Physics of Materials

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Lecture No. # 19
Features of the Fermi-Dirac Distribution Function

Hello and welcome to this the 19 th lecture in this physics of materials course that we are going through. In the last couple of classes we have derived the Fermi-Dirac statistics we have look at the basis for which we why we needed to drive it and then we went hide and looked at the derivation Fermi-Dirac statistical distribution. So, what we will do in this class is we have arrived at the expression for the Fermi-Dirac statistical distribution. So, what we will do in the classes to look at that expression that we have arrived at and try to understand what are the implications of that expression, what does that expression imply, what is sort of taken into account there, what are some feature of that expression that we can later look out for.

And also get an understanding of how it relates to everything else that we have looked at earlier on. And in what ways we have come up with some additional aspects that we are capturing here which perhaps we have not capture earlier on. So, this is the general outlook of what we will examine through this class. So, we will begin by rewriting this Fermi-Dirac expression and then we will start our discussion from there.

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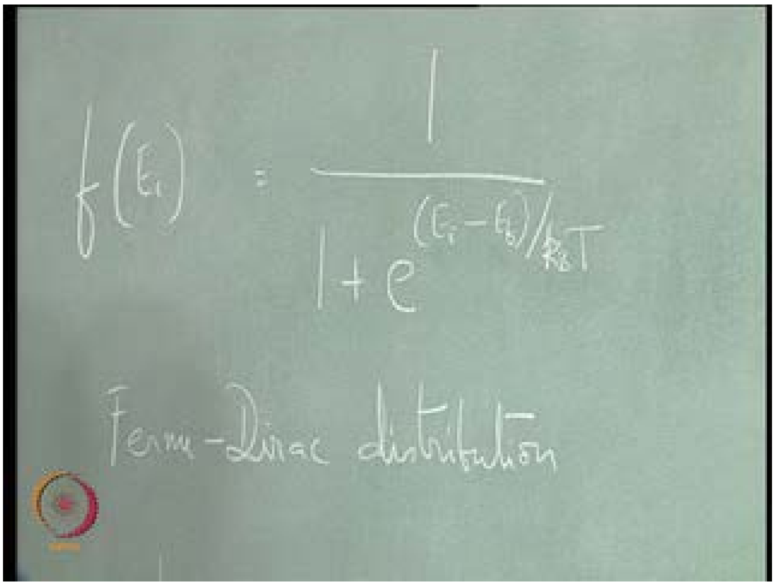

So, we designate the Fermi-Dirac statistical distribution by using the letter f so, f of E i equivalent is given by 1 by 1 plus e power E i minus E f by k B T so, this is the expression that we have come up with. So, to put it inverse, this is the probability of occupancy of a state at energy level E i and it is being given by this expression, we have 1 in the numerator and 1 plus e power this E i is energy level that were looking at. So, therefore, as you evaluate this expression, the value of E i can change, for every energy level if going from zero energy to let us say, some very high energy level.

For every energy level that you can select here to check for its probability of occupancy that energy level that show up here E i. Then we have minus E f, E subscript f which at the moment we have not really discussed in any grade detail except to say that based on how we can derived it, this value of E f is a constant for that the system. It is going to vary from system to system based on what we are looking at, that for a given system this is a constant, this E f is a constant which we can utilize that point in time. So, in this E i minus E f, E f is a constant, E i is something that we can change for all calculation purposes. k B here in the denominator is the Boltzmann constant so, by definition it is a constant so, that is nothing that that is nothing that we can do it is a constant that.

And T is the absolute temperature so, in other words and that of course, is a variable so to speak, in the sense that when we evaluate this expression, we will do show at a specific temperature. So, in other words we will specify the temperature and for that specific temperature we can evaluate this expression for a variety of energy levels. Then you can change the temperature and I am say; let say we are starting at absolute 0, at absolute 0 we can evaluate this expression or as the temperature tense to absolute 0 we can evaluate this expression.

And then we will have something that we will see is behaviour of this expression. then we can raise the temperature and at higher temperature, we can again evaluate this expression and see how the behaviour as mention. So, this is the general what should I say, the general way this expression works out in terms of what is it that we can change and what is it that remains constant. And therefore, in what contacts we can actually evaluate this expression and expect to see some data corresponding to that expression. So, so, this is the Fermi-Dirac distribution function.

(No Audio From: 04:31 to 04:44) So, this is the; and it of course, credited to two people and have contributed towards coming up this function. So, I will just briefly we will also now we will make a plot of let say as the temperature tense towards absolute zero, we will make a plot of this function for variety of energy level.

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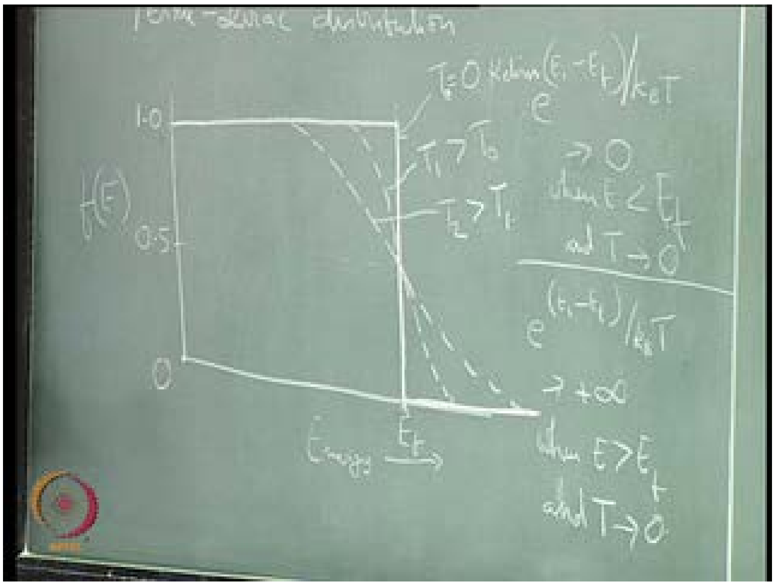

So, on the y axis we have f of E so, at different energy levels. And this is the energy (No Audio From: 05:15 to 05:22) so, on the x axis we have energy, a variety of energy levels that we can look at. On the y axis we have that function Fermi-Dirac distribution function for that for each corresponding value energy that we are going to look at. I mentioned in this expression that there is a constant which is; which we have not really looked at in great details so far, but this constant is some energy value. Some particular energy value is what this constant and it remain fixed for this system.

