Physics of Materials

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Lecture No. # 22

Confinement and Quantization: Part 1

Hello, welcome to this the 22 class in the physics of materials course. In the last class we examine the idea that you know when an electrons goes through a solid, it is not really reasonable to accept or belief that it has no interaction with what is present within the solid or that the interaction is featureless; that in other words the potential across the entire solid is exactly flat and uniform. What we realized is that the ionic cores represent locations, where the energy of electrons goes down very significantly, because the opposite charges are attracting. And therefore, there is a certain feature, certain shape to the potential was a position curve, that that is reasonable to expect with in a solid.

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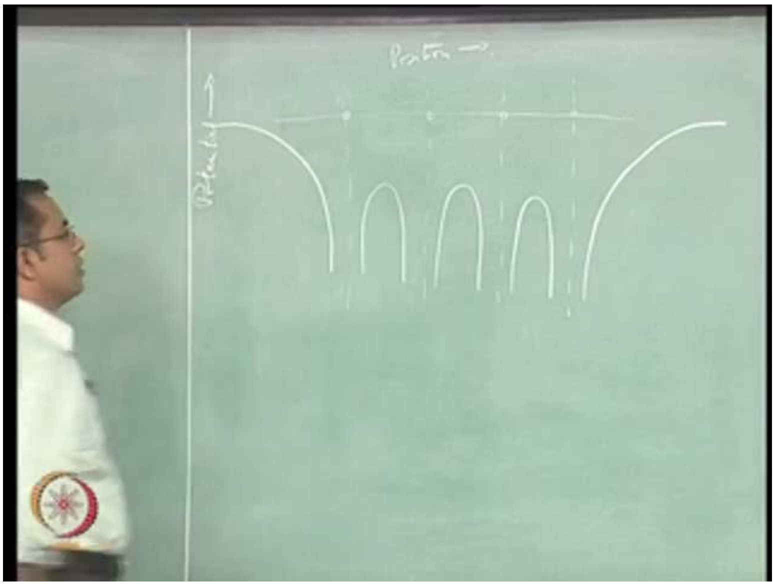

So, what we drew last class, we found that if you had the ionic cores; those are the position of ionic cores. (No audio from: 01:19 to 01:26) Then, we found that the potential was position is more in detail manner look something like this.

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So, the potential versus position curve looks something like this, in a more; so this is position and this is potential. So, the curve looks more like this and it is much more reasonable to actually believe that the electron experiences something like this as it goes through the solid. We also finished off our class by saying that this is perhaps a little too detailed for us to utilize and to some degree may not, it may not be necessary to capture this picture in exactly this level of detail to utilize it in our understanding of the experience that the electron has. So, we made an approximation to this picture and so we finished off last class with that approximation. The approximation looks like this.

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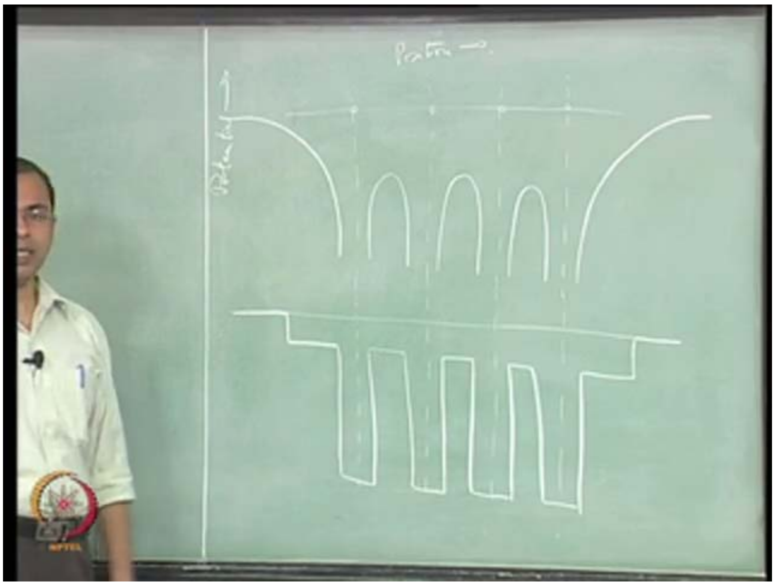

The same positions are still utilized and some 0 level is set for the energy. And, we said that when you come from infinity, you see 0 potential; then it there is a potential drop.

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And, so the electron actually can be approximated to be experiencing this kind of a potential versus position distribution inside the material. So, where in, the basic idea that once you get close to the material there is drop in potential is captured by this potential drop here. There as you get close the ionic core, there is much more significant drop in potential; in a region very close to the ionic core. So, those two are the important features of this diagram and so those two features have been captured this, in this approximation.

And, but otherwise it is been simplified in the sense that these have been made into square steps; so that we have some idea of what is that width or at least we can do something about, we can try to work with the kind of information that this diagram provides.

So, this is what we have done as an approximation to what what is actually present in a solid and it is a reasonable approximation. The other aspects of this diagram, this is of course a 1 dimensional case that I have taken. You could in principle draw a diagram which captures a basically the same information that I have shown here in two dimensions or three dimensions; the diagrams may look complicated, but the basic information they are providing is essentially what you are seeing in this diagram. So, since the information is here and most of the interaction kind of circumstances are also captured in this diagram, it is sufficient that we confined ourselves to this diagram and do our calculations with respect to this. And, the results that we will get, well in principle (( )) for the solid in its more complicated three-dimensional assign, alright. So, therefore, this is an adequate enough picture for us and we will work with this picture.

Now, additional features that this diagram is capturing, is the fact that when we use these terms saying a free electron and I mentioned last class as we finished off that a free electron is one as strictly as something that you call a free electron. A strict definition for it is that it is an electron that has escaped to the solid. In other words it is an electron that no longer has any interaction with the solid; it is just gone from the solid. And, nearest that we can think of in an experimental sense is say a photo electron; so you have some light falling on it. So, you will see photo photo electric effect and electron leaves the solid. So, at that point it has been ejected from the solid; so, it is no longer is part to the solid.

At that point position wise, it is actually passed this location; so, it is passed this this point. So, it is no longer interacting with any of the features in the solid; it is out of the solid. So, that is the nearly, that is the truly free electron; it has left the solid. That would be true also here, anything pass that position is also out of the solid; it is no longer in the solid, fine. So, then we are also talking of; so that is really a truly free electron. So, therefore, in our previous discussion that we have had, that when we have spoken of free electron gas and I also kept using this term so called free electron gas. The reason I kept saying so called, so called so many times is simply because originally that was thought of as the electrons which were running inside the solid were thought of as free electrons. So, originally the picture was that ionic cores where there, each ionic core had released one electron or whatever is the natural valence of that metal. And, those collective bunch of electrons are running around through the solid and they were being tabbed or called as the free electrons.

