Physics of Materials

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Lecture No # 23
Confinement and Quantization: Part 2

Hello, welcome to this, the 23rd class in our physics of materials course. In the last couple of class we have looked at couple of different aspects, the first is that we try to look at what is realistic way of representing a solid, with respect to what is the potential that an expel the electron will experience as the travels through that solid. So, that is the first concept that we looked at that, what is the realistic way of laying out the information, which is potential versus position for an electron, as a; that an electron will experience as a travels through that material.

So, this is the first information that we put together then, the second thing that we did was we made an approximation of this information. And we wanted to understand what is the impact of such a potential versus position diagram, all the behaviour of an electron are what it is we can expect from an electron. So, to begin with in the last class we did an approximation of this, we took much more simplified approach to understand what happens to a wave in general when it is in any way restricted.

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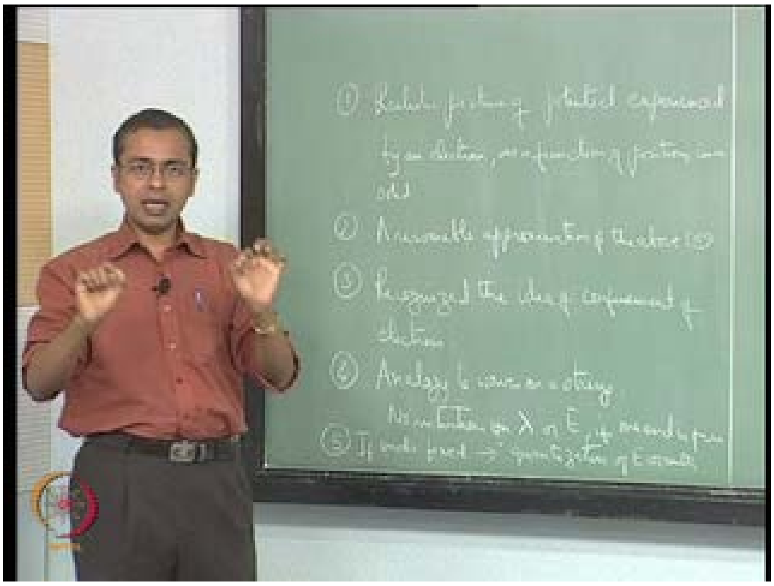

So, the concepts, we are throwing it together are as follows the first is that.

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The first is this realistic picture of potential as experienced by an electron as a function of position in a solid, for this we also used simplified one dimensional solid. So, we have not really looked at the three dimensional solid in all it is details, but it was sufficient for all purposes of discussion to look at the one dimensional solid. Then, we made an reasonable approximation (No Audio From: 02:29 to 02:36) of the above, the point number one. So, we made a reasonable approximation of this. So, where we had very clear curvatures that we could see, we saw de potential well developing we converted that two square shape potential well.

So, which more or less captured the basic concepts that we potential well was bringing to the picture so, this we have done.

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We recognized the idea of confinement of electrons. So, what we have basically recognized? Is the fact that only when an electrons completely leaves the solid and goes away it is a truly free electrons, there are number of electrons which are actually trapped within the solid. So, there are not truly free electrons, but amongst the electrons present within the solid, there are electrons which can run through the extend of the solid, but cannot leave the solid. So, they are confined within the extend of the solid. So, the; so, those are electrons which is what we are looking at when we are discussing electronic conduction and so on. They are electrons which are within the solid, they are not escape from the solid.

And so, therefore, we describe them as being electrons which are confined within the extend of that solid, whatever they extend of the solid, it is can be meters, it can be centi meters, it could be a millimetres does not matter. It is a physical constraint we are placing that we it; the electron has to stay within that physical constraint, which is what? We are experiencing in the real world terms it is a physical object that we look at. So, those are electrons which are confined so, we recognised immediately, there is this concept of confinement of an electrons. And in terms of the electrons that are participating in the electronic conduction process that we believe participate in the electronic conduction process, they are confined by the extend of the solid.

We also realised that in terms of the confinement there can be an even more severe level of confinement. And those that is the level of confinement that effects all the other electrons which are trapped with the ionic core, those are the bound electrons. So, to speak which belong to every single ionic core present within the solid. So, those are electrons which are trapped within the dimension of that ionic core, they are not to even free to run the extend of the solid. So, if you have the meter long solid? Last majority of the electrons can never access that the meter long distance, they are stuck to within one Armstrong of the location of where that ionic core roughly speaking.

So, that is also confinement, that concept is the same, that there is the physical region within which the electron has to stay. The only difference is the extend of that physical region. So, the nearly free electrons run through the or confined to the within in the solid which is in the scale of the meters, the bound electrons are confined to that ionic core which is in the scale of an Amstrong. So, that is the difference, but the concept is that they are confined. Then, we recognised that to understand the behaviour of electrons, under those circumstances where they have been confined by whatever degree of confinement. We would like to look at the quantum mechanical aspects of the effects of confinement of all those electrons, we took an analogy to waves on a string.

And we looked at two possible cases, in one case the string was such that one end of the string was free, only one end of the string was held. And therefore, we; and we realised, we noticed when we discussed that example, in that circumstances the wavelength that could be supported on that string. There was no restriction on that wavelength, you could support just about any wavelength you wish on a string, which was held only on one at one location. So, under those circumstances we say that there is no variation, there is no gaps in the wavelength that there is our work there is no wavelength that we can say affront cannot be supported by that string.

