fine detailed is there of what is the allowed energy value and what is the not allowed energy value, that details come from this level of spacing between to some degree comes is; come insulate, this level of energy. So, here the spacing between the allowed values is of the order of 10 power minus 10 electron volt so, 10 power minus 10 electron volt is the kind of spacing that, we see there. And the kind of spacing that, we see here is of the order of several electron volts to 10 of electron volts. (No Audio From: 10:55 to 11:00)

So, several electron volts to several 10 of electron volts or even hundred electron volts of that order, it is a huge difference between this and the spacing there. And when we talk of; just give you idea of k, when we talk of band gap, which we are not come anywhere close to that concept yet, we are not really talking of that yet. Just to look at comparative values, band gap are of the order of electron volts, 2 electron volts, 3 electron volts, 4 electron volt that is the kind of value we are looking at, 1 electron volt. So, of the order of 1 electron volt is the band gap, of the order of 10 power minus 10 electron volts is the spacing between these energy levels and these spacing between these energy levels is several times of electron volts if not more.

So, that is the; so that is just I have put this down so that, you get a sense of course, one how we have gotten here, how did; how we are understanding now, that confinement of electrons lead to quantisation and what quantisation of energy values. So, confinement of electrons leads to quantisation of energy values that are allowed for those electrons that now, they cannot any longer accept or assume any energy levels that they wish. But they the general form of the energy versus wave length that are wave vector, that they that is there, still the same, it is just that only the specific value of k are allowed therefore, only the specific value of E are allowed so that is the thing.

So, and; So, when we make a plot instead of having the continuous plot, there are only specific points along the plot that are the actual allowed values, the rest of; in between those points there are values which are not allowed. So that is the picture, later we will use this picture and I just want to perhaps this highlight one point, again simply because these energy values are very large, spacing here is large, spacing here is very small, if you draw that to this scale, you will almost see continuous energy values it look like that. So that is the concept that I want to keep in mind. We will later you said when we couple of classes down the road or few classes down the road we will using, at that time I will alert you to it once again. But to this scale it look almost continuous so, but actually they are all discrete, all these energy values all these so this is the picture.

Now, we write E equals h bar square k square by 2 m, but we also recognised that, k is n pi by a. So, we can actually put this value of n pi by a here. So, we can also write E equals h bar square n square pi square by 2 m a or you can put it as h bar square pi square n square by 2 m a. So, energy versus k relationship is there and we are now, converted into energy versus n relationship. In the language of physics, when we look at this kinds of topics and we address them using the language of physics, what the way in which the strings are described is as follows. When you draw something in using the using access corresponding to the particular quantity, we say that, we are discussing the topic in that is space so that is the kind of language that is used.

So, for example, this is the vector, k vector I done the unit one dimensional case that is why we simply have k. In principle, we could have k x k y k z so that is how, we could have. So, we can discuss this entire conversation as though, we are plotting everything in the coordinate axis of k x k y and k z and this all the energy information corresponding to the system can be plotted in that axis. Because the equation is there, anything you can plot using energy, you can convert and plot using k, because they are related to each other. So, you can actually plot them in k x k y k z axis using those axes, when you do that, we say that we are representing the information in k space, that is the way is the description is used made, we describe it by saying we are representing the information in k space that is the description.

So, whenever we say, we are in case space this is simply all it means, it may seem very complicated, it may seem like we are talking of something very fancy, in some sense it is not, because it is a very straight forward relationship. As long as you know the relationship, you can always plot it in; if you know the relationship between two quantities, you can plot the information in terms of one quantity or in terms of the other quantity it does not matter, because you know the relationship. But, because of the relationship the picture may look different, the physical, the final plot that, you see may have the certain shape, if you plot it using one variable, it may have the certain different shape, if you have plotted using the other variable.

Simply because of the way in which way they are related, but the information that you get out of the tool should be the same, because they are the same; this exact relationship has been used, that is the why the information you get should be the same. It turns out that, in some cases, one set of axis is more convenient to use, because of the manner in which that variable is behaving and therefore, we will switch from one space to another space as necessary whenever it is convenient. So, when you plot using k it is called k space. Similarly, you can plot using n, in this case n is a quantum number here, because it is in steps of n that the system is changing.