But we just now recognize that only when an electron truly leaves the solid, thus it become a truly free electron. So, therefore, calling electrons which are present within a solid as free electron is a is a bit misleading; there are not really free, they are not free to go where ever they wish. They are free to go where ever they wish within the confines of that solid. So, there is a limit within wish they have to stay and therefore, they are not truly free; so, and so they have not really left the solid. So, therefore, we recognized now that those electrons have to be treated somewhat differently; they are not or at least they cannot be termed the same way. So, those are the electrons that are actually struck within this potential well; this potential well slightly shallow potential well which extreme throughout the solid, represents the potential well corresponding to the idea that electrons are within a solid.

So, it represents the boundaries of the solid; this potential well represents the boundaries of the solid or it represents distance around around those ionic cores, around that collection of ionic cores that now represents the extend of a solid. If you are passed that point, you you say that the those ionic cores no longer are interacting with that electron. So, if you have passed the point, you are out of the solid; if you are within the that range then you are at least, the electrons are at least partially being impacted by those ionic cores that are present there. So, to the extent that they are even partially being impacted by those ionic cores present there; they are now within the solid, within the extend of the solid.

So, electrons that we have previously been talking off which are those electrons released by the ionic cores, but are running within the confines of the solid are actually saying within the potential well. So, in this diagram we have to recognize that; that is what this potential; the first shallow step that we see here is exactly that. It is that idea and that concept, that the electrons which are now within this region, within this starting from here up to here. They are electrons which are, but are within this potential, within this shallow potential well. So, this is the term well is used simply because it is deep; it is going deep inside this material. So, in terms of depth, I mean it is not deep within the material, in potential sense it is it is a depth. So, therefore, the term well is used in keeping with some physical temperature.

But the basic idea being it if electrons are within the shallow potential region and therefore, struck within this solid, they are now refer to as nearly free electrons. They are not truly free electrons; they are simply nearly free electrons; so, the electrons that are here are nearly free electrons.

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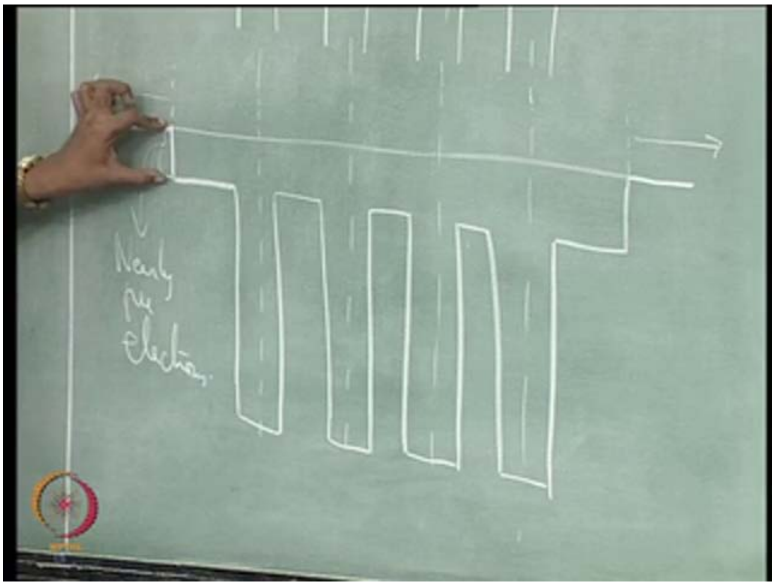

So, nearly free electrons are confined to stay within the solid due to the presence of this small potential well; so, that is what nearly free electron is. Then, we have electrons, all the remaining electrons in the solid; I mentioned we have always discussed so for that we have only the ionic cores or the atoms to begin with, have released one electron each. And, therefore, now there is an electron running around through the solid plus the remaining electrons stay along with that ionic core or the atomic core, the core of the atom which now is now ionic because it is positively charged; so, we call it an ionic core. So, all the remaining electrons are now still stuck with that original location of that atom. So, in other words when I mark these out as the location of those crystal lattice so to speak and therefore, there is an atom at each of those location, assuming that is the crystal structure that we have, we are discussing.

Then, all the electrons remaining, whatever the electrons; if there are 50 electrons, then 49 of those electrons are stuck to this, struck to that that general location. Only one electron is left to run free across this solid, for every ionic core that is present. So, those 49 electrons, all the remaining electron of that atom are actually struck to this much much deeper potential well. And, therefore, they are also struck to this narrow region in space.

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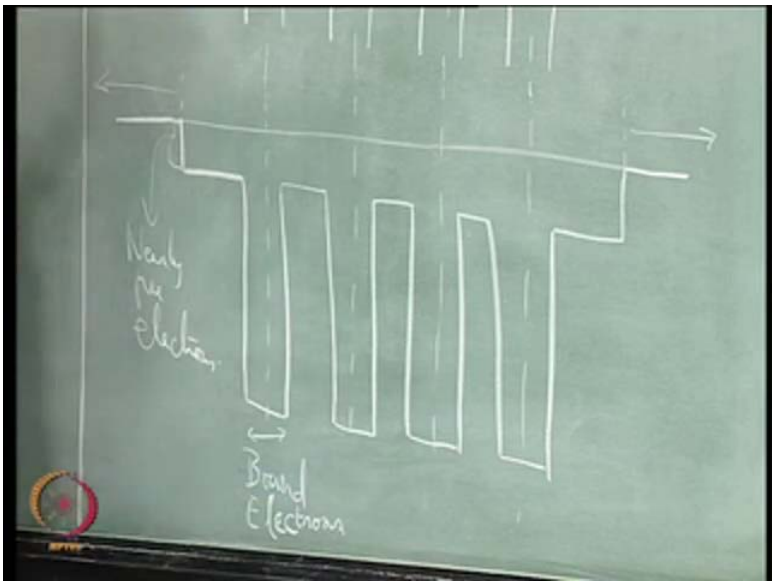

So, these are called bound electrons. (No Audio From: 11:25 to 11:35). So, bound electrons stay within a very small confined, small region along on either side of that location of that ionic core. And, they are sort of struck to that ionic core; they are not free to run even the extent of the solid. So, this so ionic core is 1 Armstrong across let say or even less; say 0.8 Armstrong across or whatever some such small dimensional. So, 49 electrons, if we assume an arbitrary number like 50 electrons in that pair atom, 49 electron are struck within that 1 Armstrong of that location. That 1 remaining electron is struck to the remaining is left to run across the solid; so, that is the difference, fine. So, that is the in terms of a dimensional skill that is the difference.

So, a bound electrons stays within that region that we would normally call as ion or an ionic core or an atom and so that is they are struck to this deep potential well. The nearly free electrons are struck to the extent of the solid, they are free to run across the entire solid, they are struck to the extent of the solid and the truly free electrons have left the solid, fine. This is these are the three definitions that we are using. So, that is the first thing we need to recognize. The second thing I will also point out something that I just mentioned, but I will elaborate on it. This size scale here of this order of 1 Armstrong; so, if the potential well we are talking of that Armstrong scale. So, that is the very very small dimension 10 power minus 10 meter.