And as long as you have an energy relationship to the wavelength and in; as long as you have the equation which relates energy to the wavelength, as long as the wavelength can take any value the energy can also take any value, because there is direct relationship right. So, if you can have continuous values of wavelength permitted in the circumstances, continuous values of energy are also permitted in the same circumstances. So, we found that the waves on the string, where it is side only on one end there is no restriction on the wavelength that can be supported by that string and hence no restriction on the energy that can be supported by that string.

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So, analogy to waves on the string, we found that there are no restriction on the value of wavelength or energy that can be supported by the string if one end of the string is free. And the last point that we noted down.

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If both ends of the string are fixed, if both ends of the string or which only fixed then, we find that, the both of the things change, why previously any value of lamda was alone? Since now, both ends of the string are fixed, only specific values of wavelength can be supported by the string. Only specific values of the wavelength or consistent with the fact that two ends of the string are now which only fixed. They are not in a position to hold any location that they wish, there is the specific coordinate where the start of the string should be; and the specific coordinate where the end of the string should be regardless of the wave that establing through it, when that happens only specific waves can be supported by that strings.

Only specific waves at consistent with that condition of that of that strings. And therefore, only specific, when we say specific waves only waves of the specific wavelength can be supported by a string under those circumstances. And since the wavelength has the direct relationship to energy, we end of with situation that since only particular value of wavelength are permitted on that string, correspondingly only those particular values of energy are permitted on that string. So, we now, we have move by simply placing one additional restriction of the string, where from; by moving from a situation where only one end of the string was fixed, two situation were both ends of the string are fixed.

We have moved from a situation where all values of energy were permitted to the system, two situation were only specific values of the energy are permitted in the system, in between the energy values are not permitted in the system cannot be supported in the system. So, there are specific values of the energy that are permitted and other values of the energy are not permitted. So, the last point if ends of the string are fixed both ends of the string are fixed quantisation of the energy results. This basically captures the idea that when you have confinement. So, the idea that there is potential well and the electron is trapped within the potential well is actually analogous and corresponds to the situation that both ends of the string are actually tied.

Because there is potential well, the electron cannot come out on either side of the potential well it is stuck in the potential well, as long as that is the deep potential well. As long as that is the deep potential well, basically the electrons is a stuck in the potential well. So, at least at one level of analysis we can look at it that circumstances, when an electron is trapped in the potential well. If you treat the electron as a wave, as supposed to as the particle then, the wavelength corresponding to that electron will have to be such that it will behave in manner very similar to a string that has been tide on both ends.

Because the potential well is there on either sides, it stuck within the potential well, it is very similar to a string that has been tied on both ends of within the potential well. Therefore, whenever there is confinement of an electron, in other words whenever an electron is trapped in the potential well, there quantisation of it is energy occurs. So, that is the very fundamental principle that we discussed and we finished out last class with that the confinement.

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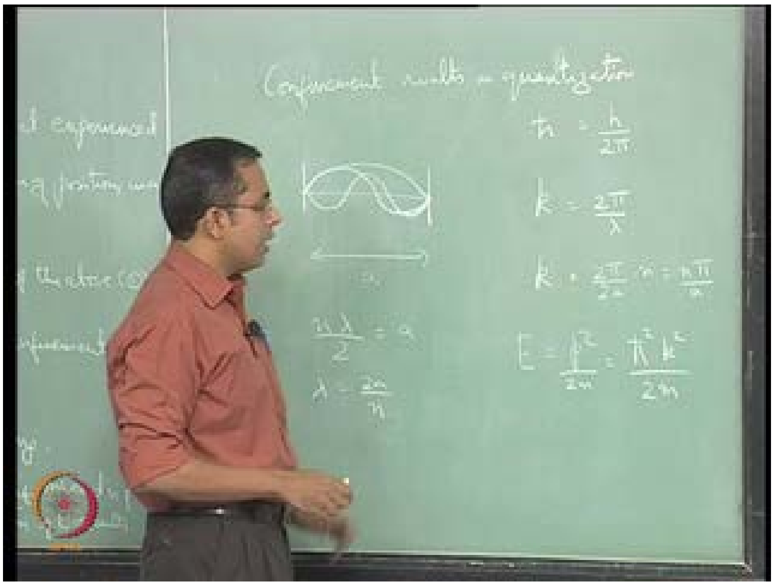

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So, we finished of this concept that confinement results in quantisation. So, when the electron is not confined, it corresponds to the string that is slide only on one side and in which case it can occupy, it can demonstrate any energy that it wishes to demonstrate. Or it can possess any energy there it wishes to possess, or comes under the circumstances where can hold at energy. But when it is confined to a potential well, the very act of confining it and the fact that it has wave like nature forces it into a situation, where it can demonstrate only a specific levels of energy. It can possess only specific levels of energy consistent with those circumstances that it is face to it and it cannot demonstrate any other form of; any other level of energy so, this we have; this is what we noted.

Now, what I would like to do, is actually we did as a as a physical picture where we basically said that we have a string and the distance between the location, where the string is tied is some dimension a and we said that is the string. So, we said that, if the distance between the location where the string is tied is a then n lamda by 2 should equal a. In other words a should be some integral multiple of half wave length. For example, I am just showing one half wave length, it should be; it can either be one half wave length or it can be two half wave length, or it can be three half wavelength and so on.

