Maxwell-Boltzmann Distribution vs Fermi-Dirac Distribution

Prof. Dr.Prathap Haridoss
Department of Metallurgical and Materials Engineering
Indian Institute of Technology, Chennai

Lecture No. # 20

Hello, welcome to this 20 th class, in our course physics of materials. So, we will take off from where we left of last class. So, in the last class, we actually look at the Fermi-Dirac distribution function we looked at the plot of Fermi-Dirac distribution function. And we try to understand, what are the various features of that function. What is trying to convey to us in other words, having arrived at that function, what is said we can interpret haves we look at the plot of that function, what are specific features associated with that function? Where is said that we see some changes behaviour of electrons. What does that represents, and how is said the electros are distributed across various energy levels at given temperature.

What variation we see the also as we change that temperature? What is the variation that we see here? So, we have discuss, the seminar is reasonable amount of details, what we will do is we will look at fact that know we have come here for a particular reason we have look at this Fermi-Dirac distribution for a particular reason. The reason being that original model that, we started off with apparently the trued model the trued Laurent's model seen in adequate some way. So, that is why we arrived at the we looked at alternate model, because given to as by so more field and it is called trued so more field model and in that contacts we came up the Fermi-Dirac distribution.

So, we will now look at it today's class we start of first by looking at what is it that we have completes it is process by coming over to the trued. So, more field model and hence having used Fermi-Dirac distribution. So, most specifically we will compare the Maxwell Boltzmann distribution, which was used for the classical model of trued model with the Fermi-Dirac distribution, which is being used for the so more field model. So, will get back we will look at the plot of the Maxwell distribution Maxwell-Boltzmann distribution function and generalize in certain way compare it with the plot that were obtain for Fermi-Dirac distribution.

And see, what is see that we are additionally able to now or what changes we have actually, incorporated in the system by assuming that it is the Fermi-Dirac distribution that now operates for system. So, we will step back and we will write down the same expression that we wrote for the Maxwell-Boltzmann distribution few classes ago make some modification to the expression. So, that conveys the same information slightly different in a way normalize way to speak and then compare it to the Fermi-Dirac distribution so, we will do that.

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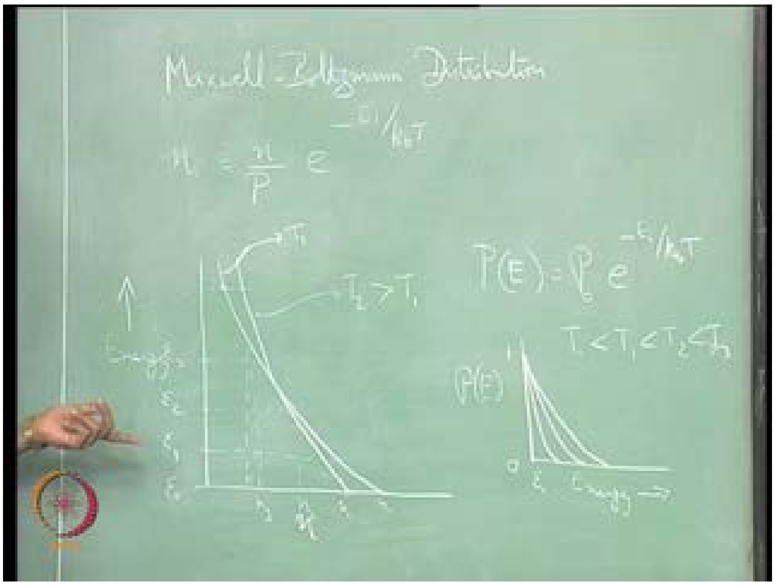

So, we look at the Maxwell Boltzmann distribution, (No audio from: 02:56 to 03:11) we had an expression that look like this n i equals n by P e to the power minus E i by k B T. So, this represented the number of states at energy level E i the number of electrons I am sorry number of electrons in energy level E i that is given by the total number of electrons in the system by a function here called the partition function P and this term. Here, which is E power minus E i this E i is energy level that we are looking at by the Boltzmann constant time T. So, this is what we ended up getting.

So, if you if we made a plot of this the plot look something like this (No audio from: 03:53 to 04:04) and this is energy. So, energy levels E 0 E 1 E 2 E 3 and so on and corresponding to E 0 we had certain n 0 number of electrons corresponding to E 1 we had an n 1 and so on and so, we had actually curve that look like this. So, this is how get all the various values of this is the actual number of electrons go to speak sitting at those various energy levels that we have. And we found that you as the so, we would have smaller and smaller number of electrons occupying energy levels that are higher and higher energy alright. And we said you know at a given temperature T at some temperature T. We also indicated that if you raise this temperature.