So, in this plot we will just mark up one location of energy and I have just arbitrarily marked it up here, it is just schematic of what we are going to get. So, I will just arbitrarily mark a location on the x axis and say that for some for hypothetical system that we are examining that happens to be the constant value of energy that corresponds this constant here E f so, I will mark that appear as E f. So, we have mark this constant E f here, all the other energy values are available for us I mean these are the energy values available for us to do something and look at this expression.

Now, I clearly given that there is a certain value for E f, there are values for energy that are less than E f and then there are values of energy greater than E f and of course, there is a value of energy equal to E f. So, E as we look at, the energy value that we look up here can take all of those values. So, at or at least you can evaluate this function, this function can be evaluate for energy values that are less than E f, equal to E f and greater than E f. So, this entire range of energy values we can evaluate, fine. So, what we will do is, we will look at this expression see what is work out to as you evaluated for different values of energy.

And we are specifically looking at a situation where this temperature is tending to zero. So, we are looking at situation where the temperature is very nearly absolute zero of in our scale of temperature, fine. So, when you had E minus E f and energy so, we will start from or; from the original forward so, for all these values of energy before you reach E f, E is less than E f. So therefore, if you write E minus E f that is a negative quantity, it is a negative quantity divided by k B T where T is tending towards 0. So therefore, this quantity here so, e power E i minus E f by k B T tense towards 0 as tense towards 0 or yeah tense towards 0 when E less than E f and T tense to 0.

E less than E f means, this is a negative quantity, negative quantity divided by a value that is approaching 0 would mean, this is tending towards negative infinity. So, negative infinity means e power negative infinity will make it tend towards 0. So, E minus E f by k B T will tending towards negative infinity when E is less than E f and T tense to 0 and therefore, this quantity is e power minus infinity is tending towards e power minus infinity. So, that is 1 by e power infinity which is 0. So, therefore, this quantity tending towards 0, then E is less than E f and therefore and when T is tending towards 0. So, under this conditions, this is tending towards 0 so, in your expression here, this is tending towards 0. So, in this expression this term tense towards 0 as long as the energy value E i is less than E f and temperature is tending towards 0.

So, in the limiting case when temperature is absolute 0, we will say that this is negative infinity and therefore, I am sorry this exponential term up here with 1 is tending towards negative infinity therefore; this entire term here is tending towards 0. Therefore, the value of this function evaluates to 1, because this whole term tense; become 0. So, if you look at, if I just draw a line here a dotted line corresponding to the value of E f and on the y axis I have 0, I have let us say 0.5 and 1. So, when E is less than E f, f of E is equal to 1 at T equal to absolute 0.

So, at for; and this is going to be true for all energy values regardless of energy value that you pick here, as long as the energy value is less than E f, this description that I have just given you will whole true. That it is going to become minus infinity and therefore this term going become 0, therefore this going to become 1. So, for all values of energy less than E f, the value of this function going to be 1 when temperature is absolute 0. So, therefore, this function will show behaviour, it puts like this till E equal to E f. So, till the temperature; I am sorry till energy level E f is attain, you are going to have this function evaluate to 1 for all energy values less than E equal to E f at temperature being equal to absolute 0.

So, now we will look at energy values which are greater than E f. So, greater than E f you have energy values that are greater than the value of E f which means the numerator here now positive, divided by a value that is 0. So, therefore, this value of here tense to plus infinity and therefore, this is tense to plus infinity therefore, this whole function tense to 0. So, e power E i minus E f by k B T tense to plus infinity.

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We have seen these two cases here. So this is it is tending to 0 when E is less than E f and tense to plus infinity when E is greater than E f and when it is 0 the function evaluates to 1, when it becomes infinity the functions evaluate to 0. So therefore, for all energy values greater than E equal to E f, the functions simply coincide to x axis and at E equal to E f we sort of have discontinuous change going from here to here. So, the function in fact shows your behaviour so, this is at T equal to we just mark the temperature here. (No Audio From: 12:37 to 12:44) So, at T equal to 0 Kelvin, we have seen this behaviour.

We see that the function is simply 1 till E equals E f it drops to 0 at E equal to E f or changes from 1 to 0 when you are at E equal to E f and then remains 0 beyond this value of E f. So, this is the behaviour of the Fermi-Dirac distribution function. Now, in fact if you evaluate this for variety of different temperatures, you will feel; you will find that we have now evaluated it for T equal to 0 Kelvin. We can evaluate this same function for variety of different temperatures, what you will see is that as the temperature increases, you will see a behaviour that looks like this.

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So, this is some temperature T 1 greater than I call this T 0 which is 0 Kelvin greater than T 0 and this is some temperature T 2 which is greater than T 1 so, as we get higher and higher temperatures, you see change in this function. Basically in this distribution what it shows you is that it is equal to 1 up to some energy level, after that it begins to drop from 1 and gradually drops to 0. Whereas at 0 Kelvin it abruptly drops to 0, the transformation from 1 to 0 is more gradual if you go to higher and higher temperature so, that is the behaviour that you see for this transformation as change the temperature. So, this is how you see this Fermi-Dirac distribution function and the way in which it behaves.

Now, let us get an understanding of this E f, because it is constant I just mentioned to as a constant and we will see what it represents. We need to keep in mind that when we looked at the Fermi-Dirac distribution when we started at whole expression on Fermi-Dirac distribution, we basically said that we are now looking at situation where first of all at you have specific energy levels and each of those energy levels there is a fixed number of states. So, that was a constraint which had not been in our discussion before a prior to when we started Fermi-Dirac distribution process so, are the function discussion of the Fermi-Dirac function. Before we started this process we never placed any restriction on the number of states available at on energy level.