So, we could have the following wave supported and so on, we have any number of waves that we can support. So, this is the kind of some integral number of half wave length should equal to that distance. And that is how we came up with this; and that is something that we noted. So, if you want we can write this again as lamda is 2 a by n and I mention that we in; while discussing the physics of material we take talk in terms of wave vectors. The wave vector is simply defined as this vector k, which is equal given by 2 pi by lamda. So, now lamda has the following value in this set of values a is fixed right.

So, therefore, two the numerator is fixed, you only have the choice of the denominator n can be one two three four so on. So, 2 a by 1 2 a by 2 a by 3 so, that is the kinds of values of wavelength that can be accommodated here. So, I can be substituted there. So, therefore, the corresponding values of k that are permitted will be 2 pi by 2 a times n equals n pi by a. And we also said that the energy itself is the square of the momentum by 2 m equals h bar square k square by 2 m. So, this relation anyway always polls E equals h bar square k square by 2 m, so that relationship polls for the electron. The only question is, what the value E? In this again h is the constant and therefore, h bar is a constant, because h bar is simply h by two pi so, h is; I am sorry h bar is h by 2 pi.

So, h is a universal constant plank constant therefore, h bar is a constant. So, the numerator h bar square is fixed, the mass m is a constant for electron that is also the fixed. So, the real relationship is between E and k that really the relationship that is there. As long as k can have continuous values, E can have continuous values; if k have discrete values and E will have discrete values, that is basically all right. So, we found that when the string is confined tied and both ends, k takes only these allowed values n pi by a, where n can be 1 2 3 4 and so on.

So, there only specific values of k that are permitted therefore, correspondingly you can substitute here only specific values of E will be permitted so, this is what we found. Now, we did this as a exercise where we just looked at wave from the string and came up with this answer. Given the scope of what we are looking at it will; it also interest to see if you actually, when target and immense yourself in the more quantum mechanical way of looking at things, will you end of getting essentially the same answer that is the basic thing we would like to see. And then; and that is in fact, the more accurate way of doing it, but we are just taken as singular step here. So, we will look at the more quantum mechanical way of doing it and compare our results and then proceed.

So, in the quantum mechanical way of doing it, we basically recognised that in any system, as we; when we discussed this and when we looked at the history f quantum mechanics, we recognised that once you have a wave like behaviour answer. It is the Schrodinger wave equation that captures the details of the system in terms of the wave function. So, that is the wave function which captures all the details of the system and we can extract all the properties of the system from that wave function. And the wave function comes as the result of solving Schrodinger wave equation for the conditions that are placed on the system. So, that is how it is done?

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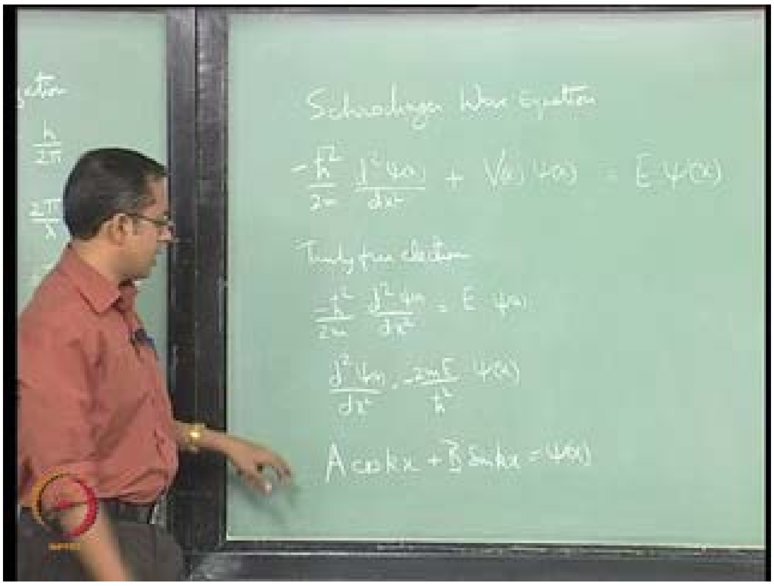

And (No Audio From: 17:44 to 17:56) so, we will have the Schrodinger wave; will write the Schrodinger wave equation and for our purposes we are looking at the one dimensional solid. So, we will write the equation also in one dimensional. And then, the as indicated before the form of the result is essentially what we are interested in and the form and significance of the result is what we interest in. You can extend to two or three dimensions and get yourself for more realistic results with respect to three dimensional solid, but the basic form of the equation is what will come out of the this analysis.

So, we have the Schrodinger wave equation is simply written as minus h bar square by 2 m d square psi of x by d x square plus V of x psi of x equal E of psi of x, where psi is the wave function and represents the details of the wave, that the system is corresponding to. So, we are actually looking at the wave function of the electron essentially in this case, the electron under, whatever circumstances that we are placing it under so, we have this situation. Now, the first we have to do, what we have to do? Is the equation is we have; as I mention earlier one of this equation is not something that you can derive it is the original; it is like the fundamental principle.