So, this is let say this is T 1, if you went to on temperature at T 2, which is greater than T 1. The general tendency of this function is to create the situation, where now the lower energy levels will now have little less number of electrons an as you get to higher and higher energy levels they will able to accommodate more and more electrons. So, the curve would look something like this (No audio from: 05:45 to 05:55). So, the second curve is T 2, which is greater than T 1. So, at the lowest energy level you do not have little less number of electrons occupying that lower energy level and as you go to higher and higher energy level.

So, if you see look at this energy level some higher energy level here you can actually, see that more electrons can now sit that energy level then (then) the edit before. So, you can have this many number of electrons relative to what you had before. So, as we get higher energy levels more electrons can here the and this is consistent with the fact energy of system has gone up. So, since energy as a system as gone up the higher energy levels have to have more electrons that the basic given that you are talking of system as that energy levels and so on. That is what this consistent with the (with the) information that energy of the system has gone up overall. And when we say temperature of that is what we are say temperature as gone up means, energy in the system gone up and energy is contain those electrons.

So, therefore, those electrons have to said that higher energy levels I mean assuming there we are only talking of electrons in that solid we are not looking at other information in the solid. So, this is how this information is plotted, this is talks of number of electrons and that is way it is done the same function can be written somewhat slightly differently simply to make it look like so, that it is normalized and it now conveys probability distribution. So, we simply rewrite this in a slightly different way, this is same as assign P of E equals P of some P 0 e to the power minus E i by k B T. So, all we are saying this is some constant P 0 and this is probability of occupancy of an energy level at of probability of occupancy of the energy level E i.

So, and this is the in can basically say that you know, if we put n i by n that gives you a fraction here and. So, on then you can integrated this is set of so, that integrated across all energy levels it will become one. So, (so) in that is since, we are normalizing it. So, that integrate overall energy levels this should become one. So, we can plot this slightly differently all I am going to do is I am going to now plot energy on the x axis. So, I am exchanging the axis now here we get energy on the y axis and the number of states on the x axis, what I am plotting now is simply as slightly swap version of this, where moving the energy to x axis. But I am not really plotting the number of states on the y axis and short of plot this in the probability of those states.

So, that is what I am plotting? So, this P of E is plotting here, and this P of E will such that it will go between 0 and 1 and if you basically see, the way this function would behave this at 0 Kelvin at absolute 0 the lowest energy level available in the system. So, if you say E 0 the probability of occupancy of that lowest energy level would be 100 percent at the lowest at absolute 0 and it will be 0 for the all energy level and as you raise temperature it will begin to look like this. And so on. So, this is T 0 less than T 1 less than T 2 less than T 3.

So, this is so, I am putting this two on the same nearby so that you get on idea of how they short of get each other this is how it will be with respect to energy and this is just modified version of this plot it is same equation I just modified. So, that it is (it is) normalize you can take the integrated so that it y tense of at 1 across all energy levels. So, it is just slightly modified version of this equation, but it basic form basic information it is conveying the trend it showing you is exactly the same, nothing none of those aspect having changed any I can manner it is simply being it become convenient for use later on that say.

So, this is how we derived it I have modified it mean with respect to the plot I have shifted the energy axis to the x axis there here it is an y axis it is become x axis there and y axis is the probability of occupancy so that is the function. So, this is what the Maxwell Boltzmann distribution is giving you both of them are same information Maxwell Boltzmann distribution. We will now compare this distribution with what we obtain for the Fermi-Dirac distribution. And we will put this information down and we will compare that two of them.

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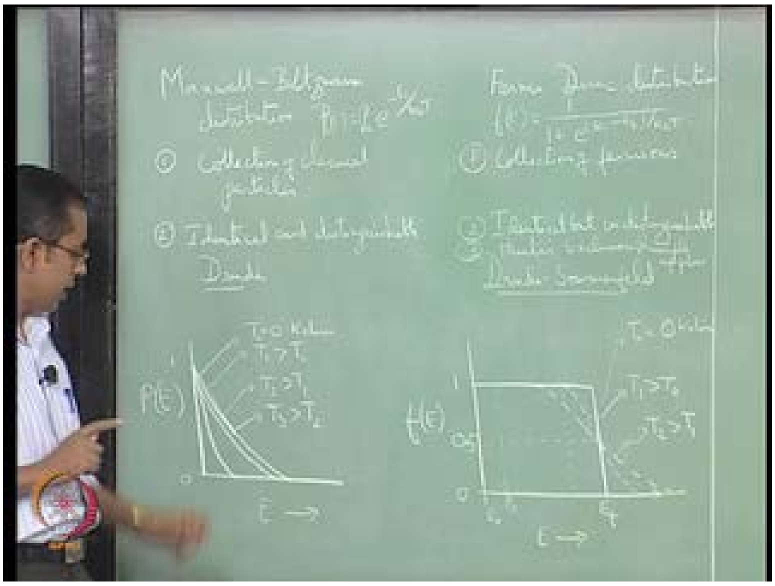