Once; one of the fundamental aspects of the whole discussion regarding the Fermi-Dirac distribution function was this idea that at a given energy level we do not have infinite states, we have a finite number of states. And therefore, the distribution process has to some in some manner keep track of for accommodate these number of states. More specifically we also said that poly exclusion principle applies and therefore, not only we have finite set of states, we also have a limit on the number of particles we can place on those states. Even prior to our discussion on the Fermi-Dirac distribution function, we might have considered a case where number of states was finite, but there was no restriction on the number of particle we could place on those states.

So, even if you had one states, you could place million particles on that states. Whereas, now we have this discussion has progress to appoint where we are saying the situation where considering or examining is one where we have a finite number of states at given energy level and we have this restriction we can only put a finite number of particles on those number of states subject to the policy exclusion principle so. In fact, if the state is has all the details in its all the quantum numbers specified for the E f for each state you can. In fact, put maximum number 1 particles for that state. So, we have those restrictions shown in to our discussion we entire distribution the derivation had this a very fundamental idea and which we solve in the last couple of classes.

So, what it means that if you actually arrange the energy levels and in realistic situation were we are saying that particles in generally the nature tense towards energy levels that it can attain. If you arrange the energy levels in increasing order of energy and you take a set of particles when you try to fill up this energy level, what will happened is that, we will first fill up the lowest set of energy level. So, let us let us consider this discussion to be occurring at 0 Kelvin which corresponds to this straight line curve; straight line that have drawn here; set of straight lines that have drawn here. So, it comes up to E equal to E f drops again goes to 0; space at 0. So, at 0 Kelvin you take all the energy levels that are available in the system, the lowest energy level and subsequently increasing higher number of energy levels.

And you start filling up the particles subject to the constraints that are prevalent in the Fermi-Dirac distribution function which is that finite number of particles should go on the states and not more than 1 particle per state. So, when you do this; obviously, since you have a finite set of states, if once it starts filling up one particle per state when you fill up all the states you have done, you cannot even though make a lot of additional particles available with you. You cannot add anymore particles from the lowest energy level. So, E 0 if you say as the lowest energy level, let say hypothetically it has 50 states and we put place the restriction that you can put only one particle per state.

Now, once you put 50 particles at E 0, you are even if you have million particles still left that you are; that are available in the system that you can done place at energy level, you do not have the option of placing any more of those particles in those 50 states. Once those you 50 states are fill your force go to the next higher energy level please remember this is a 0 Kelvin. Conventionally, we say at 0 Kelvin every think at lowest energy level possible, that entire system is in it is ground state and we have had a tendency to think the ground state means energy level possible. Immediately our discussion shows as that the moment we assume that the system is following Fermi-Dirac statistics, the moment we say that the system consist of particles that are Fermions which are following Fermi-Dirac statistics.

We are immediately face the situation that the ground state does not imply that all the particles that sitting at the lowest possible energy level in the system. What immediately happens is that, there is a finite number of states they get fill first, then you go to the next higher energy level E 1 that will also have a certain finite number of energy states, they also get filled up. You go to E 2, you filled at state E 2, go to E 3 filled up state E 3 and continue upwards in energy level. So, you keep filling, you go the next higher and next higher energy levels and fill all the state available at those energy levels. Steadily you fill those states and you go higher and higher and higher energy levels, this is at absolute itself.

So, we have not; we do not have the option of trying to take all the particles and placing them at E 0, simply because they are Fermions and they will themselves not permit you to do so. So therefore, you are cannot fix the molarity 0, you are force to use higher and higher temperature even though temperature is absolute 0. So, when you do this, you also recognize that in our typical system let say in the kind of system that we are talking of which is set of free electrons present in a solid. The number of electrons we may have is a large number, it may be some 10 power 23 electrons or whatever, where if you looking at 10 power 28 electrons per unit volume per metre cube or ((.)) number. It is a large number that is true, but it is not infinite, it is also finite number so, it is a large, but finite number.

So, when you do this process of filling up states steadily from a lowest energy level and proceeding to have a higher and higher energy levels. You will eventually reach one energy level which we are; which will just for the moment designate as E n some as we start from E 0, E1, E 2, E 3, E 4 we reach some higher energy level E n E subscript n where the last set of particles are available in the system, fill the state that are available in that system at E n. At energy level E n also there is a finite number of states, the last set of particles that you have available in the system, last set of Fermions we have available in the system fill that set of states. Once you have filled those set of states you have now run out of particles.

So, the system may have capability to go to even higher energy levels after E n, E n plus 1, E n plus 2, E n plus 3 energy levels may become available; may be available, may be present in the system may be defined for system, but you have now run out of particles. So, therefore, all the energy levels above E n will now remain empty. So, you have seen that if you take a set of Fermions and we start filling up energy levels, we will go from E 0 up to some number E n energy level E n where you now run out of all particles. So, initially we started E 0 you run out of states, you go to E 1 you run out of states, you go to E 2 run out of states continue this process till you reach E n where as you run out of states you also run out of particles, one important is you run out of particles.

So, any energy level above E n, there are no particles available to fill those states so, they all remain empty. So, this highest energy level that you fill which I have now described to you as E subscript n that highest energy level that you fill at temperature being equal to absolute 0 at temperature that highest energy level is now this value E f. So, E n which used general descriptive number so, that you can just follow this series that we did E 0, E 1, E 2, E 3, E 4 that is actually this value E f. So, up to the energy level E f, all particles; all the states that are available to you get filled, completely filled at 0 Kelvin. Please remember, this is at 0 Kelvin this discussion; this description I am giving you at is at 0 Kelvin.