So, you have I mean n can take values 1 2 3 4 and so on, so that is quantum number. So, this is actually quantum number space, if we plotted versus n. So, this k space, this is quantum number so that is how it is and again this is also done only in one dimensional sense right now, it strictly in three dimension n square equals n x square plus n y square plus n z square so in three dimensions this is how it would be. So, if you want to write it again in three dimension this is the same as say E equals h bar square pi square by 2 m a (No Audio From: 17:20 to 17:29) times n x square plus n y square plus n z square. I am sorry this is 2 m a square here, so this is a square because n square pi square by a square pi square n square by a square.

So, k equals n pi by a so, k square we have plotted. So, it is n square pi square by a square so that is what we have. So, now, the point, we have to once, we put on this relationship the point I wish to I want to draw your attention to is that the way is related to n is. Such that all values of n, this combination of n x n y and n z that result in the same amount of energy, all values of n that result in the same amount of energy will lie on the surface of the sphere.

In other words when energy is the constant, where any given value of energy we take, let says the one constant value of energy we take, it means that, n x square plus n y square plus n z square has to be a constant that is the equation of the sphere. Spherical it simply means any vector see, if you have a vector n which consists of; which the; where it is components along the x y and z directions are n x n y and n z, the modulus of n is simply n x square plus n y square plus n z square. And as long as the modulus is the same, if it keep changing this and; but the modular works out to be the same, you get the same value of energy. What happens when you change n x n y n z? Such that the modular remains the same, you are actually tracing the sphere in all directions, you are moving but, you are tracing a sphere.

So, you can look at it physically, you can look at it mathematically, mathematically simply it means n x square plus n y square plus n z square is a constant and if it is a constant, we are talking of the sphere. So, all values of all points in k space sorry all points in quantum number space which form the sphere represent the same energy, that is the important things of information that, we wish to have. So, this is the some information we have, we will keep this here for the moment, from here, we will do something slightly different. Because, we have actually comes for enough that, we can actually go back to one of things issues that we were trying to discuss earlier, but we said that stage that, we do not yet have all the information and all the tools in hand. So, we need to develop certain things and then come back to that point. So, we have developed certain things already which is where we have this equation here.

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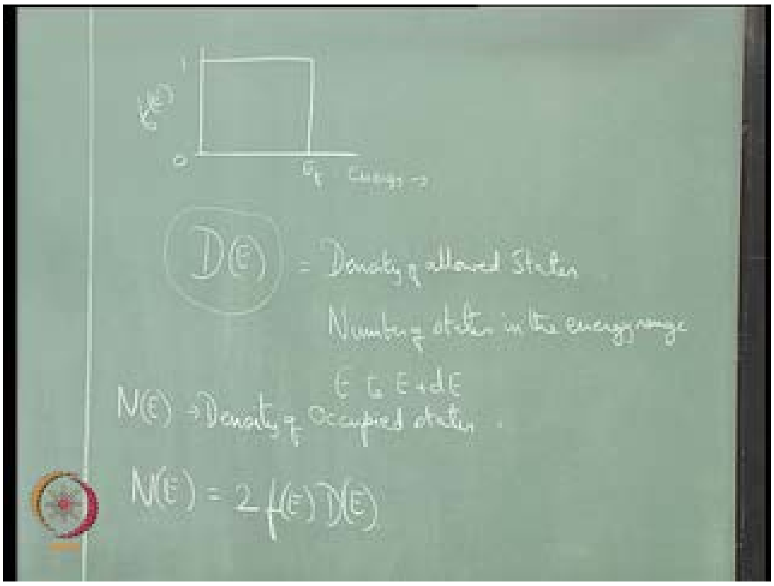

$D(E)$ = Density of allowed states

Number of states in the energy range

E to $E + dE$

$N(E)$ → Density of occupied states.

$N(E) = 2 f(E) D(E)$

So, we can now, go back and actually, look at little more of a detail of something that we left open, open it in that point. And the detail was simply is this, I basically put the Fermi direct distribution f of E (No Audio From: 20:23 to 20:31) like this, this is energy and that is the probability of occupancy. Now, when we discuss this I told you that, this picture is an incomplete picture, because it is not really telling us how many states are actually available at each energy level, it simply says what is the probability of occupancy. So, we will now define so, in other words we may have only five states here, we may have five hundred states here.