The length of the solid, the extend of the solid is something that is huge, relatively speaking; you can have a block of metal that is 1 meter long. So, 1 meter would be a distance that is this large roughly, that is pretty large distance fine. So, when we talk of a nearly free electron it is it is free to run this distance of the order of 1 meter, in a straight line it is of the order of 1 meter distance. It may take any other convoluted path, but its straight line distance displacement that it could have is 1 meter; that is the distance that it can travel. So, that is 1 meter. So, we can talk of this distance being 1 meter or you know half a meter or a millimetre or a centimetre; it is in that size scale, 10 power minus 1 to 10 power minus 2 meter is the kind of size scale depending on the object that you are talking of it is a wire or any such thing. So, that is the thing, you could have very long wires also; so, we could have several meters long.

Whereas, this bound electron is confined to a size scale of the order of 10 power minus 10 meters; so, it is an Armstrong. So, that is a massive difference in orders of magnitude in the confinement level; we call this idea confinement, that is that the electron or whatever is the species that we are talking off in this case an electron, it has to stay within certain region. So, it has to stay within a certain region because of the conditions that it is experiencing; this idea that an electron has to stay within a certain region is called confinement. So, that that basic it is it is it is just straight forward description, it is a confinement; the electron is confined to stay within that region.

A bound electron is confined to stay within about an Armstrong of that location. A nearly free electron is confined to stay within a meter of that material, just to give an order of magnitude (( )). A free electron, a truly free electron is really free, it can there is no bounds on it; it is not confined, it is free to run across the entire universe, so to speak. So, that is the way we look at the description; so, that is the other information we have. Now, the third thing that I also want to highlight here when we while we still have this picture, before we proceed forward; is that in all this time we have done calculation. First we started out with (( )) model of and we called those electrons as so called free electrons, did some calculation; that did work out completely acceptable to us. So, we changed this statistical distribution from Maxwell-Boltzmann statistics to Fermi-Dirac statistics and read it the calculations, we came with some term such as Fermi energy and we came up with a new distribution.

Now, this is the also an energy versus position curve. So, we have energy here in this y axis, we have position on the x axis; this is what we have put down here. So, potential energy I have put here, but basically it is a energy, energy versus a position; this is what we have here. So, in this diagram, the first thing I want you to understand is that, I would like to highlight what it is that we have been doing the calculations for? So, our calculations were not for those, this arbitrary example I have take of 50 electrons being atomic number being 50, our calculation was not with respect to these 49 electrons that are struck in this deep potential well. We were not bothered about this 49 electron which were the bound electrons. So, our calculation did not really look into this, this picture we did not bother about.

Why we did not bother about it? Because in principle for the properties that we are interested in, those are not the electrons which appear to be contributing, in any significant way to those properties; so, that that is reason why we did not look at it at that point in time. There are other phenomena for which these are these electrons also make a difference. So, there therefore you would have to look at that which we which we have not done so far; so, that was not of relevant to us. So, we did not really bother about these 49 electrons that were here (( )). On our calculations were with respect to that 1 one sorry 1 electron per a ionic core, which contributed to this electron so called electron cloud which was running across the extend of the solid. So, therefore, all our calculations where only with respect to this nearly free electrons.

All the calculations we did so far and whatever analysis we try to do, whatever results we try to get, whatever prediction we try to make; there only with respect to behaviour of these free electrons, nearly free electrons; that is the thing that we need to, I would like to emphasize here. We did not read bother about the bound electrons, but they are their; it is not that they are not there, they are definitely their within that solid, but we did not really bother about that. That was not because they did not really seem to impact the property we were interested in. We focussed our attention on these nearly free electrons, we called them so called free electrons etcetera and then but we did our analysis only with respect to nearly free electrons. So, whatever parameters we came up with and whatever analysis we did was with respect to this, alright.

So, what did we come up with? We came up with something called Fermi-energy. We said that you take the energy levels, from the lowest energy levels upwards and you start filling them up. You will reach a certain high energy level at which point in time you will run out of those nearly free electrons; let us keep calling them nearly free electron now, because that is what they are at this point. So, we will run out of this nearly free electrons, that highest energy level is then called the Fermi energy level. So, now with respect to this picture, so this is the energy and that is the direction in which it is increasing.

The way this picture is drawn is based on the convention that when an electron is very far from solid, it is potential energy is 0 with respect to that solid. So, when it has no interaction, it is 0 and because it also oppositely charged with respect to those ionic cores that are present there. So, therefore, its potential energy 0, as you bring it closer and closer to that solid, its potential energy becomes more and more negative; it is or in other words it is decreasing in an energy, alright. So, in terms of our convention here this potential energy would then correspond to 0 because you are going further and further away from the solid; these are all negative potential energy. So, this is 0; in this energy scale, right. And, this is decreasing energy, alternately this is increasing energy, but it is increasing from a negative value all the way up to 0.

So, in this picture when when we say that you know we are talking of energy levels of the nearly free electrons and we are starting from the lowest energy level possible and filling them up. What we are basically saying is, in this range of energy; first of all we recognize that this is the range of energy that we are talking about and in this range of energy we are talking; we are saying that from their lowest energy possible we are filling the electrons up. So, we are definitely going up in an energy level. So, we are adding energy, as we fill the electron up in the various available energy levels, we are definitely going up in an energy level.

However, with respect to with respect to with with respect to this what shall I say coordinate system that coordinate axes that we have used here and with respect to this concept that far away from the solid, the potential energy is 0. These are all still negative energy levels, negative in energy. So, that is something that I just want to highlight so that you are at least aware of how that because when you do the calculation it may not appear that way. But when we try to put the information together in a single picture, we have to be sure of where each one comes with respect to the other. So, we have to know the relative positions of all these energies.

Therefore, we need to pay attention to this detail; that in the normal convention of how these things are done, the energy of an electron far away from the ions is set at 0. And, therefore, all the other energies that were drawing on this picture are negative; that is number 1. Number 2, the nearly free electron are struck to energy values which are in this range with respect to this picture. This range is also still consisting of negative values on d with respect to the convention that faraway is 0. But within these negative values, you start at much lower energy levels here and you start filling it up, filling up all the energy levels and it then and some energy level you run out of you run out of electrons.

So, we will assume that this is the energy level at which we run out of electrons, run out of the nearly free electrons. This is this has been attained by first taking all the states are available to us, filling them up one by one one by one and then we finally, run out of all the nearly free electrons, when you reach energy level, alright. So, therefore, this energy value is now what we described in our previous calculation as the Fermi energy level; so, which we which we edited as E subscript f. So, this position here is E subscript f; so, this is the Fermi energy. So, the calculations that we did previously refer to this set of electrons which are the nearly free electrons. In that calculations we came up with something called Fermi energy, that Fermi energy in this picture will show up something like this.

