Physics of Materials

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Lecture No. # 18

Fermi-Dirac Statistics: Part 2

Hello, welcome to the 18th class in the physics of material course. In the last class we look at the Fermi-Dirac statistics and we started deriving the expression for Fermi-Dirac distribution. So, we progress to some extend, we will continue from that today and complete this derivation and try to understand what this derivation implies so this is what we will do. So, edges put down in words what we have done the I put down inverse the major expression that we have; we arrive that from where we actually going to narrow down the solutions that we are looking for.

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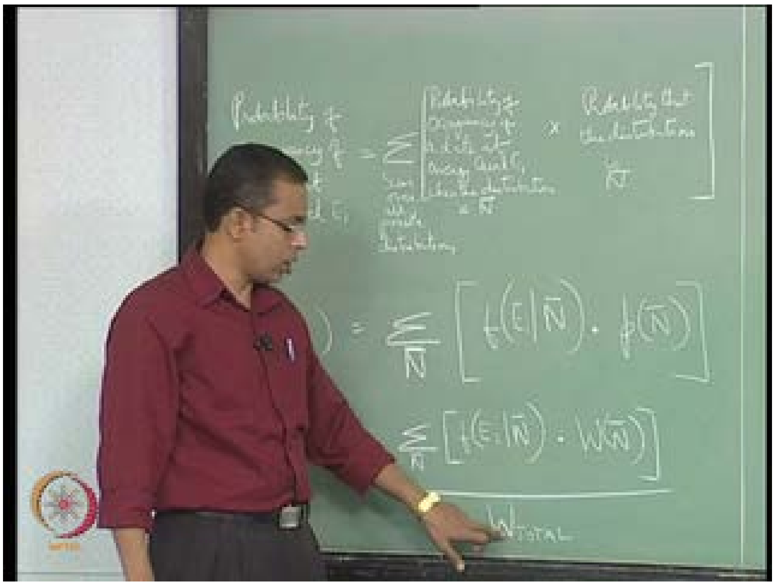

So, this is the expression that we got in words that comes like this (No Audio From: 00:56 to 01:03). Probability of occupancy of a state at energy level E 1 is equal to, we will get sum so sum over all possible distributions of (No Audio From: 01:46 to 01:55) probability of occupancy of a state at energy level E 1 when the distribution is (No Audio From: 02:19 to 02:29) is certain distribution that we are falling as N bar into.

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Alright. So, this is the general expression that we derive, I will just put this down in the expression form that we just that equation form just moment. So, we just what it means probability of occupancy of a state at energy level E 1, that is the information where looking for, that is the Fermi-Dirac distribution expression ok this is what looking for.

What we have recognize? Is that given a bunch of energy levels number of state that exist at this energy levels, there are various distribution which can be used to distribute the electrons across all of those energy levels. We say there is a; we can pick them each of those distribution one by one so, at given instance, we have pick the particular distribution N bar out of all possible ways which we can do this, we have pick one particular way we can do it. We see, what is the probability that it is that distribution so that is the first piece of information we need to know. What is the probability, that it is distribution we have closure given there are so many possible distribution, what is the probability that it is that particular distribution that we have currently closure solution.

If you know that probability, then within that distribution what is the probability of occupancy of a state at that energy level E 1 when the distribution is that N bar. So, this is therefore, these two together then becomes the probability that it is occupied that energy level at E i is occupied, given that the distribution is N bar finds the probability that distribution is N bar. If a sum (( )) over all possible distribution right so, this is what kind of, this can change and this can change so for all possible values of N bar, you calculate also, what is the probability of occupancy of that state at energy level E 1, given that it is that particular N bar. You summit over all possible distribution that we can possible we have, when we get the over all probability occupancy of state at energy level E 1. So, this is the overall piece of information that we have looking for so, we will put it down in the equation form that we did

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So, this is all it is, this is the; this is the probability that I mention, this is the probability that particular distribution actually exist, that that we have actually arrived at distribution so that is the probability. In this distribution what is the probability that state is occupied, so you companied two of them and sum them over all distribution you get the general probability that the; that state at energy level E i is occupied. So, will mark this E i so that is clear this particular energy level. So, this term here is also refer to in the last class I mention, this is also refer to as the conditional probability, that state at energy level E i is occupied. And when we say conditional the condition that we are imposing is that instance we overall distribution is also N bar, here that condition is not there, there is no condition and what the distribution is, because we have sum overall distribution.