So, at 0 Kelvin up to the energy level E f, the states get steadily filled therefore, since all of those states are completely filled the probability of occupancy of state is 1. Because if you have 50 states and you have 50 particles sitting in those states with the rule, that can be only one particle per state. It means that you have guaranteeing that every state is completely filled that is the meaning in saying that you have 50 states and have 50 particles and and the that is only way you can be consistent with that description that have 50 states 50 particles and you are permitting only one particles therefore, every state has to have one particle. Therefore, if you looking at the number of particles by number of states, you have 50 by 50 which is n i by s i and therefore, that is equal to 1.

So, you have 50 particles 50 states all of them occupied, probability of occupancy is 1. So therefore, at the lowest energy level that you can go, whatever is the number of states you have you have that many number of particles therefore, probability of occupancy is 1. You go the next so, if you mark up here E 0, E 1, E 2, E 3, E 4 and so on, for every one of them you go to E 1 if the situation is exactly the same. You have a certain number of states, you have exactly that many number of particles and therefore, probability of occupancy is 1. E 2 it is a same thing, E 3 it is a same thing, you continue all the way up to E f, up to that whatever is the number of states available at those energy levels they are all completely full.

At E f we sort of have a discontinuous change going from 1 to 0 you know it abruptly changes from 1 to 0 and then from there on states of 0. So, at E f you have used at the first time you actually run out of particles, you have certain number of states certainly run out of particles you fill those so, last energy level that you fill is then therefore, this E f. Above E f, you have states available in the system which can; which may be defined for a system that there are no further particles available to fill those states. Therefore, whatever be the number of states let say there are 38 states and this giving us some arbitrary number let say that number 38 states possible above this energy level at the first energy level that is available above this E f.

Since there are 0 particles fill those 38 states if probability of occupancy is 0 so, therefore the probability abruptly drops from value of 1 to value of 0 at temperature being equal to 0 Kelvin. So therefore, this is a very clear is very clearly visible how this function actually results in this distribution then temperature is equal to 0 Kelvin. And there also able to see the meaning of this value E f this E subscript f is the highest energy level that is occupied in a collection of Fermions at 0 Kelvin, this E subscript f is called the Fermi energy.

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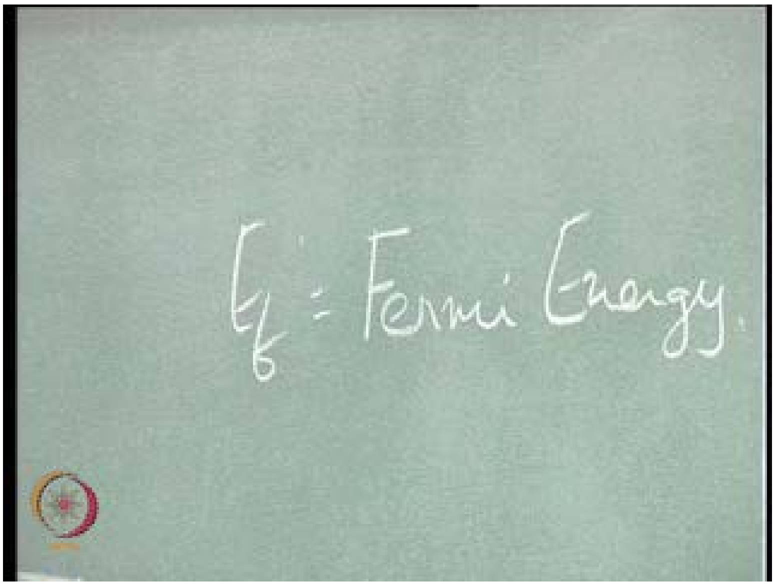

So, E subscript f is refer to as the Fermi energy, Fermi energy is defined there are actually two definitions for it, the first definition is at 0 Kelvin. At 0 Kelvin, it is the highest energies level that is occupied, the highest energy level that is occupied at 0 Kelvin is defined as the Fermi energy. So; see, even though we need to keep this in mind, one of the things we need to understand about couple more aspect of more distribution like to point out. The first is that since we have a finite number of states, we are forced used energy levels above the lowest energy level available in the system so, that is point number 1.

At the same time, still nature would like to occupy the lowest energy configuration that, that is possible. Therefore, it tries to keep filling all the states as much as possible at the lowest energy level that it is possible to fill. So, only when you reach E f and we first run out of particles, all the particles occupy only up to E f, they do not occupy any energy level above E f at 0 Kelvin. So, in the ground state of set of Fermions you find this kind of distribution, ground states being state of lowest energy that is possible to attained for that set of Fermions and the; and therefore at 0 Kelvin.

In the ground state we find this distribution that continuously it occupies all the states available to it till the; till it runs out of state and runs out of particles and therefore, actually runs out of particles more specifically. And then, it occupies as many states as particles and then finally runs out of particles and that energy value is Fermi energy value. So, the Fermi energy is therefore, defined as the highest energy level that is occupied in the ground state therefore, the highest energy level that is occupied when T is equal to absolute 0. So, the highest energy level that is occupied in a set of Fermions as the temperature is absolute 0 is then described as the Fermi energy level.

So, therefore, if you take set of electrons and you say that electrons in a solid I am going to behave like Fermions, you will find that this is true, you will find that they are going to have a set of energy levels available to them. And at absolute 0 they will fill all of the states of up to a certain high value of energy relatively high value of energy and that point you run out of electrons and that will then become the Fermi energy level. Incidentally, for a typical metal this is of the order of 2.5 electron volts so, of that order you are looking at values.