So, once again, will write it here (No audio from: 11:22 to 11:34) Maxwell-Boltzmann distribution and we will have the Fermi-Dirac distribution here (No audio from: 11:44 to 11:55). So, these two are we are going to compare in terms of T aspects of the distribution that are significantly different between two of them. These are the specific points where the differ first this applies to classical particles collection of classical particles (No audio from: 12:14 to 12:25) this applies to a collection of Fermions (No audio from: 12:28 to 12:38). So, these are identical and distinguishable, (No audio from: 12:41 to 12:56) these are identical in distinguishable (No audio from: 12:58 to 13:13).

So, these are the major aspect that differ between two of them and in the manner in which we have progress in our analysis so far is started of by treating electrons as classical particles. And therefore, employed in Maxwell-Boltzmann statistics see how they behave and that basis derived their derived (derived) equation to see, what is it represents in terms of the properties of the of a solid that contains electrons. And then we are now moving to system, where we are we are saying where we have some problem by assuming this making this assumption how, what we try this assumption. Because electrons are meet call this feature that they are called Fermions.

So, they are Fermions and they have identical and in distinguishable and meet all the criteria for being of Fermions so we have consider this possibility. So, in terms of model this is the old Drude model and this is the Drude-Sommerfeld model. So, these are the two; so, in terms of the plot we will just make the plot here, so we just tick this moment ago. So, this is at 0 Kelvin this is coincides with the y axis I will call it T 0 this is T 1 greater than T 0 and this is T 2 greater than T 1. We will just schematic this is not really I mean; you doubt, actually exact kind of plot.

So, just along as you understand, the form of what we did that this drawing that should suffices, if you want exact plots we need to draw the map to see what did that we are actually getting so, this is are T 3, which is greater than T 2. So, this is the behaviour of this is not going to below the axis. So, this is the behaviour of this function as we see. So, that is how we see it T 0 T 1 T 2 T 3 and T 0 being 0 Kelvin, when it is coincides with the y axis the Fermi-Dirac distribution we derived last class and we designate that f of E.

(No audio from: 16:02 to 16:20)

We recognized at that point there is a certain value E f. So, that is given by it is plot and so, if you want write down this equation we can those write down equation that down here. So, that we can follow, what it is that we have plotting P of E equals P 0 E to the power minus E i by k B T that it is the plot that, we have make here and this is f of E equals 1 by 1 plus E to the power e i minus E f by k B T. So, this is what this is the Fermi-Dirac distribution function, this is the Maxwell-Boltzmann distribution function. So, these are two that are they in our system that we are comparing and this is the plot for this function at as a function of so, assuming T is fixed at 0 Kelvin we get one plot. You raise temperature you get another plot raise it further you get plot raise it even further you get fourth plot.

So, that is function how we have this plot, the same thing we will do here f of E and so, we have plotted this as function of energy at 0 Kelvin this is the plot that would get. So, at 0 Kelvin this is at 0 Kelvin T 0 equals 0 Kelvin at T 1 I just mark 0.5 here, T 1 we would get this plot (No audio from: 18:18 to 18:35) this is the T 2 greater than T 1. So, this is how it progress this? So, we have at various different temperature you would have the probability of occupancy dropping to less than 1 at energy level, which are lower and lower as you raise the temperature as the temperature going up this is, what we are beginning to see in the system.

So, we recognize that in the Fermi-Dirac distribution there is this value E f, which represents the Fermi energy, which is what will see here (here). So, that is the highest I described last class that it is highest energy level occupied by electrons at 0 Kelvin and at any other higher temperature higher than 0 Kelvin it is pro-longer as the highest energy level that is because energy level above E f are also occupied, but then the definition for the Fermi-energy you can see that here at higher energy levels the probability of occupancy of energy level higher than E f it is non-zero.

So, therefore, energy levels higher than the Fermi energy level are being occupied at higher temperature. So, therefore, it is no longer the highest occupied energy level, but the way is defined, it would be the energy level where the probability of occupancy is 0.5. So, at 0 Kelvin it is highest energy level is occupied at higher temperature it is energy level, where the probability of occupancy is 0.5. So, that is the definition for Fermi energy. So, now, we have these two function whatever, we have discussed, when we derived the electronic conductivity and thermal conductivity as we (as we) aided when we assumed that Druede model was working.

And therefore, we assumed that Maxwell -Boltzmann statistics as working effectively, what we have assumed? In other words, where we you know the calculation in we were doing somewhere, did on calculation the kinds of parameters we use the way will relate the parameters each other. They were all somewhere in very fundamental sense were using this information. And the major issue that we found was the contribution of the electronic contribution to specific heat that the Drude model predicted was about two order of magnitude higher than, what was actually observed experimentally.