Since, your sum in overall distribution there is no further condition placed here, whereas here within this bracket there is a condition that it is a particular distribution and where multiplying by their probability that it is that distribution. So, there two probability is being multiplied here, a probability that a particular distribution is arrived and within that the probability that in that distribution what is the probability that particular energy level occupied. So, we also wrote this in; by recognizing what the term represent, we wrote at like this.

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So, p of N bar which is the probability that we have arrived at particular distribution or the probability that we do have the distribution. Simply number of a's in which we can arrived at distribution divided by the total number of ways in which can arrived at all possible distribution. So, that is by definition of what a probability that is what is it. The number of ways which we can do in something by the total number of ways which we can do everything is the probability that you can do that something right. So, this is the number of ways in which you can arrive at a particular distribution divided by the total number of ways in which you can arrived all possible distribution. So, this is; the denominator is actually a sum across all the possible; all such terms over all other distribution. So, we arrived at this term, then I also indicated the fact that we are using the approximation that is typically used for such statistical distribution which is the statistical mechanics approximation which basically says that, there is one particular distribution N bar, let us say N bar not.

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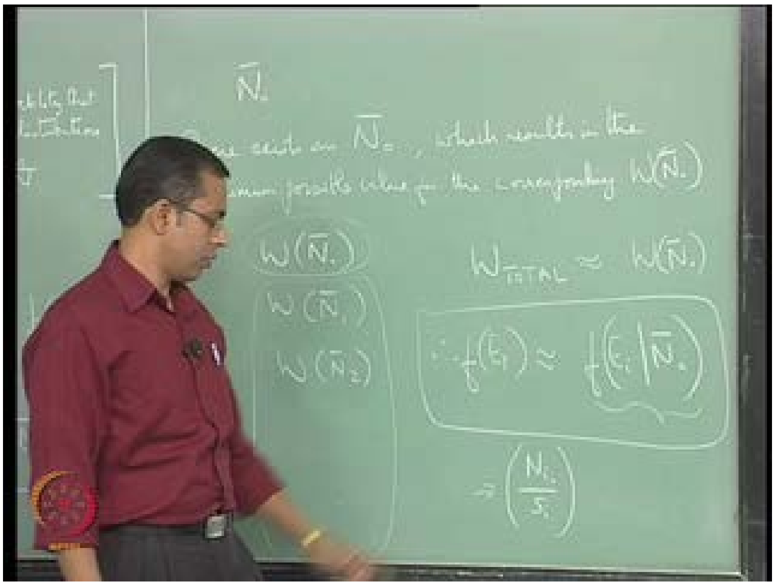

So, that some particular distribution N bar not. So, there exist an N bar not, which results in the maximum possible value (No Audio From: 08:39 to 08:47) for the corresponding (No Audio From: 08:49 to 08:58). So, we are assuming that there exist an N bar not which results in the maximum possible value for the corresponding W or omega N bar not, if you capital omega N bar not, capital W N bar not if you want, call it. Which is where, we are essentially saying that there is a particular distribution, where if you actually figure out the number of ways in which that distribution can be arrived at. So, if you just look at the number of ways, we can arrived at distribution that value will be a huge number relative, even the very next number that you can arrived for the very next distribution.

So, in other words should take all possible distribution, you arranging them decreasing order of occurrence. So, maximum, so this is the maximum occurring distribution, so this is on top W N bar not, so that say the next most occurring distribution is N bar 1. So, you will have just for example, let say this is W N bar not and let say next this occurs the maximum number of times W N bar 1, W N bar 2 and so on. Where N bar 1, N bar 2, n bar 0, N bar 1, N bar 2, N bar 3 and so on or different possible distribution so for each distribution we have done the math and figure out how many ways we have arrived at distribution. So, we find that and we arise that mean decreasing order.

So, therefore, maximum one is sitting write down top, which is W N bar not and what we are and we find that. So, there exist certain N bar not which results in maximum possible value for the corresponding W N bar not. Statistical mechanics operates on the principle that this number is much much larger than W N bar 1, it is much much larger than W N bar 2, it is much much larger than all the terms below it. Because after all there in decreasing order so if it is significant larger than W N bar 1 it is that therefore, significantly larger than all the other terms also. But more than that it is significantly larger than the sum of all the other terms, it is not just significantly larger than the next term, it is significantly larger than all of these terms together, taken together.

So, therefore, when you calculate W total which is the term here, in denominator here W total, W total is in the denominator, that is, the sum of W N bar 0, W N bar 1, W N bar 2, W N bar 3 like that sum all of them, if you sum all of them you arrived at W total. But I am also saying we are recognizing the fact, that in all those distribution that are available to you and the number of ways which all of them be calculated. We are finding that W N bar 0 is significantly larger than all the other terms and it is significantly larger than the sum of all the other terms. Therefore, when you take W bar total since, it is all of these terms together and we find all of this are smaller than this, it is in fact almost equal to W N bar 0. So, that is the very important approximation where making, an approximation that which basically says that the W N bar 0 is so large and we did this.