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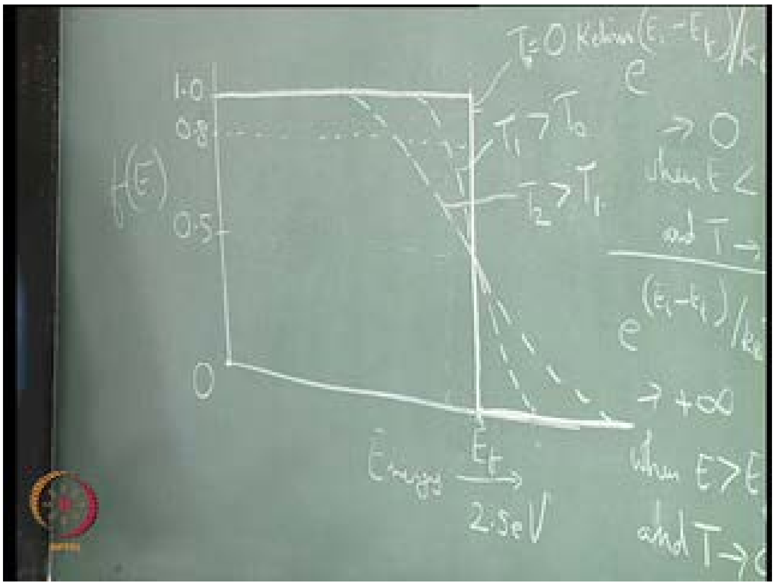

So, we will look at that little later, but that is the kind of number that we are looking at so, that is the first thing. The other aspect I want to alert you to now which we will discuss little later in greater detail, but I will simply alert alert you to this aspect right now is that, is not that this is function f of E is the probability of occupancy. It is not; it in fact does not tell you anything about a number of states available at that energy level these are two different pieces of information. Even in the description that I give you so far, I kept mentioning that may be there are 50 states at some energy level and all the 50 states are full, the next energy level may have some other number of states and so on.

The point you remember is that as you change the energy level, the number of states that exist in the system may be different. This diagram gives you if you are not careful, it gives you the misleading information or misleading idea that perhaps the number of states is the same at all the energy levels, that is, not the case. Because this is got this is not this function does not talk say anything about the number of states, it nearly tells you about the probability of occupancy of those states. More specifically, let us as realistic system might have I mean hypothetical system may have say 50 states at E 0, may have 75 states at E 1, may have 110 states at E 2, may have 150 states at E 3 and so on.

So, the number of states available at E 0, E 1, E 2, E 3, E 4 etcetera need not be the same and are typically not the same. In fact, in general system it is not going to be the same, those numbers are not going to be the same there is a certain pattern for those numbers, there is a certain function which tells us what those numbers are which will look at later. So, this plot here thus not tell you any information about the number of states available in the system at any given energy level, it only tells you probability of occupancy of those states. What is the difference? It simply says, I just said here example let say that E 0 has 50 states, probability of occupancy is 1, it means there are 50 states and all 50 state are full that is all it says.

The next energy level E 1 may have 70 states, again probability of occupancy is 1, that means 70 particles are on those states and therefore, those states are full. The next one may have 100, 100 states available to you, next energy level all 100 are full, because probability of occupancy is 1. So, you have 50, 70 and 100 particles continuously filling those 50, 70 and 100 states that are available to you. So, probability of occupancy is 1 that is the meaning of this function, it does not tell you anything about number of states. If there are 5 states and probability of occupancy is 1 it means there are 5 particles there, if there are 1500 states and probability of occupancy is 1 there are 1500 particles there.

So, at every energy level, there is an additional piece of information not captured on this plot which is the number of states available to you at that energy level, that is an information there is not been captured here. So, that is an information we will add on later and that is the more complete picture of what is there in the solid, this is only the probability of occupancy. So, you need to know the probability of occupancy and the number of states at that energy level to taken together that tells you the actual distribution of electrons in that system. So that is; so, the full information is not yet here so, that is something we should be allowed to, this is simply the probability of occupancy.

So, when you continue in this form, we know there are 50 states and 70 states and 100 states 1500 states whatever, you may eventually reach a point there are I am just point taking some arbitrary value of energy just above E f. And let say there are 3000 states available at energy level just above E f, E f plus 1 whatever, if we spoke of E 0, E 1, E 2, E n, E n is E f E n plus 1. So, the first energy level above Fermi energy level let say hypothetically that there are 3500 states, probability of occupancy 0 it means there are 0 particles sitting in those 3500 states. All those 3500 states are empty fine that is information so, that is how the actual distribution in the system is going to be.

So, the number of states available at each energy level is a separate piece of information which we have not incorporated in to this plot, you only incorporated regardless of the number of states available at that energy level, what is the probability of occupancy of those states? Is it 50 percent? Is it 100 percent? Is it 90 percent? That is the information that is available here. We will do the complete picture little later, because we need to do some more calculations to get them, but this is the very important piece of the overall picture so, that is by we are focussed on it. So, this is what it is.

So, we are look very carefully at what happens at 0 Kelvin. At higher temperature what you see is that, you see that certain number of states very closed to the Fermi energy level, the probability of occupancy at decreases from 1 all right. And certain number of states which are at energy level above E f actually have a probability of occupancy greater than 0 compare to the what you see at the 0 Kelvin. So, if you actually make a plot of these slightly higher temperatures and then you do this look at E 1; E minus E f and then you do this calculation. When; obviously, when E is very small, this will still follow the same kind of behaviour that we described as E gets very closed to E f on either side of E f, it shows you slightly different behaviour, because this T is not exactly 0 T is actually finitely above 0 Kelvin.

And then so, this is very small number, but it is a nonzero number so, when you have a small number here and you have a small number here, this is not going to evaluate to either plus infinity or minus infinity. So, you are going to have some finite numbers there, because you are going to have some finite numbers here, this is also going to have numbers between 0 and 1 that is how this function going to change. And as the temperature changes to little higher and higher values that effect of that range of energy values over which we see this gradual transformation from 1 to 0 is going to increase. So, you see that at small temperature just above T 0, you see that the energy values over which this goes from 0 to 1 is actually here so, this energy to this energy. So, in this range, the values are going from 1 to 0 for example here.

If we go to the slightly higher temperature, it goes from 1 to 0 this range of energy values so, in this slightly larger range values the transformation goes from 1 to 0. So, what it means is that, if you compare with respect to E f at values of energy which are less than E f you have a probability of occupancy that is little less than 1. And at values of energy greater than E f, you have probability of occupancy greater than 0. So, this is the kind of distribution that you see for coming out of the Fermi-Dirac statistics as a function of temperature now. So, as a function of we have seen as a function of energy, these different curve for function of different temperature so, that is what we are seeing.