So, that was one piece of information we have. So, what is it that, when you look at this picture, what is it that the picture conveys to us in terms of the specific heat behaviour that we are looking at. Now, what it is conveying to us is that fact, that in the Maxwell-Boltzmann distribution will you raise the temperature. When you raise the temperature (when you raise the temperature) all the electrons that are present in the system or in a position to absorb some energy and go up to the, such that the overall temperature of the system goes up. All the electrons in that system are able to absorb energy such that the overall energy of the system goes up and hence therefore, the temperature goes up.

So, this is the (this is the) manner in which the system is responding to raise temperature. So, therefore, when you have a large collection of electrons, you have to provide a enough energy that all the electrons are in a position to pick up energy and only than the overall temperature of the system goes up. So, that is the basic idea in the Maxwell-Boltzmann analysis or other the when we assume at Maxwell-Boltzmann statistics holds that is what is happening at a fundamental level So, all the electrons the way this statistics works is basically that there is no restriction placed on the number of electrons that can be attaining given energy level.

So, therefore, all of them can be at lowest energy level and as you raise the temperature all of them can attempt go up in a energy levels, because I hate of them there are all vacant states the and no restriction how many electrons can sit in that states. Therefore, they can all move forward in energy and therefore, all attempt to pick up this energy level you see a distribution such that average distribution the average energy corresponding to all those electrons corresponds to temperature of that material. So, this is what we are seeing here, and when you doing it this manner in the manner that I have described you it appears that we are over estimating the amount of energy we have to provide to raise the temperature by a certain amount.

So, that is the specific heat specific heat simply says, what is the amount of energy, you have to provide to the system to raise temperature of the system by a certain by an incremental value. So, let us say 1 degree Kelvin you want to raise go from 0 Kelvin to 1 Kelvin or 10 Kelvin to 11 Kelvin, if you want to this, if you want raise the temperature of the system by 1 degree Kelvin, what is the amount of energy? You have to provide according to this model, when you make this kind of an assumption and you find the this is the behaviour of the system you raise the temperature of the system. This model says that all the electrons in that in that material can (can) gain energy and therefore, will attempt that energy as you raise the temperature of the system.

So, you have to provide energy sufficient for that process. So, huge number of electrons other you have to provide energy sufficiently. So, that all of them can gain that energy such that the average temperature of system goes up, if a very large fraction of electrons sitting at a low energy level and with only very small number of them gain energy. You are average temperature is not going to go up by 1 Kelvin you have to provide energy of all electrons to move up in energy level. So, that the temperature goes up to 1 Kelvin goes up by 1 Kelvin. So, that is what you are assuming here, if you shift to the Fermi-Dirac distribution, what we find is that the at 0 Kelvin the probability of occupancy of states is one all the way up to Fermi energy and then, if drape to 0.

Now, the when you plot this function of temperature, what you note is that as you raise the temperature not all of the electrons participate in the process of gaining energy. So, that is the very important point that we need to notice that in this (in this) distribution not every electron participate in the process of gaining of energy. As you raise the temperature of system not every electrons is participating in that process why is that (why is that) case, if you look at this distribution the fact is that we have also assumed here for example, that the information that, which is very important here is that polis exclusion principle applies (No audio from: 24:56 to 25:10). And in fact, (that is) that is why, it is a Fermions it is a Fermions, because it meet all those it is characteristics polis exclusion principle applies.

So, when polis exclusion principle applies, we bend in to this statistical distribution, we accepted as we this idea then there are first of all only a limited number of states at given energy level, you do not have an infinite number of states. On top of that, we accept the fact that since polis exclusion principle applies with limited set of states we can only put a electrons on those states you cannot arbiterly put twice as many electrons five times as many electrons thousand time as many electrons on those states. There are states they represent actually; when you go down to the more fundamental understanding of the system they represent all the quantum mechanical numbers of those of the system of the particle.

And therefore, when each state can occupied only by one electron, you cannot put more than one electron per state. So, when you have finite number of states at low at low energy level you can only put a finite number of electrons on that on that states. You go to the next energy level, but you have a large collection of electrons (you have a large collection) of electrons. So, you have to start building of system that way, you take the lowest energy level you fill it up with electrons then, you find you still have additional electrons available. You take the next higher energy level you fill it up with electrons you still have more electrons available go to E 3, you have fill it up with electrons and go on till you reach E f when you finally, run out of electrons.

So, we need to look little bit more carefully, but what this picture represents when T 0 is 0 Kelvin, when the temperature is 0 Kelvin all the states that is very important note, all the states in energy levels from E 0 up to E f all of them having filled there is no vacant state available between lowest energy level that you have available here and the Fermi energy level that is there. Every state that is available between these two energy levels has been filled completely that is why the probability of occupancy of those state is one, if there are fifty states there are fifty particles at that energy level they have full.