You know very specific example of two particles, three particles and four particles, we found that progressively the more, the most probability distribution is occurring more and more is becoming more and more probable, then all the other distribution combined or I think certainly second second most probable and so on. So, if you look at it, when you go to large systems not just to four particles, when you go to ten part, twenty three particles this approximation begins we look very good. Where you have the most probable distribution or the distribution that occurring at most number of times being more or less equal to the entire total of all the distribution together.

So, therefore, what will say is that our entire calculation in our approach is based on this, is based on this idea which is reasonable for the system that we were dealing with. So, it is not just idea arbiterly with one, it is reasonable idea for the system were dealing with. So, where assigned there exist N bar 0 which results in the maximum possible value for the corresponding W N bar 0 and which creates the situation that the W total is almost equal to W N bar 0. Because it is not just marginal maximum it is maximum by long sketch, it is much much larger than everything else combined. So, therefore, if you go back to our original expression, if this is W N bar 0, then this term by this term essentially equal to 1 more or less, because their work more or less same. So, therefore, if you are able to identify this maximum distribution we have the following situation therefore.

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So, we find that by original had, if you go back are original think, we have several terms in this in this equation, because we do not know how many distribution we have? We have huge number of distribution possible. And therefore, this had several terms here, but we found, when we reach a maximum which is, it is not just arbiterly N bar, but it is N bar subscript 0, when you do that, the term here is almost equal to at the term here denominator so therefore, this reduces to 1 or can be approximated to 1.

Therefore, now the contribution of this term to this f E i is almost entirely 100 percent. So, therefore, we find that are probability of occupancy of a state at energy level E i which is the general probability of occupancy of that state at energy level E i. Is the conditional probability that that state at energy level E i is occupied subject to the condition that we have reach the distribution that is the maximum possible distribution in that system of most probable distribution system right. Therefore, we reach this and with this approximation we realize that it is enough if we focus on this term, in fact more than this and therefore, this is what we are really interested in.

So, in fact all we are looking for is N i by S i at 0, S i is anyway fixed so, N i 0 by S i in fact begin it call it N i by S i, N i 0 by S i 0. Where N i is number of particles, where N i 0 is number of particles at energy level E i under the condition that we distribution is N bar 0. And therefore, when you get N i 0 by S i which is the number of state at energy level E i, here you already got this term here and it also terms out to be this term here. Therefore, all the mathematical approach that we are doing is simply to get us this term and once, we reach here, we have got the answer here looking for right, this is what we will do.

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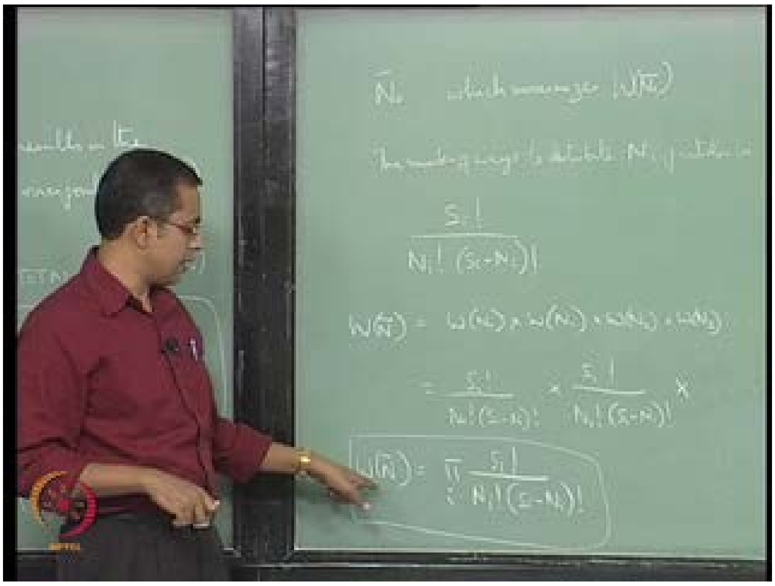

So, we also; so we now, said that we have now interested in a looking for the distribution N bar 0 which maximizes W N bar. So, to do that we start out with the number of ways to distribute N i particles in S i states. So, for a given distribution we say that there is a N i particles sitting S i state, how many ways can you make, how many ways you do this arrangement? It is simply S i factorial, S i factorial by N i factorial minus in times S i minus N i factorial. So, this is the same as same you have N i particles and which are all of the same kind and you have S i minus N i empty spaces which are all of this same kind so empty states. So, how many ways you can arranging? This is the number of ways you can arrange that mathematically right.