So, that is how the information that we have, the concept that we did we have discussed some of our earlier classes relate to the concept that we have discussed in the last class and where discussing now. So, that is how they relate; so, this is the Fermi energy. In this also having come this far it is of interest to put in one more piece of information, before we proceed; which is simply that when we talk of say the photo electric effect. We talk of something called a work function; that is the amount of energy that is required to pull an electron half of that solid, that is the work function. Now, in this picture the work function is now the amount of energy required to pull the highest energy level; the electron containing the highest energy. So, in other words, in the solid whichever is the highest energy electron, what is the energy required for you to pull it out of the solid.

The highest energy level electron is sitting at Fermi energy, right and to get it out of the solid, to make it escape from the solid you have to get it up to 0; in this scheme of thing this is 0 energy, this top most level is 0 energy. So, you have to get it to 0 at that point in time you can legitimately say that the electron has escape the solid. So, therefore, this difference here; that difference that difference is the work function. So, from this here is the work function which is typically designated or denoted by phi. So, this is the Fermi energy and that difference in energy between the Fermi energy and this 0, that we are setting for this scale; then is the work function.

So, so this is how several of concept that we have discussed with respect to quantum mechanics, with respect to Fermi Dirac statistics and with respect to what a solid is, how the electrons in the solid are, how the ionic cores are, what is the impact of all of this on this picture and so on. This is how all of them come together and this is how they relate to each other. So, therefore, this is the picture that is useful to have in mind and it is useful to understand, what is the significant of all of this information, alright. So, now, temporarily we will conclude this particular discussion; we will now proceed to another discussion. Wherein we will start using these features, we understand how this picture came about; we will use this picture now to see what is it that we can predict about the material, so we will start using this picture.

So, to do that we will actually, we will do it in two steps. First we will look at electrons that are truly free; in other words electrons that have escape the solid. So, that is what we now mean by saying truly free electrons; they are free electrons, they have escaped the solid. Then, we will look at electrons which are trap in a potential well. So, such as electrons which are trapped in this shallow potential well here or the electrons are trapped between deep potential well; so, both of these we will look at. So, we have to look at in fact 3 cases effectively, but actually strictly only two cases; either it is out of the material where it is completely free or it is in a potential well.

And, we will make some classification on how deep the potential well (( )) surprise in a bit, but that will assist us in our calculations. But these are the two that we look at, free electrons and bound electrons; regardless of whether they are totally bound or they are just nearly free or whatever. They are deep bound in some potential well; so that is what we will differentiate between. Now, to look at the behaviour of those electrons, these free electrons and bound electrons or free electrons and nearly free electrons, look at their behaviour, to differentiate between their behaviour we have to actually utilize all the quantum mechanical principles, alright; so, which we will do. But before we do that, we will first do it, we will look at this this concept in this picture from much more simpler approach, a much more simpler approach, a much simpler approach which will still give us essentially the same result.

So, what what we will do is we first look at this in a much simpler approach and then we will use all the quantum mechanical much, a more rigorous quantum mechanical approach; both of them will give us the same results. Therefore, it is acceptable to us, but by starting with simpler approach you will get a better feel for what it is that we have done, alright; so, that is what we will do. Now, the what we are trying to do? At the end of it we are trying to look at the behaviour of the electrons, we would like to see what they can do, what they cannot do and understand with with and that is with respect to the conditions that they are facing; may be the potential well they are facing etcetera.

So, with respect to the conditions we want to know what can the electron do, what the electron cannot do; that is the basic question that we are trying to answer. So, to do that we would like to see what is the behaviour of the electron; so, we will look at it. So, quantum mechanical description says that we have a wave particle duality which is something that we already look at, we discussed. So, we can look at electrons as particles, we can also look at it look at electrons as waves. So, for the moment we will look at it as waves and then see what it is that we can obtain from this picture.

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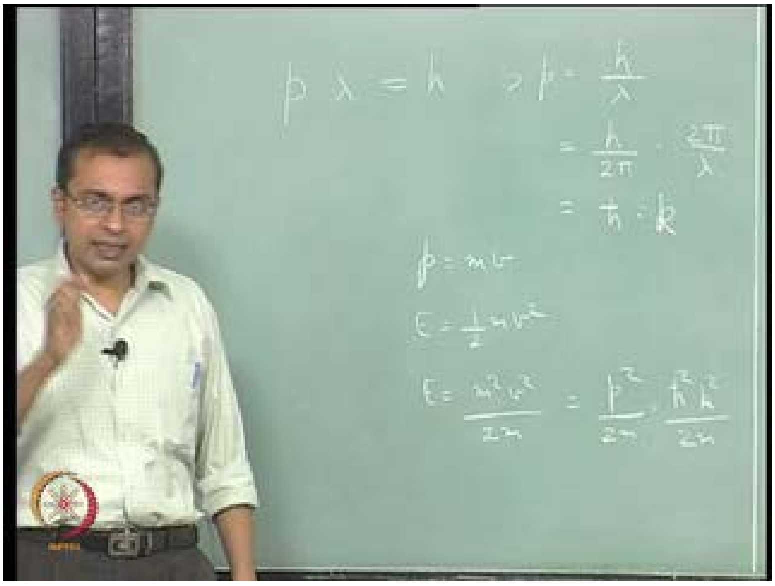

Also, we wrote the De Broglie, De Broglie wave length as of of of particle having momentum p as p lambda equals h; so, this is the De Broglie equation. We can rearrange this marginally, we will have implies p equals h by lambda and for convention sake impact we tend to use h by 2 pi. So, we can write this as h by 2 pi and also this as 2 pi by lambda. For convention sake this is done, we will see that that convention later, but basically, it conventionally this is the way this is done; this h by 2 pi is also designated as h bar. So, if you see books you will suddenly see a, sometimes you will see h bar being used, sometimes you will see h being used; this is how they relate, h by 2 pi is the same is what is called h bar.

So, h bar is (( )) this is and this quantity here lambda 2 pi by lambda, it has the dimensions of 1 by length because lambda is in denominator, lambda is wave length; so, lambda is in the denominator. So, here p is momentum, h is Planck’s constant, lambda is wavelength and lambda is in the denominator. So, this quantity here has the dimension of 1 by wavelength and is actually referred to as a wave vector. So, this is referred to as wave vector and there is designated as k. So, this is how they relate, when you talk of k your actually it is the inverses of wavelength and it is a wave vector in the sense it is talking of it is just not just the length of the wavelength, it is also direction of wave; so, therefore, that is an important quantity.