So, just looking at the square diagram like this, if you are not sure what you are talking of, you may get a miss reading idea that the number of energy levels is the same at all number of states available at every energy level is the same that is not the case here. Because this is not the number of energy level, it simply use the probability that, those levels are occupied. There may be five states here it means, all five states are occupied, there may be fifty states here it means, all fifty states are occupied, there may be five hundred states here it means, all five hundred states are occupied, that is the basic information then will give you, it is not telling you how many states are there that is the independently we have to find out.

So, independent of this picture, we have find out how many energy states are available as a function of the energy and taken together, you will get the full picture of the sys situation, which is how many occupied states are there in the system. So, what we are really looking for is a parameter that, we represent as D of E and D of E is called density of allowed states. (No Audio From: 22:03 to 22:11) Effectively it is the number of states in the energy range E and E plus d E to E plus d e, if you know the number of states. (No Audio From: 22:15 to 22:22) In the energy range E to E plus d E, in the small d E between E and E plus d E that is this density of the allowed states and if you define N of E as the density of occupied states. (No Audio From: 22:57 to 23:06) So, density of occupied states then N of E is equal to 2 of f of E D of E.

And I will just explain it this 2 factor comes from the fact that, if you say that, would if you not accounted for spin then, in the given state, you can give a spin up and spin down. So, you can have two electrons per state, if you want, if you that is the way you done your calculation, that is why this factor of 2 comes. But, if you set aside this 2 even if you do not bother much about this 2, the number of; the density of occupied states is simply the product of the density of the allowed states, times the probability that those states are occupied that is all that is quite straight forward. This is the density of occupied state, this is the density of allowed state, the fact that these are allowed does not mean they are necessarily occupied, they may be occupied, they may not be occupied.

Whether or not they are occupied is decided based on the probability of occupy, if it is 100 percent then all of them are occupied, if it is 50 percent there only the 50 percent of them are occupied. So, the number of; the density of occupied states is the density of allowed states times the probability of occupancy and of course, since it is the density we are still talking of E to E to we are talking of this range E to E plus D E fine. So, this is what it is the density of occupied states is the density of allowed states times the probability of occupied. For us, it is of interest to get an expression for this we would like to get an expression for this.

So, what we will do is actually, we will approach this problem from two different directions and so, we will have some two different expressions that, we will have and we will be able relate those two expressions and in the process, we will get this d of e. So, we will do that.

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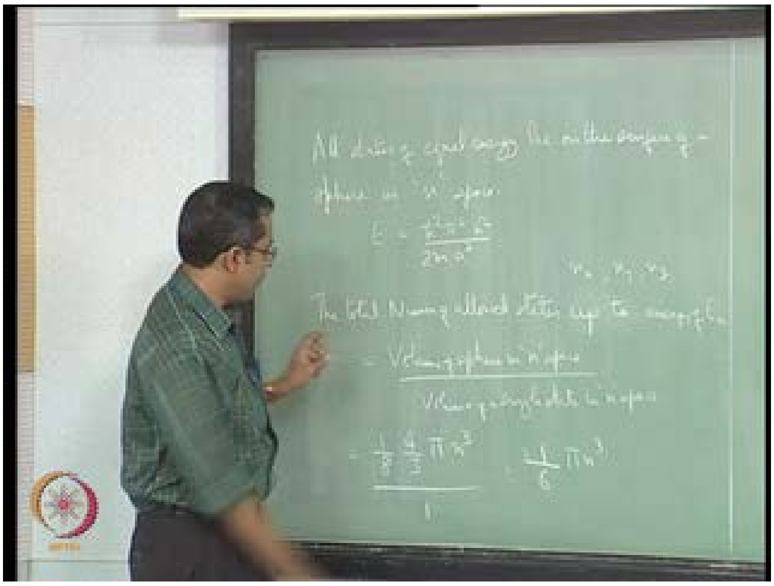

So, now the first approach we will take is that, we wrote this, we noticed that basically all states of equal energy lie on the surface of the sphere in n space is, this is something that I just explained you, if you write and so, we have just discussed that. So, we you can you have understand now, since it is simply E is equal to h bar square pi square n square by m a square 2 m a square. So, simply because you have this situation, all the states of equal energy lie on the surface of the sphere in n space. Now, we also want to; we also wish to point out that, n is an integer so it means, it can only take positive numbers positive values. So, n can only be plus 1 plus 2 plus 3 plus 4 and so on and if you write it as n x n y and n z, the values of n x n y n z can also only be positive numbers they can only plus 1 plus 1 plus 2 plus 3 plus, 1 plus 2 plus 3 like that. So, they can only positive values.