So, therefore, if we look at the number of ways which we can attained particular distribution N bar, it is the number of ways in which you can arrange.

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The number of ways in which can create I mean distribute N 0 particles in S 0 states at energy level E 0, times the number of ways which we can distribute N 1 particles is S 1 states at energy level E 1 times N 2 particles in energy level E 2 in S 2 states and so on. Each of this terms will look like this S i by S i factorial by N i factorial times S i minus so just write it down we will put it as S 0 factorial by N 0 factorial times S 0 minus N 0 factorial times S 1 factorial by N 1 factorial times S 1 minus N 1 factorial so you have such terms. So, this is what; this is the total number of ways in which we can arrived at a particular distribution N bar, because in each of those state we can do all of this arrangements.

The states a minute you say N bar that means also you fixed values of N 0 N 1 N 2 N 3 N 4 and so on already as I mention for this kind of system we were dealing with the E i in other words E 0 E 1 E 2 E 3 etcetera or already fixed for the system. S i are also fixed which is number of states in E 2 as energy level S 0 S 1 S 2 etcetera, only the number of particles are possible in each of this states it is a variable through the entire problem. But when you fixed particular N bar even that becomes, for that particular distribution the values of N 0 N 1 N 2 N 3 and so on or also fixed. Once, all of these three of fixed you can make a calculation for these terms, so like this you can make a calculation therefore you come up this.

So, therefore, actually if you want write it in a simplified notation, we simply write W N bar is pi over i which we are; in just way you do sum sigma for sum use pi for a product S i factorial right N i factorial times S i minus N i factorial so, this is the term we have. Now, our purposes given that this we have this expression, we would like to understand what can we do to maximize this expression. Because that is all problem is once, you maximize expression we have got N bar 0. So, our intension is to get N bar 0 therefore, you just find way to maximize expression. And it terms out that the form of W N bar is such that if you maximize W N bar it is the same as maximize; maximizing lon W N bar both those functions behaves in essentially the same way they were all monotonically increase how to speak. So, therefore, W N bar if we can maximize it is the same as maximizing lon W N bar. So, and mathematically if easier to handle lon W N bar therefore, we just do that lon W N bar.

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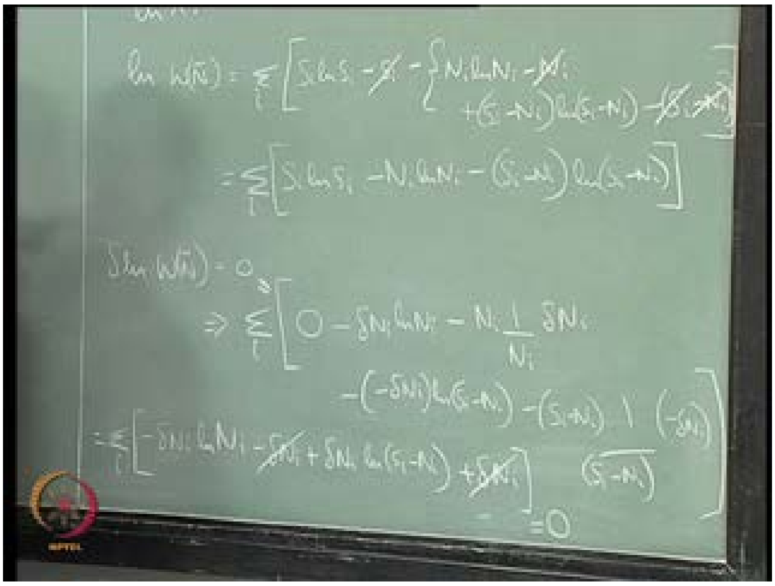

So, therefore, we write lon W N bar, we write the expression for it and that is simply what is the product will now become a sum, sigma over i lon S i factorial minus lon N i factorial minus lon S i minus N i. So, this is the; if you write take a natural logarithmic logarithm of W N bar then this is what you get right. So, we also recognize sterling approximation i am sterling approximation for factorial is simply that lon X factorial is X lon X minus X, we did that same kind of approach for Maxwell Boltzmann statistics as we do this Fermi-Dirac statistics, mathematically similar step were involve. So, you can always look at our derivation for the other distribution and see how they compared. And we will find therefore, that we can write lon W N bar equals sigma over i S i lon S i minus S i minus N i lon N i minus N i plus S i minus N i lon S i minus N i minus S i minus N i. So, this is what we will get.