So, when you write h bar k, it is same as writing h by lambda, because simply we divided by 2 pi, multiplied by 2 pi; so we have made no difference to this equation, this is divided by 2 pi, this is being designated as h bar, that is being designated as k. So, h by lambda is nothing but h bar k; so, that is the thing that we need to understand. In much of our analysis we will tend to keep using h bar and k. So, this is it is important to at least be alert to the fact that these are (( )) straight forward the associated with the quantities we are already accustomed to. So, p is this and we also say that you know we also write p is mass times velocity; energy is half m v square, right.

So, these are the conventional definitions for momentum p, linear momentum p is mass times its velocity and energy is half m v square. So, if you therefore if you relate, if you try to relate energy and momentum, it is simply energy is m square v square by 2 m, right. If you do this you will get the half, you can multiply; essentially all I have done is I multiplied numerator and denominator by m. So, m square v square by 2 m is what this equation is and the numerator is now p square; because p is m v. So, therefore, v equals p square by 2 m. So, this is this is just a general relationship between energy momentum and if you see p as h bar k, this is h bar square k square by 2 m. This is just a relationship between energy and momentum or energy and wave vector, momentum and wave vector, all this relationships are there; as necessary we will utilize them. So, now we need not, we just need to be aware that this is this is the way we need to look at it.

Our immediate, actually our immediate concern or our immediate task is in fact to understand for electrons that are present with in a solid and to the various circumstances that they are present within a solid or under the various circumstances that they can exist. What restriction are there on the wave lengths that they can adopt? So, that is the piece of information that we should find out; what are the restriction of the wavelengths that they can adopt. At this time it may seem like, I mean somewhat detail piece of information that we are looking for, but later as we utilize that information we will see why why it is of used to us.

So, that we will see little later because it basically says, what is the energy levels that are allowed for the electron. We have always said that you know energy values, that we said that there are energy levels E 0, E 1, E 2, E 3, E 4 and so on and that is how we fill those electrons, right. We took the electrons, the nearly free electrons and fill them across those energy levels. Now, the point is we did that sort of in an arbitrary way; we assumed at E 0, E 1, E 2, E 3 etcetera existed with the solid. We have not really looked at how they may exist or why they exist, right; we have just assumed that they exist. So, why should they be E 0, E 1, E 2, E 3; why cannot it be continuous energy values? So, that is an important piece of information that we need to understand.

First of all why is it that it energy cannot be continuous and or is or at least we are saying that in our system it is not continuous; we just assumed it is not continuous. We would like to see why it should not be continuous; that is number 1. And, when it is not continuous what are the values of energy that are that are those values. So, far we simply said E 0, E 1; what is E 0? What is E 1? What is E 2, that is not something that we have discussed. But they are very important features of all the calculations we have done. So, our task at the moment is to actually explore that region of our discussion and then utilize those results as we see if it, in the subsequent discussion.

So, in other words when we would like, when I say what is that value of E 0, what is the value of E 1 etcetera, I would like to know and E is related to the wave vector here in this manner. The h bar is a constant because it is a Planck's constant, mass of the electron is a constant; so, that is also constant. So, really when you say E has specific values, E 0, E 1, E 2, E 3 etcetera, when you say that E has specific values all we are saying is in this relationship the allowed values for the wave vector. Therefore, the allowed values for the wave length of the electron are also only specific values, right. Because h bar is a constant, 2 m is constant; therefore, this has only specific allowed values, this parameter can also only has specific allowed values and this parameter is the wave vector which is one by wavelength.

Therefore, wavelength has only specific values, allowed specific values. So, we want to understand, why this is the case? Why is it that electrons in a solid appear to have only certain allowed wavelengths and then they are not allowed to have whatever the wavelength they wish. So, the that is the question we wish to answer and as I said strictly speaking, the way to answer that question is really look at the quantum mechanical behaviour in the quantum mechanical approach and then see what is the answer. And, we will do that, but we will take a much easier approach to answer that question and then take the quantum mechanical approach to answer that question.

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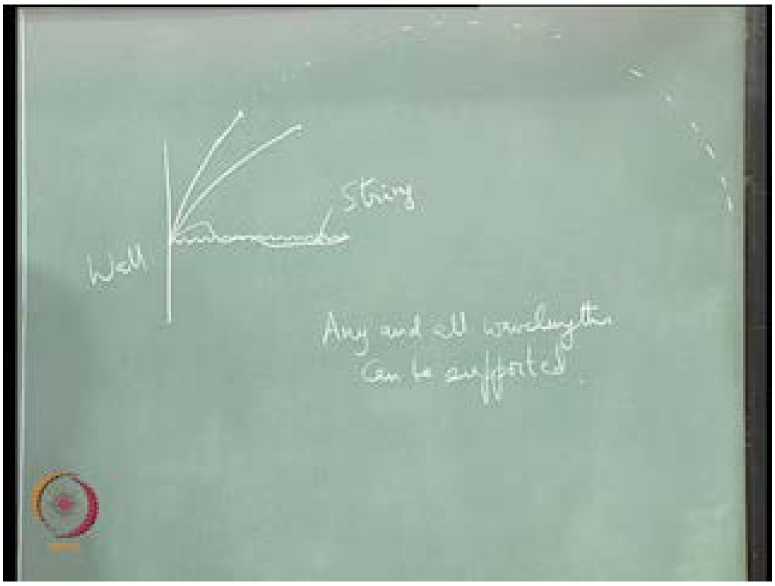

So, let us look at this way, we are aware of waves on a string. So, I just say that we have an wall here and we have some string. So, this is a string, that is now attached to a wall at one location and it is sort of lying loose, loosely lying there, alright. So, now in this situation so this is wall, this is some string. When you have a string that is tied to a wall at one location and the other location is free to hanger on. What are the question that we need to answer is, what are the wavelengths that can be supported by this string? So, what are the wavelengths that can be supported by this string? In this particular situation where one side of the string is tied, the other side of the string is free to hang as wherever it wishes. Now, in this situation since this end of a string is free, the answer is; it can support any wavelength or any and all wavelengths.

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In this case, why do I say that? Because this end is free whatever wavelength is say; supposing I say wavelength is very large wavelength, this string will take this part of that wavelength. If it is a very very very short wavelength this string will take this and we will assume this are all sinusoidal waves. So, it can be a small wavelength, a very tiny wavelength or a very large wavelength. Because this end of the string is free, it can support that wavelength; there is no problem. It will illustrate what ever part of wavelength that it can with respect to this length and it take that wavelength. It is (( ))

wavelength it will take that; which means string sort of ends here and it is part of that very very very large wavelength that is there, alright.