So, what we will do, we see we will look at the; we will say; we will see, if we can look at what is the total number of allowed states? The total number of allowed states below an energy value E subscript n. So, the total number of allowed states up to an energy value of up to we say, below is same as that, up to an energy value E subscript n (No Audio From: 27:41 to 27:50) up to an energy value of E n, is simply given by the volume of. So, you have a; so, it is given by the volume of sphere in n space divided by the volume of the in single state. So, I will write it down then I will explain it, equals volume of sphere in n space.

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And that is quite straight forward, you have the number of states each of it is has some unit volume and taken together they form the sphere. So, you take the volume of the sphere, you divide by the volume of the single state, you get the number of states so that is what it is. Except that here, it says volume of the sphere, but I just pointed out the that, we are only allowed positive values of n x n y n z. Therefore, we do not take the entire spheres volume, we take only the volume of the sphere in the first of ten, if we look at the sphere and you divide it into x y and z.

So, only one-eighth of the volume of the sphere you take. So, the numerator is simply 1 by 8 4 by 3 pi n cube that is the one-eighth of the volume of the sphere, 4 by 3 pi n cube will be the volume of the sphere of the radius r, it is cube here the radius is being measured in terms of n the quantum number. And so therefore, this is the volume of that sphere generic sphere of radius n, but, only one-eighth of the volume is in the positive quadrate which would corresponds to positive values of n x n y and n z which total up to this n, so that is what it is.

In quantum number state, the volume of the single state the step size is 1. So, n can take values 1 2 3 4 and so on. So, the step size is 1 the step size is n wise in n x direction is 1, in y direction is 1, in n z direction is 1 so, the unit volume per state is 1 so, this is 1 divided by 1. And please note, we are talking now of the number of allowed states this is still not occupied, we are said nothing about occupancy at this stage, in this calculation the way I am doing it right now, it is only the a number of allowed states, nothing has been be said about whether or not they are occupied. So, this is simply whatever the numerator is 1 by 8 or this is we try to 1 by 6 half 1 by n cube. So, this is all it is.

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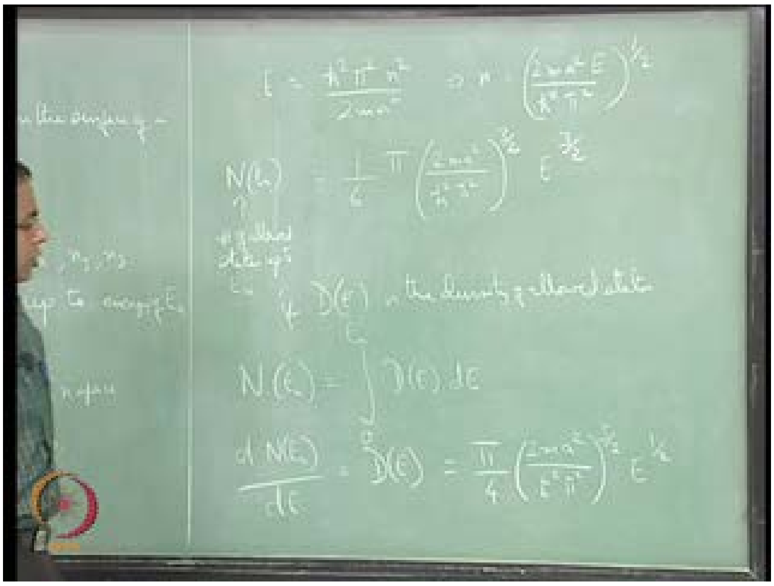

So, we will take it from here.

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We wrote energy as h bar square pi square n square, we written it there by 2 m a square. So, we now have a relationship between energy and we already have the relationship between energy and the quantum number. Now, we have the relationship between the total number of allowed states up to upper limit of energy E n and the same quantum number so, we have those two relationships. So, if you use the previous relationship and substitute here, we will now have the relationships between the number of allowed states up to the energy value E n and the energy. Because we have the relationship with quantum number and quantum number and n energy relationship is there so, we just substitute there. So, N which we will call us N of E n, number of allowed states.