So, there is no way states between the lowest energy level that you have here, which we fall E 0 and the highest energy level that you have here, which is E f at 0 Kelvin or 0 Kelvin everything is full. Now, when you raise the temperature of the system I said that you know we need to understand raising the temperature of the system is that electrons in the system or gaining energy. That is what will we need then we say energy of the system as gone up for an electrons to gain energy it will represents in this picture, the idea that the electron has to boot higher energy level.

So, it is previously at say E 10 is energy level it is sitting at you now give it some energy it has to move to E 11 or E 12 or E 13 somewhere it has to move only then that electron has actually gain that energy. O r it is able it is only then picture as consistent with idea that electron is gain energy if you are arranging the energy level increasing order if it is sitting at E 10. Now, you given at little bit of energy depending on what quantum of energy you have given it what mount of energy you have given it has to go to E 11 or E 12 or E 13 somewhere there it as go for it gain that energy. And stay within a constraint of the system, because cannot just gain energy these somewhere in face it has been within that system.

That system has already, placed all these constraint that there are fixed number of states available within the system. And those states have a restriction there can only be a fixed number of electrons on those states that is the system within these electrons have to operate they cannot operate outside of this system. Now, if I take an electron sitting here, so, let me say some arbiterly E let say E 3 an electron sitting an E 3. I try to raise the temperature of the system, when you try to raise a temperature of the system this E 3 let say it can go up to E 5 let say the amount of energy you have to provide to the system is sufficient for E 3 go to E 5.

That is the amount of energy you have providing to the system let say that that is all you have doing, when you do that this electron at E 3 attempt go to E 5, what is a problem it faces? The problem it faces is that at 0 Kelvin E 5 is already full, it has certain number of states available all those states are already full. There is no weight can state available at

E 5 therefore, the electron at E 3 even though it principle it have picked up that energy the delta energy that you have provided in to the system. Some you know, so many electrons holds of energy you have to provide in to the system. Even though, in principle you can say that it is sitting at some energy level you have given let say three electrons holds to it.

So, it can move to new energy level which is E 3 plus 3 electrons holds. Let, say (let say) that is the kind of picture you are looking at, but at E 3 plus 3 electrons holds you have a new energy level which is let say E 5, E 5 already the states are full. Therefore, the electron at E 3 is in capable now in to in participating in this energy gain process, it is prevented from gaining that energy. Because to gain that energy it has go to new location that new location is already full, there is no way can participate then new location. So, even though, the energy coming in to a system this electron at E 3 cannot participate in the gain of that energy.

So, therefore, electrons at this lower energy levels in a similar manner or prevented or not in a position to accept the energy being provided to the system from the external source. So, when you raise the we have just started from extreme case that it is 0 Kelvin. So, because that conveys to us picture much more clearly we will just extern the argument to higher temperature. So, at 0 Kelvin we are face to this situation that all the energy states are full up to the Fermi energy level. So, at any lower energy level we are the electrons at that lowest energy level even face the situation that I had of them at energy level just higher than where they are all the states are already full.

So, if you give the small amount of energy they are they will attempt to get only to that energy levels we can just marginal state I had of them and marginally I had of them states are already full so, they cannot move. So, therefore, they do not participate in the gain of energy. So, this continues still you get very close to the Fermi energy. Now, let us look at a electrons, which are art that very near the Fermi energy the electrons, which are art at very near Fermi energy I had of them I had of them or energy levels, where there are empty states. In fact, at 0 Kelvin at 0 Kelvin at E equal to E f you have finally, fill the states with last state of electrons that you have available and at any energy level above E f these states are completely empty.

So, the probability of occupancy is 0 at that point in time at energy levels above E f therefore, above E f at 0 Kelvin the states are completely empty. So, now, when you try to raise the temperature of the system marginally by providing a little bit of energy in to the system electrons, which are very closed to E f very close to E f, or arc E f for them I had of them the energy states are week end. So, if you provide a small amount of energy let say in again, use the same description say it is now at E f and you have provide it at three electrons hold let say. Then those electrons, which are at E f, if they look at E f plus 3 electrons hold I had of them at states are all vacant.

So, therefore, these electrons can now accept that energy. So, therefore, when you take the system following this Fermi-Dirac statistics and you look at you try to raise the temperature of that system by providing a little bit of energy. You find that the electrons, which are very closed to the Fermi energy are able to participate in that process. Whereas, all the electrons, which are below the which are distinct the Fermi energy, which are twistingly below Fermi energy level or in capable of participating in the process. So, that is why you see this picture that are if I go to the temperature T 1, which is greater than 0 Kelvin you find that.