So, a string that is tied only on one end can support any wavelength that you decide or it can take a shape that is consistent with any wavelength. There is no restriction on the wavelength that it can support, absolutely none; it is free to do whatever it wishes. You can take extremely tiny wavelength also it will do it, very very large wavelength also it will do it. Because there is no restriction on the string; especially on one end, it is free to do whatever it wishes. So, that is a very important statement we can make about a string that is in this format

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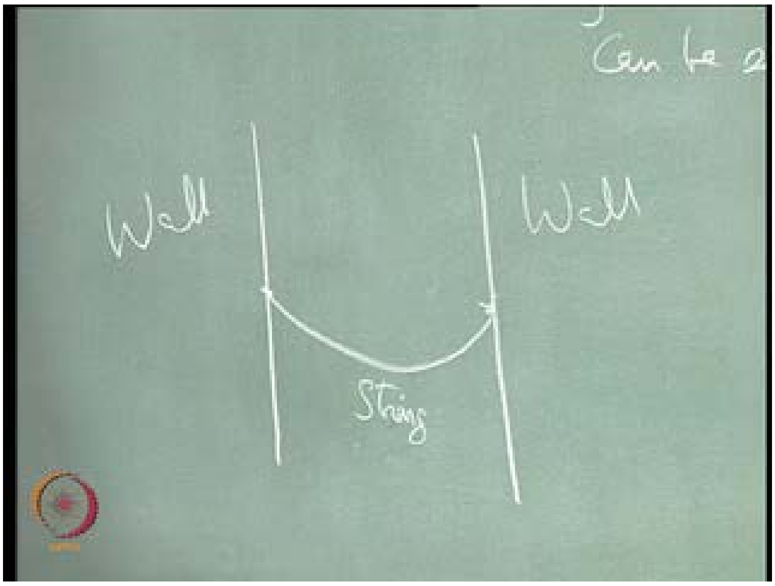

Now, we will take another string. (No Audio From: 37:36 to 37:48) So, now let say that it is now tied between two, it is basically tiled two locations on two walls. So, wall here, wall here and this is a string, (No Audio From: 38:02 to 38:09) alright. Now, we ask ourselves exactly same question, what are the wavelengths that this string can support? So, that is the question that we wish to answer, what are the wavelengths that this can support? What we will find is that because the two ends are tied, in other words when you talk of a wave.

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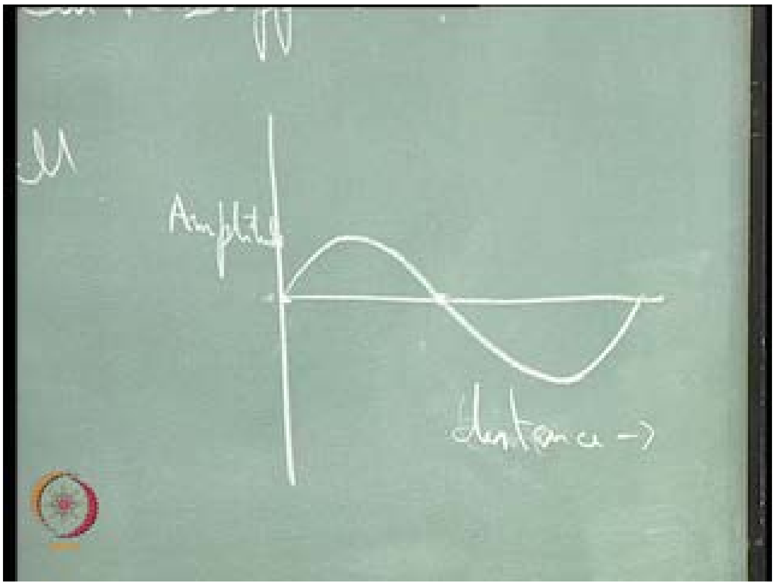

When we talk of a wave, we first of all we have some origin or about which this wave is being drawn. So, it has to start somewhere, if it is sinusoidal wave it has to reach a maximum, it has to come back to 0, it has go to a negative maximum, comeback to 0; that is the way we have to look at sinusoidal wave. So, we say it is amplitude and this is the with respect to time let say; time or distance, we will say distance or position, fine. So, it has to go to a maximum, comeback to 0, go to a negative maximum, comeback to 0. Now, when you say that two locations are fixed. So, and they have fixed like this, they are fixed at the same height.

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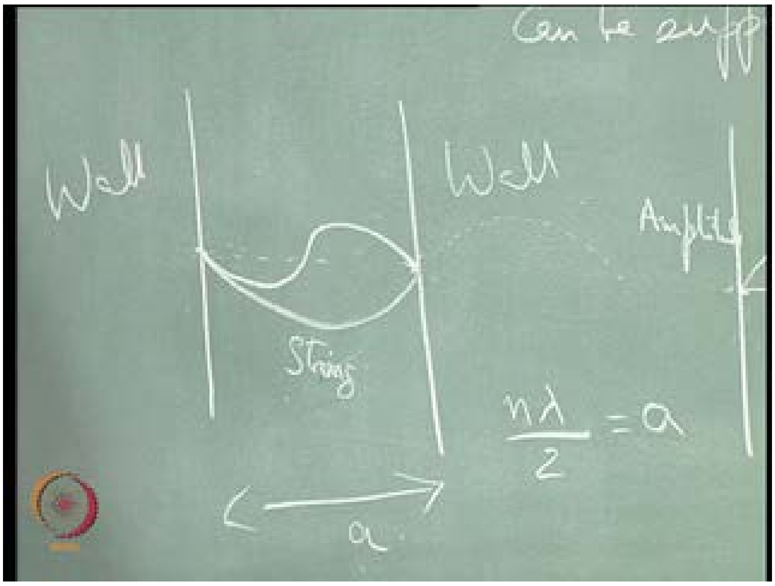

For example, let say, so they are fixed, which means that no matter what you do, this position has to be at 0 displacement with respect to this axis. So, the displacement of this position is 0; it cannot move from this position, it is struck here. Similarly, the displacement of this position with respect to the horizontal axis is 0; it cannot move from there, so it is struck there, right. So, this wave, this string can only adopt shapes, where in one end still remains at 0 and the other end also still remains at 0. So, if you if you place this constraint, the largest wavelength that it can support is one; where half the wave is what this difference represent.

In other words, as you can see here half the wave, at half the wave this is at 0, this is at 0. So, except that is on the positive direction this is an the negative direction. So, half the wave will correspond to this length, which is this length here and therefore, this string can support in principle wave wave length which will correspond to double this length. It can be part of a wave, it is part of a wave; it is, it can maintain the shape consistent with a wave where wave length is twice what this length is, that is all, that is the largest wavelength it can support. Below this also it can support various wavelengths; however, it cannot support all wavelengths below it, because you are always placing the restriction that these two ends have to be at 0. So, at x equal to 0, the value of this wave should be 0; at x equal to whatever this distance is a let say, I call this a.

If a is the distance between the two walls, at x equal to 0 this distance displacement has to be 0; at x equal to a, this distance displacement or this value of this wave has to be 0. So, those two have always got to be true. So, therefore, what will happen you can only, if you look at this those two restrictions what it means is you can have a wave that look like this or you can a wave that looks the next smallest wave that you can hold, will be 1 where it will look like this. So, in other word a can either by lambda by 2, in which case this is the lambda; a can either be lambda by 2 or it can be lambda, in other words 2 lambda by 2 or 3 lambda by 2, 3 lambda by 2 or 4 lambda by 2 etcetera. It can only be n lambda by 2. So, therefore, n lambda by 2 equals a; these are the only values of lambda that are allowed.