In fact, it is simply another way of stating the conservation of energy it is essentially, what it is telling you? It is simply tells you that the total energy of the system is some of the potential energy of the system and kinetic energy of the system. That is basically the information that is being provided through this equation. And therefore, in that sense it is the fundamental principle and you cannot really derive it from any starting point. You can nearly show that it is consistent with any other starting point. You can start with any other starting point; you will find that the expression you come up when you substitute here will work out write. So, therefore, you can start out show that it is consistent, but you cannot really derive it from anywhere it is.

So, now, what we need to know is we need to an understand, what is potential as the function of position and introduce that into the system. In the first case we will look at truly free electrons or free electrons if you want to solid. These are electrons that have escaped the system; these are electrons that are escaped solid. So, therefore, they are not in the solid and they are free chrome anywhere else and where; when we say anywhere else what exactly we mean is that the potential is zero. So, they are roaming in the region where the potential they are experiencing is zero, as a function of position regardless of where they are? They out of the solid so, they are out of the potential well that correspond to the solid, we are free to roam around the rest of the universe.

And for the purpose of our discussion and the rest of the universe is the flat potential equals to zero atmospheres. So, therefore, the V of x term is 0. So, we actually have minus h bar square by 2 m d square psi by d x square equals E psi of x. So, if rearrange it little bit, you will have d square psi by d x square equals minus 2 m E by h bar square times psi of x. So, what you see is you are essentially differentiating some function x twice with respect to x and you getting back some constant times same function. So, typical solution therefore, is a trigonometry solution.

So, cos of x or cos of some constant times x or sin of x and sin of constant times x would essentially come to the same thing. Because if you differentiate sin x you will get cos x, if you differentiate it a second time you will get back minus sin x and you will get minus here. So, therefore, this very conveniently fix with are concept of the trigonometry function. So, at simply by observation, we reach the conclusion that a sin cos type of function would satisfy this equation just write. So, the general solution therefore, is of the form A cos k x plus B sin k x, where the value of k is simply; I mean the k would have dimensions of one by length. And therefore, is consistent with the wave number dimensions and excess the positions. And A and B are some constant which represent the amplitude of the system.

So, now that you have this is a solution, we see that essentially what we are seeing is that in principle the regardless of the wave number, the wave vector we actually have the same relationship with the energy. So, if you see here this k here, if you differentiate this twice with respect to the left hand side. And therefore, you will get yourself with expression on the left hand side. If this is psi? So, we are saying that the; if this is psi and that we did only by observation. We are by same that you know that cos function and sin function will create the situation, where it differentiate this twice you will get minus times the some minus constant times this original function that is are we came up this.

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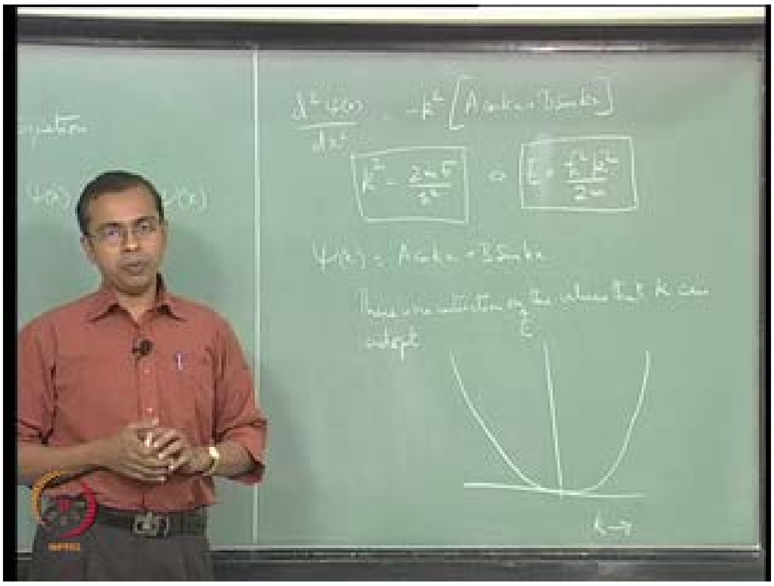

If you differentiate this twice, you will find that; this basically d square psi x by d x square will be minus k square a cos k x plus b sin k x. So, as I; as we indicated that the x to be we expected that is exactly what we are getting, we differentiate if we assumes right to the cos k x plus b sin k x, if you differentiated twice you will get minus k square times a cos k x plus b sin k x. So, therefore, k is what we saw here, if we look at this equation, you have some term here minus some term here and in the equation we just (( )) of there, we have minus k square r. So, therefore, k square is that term there 2 m E by h bar square. So, therefore, once again it will just rearrange this, you will simply get h bar square or you get E equals.

(No Audio From: 24:52 to 25:04) So, we get E equals h bar square k square by 2 m and that is the same as what we have return there on, when we have talking of we know, we just said that; we just looked at energy, we looked at wavelength and we looked at the momentum, we note p square by 2 m so, we come up with this equation. So, therefore, for an electron that is free to run across, that is really free electron where it is experiencing only a potential of V equal to zero everywhere. So, there is no specific location, all across the universe it experiences V equal to zero. When you face such an electron, you find that it is energy was out this h bar square k square by 2 m and not just that the wave the solution is return; we have come up with an equation for this solution what this psi.