(No Audio From: 32:02 to 32:18)

n cube we needs. So, here this implies n is 2 m a square E by h bar square pi square power half, just rewriting this is numerator becomes 2 m a square E by h bar square pi square so, this comes down here, this is n power half. And, we need n cube here 1 by 6 pi n cube. So, this is simply will become 2 m a square by h bar square pi square power 3 by 2 E power 3 by 2, this is what we will get. E power 3 by 2 is what you will get there, because all we have done is I taken cube of this, this is the n power half, this is this term times half and we want n cube here. So, I have cube this whole thing and brought E outside just, because we are going to just something with it and. So, this term is still here power three by 2 E power 3 by 2. So, what is the total number of states up to an energy level E n.

So, now density of allowed states so, if you look at; if D of E is the density of allowed states. (No Audio From: 33:49 to 33:57) Then, this number the total number of allowed states up to an energy level E n is simply the integral of this with respect to energy that is all it is. This is the number of allowed states between an energy value E and E plus D E, if you integrate that from 0 all the way up to upper energy level of E n, you should get the total number of allowed states up to that energy level E n that is all it is. So, that is all this to relate to each other. So, N of E n it simply integral 0 to this value E n D of E d E, it is a just based on definition that is what it is. This defines, what is the number of energy states between E and E plus D E? We want all the way, we want all the number of states, total number of states between 0 and higher value of E n.

So, you simply integrate this thing as a from 0 to E n, you should get this value or if you reverse this, if you differentiate this with respect to E you should get D E that all you have to get. So, d N of E with respect to E, E equals D of E fine. So, now we already have an expression for E of N, if you differentiate this with respect to the energy you will get D of E fine. So, what we have done? We have approach this problem from two different ways, we have looked at the total number of states available in the system and treated that as you know once one-eighth of the volume of the in n space. And then, independently, we also looked at the energy association with the system and how it relates to n. So, we got an expression between energy and n.

And since, we have the relationship now, between the number of states and the quantum number and energy in quantum number. Now, we have a relationship between the number of state and energy, because quantum number is common to both of these relationship and then, given that you have the relationship between the number of allowed states and energy, we are able to get an expression for density of allowed states which simply the differential of those. So, we differentiate this with respect to E you will get D, if you differentiate with expression with respect to E you will get 3 by 2 will come here and you will become E power half.

So, this implies so, you just differentiate this expression with respect to E. So, 3 by 2 will come here. So, 3 will go so, this is become one-fourth, this is become pi by 4 and this will remain the same, we will not disturb this in any way.

(No Audio From: 36:40 to 36:57)

So, this is what we get. So, this is D of E and incidentally a square power 3 by 2 is a cube and a cube is the volume of the system that we are talking of. So, you want write it at as volume, we can write it as volume. But the main point we make here, is that D of E as a function of E it is actually parabolic, you see this E and D of E are related like this fine.

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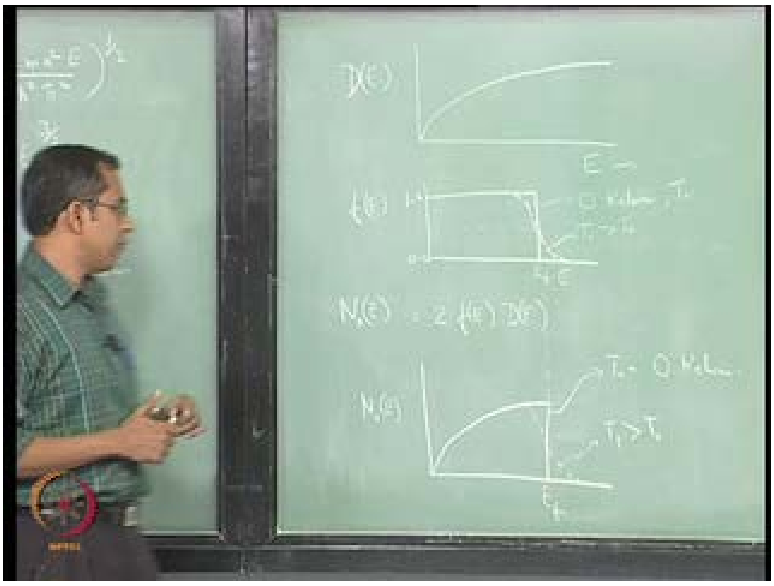

So, now, if given that, we have this relationship and we also know that, we know the manner in which the how many Fermi direct statistics function behaves so, we have.