Now, in this case, you know this energy level this band of you want let us not use the word band, because; that means, thing else elsewhere. This range of energy value (this range of energy values) corresponds to the electrons, which are now in the position shift forward to higher energy levels. Therefore, the probability of occupancy of these electrons at these energy level begins to decrease and the probability of electron levels above E f begins to increase. So, in this system in this picture we find that, if you have a large collection of electrons then very high fraction of that collection of electrons. So, if you have a million of electrons.

So, ninety nine percent of those electrons are at energy levels twistingly below the highest energy level E f only a very small percentage. Just for (just for) number say I mean, without too much specificity on the number something like very small percentage may be, one percent or something like. That or the number of electrons, which are very closed to that Fermi energy level they are the ones participate in this energy gain process. So, therefore, they move forward. So, now, and this (this) continue once you got T 1 we are reach to this situation, when you raise it for that T 2 few more electrons from a slightly lower energy level can also still participate and there able to move forward.

And (and) this is short of accumulated process, when (when) you move let say an electron at E f to an energy level higher than it than an electron, which is little lower energy level can move to an energy level E f. So, (so) they can all move forward progressively as the as it begins to move. So, that is what is happening, it is in a sense it is analogues saying you know have I view drawing this analogy to container containing you know sand or water or something like that. The disturbance that we caused that system is usually, something you see that surface the way occurs the surface, if you want disturbed.

So, it is a very loose analogy that point in time, but it short of conveys the idea to you that you know something add the surface is participating in the process, when you do a marginal when you just only marginally you know tap the surface. So, if everything is corrosion still at 0 Kelvin you marginally tap the surface only that surface participate is the way you want to look at it. So, this is the way in which this is occurring Fermi-Dirac distribution. So, now, if you compare the two of them the difference begins becomes apparent, when we use the Maxwell-Boltzmann statistic, when you try to raise temperature all the electrons in the system or participating in the process of gaining energy when you use the Fermi-Dirac distribution very large fraction of the electrons are not participating in the energy in these process.

So, to repeat in the Maxwell-Boltzmann distribution if you raise all the electrons participate in raise in the temperature or are in a position to participate in the gain in temperature. Therefore, for the same gain energy in temperature for same change in temperature you have to provided fairly large amount of energy. This certain amount of energy you have to provide for the entire set of electrons to participate in the process of gaining energy and therefore, raising temperature of the system, when you go here, very large fraction of electrons. In fact, do not participate in the gain energy gain process they are passive to the energy gain process simply, because they have no means of participating in the process.

Therefore, when you try to raise the temperature of the system most of them simply do not participate in the process very small fraction very close to the Fermi energy, if the only set of electrons that participate in the process. So, by simply and therefore, you find that you know in terms for this same change in temperature simply, because you have to a very large set of electrons participating in this energy gain process and a very small set of electrons participating this energy gain process. The amount of energy you have to provide here for this same change in temperature is significantly larger than the amount of energy that you have provide here for same changing in temperature.

So, to repeat the amount of since large number of electrons or participating this energy gain process here, the amount of energy to attain similar change in temperature is relatively large here, compare to a situation here, where a very small number of electrons are participating in the energy gain process. So, therefore, as a small amount of energy is required here, the specific heat by definition is exactly that same pieces of information it is what is the amount of energy that you have to provide to the system. So, that the temperature of system goes up by a certain marginal amount that is the basic concept that is the captured in the specific heat of a substance.

And therefore, we can see that simply, because the two system differs the number of electrons participating in energy gain process the your estimate for specific heat if you use the Maxwell-Boltzmann statistics is going to be large, because any more energy for same change in temperature. Your estimate for specific heat is going for significantly less, if we use the Fermi-Dirac statistics, because you are very small number of electrons will participating in energy gain process small amount of energy is required for the same temperature relatively speaking.

So, in this fundamental sense, this is a very descriptive concept that we have looked at so for in terms of comparison. We will try and put some numbers down to these number little later, because we have to develop few more concept before we do that. But even as (as) a concept write now it we immediately understand, how by employing the Fermi-Dirac distribution on set of electrons in a solid we are able to correct one major anomaly of the Druede model. So, the Druede model, which used which effectively use Maxwell-Boltzmann statistics in it was over estimating the specific heat the electronic contribution to specific heat it was over estimating the electronic contribution to specific heat by a factor of hundred.

So, the Druede model originally was over estimating the electronic contribution to specific heat by a factor of hundred. And we find now, that by using the Drued-sommerefeld model are able to dramatically reduce the electronic contribution with specific heat. So, and this sense Druede -sommerefeld model immediately corrects one of the major anomaly of the Druede model the original model the original Druede model. So, by shifting from a classical particle by shifting the distribution of electrons in a solid from the part of classical particle to that of a Fermions and by shifting that description, we have shifted that function applies those particles.