So, we wrote psi of x equals A cos k x plus B sin k x and we found that based on the only condition that is placed on the system, which is that V equals zero everywhere. Based on that condition this equation satisfies our Schrodinger wave equation or is a solution to our Schrodinger wave equation for all values of x and all values of k I mean; I am sorry for all values of x. So, that therefore, this have no additional recruitment have been placed on this equation, the equation has it is a written here is already a solution for the problem that we are looking at alright.

In other words there is no restriction on the values of k. (No Audio From: 26:49 to 26:57) the way it is written as long it is a cos function and sin function, it does not really matter what this value of k is, if you differentiate twice? You will get minus k square times the same function. And that is consistent with what we have with the equation and then that k will ensure the; this polls to and that poll too. And therefore, this remains the solution, there is; we have no restriction of the value of k so, this case free to protect any which values it ones. So, therefore, we find even when we employ, when we do it the formal way, which is to employ the Schrodinger wave equation and solve for the possible solution of this system in question in which case in this case it is an electron.

When we look at the free electron, which is ready to run through the entire space that is allocated to it. The solution creates brings out the fact, that there is no restriction on the value of k. And therefore, the wave length that can be adopted by those electrons has no restriction, wave length can be anything that we pushes to the; and therefore, the corresponding value of energy can also be anything that wishes to. So, all values of energy are permitted, all values of k are permitted for free electrons. Real truly free electrons all values of energy are permitted all values of k are permitted. Other important thing, which I already pointed out all I reiterate here, is in this equation h bar is already a constant it simply h pi by 2 pi m is also a constant the mass of the electron.

So, this term is a constant, this term is a constant. The only thing that is really allowed to have all sorts of value is k and therefore, this relationship is really a relationship between energy and the wave vector. And we find all values of wave vector are alone therefore, all values of energy are alone, but it still has a certain form. So, we can make a plot of energy versus wave vector and the wave can travel in the positive x direction or negative x direction given in our one dimensional case. So, therefore, k in strictly speaking k can also have positive values or negative values based on the direction, because it is a wave vector.

So, therefore, we can make a plot of E versus k and for the free electron (No Audio From: 29:09 to 29:18) for a free electron E versus k simply a parabola and what we are going to see is a behaviour that look like this. (No Audio From: 29:27 to 29:36) E versus k is this, it simply it says that E is some constant times k square. So, E versus k will give you that k square for every value of k you square it and you will get this and some constant you multiply, which is h bar square by two m you will get the value of (( )). So, the few things to note here on, this is an E versus k relationship and it is a continuous relationship. Because all values of k are alone, every value that you can think along the x axis is permitted value. Therefore, all corresponding energy values are also permitted.

Therefore, this curve consist of; so, all the valid points of energy for the system, continuously follow E versus k actually are form of; are the part of this continuous curve. There is no discontinuity in this curve there are no gaps in this curve it is a continuous curve, all the way across. Even as it is drawn it is continuous, but all the more so, because all values of k are permitted, all values of E are permitted and therefore, this curve consist of continuous points there is no discontinuity in this curve. So, that is the important information that we wish to have.

So, as we step back here, what we have seen so far is, we have used Schrodinger wave equation to deal with the problem that we are dealing with, as supposed to simply looking at a string. And we find that the solution we are arriving at is exactly the same, the energy versus k relationship is works out to be the same. And also the idea that is there is no restriction on k, again comes out of the Schrodinger wave equation also. So, our analogy which is in fact, I mean; this is the most rigorous way of doing it, the analogy is simply an analogy it terms out an analogy is accepted, because it is giving as the result of the interested so that is what we have seen.

Now, we will look at the next case, which is that we have we are now confining the electron to our potential level. And then, we will see again we will start with the Schrodinger wave equation. And see, if the solution that we are getting from the system is consistent; is also similar to what we got in the analogy that we did of the string tied it to it.

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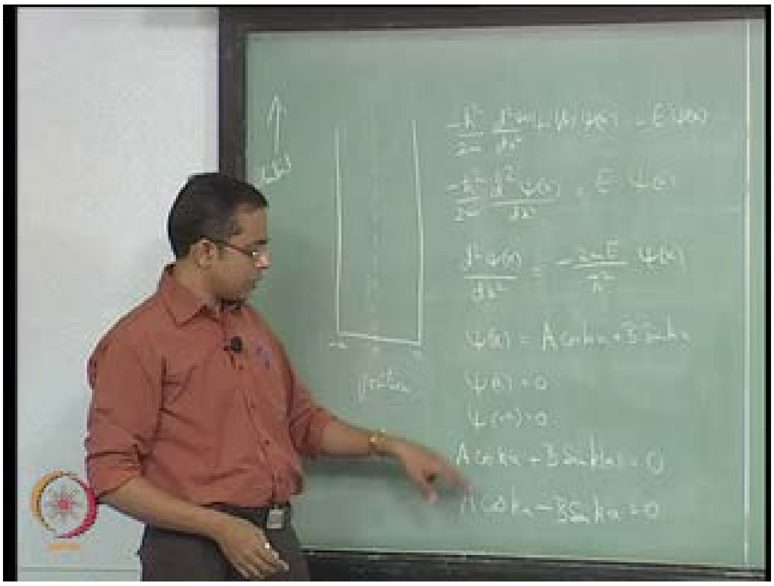

So, we will now look at the case now, where the electron is trapped in the potential well. (No Audio From: 31:44 to 31:50) So, this is potential, this is position, for ease of calculation will take some symmetric position all these origin. So, this is 0 and we can go to plus a or minus a so, we have this situation. Now, when we say it is trapped in this potential well, we will look at one extreme case where basically the potential well is so deep that we wish slowly trap there, if the electron cannot come out of it. So, when we trap for the purposes of our calculation, what we have to say is that if you try crossing this position x is equal to a, the potential experience by the electron is actually infinite.