(No Audio From: 37:38 to 38:04)

So, we know if we plot D of E (No Audio From: 38:10 to 38:24) So, if we plot D of E versus E. So, you will get the function that, have the basic behaviour look like this. So, this is the density of occupied states as the function of energy. We also wrote f of E as a function of E and that, we saw behave like this (No Audio From: 38:50 to 38:58) this is how let two of them behave. Therefore, the density of occupied states, we will say N occupied subscript o, N occupied up to an energy level E with simply 2 times f of E D of E, that plot will look something like this. If, we assuming will drawing to the same scale, we can say this is E f, this is N occupied of E. So, it will look like this, it will start of like this, it will drop up to this, it will drop down to 0 and stay that there.

So, in at 0 Kelvin so, this is the, this picture drawn at 0 Kelvin at 0 Kelvin this is the picture, we have of the probability of occupancy, as given by the Fermi direct statistical distribution, the probability of occupancy is as shown here. This function D of E gives us the density of allowed states, the density of the allowed states is given by the function here, so that look like this, probability of occupancy look like this. Therefore, at 0 Kelvin the density of occupied states look like this, it is simply the product of these two by multiply by factor of 2 which we can ignore for the moment, but it simply the product of these two. I mean, we are not ignore it, but in this plot we just schematic we are not necessarily focussing on the constant too.

But, you actually see that, it goes of this way drops to 0 and goes, at higher temperature the probability of occupancy changes. So, we see a behaviour that looks like this. So, this is 0.5 you see a behaviour that look like this. So, at higher temperature this is at 0 Kelvin. So, this is T 0, this is T 1 greater than t 0. So, we will say that this is at T 0, at higher temperature this distribution look like this. (No Audio From: 41:29 to 41:40) So, therefore, we now have a picture which is now a distinct step forward from where we where earlier. Previously, we where we came up, we understand that, you know we had to use quantum mechanical principles, because our system was search.

And therefore, the Maxwell, Boltzmann statistics was not good enough, we did Fermi derive statistics. So, we develop Fermi derive statistics and at that stage, we basically found at this is the kind of behaviour it has. So, we recognised at that stage, that when you raise the temperature of the system only electrons very close to the Fermi energy are able to participate in the process, this E f is the Fermi energy, only electrons very close to the Fermi energy are able to participate in the process. So, and therefore, we recognised that, as the concept that immediately indicated to us, why it is that let say the Drude model is in correctly predicting the electronic contribution to specific heat.

Because according to the Drude model which uses Maxwell Boltzmann statistics, all electrons can gain energy as you raise the temperature, which means there is the huge number of electrons they all attempting to gain energy so, relatively speaking that the large amount of energy. In the Drude Sommerfeld model, because it uses Fermi derive statistics only electrons very close to the Fermi energy or in the position to accept energy.

And therefore, it is a relatively smaller number of electrons that are able to accept that energy as you raise the temperature of the system. And therefore, the electronic the prediction for the electronic contribution of specific heat distinctly decreases when you use the Drude Sommerfeld model. And the reason of course, that only electrons close to the Fermi energy can participate in the process is, because all other states before it or occupied. Therefore, an electron which is way below an energy level, if it attempt to gain the energy it has to go to valid site, a valid energy level, rather valid energy level and all those valid energy levels a hide of it or occupied or all already occupied because the probability of occupancy is one.

Therefore, none of these electrons which are at lower energy levels can participate in the process, when you use the Drude Sommerfeld model. And therefore, as the concept it is immediately clear how the Drude Sommerfeld model handles the problem of electronic configuration to specific heat. But, at that is at the same time, we recognised that the this was not the complete picture, because it did not it tell us what is the number of states available as the function of the energy. And the exercise we have done today, as enable us to find that, we have looked at by; because we are accepted that, you know there is the confinement in the system, we recognised that the confinement lead to quantisation.

Because it lead to quantisation we realised that, only specialised value of d wave vector are alone which now correspond which are based on the quantum numbers n. So, those n values, which are integer values, permit only specific value of k in the system because of the confinement of present in the system. Because only specific value of k are allowed, only specific value of energy are allowed corresponding to that. Now, we also in today’s class, we said that given that situation, we understand that since, there are relationship between E and k and E and n, you can pot this information in k space, you can plot this information in n space.