Now, if you look at this, you have a minus S i term here there is a minus here and minus here so therefore, this becomes a plus S i right so therefore, this will cancel out with this term here, you have minus N i here this minus sign here minus minus will become plus so, minus N i and plus N i so, this two will cancel out. So, therefore, this is simply sum over i S i lon S i minus will remove this bracket minus N i lon N i minus S i minus N i lon S i minus N i so, this is what we have. Now, I also what to understand that in our system in the derivation the state the stage that we are in this derivation at this moment S i is also; S i is already fixed for this system as is E i that we have no choice over, but we are in the process of maximizing finding the condition for maximizing N bar.

So, in that processing what we are doing? We have actually trying out different possible combination, we are trying out you know taking some N particles we have at given energy level, we take few particles from their we move it some other energy level, then we again calculate this capital W N bar, we want to see whether we have gone up in quantity our gone down quantity so. So, the N i values, the values for N i that we have here or actually not a constant, we applying are that value to figure out what is the maximum that we can reach for this term here right. So, therefore, this is the variable that we have in our system N i, S i is not a variable it is a constant.

Now, assuming you do reach a maximum so, when you reach a maximum the one of the ways in which we describe a maximum is that if reach a maximum marginal changes in the quantity will still keep you that at the same value. So, in other words D lon W N bar by del D N N i will reach 0, D lon so, when this function we reach as maximum with this variable D lon W N bar by D N i should be 0 for another words the other way of writing it is we write del lawn W N bar equal 0 implies sum over i. Now, we have to do differential of this term with respect to N i, the first is a constant so, differential of this term with respect N i 0.

So, if you want clarify just write 0 here for the first term, the second term there are two places where N i R i so therefore, will write minus N i lon N i and then minus N i into 1 by N i del N i. So, differential of first term times second term and the differential of second term with this here times first term which is here. Similarly, minus you will have minus del N i lon S i minus N i and minus S i minus N i into 1 by S i minus N i times minus del N i this is same thing I have done. So, this first term comes to 0, this can this becomes two term, because you can differentiate the first term being second one constant or differentiating the second term being first one constant so, these two term arrive, same think you can do with a last term.

So, minus sign still here so minus del N i lon N i lon S i minus N i and minus S i minus N i by 1 by S i minus N i minus del N i so, this is the terms that these are terms that we have. So, we will just simplified here so, this is equal to so, this is equal to 0 so this is also equal to 0 so both of them there all equal to 0. So, if we write this simply mathematically simplify this, we will have minus del N i so, this is sum here, sum over i minus del N i lon N i this is going to simply minus del N i. This term here will also remain so, this is plus minus and minus become plus minus del N i sorry plus del N i lon

S i minus N i, these two will cancel minus and minus will cancel so, will have plus del N i and this is equal to 0 equal to zero so, this is what we have. So, therefore, this minus sign with minus del N i and plus del N i with cancel.

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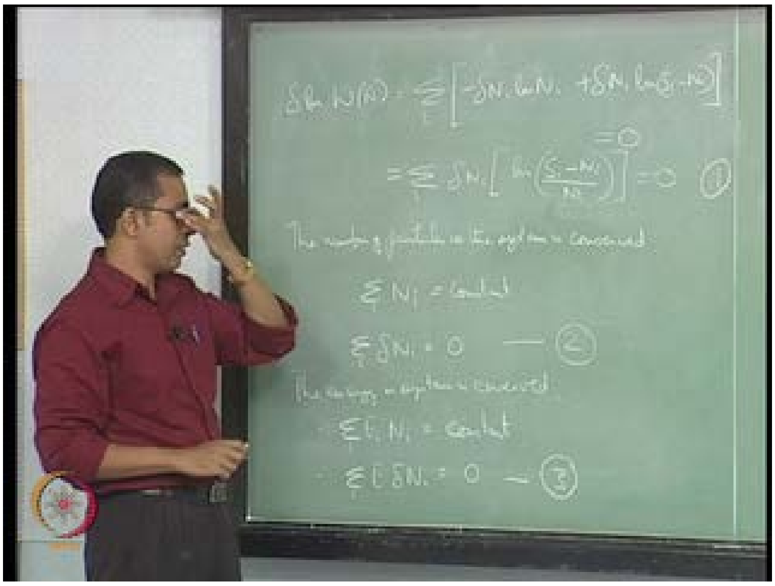

So, therefore, we now have a simple.