So, therefore, the potential well is very indeed. So, if you crosses on either side of this barrier the potential is infinite and therefore, the electron cannot exist on either exist on either side on this barrier. Therefore, the electron can only exist within this resume value of the potential well and within the potential well, we will say that the potential is zero. So, we have the potential well that is 0 here and it is infinitely high on the either side of the potential well. So, therefore, the electron cannot show itself of on either side of the potential well, it is stuck to staying within the potential well where the potential is zero alright.

So, under these circumstances, next look at the Schrodinger wave equation and seeing what the solution tells that. So, same thing we write minus h bar square by 2 m d square psi by d x square plus V of x psi of x, this is also function of x equals E of psi of x. Once again our analysis is quite simple actually as long as it is within this potential well, the potential is zero, that we experiences is zero therefore, this term evaluates to zero. There is also in well the potential is infinite and therefore, we no longer or in a position is solve it does not really exist in those conditions, because it is too higher potential for come there.

So, this term was out to zero. So, we have minus h bar square by 2 m d square psi of x by d x square equals E of psi of x. And so, we see the similar situation coming up now, which is the d square psi of x by d x square equals minus 2 m E by h bar square psi of x. So, in fact, up to this point in fact, the solution looks suspiciously the same as the previous case that we looked at where the electron is completely free. So, there is a suspiciously similar look to this derivation so far. But there is a detail here which is what make the difference. So, write now the form works out similar therefore, the same reasoning that we used previously still holds psi of the form A cos k x plus B sin k x, will satisfy this equation so, it satisfies this equation.

Now, so up to here we are exactly the same state of derivation as we derive just a short write of it, this is where the difference begins to come. The difference is that previously there was no additional condition on the system; we simply said potential is zero everywhere. Therefore, our discussion ended more or less with this equation, this equation gave us said that there is certain k and all k values are permitted therefore, all energy values are permitted and so on so, discussion actually ended with this equation. Now, we have an additional detail, the additional detail is that there are what we can; what are often describe as boundary condition.

In all these problems we talk up boundary condition, here there is a boundary condition. The boundary condition is that the potential pass this point minus a is infinite and the potential pass this plus a is also infinite. For us to be; for is therefore, the and therefore, the probability of existence of the electron on either side of this potential well is zero. For that to happen this psi of x, the function, the wave function that we are deriving here the psi of x should come to zero at exactly this point and come to zero at exactly this point. So, if it were non zero we would have the discontinuity, this was the work; which is not consistent with the physical picture of the system.

So, for it to actually drop to zero past for it to be; for us to guarantee that the way function is; the probability of finding an electron outside this region is zero. And outside this region is zero for that to happen, the wave function whatever it is in between should drop to 0 at minus a and drop to 0 at plus a, after that you can say that you know at that point the probability drop to zero and then it says zero pass. Therefore, we are saying that psi of a equal 0 and psi of minus a equal 0. So, these are two condition that we did not previously have in our derivation, we know have the psi of plus a is 0 the psi of minus a is 0.

So, therefore, there is an additional thing that we can do with this equation. So, we can write A cos k a at us when we look at the positive a condition plus B sin k a equals 0 and when you look at minus a sin of minus theta is minus sin theta, but cos of minus theta is simply cos theta, cos k a minus B sin k a equal 0. So, these are two equations that we did have previously, these two equation come about, because the system has some specific boundary condition, which previously we did not have, we know have this particular boundary condition.

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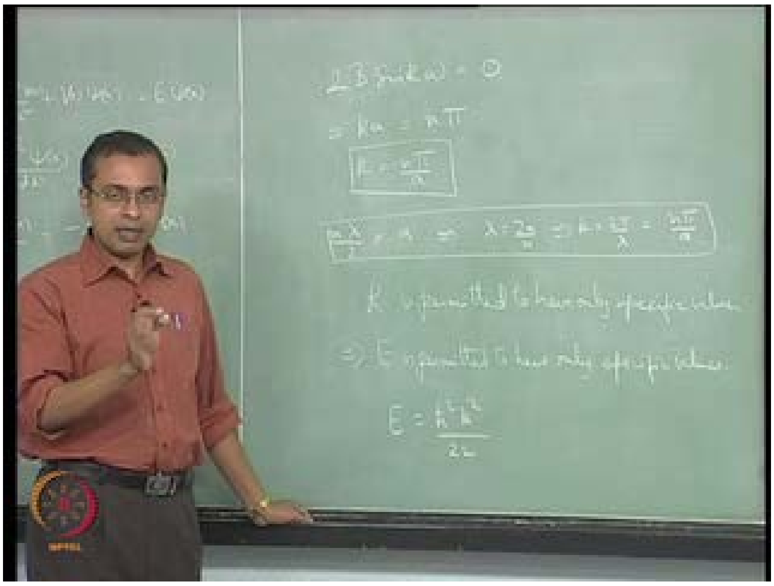

So, if you; since both of them are equal to each other, if you equate them the cos terms will go, you will simply have.