And for our calculation purposes, where one of the things we are looking for this to add more detail to this picture and which is to find out the number of allowed states at each of this energy level and therefore, the density of allowed states it is use full for us to start working with the n space. So, therefore, in that context, we looked at this n space, we said that, if we look at E versus n relationship, you will get something like this and in this system. Therefore, all values of I mean all states which have the same value of the energy lie of the surface of the sphere in n space, so that is the context the information that we put together. And in the n space given that there is a sphere which contains all the states containing the same energy, if you divide it by the volume of the single state in n space then, you get the number of such states that are there present within that sphere.

So, up to the value n all of these they contain within that sphere. So, whichever is the higher value of n energy, you take that energy you get a sphere, all the states within that sphere or energy values that are permitted corresponding to those n values that are permitted, at those specific n values whenever they added to it. And so, when you divide by the value of the single state, you get this is as the number of this is the number of allowed states. So, please note that here also we are using and E n which is the number here. So, this n of E n is the number of allowed states this is not the number of occupied states this simply the number of allowed states this value here.

So, we find that, that actually has the relationship of E power 3 by 2 with respect to energy. Then, we find that we note we noted that, if you have the density of allowed states the number of allowed states is simply the integral of the density of allowed states from 0 to E n. And when you do that or rather if you differentiate now, you will get the density of allowed states. So, we simply differentiated this expression here and we ended up with this expression here and I said for example, a power 3 is the volume of the system that we are looking up.

So, now we have the density of occupied I mean sorry density of allowed states as a function of the energy, we already and so, we plotted that, and we have the probability of the occupancy as a function of the energy. So, we are able to plot that as well. So, given those to we are now, having the complete picture of the solid which says as a function of energy, what are the number of allowed states and also how many of them are occupied all of that information is here. This is actually the density of occupied states density of occupied states. So, this includes both information, how many states are there and what extend are they occupied, clearly at 0 Kelvin given this kind of the picture all these states up to certain energy level Fermi energy level are fully occupied. So, therefore, it goes exactly up to that point, drops abruptly at 0 at space at 0.

Whereas, at higher temperature as you approach the Fermi energy there are energy, there are states of energy value less than Fermi energy which are not fully occupy. Say for example, this some number of states are not getting occupied here, but some number of states at energy values higher than, the Fermi energy are getting occupied, but these now, contained the complete picture. So, therefore, the utility of the information is of course, first of all it is the more complete picture of the system, but the more immediate utility of the system is that.

Whereas, previously when we looked at the specific heat discussion we can only descriptively say that you know the in the Fermi direct, in the Drude Sommerfeld model only energy values close to E f participate in the specific heat process in the process of gaining energy. And therefore, that reviews the; let say the Electronic contribution to the specific heat significantly, but you are able to say that only descriptively. Given this information where we actually have much better feel for, what is the complete system, what is the number of states and what is the probability that they are occupied.

When since, both this information is now held together in this picture we can actually see that, we can calculate now, what is the; we can put the numbers down to this general idea that, the Drude Sommerfeld model that is the better job of predict in this specific heat. We can put the numbers down, because we now have the background for it we do not simply have the probability of occupancy, we also have density of states therefore, we have density of occupancy. So, with this we can say clearly what is the number of; what is actually happening as you try to raise the temperature, we can get the much better idea what is actually try to happen as we try to raise the temperature.

So, what we will do is, we will halt here for now, in the next class, we will look at this particular problem and see, what is the number we can put down for the process. As you raise the temperature of the system and how well know the model which has all the details the where we arrive now we are still with same Drude Sommerfeld model, but we have much more detail of that model incorporate in to our equation. Having come this part, we will see how well we are able to actually predict this numbers of the electronic configuration specific heat. Additionally, we will also see certain features related to the Fermi energy.

So, Fermi energy so far I have sort of just stated that, it is the highest energy level available in the system and we just put it down. But there are certain things we can say with respect to the Fermi energy or certain ideas that we associate to the Fermi energy that, we keep utilising at various places. So, will at least spend little bit of time in our next class looking at the concepts associated with the Fermi energy or different ways in which the Fermi energy information is conveyed in different discussion. So, we will look at that in our next class. So, we will do that, the Fermi energy discussion as well as the specific heat discussion and see how everything that we have to learn so far, enable source to much have the much better picture for what we solid is an how it is being.

We still have some holes in the discussion, we still do not have complete detail of how something occurs in the solid, because there is even more detail that, we can through into the picture and which we will do in classes after that, for now, we will halt, we will take it up in the next class. Thank you.