Again, even in these additional cases that I spoke of which is at T 1 and T 2, this picture is not telling you anything about the number of states. I will just arbitrarily state that you know let say this is let say this is 0.8. So, all it says that, at this energy level the probability of occupancy is 0.8. So, if there are 100 states available at that energy level there are 80 particles sitting those 100 states, that is basically all the information that is captured in this plot. So, now, I said that at temperature equal to 0 Kelvin, the highest energy levels state that is occupied is the Fermi energy level.

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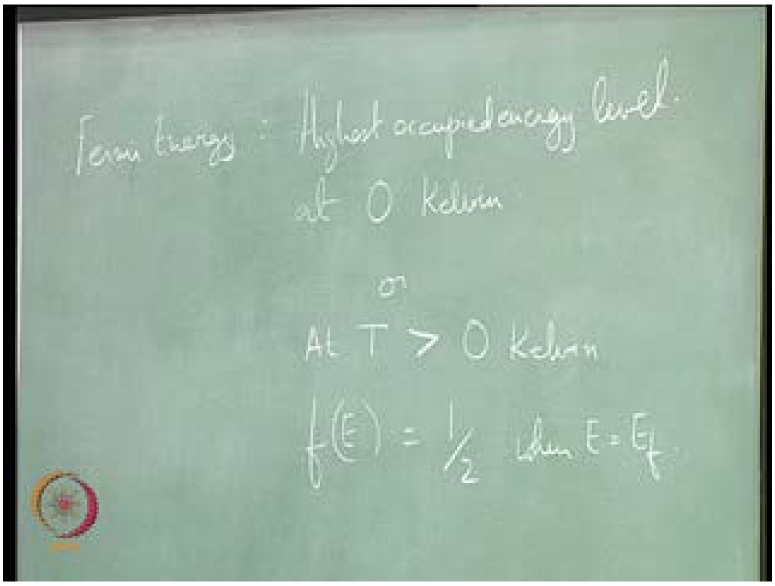

So, Fermi energy we will define.

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If the highest occupied energy level at 0 Kelvin and at the same time or the other description at temperature greater than 0 Kelvin at any temperature greater than 0 Kelvin then energy is equal to E f, E equals E f. If we look at this plot here when T is not equal to 0 and E equals E f, if E equals E f this E minus E f is going to be 0. E minus E f is 0 divided by nonzero quantity so, that is still 0. So, e power 0 is 1 therefore, this f of E will evaluate to half so, 1 by 1 plus 1, this term will become 1. E equals E f this becomes 0 so, this whole thing remain 0, therefore this term becomes 1. So, this f of E will become 1 by 1 plus 1 which is 1 by 2 so, it is half.

So, at any temperature greater than 0 Kelvin, the f of E will equal half when E equals E f so, these are the two definitions for the Fermi energy. Fermi energy can be defined at 0 Kelvin as the highest occupied energy level at 0 Kelvin; it can be also defined for any other temperature as the energy level where the probability of occupancy is half. So, in fact, I have sort of deliberately drawn it that way without highlighting it so far, now we will highlight it. If you take we have written f of E as 0, 0.5, 1 those are three points are highlighted I just put in the 0.8 here.

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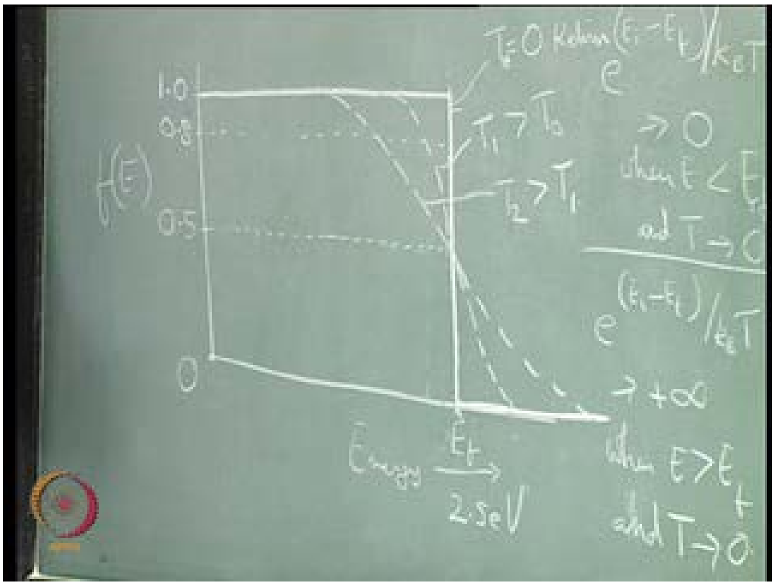

So, if you just draw this 0.5 line at all temperatures greater than 0 Kelvin, you will find that as the change as the probability of occupancy changes it will go through 0.5, it go through 0.5 when E equal to E f. So, at T 1 also it goes through 0.5 when E equal to E f, at T 2 also it goes through 0.5 when E equals E f as you go through all the energy levels available in the system. So, this is the manner in which this process is occurring so, that is how the Fermi Dirac distribution is present within this its manufacturing itself that you have all this energy levels as you change the energy level you have this behaviour.

The Fermi energy is simply the highest energy level that is available in the system at 0 Kelvin and it is the energy level where the probability of occupancy is 0.5 at any other higher temperature so, that is the thing. And also as I mentioned again I will once again mentioned that this diagram does not tell you anything about the number of states available in the system. Now, which is the other piece of information that we will look at later, what I also want you to understand from this picture is that when you have a set of Fermions and you fill them up. The kind of physical analogy that you can think of is very simply that you have some kind of container in which you are filling items of some category.

Let us just say you have a container and you putting sand in it or water in it whatever sand or water whatever it is that putting you pouring into it. The container has only finite space at lowest energy or the lowest height that you have in the container. So, as you pour water or sand into it, it fills up from bottom and moves upwards till you reach the top of; till you run out of sand or water. Assuming that the container is a very large long container which in which we can put how much ever which is larger than the amount of sand or water that you have. So, we start pouring it in, it will start filling up the bottom and moving upwards, then eventually you run out of sand or water.