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We will simply have 2 B sin k a equal 0. So, therefore, if B is a constant if basically have k a implies, k a should equal to n pi. If a sin function so, as if you are aware when whenever it is some integer times pi when the argument of the sin function which is this k a. When k a equals some into integral number of pi; sin of that argument was out to 0 so, sin n pi is 0. So, therefore, the k a equals n pi for us to get this to be 0. So, therefore, k equals n pi by a alright. And this is exactly the result that we got previously when we look at that string. We look at that string and we said that at that point we said that n lamda by 2 equals a, when we look at the string.

And therefore, lamda is 2 a by n implies k equals 2 pi by lamda is n pi by a, this is what we wrote for that string. And this is what we just arrived at using the Schrodinger wave equation. So, a Schrodinger wave equation use this string gives also gives us the same result. So, the analogy is the reasonable this was the problem that we are dealing with. So, more importantly we have a result which we have to look at. Again what it means is that only values of k that are alone as specific values of corresponding to the site where n can take values, because n is an integer. So, only then this is true, if n is not an integer this is not true, I mean; I cannot write k is some arbitrary number here times pi where that number is not an integer and then expect sin k to be equal to zero that is not going to occur.

So, this can occur only when n is an integer which means now, when you rearrange it and you write it as n pi by a, k can only have values of pi by a two pi by a three pi by a four pi by a and so on. It cannot have values of 1.1 pi by a 1.7 pi by an all those values are not permitted. After one pi by the next values there it can have can it is only two pi by a and then only three pi by a and so on. So, k therefore, now has distinct discrete values. So, therefore;

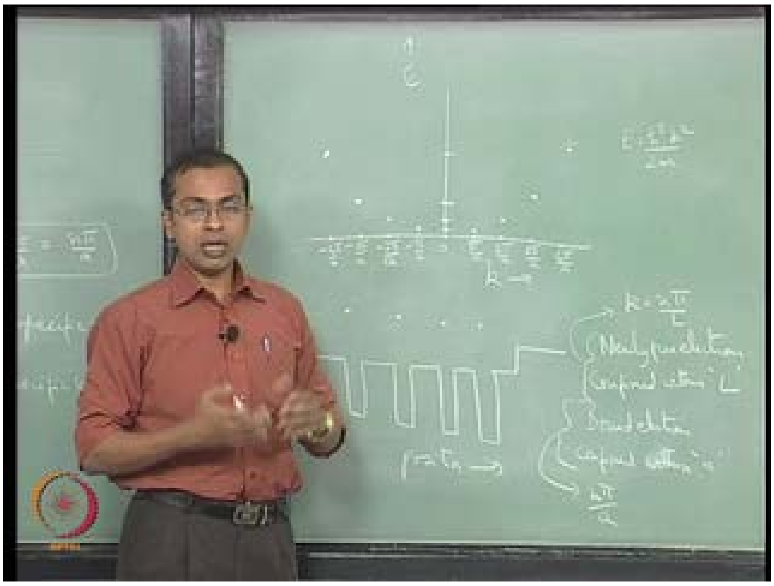
(No Audio From: 41:17 to 41:35)

K is permitted to have only specific values, implies energy is permitted to have only specific values. (No Audio From: 41:44 to 41:56) So, it is only permitted to have specific values. However, the relationship between E and k remains the same, the E is related to k using E equals h bar square k square by 2 m. So, this equation has not change, this equation still holds. Only difference between the previous case and what we are currently considering is previously k was allowed to have any value it wish to have therefore, E was allowed to have all the corresponding values of energy, corresponding to values of k.

Now, we have the situation where k is permitted only specific values. So, based on these relation E is permitted only the corresponding values, it does not permitted any intermediate values. Because k is not permitted any intermediate value between pi a and two pi by a, correspondingly the energy also permitted only the value corresponding to pi by a and the next value of the energy that is permitted is the value corresponding to two pi by a. So, that is the difference, specific values of k are only alone therefore, specific values of energy only alone. Therefore, we see even through the Schrodinger wave equation approach.

We find that when you confine an electron, the energy values that it can demonstrate or adopt or display or specific they are quantised, we are; they description that we used is quantised. It is not continuous, it is specific and discrete and that is; that process is called it is; that it has been quantised. So, confinement of an electron leads to quantisation of it is energy that is the principle and the kind of and quantisation that comes about is what we have looked at through this mathematics. Now, if you want to make a plot of this just a way will made a plot of E versus k for a free electrons.

(Refer Slide Time: 43:59)



If you wish to make the plot of E versus k for a bound electrons or confined electron, what would be the difference? We would just make the plot.

(No Audio From: 43:42 to 44:06)

So, we have E versus k, I said that the relationship is still the same so, E equals h bar square k square by 2 m. But the only difference is; now, k can have only values of n pi by a. So, I will just say this is 0, we are allowed value of pi by a, we are allowed value 2 pi by a 3 pi by a and so on 4 pi by a. Similarly, we can come here minus pi by a minus 2 pi by a minus 3 pi by a minus 4 pi by a. So, corresponding to each one of them, if you see you will find that the energy values will have some specific value here. E corresponding to pi by a will work out you use that formula, you will get some value here similarly, you will get a value here.