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So, if we look at what we have this put down, this is what we have got del lon W N bar is simply the sum over i of minus del N i lon N i plus del N i lon S i minus N i. So, this is the and this is equal to 0, because we have reach the maximum or were looking for condition for maximum and when the maximum is reach this is going to be 0. Or to simplify further we simply write it as sum over i del N i of lon S i minus N i by N i since, this is minus lon N i the N i go to the denominator so therefore, we will end of with this. So, under the condition of maximum this sum equal to 0 so we have one equation here, equation 1, I also said that you know the number of particles the number of particles is conserved.

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Therefore, the sum of all the particles equals of constant right so sum of all the particles available the system is a constant. Therefore, any changes in this number of particles should be total sum of all changes that we can make this particles, you add some particles at one location, you remove some particles another location, that total. The sum of all the changes that you make has to work out 0 right, because you remove five particles means those five particles have to go somewhere. So, if you put minus five particles some place, you have to put plus two and plus three somewhere else so that some works out 0 otherwise will removing particles from the system. So, therefore, that has work out 0 therefore, sigma over i del N i equal to 0 and you can see also equation wise if sum is 0 I mean this is constant if you differentiate it going to be 0.

Similarly, the the energy in the system is conserved (No Audio From: 31:50 to 32:00). Therefore, sigma over i E i N i equal 0, the energy; states are anyway fixed so E i is already a constant you do not have choice in it only N i can change. So, once again if the energy is conserved conserved, if you even shift particles between the various energy levels. The total change in energy that you creating, if you creating adding some energy at one location you have to remove exactly the same amount of energy from bunch of other location to be one or more other location so that process of adding energy to one location was removing energy from other location should reaches 0.

So, therefore, since i am sorry, this is not zero this is constant this is not zero this is constant just way this is constant here therefore, the changes in it has to be 0 so. So, change the sum of the change has to be 0, the actual sum has be constant it not be does not have to be0 it has just to be constant. So, this is what we have so, if we can call this conservation of number of particles has equation 2 and conservation of energy as equation 3. So, we have three equation here, the first one is the process by which we try to the equation that we arrived at by trying to maximize the number of arrangements that you can have in a particular. The number of ways which we can arrived at particular arrangement N bar in that process we arrived at one equation, the conservation of number of particles in system gives us equation 2 and the conservation of energy in the system gives us equation 3 all of this equate to 0 right.

Now, having got this these three equations, we will use the method that we did when we also try to derive the Maxwell Boltzmann statistics which is the Lagrange method of undetermixed multipliers.

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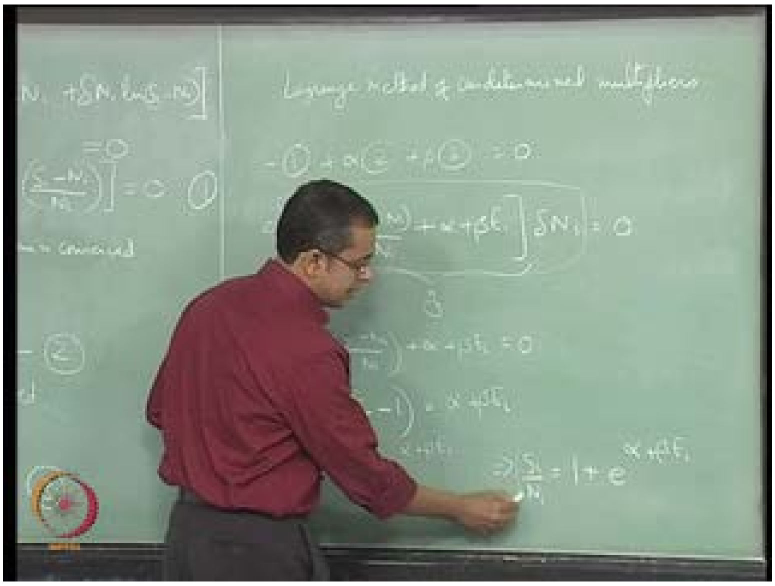

So, we have to use the lagrange method of undetermixed multipliers.

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The point is we are trying to maximize particular quantity in this case lon omega bar sorry lon N bar lon of W N bar, we are trying to maximize this. Subject to this two this constraint, that is the basic problem that we have to put it in words, we have to maximize this expression lon W N bar which means we work out which implies that we have this expression should workout be correct right. The process of maximizing this implies this equation has to whole true, but it does not have the arbiterly whole true or it cannot arbiterly whole true, it has to whole true subject to this constraint, two additional constraint apply on system which are two other equation which are valid for the system.