Therefore, there is a highest height and therefore highest energy level for this sand or water that you pour inside that container, this is the exactly to the system that we are looking at. We have energy states available in the system from lower energy state to higher and higher energy states; we start pouring electrons so to speak to in a descriptive sense pouring electrons into those states. They start filling up from lowest energy level that is available to higher and higher energy levels eventually you run out of electrons. So, there is a certain number of the certain highest energy level states that is available to you and upon till that state the all the possible upon till that energy level all the possible states that are available are completely full are completely full.

And therefore, the probability of occupancy is one above that energy level for above that height as long as the system is undisturbed in stationary, you do not have any further states that are occupied and therefore the probability of occupancy is 0. So, that energy that I described you captures this information that I have just shown to you as the Fermi-Dirac distribution. The information that is not captured in this picture which I kept talking of in terms of the number of states how to speak is to if look at the analogy that I just described of some container in which filling this water or sand, the information that is not captured that in the shape of that container.

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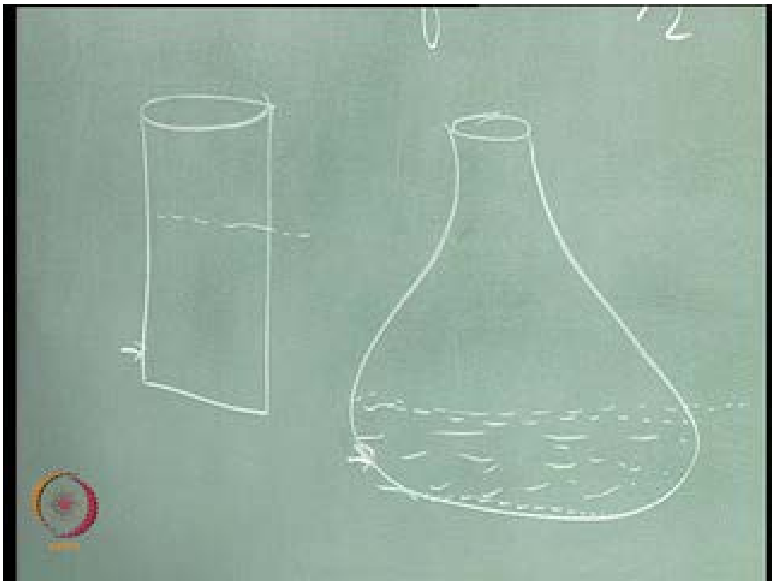

So, I could have a container that looks like this, I could also have a container looks like a part. (No audio from: 42:22 to 42:28)In this container the number of, the width of this container is constant. So, to in terms of my energy if I am pouring sand or water into this, the amount of sand that is held at the lowest energy level is the same amount of sand that is held of next higher energy level or next highest height, the next highest height and so on. The amount of sand that I am holding that all these heights is exactly same. So, I am tried to run out of sand somewhere here in this case right.

So, the number of sand particles if the sand particles were all very uniform and the way they were sitting are uniform, if all that with true. The amount of sand that I could hold in this lowest height would be the same, as the amount of sand would hold at any other arbiter height above it, at that given height, at given location. That is not true in the container. This container has different shapes, it is still filling up container the logic behind filling up the container is still the same, whatever initially I am pour some sand into it or water into it. If there is only so much sand, it will just fill up lowest available height in that container and you will just fill later and it is still there.

So, again the probability of occupancy of the locations below that height is 1, because the sand is sitting fully filled up to that point, above it there is no sand available it is all 0. But you can fill more and more sand let say, I will take same amount of sand fill it up, it may come up to this height in this container. Sand or water, whatever it is that you have filled here, will now come up to this height in this container. So therefore, the probability of occupancy this container is full up to this point. So, whatever space is available is all occupied by the sand or water. So, there is no empty states, we assume that sand sand particles are able to pack themselves up very well, there is no gape between them this a hypothetical case.

So, may be liquid is better analogy we can more easily visualize no space being there between the liquid molecules, at is from our perception of the liquid. So, we adjust filling up all those location. So, it is fully occupied fully filled in position below this highest height level and fully empty above this height level and this highest energy. So, there is a corresponding highest height here or energy level that we can talk off. So, the Fermi energy that we described is the; is the analogues behaviour in a set of Fermions or electrons, which are trying to fill instead of container, that energy states available in that system. In that kind of situation this is exactly what is happening. So, you fill up all the energy level up to this point.

And in simply we talking of the f of E distribution all it says is that f of E is 1, up to this point and it 0 beyond it, if you want look at the energy. Similarly, f of E is 1 up to this point, it is 0 above this. So, in both these cases, the general idea is still the same that the probability of occupancy is 1 up to certain point, and the probability of occupancy drops to 0 above that certain point. What significantly defers, even though the number of let us assume, I put this same amount of water in both these containers. What is significantly different about these two pictures is that since the shape is different for these two containers.

And therefore, the number of states so, to speak or the number of location in which you can place water at the lowest energy level is only so much here, for as it is much higher in this other container here. Since, these two have different shapes, the amount of water or amount of sand that you can fill at given energy level or given height is very different in two cases. Therefore, even though the probability of occupancy is the same at some height level. So, same height level I take here, with respect to the ground position.

So, if I take the same height level, in both cases the probability of occupancy is 1, in given the height levels that I have chosen, probability of occupancy is 1. That the shape of these two containers is different therefore, the actual occupancy which consists of the probability of occupancy, plus the number of states that is available inside this system. That actual occupancy is different, for these two states. So, that is very different piece of information.

So, and therefore, where it fills up, where it runs out of particles is very different for this condition. So therefore, keep in mind that the shape of the container is also very useful, is very important piece of the information that we need. And in with respect to the system that we are talking of, that is the number of states available at each energy level. That is very important piece of information that we have not included in the discussion. So, that is something that we will get to later.