Then, for 2 pi by a you will get a value that is above, you get a value that looks like that 3 pi by a you will get some value there, 4 pi by a you will get the value there. (No Audio From: 44:27 to 44:33) So, you will find now that the energy values that are permitted are the specific values, this energy is permitted, this energy is permitted, this energy is permitted, this energy is permitted. So, there are specific values of energy that are now permitted in the system, because specific values of k are permitted in the system, only these values of k are permitted. So, whatever values of the energy you are calculating from it, this particular E versus k relationship give you value of the energy which you can plot on the y axis.

So, the; if you connect this lines you will get the parabola that you got for the free electron parabola here. So, we have a free electron parabola here so, if you connect the point that we just discussed or points that lie on the parabola, because the E versus k relationship is the same that the E versus k relationship is not change. So, therefore, the point just we plotted will also fall on the parabola at specific location along this parabola. The main difference is here we have a continuous curve; there is no gap in this curve. If you go back to this plot that we just made there are gaps, I got this point here there are no points between here and here it is all gap so, all of this points are for forbidden in the system.

So, to speak so, they are not allowed in the system. So, the value of energy that is permitted in the system is a particular value then there is a gap you will get the next value of energy that is permitted in the system which is scale. Then, again there is the gap there is the third value of energy that are permitted, in this fourth value of energy are permitted, each of them corresponds to this is drawn to scale, each of them corresponds to particular value of k. So, that form is still parabolic the way, because there is no difference in the equation. The main difference is only particular values of k are permitted therefore, only particular values of energy are permitted.

So, this would still the general form which still the same form find. So, we find that now, because of confinement only particular values of k have been permitted. And therefore, because of confinement the energy has become quantised, the energies energy level permitted in the system have become quantised alright. So, this is the very important thing and we have now seen in two percept things from an analogy of simple string and the wave that it can support. And for the more vigorous way of looking at looking at our system of an electron confine within the material in using the Schrodinger wave equation. So, we have looked at it from both perspectives and we find the results, we are getting at the same.

So, our analogy is for the purpose that we are used in and it also helps the understand the situation is better. And we see the result the important results here that confinement has lead to quantisation. The one additional detail that will I add here before we close for this class is that we mentioned in our picture that we have two levels of confinement in the solid so, we had actually this picture. (No Audio From: 48:20 to 48:32) So, we recognised so, this is the potential and this is position. And we said that there are electrons which are nearly free electrons these are all the ionic core position, we said that there are electrons which are nearly free electrons which can level across the extend of this solid, they are still trap in the solid, but they run across the extend of the solid.

So, they are also confined electrons except that the length scale over which they are confined is the length of the solid. Just for clarity say, we will say that the nearly free electrons confined within L, where L is the length scale of the solid and bound electrons confined within a. So, bound electrons are confined within the width of that; this potential well which is a and the free electrons are confined within length scale of L. So, the extend of confinement varies between whether it is bound or it is free or nearly free. The nearly free electrons are confined with an length scale of L, nearly bound I mean bound electrons are confined within the length scale of a. And a is very small a is of the in the Armstrong level, L is in the meters level, this is that is the difference of ten power ten or ten orders of magnitude show to speak.

So, if you look at the result, what you will find is that for the nearly free electrons the values of k that will be permitted, the entire analysis we look the same, but the values of k that will be permitted will be n pi by L. So, n pi by L is what we are looking at so, k will work out to n pi by L for is nearly free electrons allowed values of k and for bound electrons it was out to n pi by a. So, k values here are n pi by L, k values here are n pi by a the form is the same because it is the same kind of derivation and that is all we arrived at those to values. The difference is that, you have very small number in the denominator here, you will have the very large number in the denominator here. So, the k values of space very close to each other because of this large value of the L in the denominator the impact of n is relatively small.

So, the L is the large number or relatively speaking the k values. So, one value of k that are permitted and the next value of k that is permitted or very close to each other, because the denominator is very large here. Because the denominator is very small one value of k that is permitted and the next value of k that is permitted or reasonably well separated. So, what happen is you still see the discrete nature of E versus k relationship for both the nearly free electrons as well as for the bound electrons, because both of them are experiencing confinement.

The difference is that, because the nearly free electrons are confined over the larger length scale, the spacing of this point is closer it is much much much closer. Because it is pi by L 2 pi L 3 pi by L 4 pi by L and L is a very large number therefore, the spacing between those points become closer where as when a is when you look at the bound electrons. Because a is the very small number pi by a 2 pi by a 3 pi by a 4 pi by a are separated apart more. So, in both cases the E versus k now results in discrete points it is no longer continuous curve as we saw in the case of truly free electrons.

The main difference is the spacing between the energy levels, the spacing between the k values and hence the spacing between the energy levels is extremely small when you looking at nearly free electrons. Where as the spacing between the energy levels and the spacing between the allowed k levels is relatively long when we looking at the bound electrons. So, this is an idea that we should keep in mind, we will look at this idea we will halt here for this class, in the next class we will start by examining this idea little bit more and the consequence of the idea little bit more. And then we will proceed from there to see what we can say what additional details, we can say about the solid with this we will halt for today, thank you.