And I mentioned last time we discussed lagrange method of undeter mixed multipliers that basic idea is it is like same you are trying to walk along a hill and trying to find maximum point on that hill. So, you have a lot of uneven surface going up and down and so on, you wish to find the maximum point on the hill. But our problem is not simply to find the maximum point on the hill if it where only to find the maximum point on the hill, only the first equation would whole that is in other words it is necessary only to solve the first equation. But additionally we are saying it is not just to find the maximum point on the hill, but your subject to the constraint that you have to walk along a path. So, your walking along a particular path and that is a kind of constraint we are creating we are putting in additional equations.

We are putting in additional equation, we have to seeing we have to say along particular path which could be in some arbiterly path we were saying. So, some arbiterly path moving alone in that path what is the maximum that you are reaching that hill? Clearly that need not be highest point that hill right. So, you may you may walk cross in such a way that half the height of the hill is highest point that your reaching along that path, along that I think trail go to speak. So, the maximum point that you reach when you travel along a particular path need not be the maximum; absolute maximum point of that hill, if it is some point subject to this some maximum subject to this constraints. So, when you do so; that is a kind of problem lagrange method undeterminant multipliers try to solve, it simply says that for you to we able to solve this like this.

We will assume affront there is certain constant that which we can multiplied this equation and another constant by which we can multiplied by this equation. And such that, we can add all of them together and that would equal to equate 0, it is same as saying that now trying to separate out variables, separate out to del N i. So, in other words what we have saying is that? We are basically saying, we going to equation 1 and we just for convention seek you just going to take the negative of that equation, because it works out conveniently for calculation plus equation alpha time equation 2 plus beta times equation 3 and equate to 0. Anyway all equate to 0 so therefore, you can always do this, but we assume that there is alpha and beta which is enable as separate the various N i del N i.

Therefore, we are going to say this implies sum over i of.

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Equation 3 had E i del N i sum over sum of E i del N i equal 0 I simply pull E i and kept del N i out multiplied by beta so, beta shows appear. Equation 2 simply said sum of del N i equal 0 so, I multiplied it; that by alpha I just pull out of the bracket, because it common term so, you just get see alpha here, so you should actually been alpha del N i. Equation 1 it say what we already have which was sum of lon S i by minus N i by N i equal 0 at times del N i equals 0 I again put del N i out and we and have put in minus sign here, so just for convention sake, because answer then in works out in manner in which explain in better there. But, there is no harm anyway we are not done anything inherently wrong, because anyway the sum is equal to 0 so, whether the sum is equal to 0 or we put minus sign in front of it makes no difference it is still this is 0. So, there is no nothing particular wrong that we have done right.

So, and all we are saying is that we assume that there exist an alpha and beta which and which we do not know in front then do exist we assumed that they exist. In such that, if you write this equation of the variables the each of the N i is now separate variables, we are able to separate variables out. You can perhaps look at mathematical book for lagrange method of undeterminant multipliers and see, if we can understand the manner in which this is done, what I am explain you is the method in which done and the reason will behind it. But there are several book which explain it so, you can look that to see, if in case you are not understanding it any particular reason, you can look up reference mathematics engineering mathematics books or any other mathematic book to specifically look at this lagrange method of undeterminant multipliers.

So, if we do this we are saying that for this to be generally true, regardless of value of del N i right for this to be true regardless of the value del N i, because you have wide variation of del N i available right. So, when you assign that you are reaching a maximum, you can change del N i, you can put in more particles in our location, less particles in another location, the variety that you have one del N i is quite large. You can always do this in definitely, but you are saying that sum should always equate to equate to zero right. We are saying that individually we have understand why this term should equate to 0, why this term del N i should equal 0, why this term should equal 0 and why this last term should equal 0 equate to 0. So, this we understand individually.

We also understand that this sum, because simply, because multiply by some constant and each of these terms is in the sum equal 0, we understand that therefore, this whole equation should always equate to 0 so, there is no controversy there. Where we are not taking as step forward is saying that the del N i or values that you can change in a significant manner through the system you have no real control to del N i or you would like that be free, you would like to be change that del N i independently, you should able to add more here less here and so on, you have real control on del N i, except to say that overall sum has to be 0, but you can do more or less and so on.

Therefore, to always ensure that the sum, to ensure that the sum always equals 0 regardless of values of del N i that you have or employing you are in the system, what you are actually requiring is that the first term should equal to 0, should equate to 0. The only way you can guarantee that the entire sum is equal to 0 regardless of values of del N i that your selecting is to guarantee that this term is individually equal to 0, if this term individually equal to. If this term always equates to 0 then does not matter what you doing this term you will always get 0 right. So, if this term equates to 0 does not matter what this terms are and does not matter taking as sum over all i you will always get 0, because this term is guaranteeing that will be 0.