And, because this function behaves this way we find that one of the parameter that it is prefix is now significantly less than the prediction that was made here and it turns out this kind of a prediction much more accurate. So, therefore, the purpose of exercise, which was to address the difficulty that the Druede model had that was the purpose of this exercise. We found that it was having some short coming including this prediction of this specific heat and therefore, that was the purpose of this exercise to see, if there was better model and that is how sommere field tried out this better model he took the basic idea to the Druede model.

The basic approach of the Druede model still maintained, but you just basically recognize the fact that it is not it may not be appropriate to read the electrons is the classical particles. Because for all the character characteristic that they have they are not really classical particle based on the characteristic they have the display the quantum mechanical behaviour much more significantly. So, there is a tendency for them to show on quantum mechanical behaviour much more significantly as I mentioned before we looked at history of quantum mechanics in a sense, you can try and look at the quantum mechanical behaviour of even larger object day to day objects and so on. It is just that, when you look at larger and larger objects the effect of the quantum mechanical calculation becomes very mini cube.

Therefore, when you treat them classical object we treat them as classical object that we can identify we can locate we can move and handle and so on. The that it is (it is) in a sense it is approximation, but it is very very good approximation, because the quantum mechanical effects are negligible to non-exist I mean nearly non-exist that state from our perspective. So, therefore, it is ok when you plot of, but when you go down to the electronic scale, when you go down to a sub atomic scale and you look at electrons then the quantum mechanical effects seen to become very significant.

And therefore, the phenomenon that we see at an electronic level in term of how it gain energy how it loses energy how it interacts surrounding and so, on. When you look at the phenomenon, if you simply apply classical type of understanding on it the prediction it makes turn out round the prediction that we find or turn out to the round. So, therefore, we are force to accept the fact you know that level quantum mechanical behaviour is much more significant and therefore, whatever, equation go along with the quantum mechanical behaviour we have to use utilize. Only then the only then we are capturing the fax it of the behaviour of those particles appropriately only.

And therefore, only then when (when) those equation work out whatever, result we get only then it is likely that final result that we get will be accurate and will be matching the experimental data that we have much more accurately fine and so that is the basic idea. So, that is what some more field is done Sommerefeld recognized that it is not really appropriate to use classical mechanics. So, you just find out quantum mechanical approach and it is around that time that we Fermi-Dirac distribution had also been proposed. So, this is in the 1927 time frame.

So, around that time is Fermi-Dirac distribution had been indicated independently. He just took the Fermi-Dirac distribution and imposed it on collection of electrons, which are sitting in the solid that is basically what we did. This was much older piece work Maxwell-Boltzmann distribution is 1870 (( )) it was a work. So, this is about 50 years 50 60 years apart in time frame in terms of the distribution itself and then in terms of usage this is what has been done. Now, I also mentioned that you know, so we see in now immediately how (how) it is that this model is an improvement on this model how the Drued-Sommerfeld model becomes on improvement model immediately from this plot we can see the difference.

However, I also pointed out that this is not the complete picture; because this simply talks of the probability of occupancy it does not tell you how many states are available at each of those energy levels. For you to actually, put a number down for specific heat we have to that we have to an idea very descriptively telling you that you know number that electrons very close to the Fermi energy or participating in energy gain process. That is descriptively it is fine and it does convey this idea this model is (is is) refining something that is not visible here, so, that is last here in fact.

So, this model definitely refines that and you (you) see able to captured that particular sterility of this process, but what this model the way it is currently shown to you, what it is not capturing, what it is not indicating to you is the number of states that are actually available at each of this energy levels and that is the very important piece of information. Because only when you have that can you actually, put numbers down to how many electrons have done this participated in this process. I can only tell you now that electrons closed to this participated, but what is that number I mean, if there are only five states at E 0 there are fifty states at E 1 and there are five hundred million state at E f.

Then obviously, been the contribution of those state and there are fully occupied then (then) you will find that out of you know five hundred and million plus ten five hundred and million are participating in the process that is not really useful for us, because that is that still that help us understand the process. So, we need to know how many states are available at each of this energy levels and therefore, get an understanding of how many states are actually available very close to the Fermi energy level or at the Fermi energy level. And we need to understand that this picture is already there with us that tells us the probability of occupancy, when you have the number of states also thrown in to this picture.

Then we can actually, do calculation, which says that now you raise the temperature of the system, what is the amount of energy that has been absorbed by the system to raise the to attain that same temperature change and what is the number of electrons that have participated in the process. So, that is the piece of information that we need to do that is (that is) the detail that we have to provide. So, in the next few classes that is we will attempt to do next few classes you will try and get in this picture. To do that what we are going to in the immediate next class is to we are going to actually, see that you know we have to refine the picture of what is it that solid is.