But still the definition for Fermi energy is something that is, that we are now clear about. It is the highest energy state that is occupied at temperature being equal to 0 Kelvin and at any other temperature it is the energy state where it is value of energy at which the probability of occupancy is 0.5. So, these these are the two definition that go with the Fermi-energy and and something that will become familiar with. The Fermi energy is of important, because it represents the highest energy level available in the system, which is occupied by electrons at 0 Kelvin or probability being half at any higher temperature. And therefore, in some ways is the energy is the energy value around which those electrons are now available to interact with higher energy levels.

So, so for example, we will see in our next class, we will discuss in little greater detail. As you raise the temperature as you raise the temperature, you see that only it still remains probability of occupancy it still remains closed to one or very closed to one. All the way to some energy level, that is closed to the Fermi-energy level. Only at energy values which are very closed to Fermi-energy level, you start seen probability of occupancy the drops from 1, for values below E equal to E f and goes up from 0 for values above of energy above E equal to E f.

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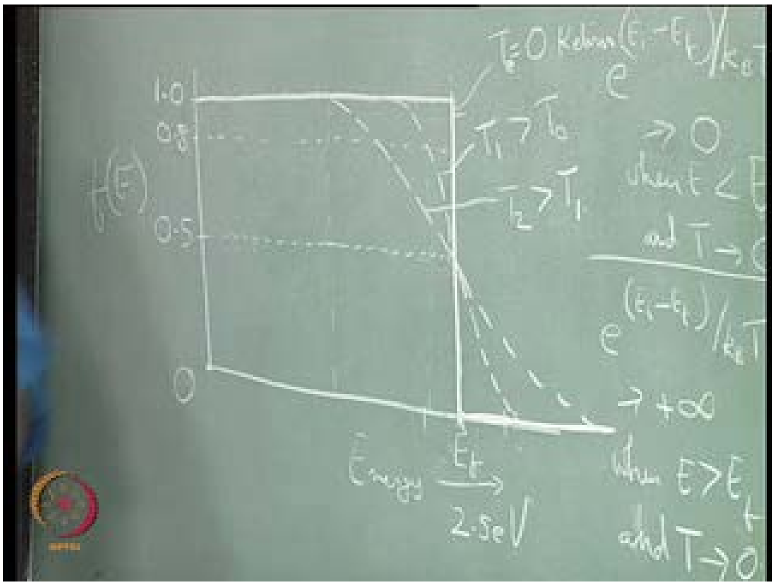

So in fact, when you raise the temperature, it is energy values which are closed to E f that sort of participate in the process of the change change in temperature. And that is the very important information that we will look at. Not all energy levels seem to participate in the process of the gain and temperature, temperature of system as gone up from T 0 of 0 Kelvin to some other temperature T 1, which is greater than 0 Kelvin. So, the temperature of the system is gone up, but if we look at this figure, the way I have drawn this figure, which is consistent with this function here.

So, if you take this function and you plot it up in any software that you that you are comfortable with, you will find this is true, this is kind of picture that I am drawn is essentially correct. So, what we find is only energy values which are closure to the Fermi energy level, seen to be participating in the process of change in temperature. The greater the change in temperature, more the number of energy states on either side of E f which seen in to participate in process.

So, that is the very important piece of information, more specifically there are large number of states which are away from a value of E equal E f, which do not in any manner any significant manner participate in this process of change in temperature. So, they all the energy values which are below this for example, at this at T equal to T 2, the probability of occupancy is still 1. And as long as probability of occupancy is still 1, they are not actually participating in this change in temperature. So, they are largely oblivious to the change in temperature change in temperature for the entire system. So, that is how this behaviour is so, most of these energy values are not participating in it.

Similarly, here it is at all values below this energy level, are not participating in change in to it. So, for set of Fermions that is very important characteristics of a set of Fermions. The set of Fermions, collection of Fermions, if you have and which is what we are talking of as the collection of electrons present in the solid and if you take that collection of Fermions. Given the characteristics of Fermions they come up with they are following all that things that we have just described here and therefore, this diagram. And therefore, this diagram represents what behaviour they are demonstrating to us.

And when you have such a diagram, it is telling as those Fermions show all this kind of behaviour, that not all of them participate in a energy change process, only some of them participating. So, so this is Fermi-Dirac statistical distribution, we have derived it last couple of classes, today we have looked at various features of the Fermi-Dirac statistical distribution. We have looked at Fermi energy, we have looked at plot of Fermi-Dirac distribution, we have try to see what that plot implies as a function of temperature, what the plot implies as a function of energy, what is the significance of the highest energy level that is occupied.

We have designated that as Fermi energy, we have briefly considered what is said that I mean, we have designated something as Fermi energy so what. I mean, why I said of any significant, we said that we know it is the border line between what is already fully full and what is empty. And therefore, that represents the border over which across which all the transaction are occurring so, to speak. So therefore, therefore, it is of some significance, we will discuss in the next class what is the; although I have described to you here in this plot that what is happening across that Fermi energy level as you change temperature.

What is the implication of it, we will see in our next class, but that is the characteristic of it. So all the major features of the Fermi-Dirac distribution, we have looked at and we have examined. I have also highlighted to you the information that is not available in this Fermi-Dirac distribution, which is the actually the actual number of states at each of those energy levels, which is corresponding to the shape of the container, if you are looking at an analogy of pouring water into it container or sand into a container. So, this then is the summering of what we have discuss this class.

Next class we will take up, we will we have done all this discussion, because we want to make in an improvement to the Drude model. And hence came up and decided and look at Drude-Sommerfeld model which came up as the next improvement on the Drude model. Which use the Fermi-Dirac distribution and therefore, we have looked at this. So, in the next class we will look we will directly compare the Maxwell Boltzmann distribution, which was used by the Drude model with the Fermi-Dirac distribution which has been used by the Sommerfeld model. And in this comparison we will understand in what way Sommerfeld model has actually improved on the Drude model. And from there, we will look at further possibilities of improvement in the other features of the system. So, that will halt for today. Thank you.;