So, I have just done some calculation, let us understand significance of the calculation. The calculation says that is you tie a string on two ends, you are now faced with a situation where the wave shapes that the string can demonstrate are once where the two ends are fixed. The only waves those two ends can be fixed are when they are at one is at

0, the other one is either at lambda by 2 or at 2 lambda by 2 or at 3 lambda by 2 etcetera. It has to be some integral lambda by 2; at lambda by 2 it reaches 0, right. Since, it is at since it reaches 0 at lambda by 2, only at subsequent if you add on half a wavelength, half a wavelength, half a wavelength etcetera; only then you will keep coming back to that 0 position at the other end, given that one end is fixed.

So, but you still have only a total length of a. So, when I, so when I am adding up lambda by 2 the total length has still got to be a. You cannot so, you can in other words the wavelengths can getting smaller and smaller and smaller, such that as you go through a entire wave at some lambda by 2, you arrive back at this point. You start with simply lambda by 2 arriving back at this point, this is lambda by 2 plus lambda by 2. So, now 2 lambda by 2 you arrive back at this point or you can do 3 lambda by 2 which will bring you bring back to this point, 4 lambda by 2 you will come back to this point and so on. So, you can keep on doing this such that the total number of half wavelengths that are present within this region is some integral value; some integer value, some 24 lambda by 2, 36 lambda by 2, whatever. So, some integer value of lambda by 2 is what this the set of constraints permits on this string. So, we will look at the, we will now put this information down.

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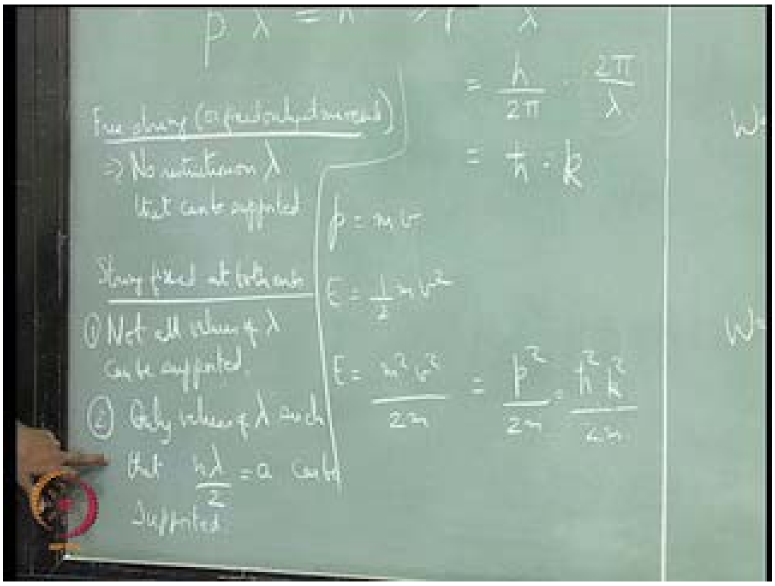

So, we will come back here we have a free string, implies no restriction on lambda that can be supported, right. For a free string there has there are no restriction on the value of lambda that can be supported; that is what our diagram showed us. So, that there is no restriction on the value of lambda it can support, any value of lambda that can be taken. String fixed at two ends, in our case a free string also (( )) or fixed only at one end. (No Audio From: 44:55 to 45:04) A free string or fixed or something that is fixed only at one end, no restriction on the value of lambda that can be supported. A string that is fixed at both ends, a string that is fixed at both ends we find that first of all, not all values of lambda are supported. (No Audio From: 45:29 to 45:38) That itself is a very important piece of information.

The fact that not all values of lambda can be supported; right there this situation is different from this situation. In the top here any value of lambda can be supported, first thing we find out is that when you when you tied on both ends of the string, not all values of lambda can be supported. So, that is the very first piece of information that you find. Over and above this information we also realize that we are able to find out the condition for the lambda for it to be supported by the string only; so, that is number 1. So, number 2, only values of lambda such that, n lambda by 2 equals a which is the distance within which the string is confined can be supported.

So, we see immediately we understand that there is a difference in the two cases that we have just looked at. So, this is, put it down here because you are going to relate this information here; that is the reason why have put it down along with this information. We realize that not all values of wavelength can be supported and we realize that it is not just that more specifically only by n lambda by 2 is equal to a which is that distance within that range is confined; only those values of lambda corresponding to this can be supported. So, you can write this other way; therefore, lambda equals 2 a by n, those are the values of lambda that are permitted. So, that is the restriction that we have now recognized in this situation.

Now, if we look at our equation here, we have energy and we have wave vector k which is simply 2 pi by lambda. So, so we already have a relations given that there is a wave given that there is a wave, we have a relationship between the energy of that wave and the wave length of that wave; we have that relationship. So, that is an independent relationship, it simply says that this is the wavelength, this is the energy; that is all it says. What we are trying to look at is that given our system, we find that we have a certain set of possibilities. We have a possibility were string is free or at least we tied only at one end; which means it can adopt any value of wavelength. So, there are no restrictions on the value of wavelength for this string.

If you now take that information and imply it on this equation, where E equals h bar square k square by 2 m. The consequent is that we find that, since there is no restriction on the value of lambda, that any value of lambda can be accepted or sustained by that string; there is no value, there is no restriction on the value of k which is simply 2 pi by lambda, alright. So, since there is no restriction on the value of k which is 2 pi by lambda therefore, there is no restriction on the value of energy. Because in other words continuous values of energy can all be demonstrated by that system. So, the in a for a case of a string that is tied only on one end, all values of lambda are allowed. Therefore, all values of k which is equal to 2 pi by lambda are allowed; therefore, all values of energy which are directly which is directly related to k by this equation, E equal to h bar square k square by 2 m, all values of energy are permitted. There is no restriction on the value of energy, that can be demonstrated by a system which is string which is tied only at one end.