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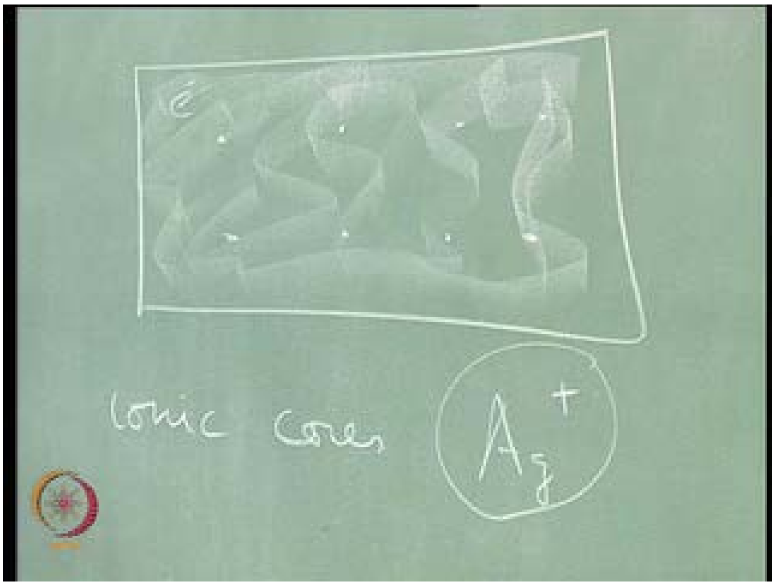

So, so for we have said that it is simply that you have a ionic core. So, we have a set of ionic cores, which is simply what I mean, if you have silver (silver) atom we are saying we assume that let say silver atom behaves a univalent manner we just on assumption that we may base on it is standard behaviour. So, we have silver we say that all these location based on the crystal structure. So, whatever, any metal that you take, you look at it crystal structure. So, at those crystal lattice location you have those metal atoms and they are reject fixed we are saying that there are metal atoms there actually metal (metal) ions. And so, therefore, and that is the core they ionic core and the electron that it us release to free run around through the solid.

So, the electrons in the ionic around in the (in the) solid for this is called electrons in the ionic around. So, this is the model that we have used physically this is the model is used. The electron are freely run around the one electron that every atom has to be freely run around at those location we have ionic cores, which corresponds to this A g plus ions. So, these A g plus ions at all the lattice location these electrons are now running across. In our Druede model original model we basically, said that it is a plot potential throughout this system there is no (there is no) preference for this electron put any one location or any other location as it moves around it does not it only sees and overall average to behaviour overall averaged atmosphere.

So, to speak and it has no greater tendency to be at any one location no greater tendency to be at any other location you just freely runs around across this entire solid. What we need to recognize is that, when you have ionic cores the if you take even single line of this ionic cores as you move from left to the potential that electron will experience at each location is going to be somewhat difference. Because this ionic cores, because you are going to get closed to an ionic cores then move away from ionic core get closed to ionic core away from a next ionic core and so on.

So, therefore, it is not really accurate to say that there is no variation in the potential in the system there is we just we simply have to accepted we cannot ignore it beyond upon. So, what we will do in our next class is try and built very real much more realistic picture of how the potential exist in the solid. So, and we will (we will) see that there is a way in which we could probably draw we will take a linear collection of ionic. And we will see what is the more realistic way of debiting the potential as you move from location to location the potential that electron will experience as it move to location to location, when you when it is facing a set of ionic cores arranged in a linear fashion.

So, we will see realistic picture of this potential was is position then we will (we will) find that it is not necessary that fact detailed picture, what we will do is we will (we will) recognized that we need not use the compete detail as it is, but we can make a very good approximation of that same potential (potential) was in (was in) position picture. We could make a very good approximation of a potential was is position picture. And we will make the approximation such that for our analysis we will find it much easier to handle.

So, we will make that approximation and since, that will make our analysis easier could able to write the mathematics corresponding to that and come up with the with better understanding of better prediction, what the electrons is going to do depending on where it is and depending on what it is capable of doing. So, that is the picture, we will have to draw. So, we will start next class by looking at an attempting to come up with our much more realistic picture of potential was a position was that on electron will experience and we will come up with a more sensible more acceptable approximation of that picture. So, that the main details of that potential process position function is captured main details are captured, but at the same time computationally and conceptually much easier for us to follow, what is happening with in this system.

So, that we will reduce a level of detail a little bit, but at the same time will captured all the major details. So, we will do that and only (only) after we do that we will start we will get in to a position, where we can start answering this additional question that we have now put just how many states are at an given energy level. So, that is on information we do not get have, we will need to get there to do that as a first step we will need this potential approximation for potential in a in a solid. So, in the next class, we will do that and from there we will take it. So, with that we will halt for today. Thank you.