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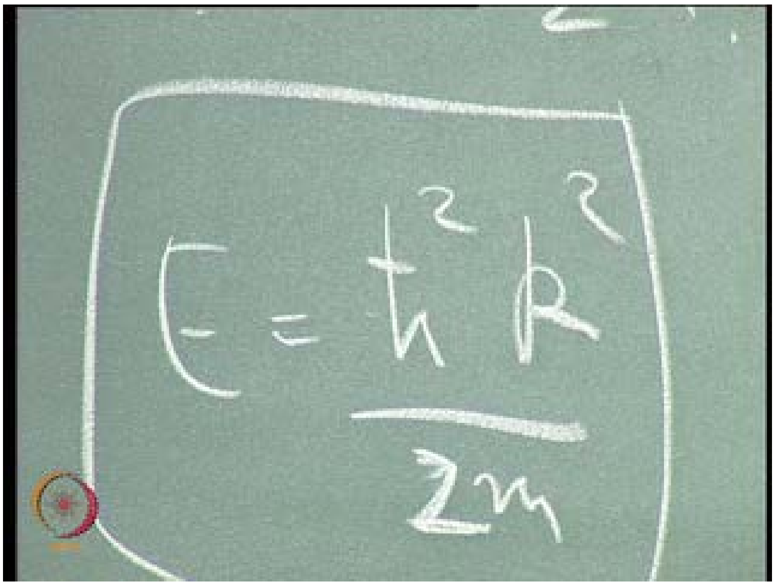

$$E = \frac{\hbar^2 k^2}{2m}$$

So, that is that is the significant of discussion that we have had E equals h bar square k square by 2 m. So, in our first case of a string that is tied only at one end, all values of k are permitted; therefore, all values of energy are permitted. Therefore, if you make a plot or if you list all the values of energy that can be shown by a string tied at only one end, you will find that all the points are continuous; I mean there is no gap, any it is just a continuous set of energy values. There is no, you do not have to jump between energy values; it is all continuous, all energy values are permitted by the system.

On the other hand, when you look at a string that is tied at both ends, a string that is tied at both ends we find that, immediately we find that not all values of lambda are permitted in that system. Because that is that system cannot consistently show you all those values of lambda, subject to these two constraints. Only specific values of wavelength are permitted, which is related to the distance with in which that string is confined. So, if a is that distance within which that string is confined, the values of lambda that are permitted are such that n lambda by 2 equals a for n is any integer 1, 2, 3, 4, 5, 6; all the integers you can use. So, only values of lambda that are permitted are 2 a by n, if you just rearrange this.

So therefore, those are specific values, those are not continuous values. So, when I say 2 a by n you cannot have 2.1 a by n, you cannot have; you cannot have 2.1 a by n. You can only have 2 a by n and those n values can be 1, 2, 3, 4 that is all you can have. So, so therefore, you cannot have yeah so you cannot have 2 a by n, I mean you cannot have 2 a by 1.1 n that you cannot have, right; so, all those things you cannot have. n has to be an integer; so, that is the bottom line, n has to be an integer. So, only specific values of wavelength are allowed and so, you can make the table you can make the table of the specific values of wavelength that are allowed. What would be those wavelengths? It will be 2 a by n equals 1; so 2 a, 2 a by 2, 2 a by 3, 2 a by 4, 2 a by 5 and so on and a is some fixed value, a is some distance. It could be 1 meter, it could be half a meter, it could be 1 centimetre, it could be an Armstrong, whatever it is, but it is the fixed value. So, it is a fixed value and it has the units of length. So, within which that string has been confined.

So, now that is the fixed value; so, the only values of lambda that are allowed are two times that length divided by an integer. So, 2 a by 1, 2 a by 2, 2 a by 3 such are those values that are now permitted in the system. And, those are discrete values; so, you can actually write a table of those values. For each of those values you will have a certain energy given by this equation. So, for every value of wavelength that is permitted, you can write 2 pi by lambda; therefore, there is a specific k that is permitted, k vector, wave vector that is permitted; therefore, there is a very specific value of energy that is permitted. The next value of energy that was permitted will not necessary be adjacent to this energy level, you have to find out the next value of lambda in your table.

So, first value is 2 a by 1; so therefore, it is simply 2 a, for that you have a energy value. The next value of that half wavelength is permitted is 2 a by 2. So, for which you will have a another energy value; in between there is no energy value that is permitted. You have the first energy level, the next energy level, in between there is no energy level that is permitted. The same thing will happen happen next, again go to 2 a by 3, you will have a particular value of energy. Once again between what you got for 2 a by 2 and 2 a by 3, in between no energy values are permitted.

So, the important idea that we have to take from this class, the very important idea that we have taken from this class is that in our analogy of string that is being tied to location. In other words, a string that is confined, the confinement of the string causes a situation that all energy values cannot now be demonstrated by that system; only discrete energy values can be demonstrated by that system. In other words, the most important idea that we will we have to take from this class is that confinement leads to quantization.

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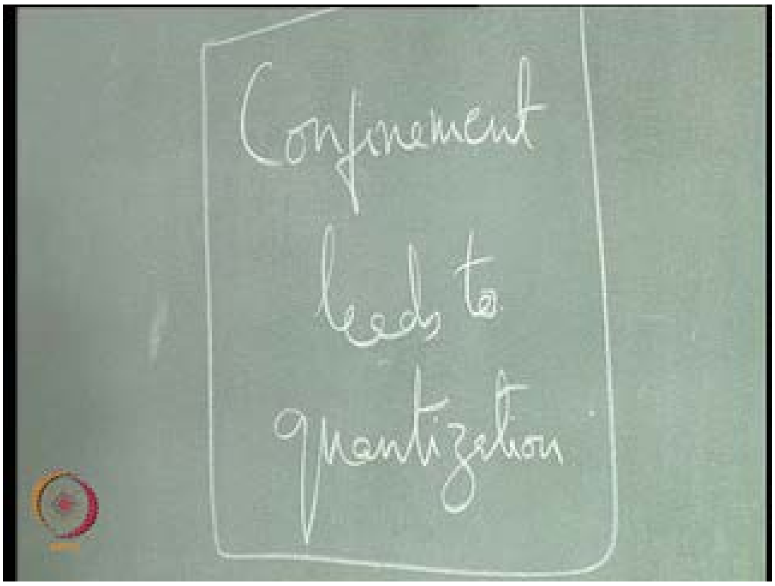

So, we will write that down, we will write it here.

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The idea that is only specific energy values are permitted is what this quantization is all about. That you have a certain energy value; if you leave it, the next energy value is a is a some distinct step away from it. It is not immediately next to it, it is some distinct step away based on the allowed wave, next allowed wavelength. So, that idea is quantization; you have one value, then another value, then another value and so on. Those are discrete values and this is quantization; that has come about only because you have confined the string, to stay between those two locations, tied between those two locations. Because it is confined, this is this is occurred; when it is not confined, you have continuous values, you do not have quantization. So, that is the difference between free string and a string that is tied in two locations.

So, confinement leads to quantization. So, this is the most important idea that we have for this class which we have discussed. We will halt here, in the next class we will actually look as I said we have done this with a simplified analogy of a string. We will actually do it in the in a more puristic quantum mechanical way. We will look at the quantum mechanical approach of handling, exactly the same problem and see whether that result that comes out of it actually is similar to, we will find that is in fact identical to this special that we have obtained here. And therefore, this picture that we have used is actually useful for us because it is giving us result, that is sense reasonable with respect to what we have looking at. So, we will halt here, we will look at we have concluded that confinement leads to quantization; we will explore this idea little bit more in the next class and then take it forward from there. Thank you.