Therefore, the only way to guarantee that this is 0 is to ensure that the first term equates to 0 so that is the important step that we are additionally taken. Therefore, we are saying that this implies that minus lon S i minus N i by N i plus alpha plus beta E i equal 0. So, you just simplify this marginally we will just move this term other side. So, therefore, we are saying lon of S i by N i (No Audio From: 42:17 to 42:29) minus 1 equals alpha plus beta E i. Now, please recognize the fact that this is the equation we are arriving at when we have reach the maximum the distribution occurs the maximum number of times. In other words when by the time we arrived here we are not taking of any arbiterly distribution, we are talking of maximum distribution. So, in fact N i is not arbiterly N i it is N i 0 so to speak corresponding to N bar 0 so that is where we have arrived at in the process of all these mathematic, but we will mathematics we will get that just moment.

In this equation therefore, we are saying that S i by N i minus 1 equals e power alpha plus beta epsilon E i. So, therefore, this implies S i by N i equals 1 plus e power alpha plus beta right, I am just re arranging the terms here this log becomes since, I remove log goes to exponent so that is why we have e power alpha plus beta E i and S i minus N i just used S i minus N i minus S i by N i, minus 1 goes to the other side it is 1 plus e power alpha plus beta E i. Now, note a couple of things we have arrived at S i minus N i I started of saying we need N i by S i so, we need this inverse of this right. So, this is always we were looking at. So, we will just put down the expression for that.

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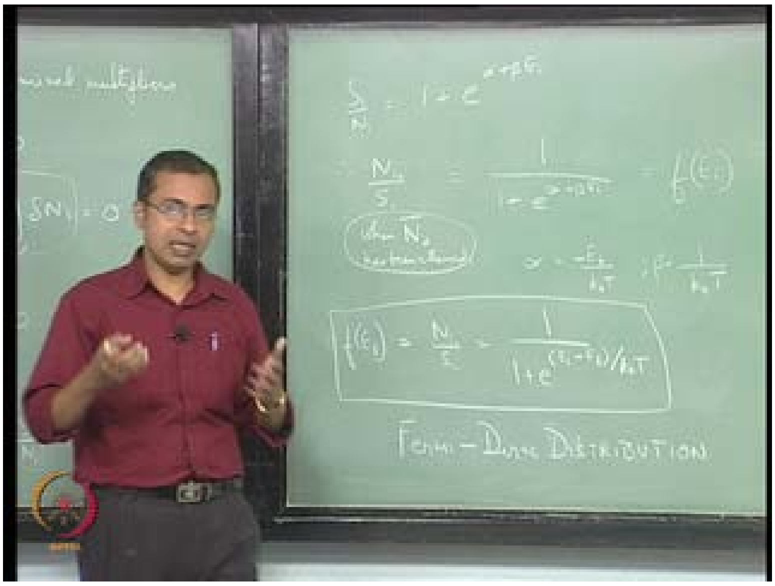

So, I will just put it again S i by N i equals 1 plus e power alpha plus beta E i therefore, N i by S i when N bar 0 has been attained. So, when N bar 0 has been attained so N i by S i, we can in fact designate now, by N i 0 subscript 0 indicating that we have reaching maximum possible distribution it is not some arbiterly distribution it is simply 1 over 1 plus e power alpha plus beta E i. So, this E i or energy levels that other in the system there are several different energy levels, for any given energy level the number of the probability of occupancy. So, this is f of E i I told you affront that f of E i is the general probability of occupancy of state at energy level E i, but it also equals to the probability of occupancy of the state at energy level E i subject to the fact that we are only working with distribution that is the most probable distribution and that is what the N bar is, N bar 0.

And we have attained this mathematical expression we have reach in accordance with all of those constraints that we have placed. And these alpha and beta, alpha works out to E F some constant e f by K B T minus E F by K B T and beta works out to one by K B T where t is temperature and kb is Boltzmann constant this is E F here is constant so alpha is also constant E F is also constant what it represent we will see a did later. So, the f of E. So, I will just write here f of E i or E F of E in the general case I would be some particular energy level which is what we are designated by this energy value E F of e is simply N i not by S i not you want you can write E i you will specify that specify that think later generalize it later. So, we have simply one by one plus e power E i minus e f divided by K B T so, this is the probability of occupancy of a state at energy level E i. And this is given by this expression 1 divided by 1 plus E power E i is that energy level E i that were talking about minus some constant e f which we have not it describe in any grade it at some constant the whole think both e minus e f or divided by K B T which is the Boltzmann constant times t temperature t so, this is expression here is the Fermi-Dirac expression. So, this expression that we have arrived at here is the Fermi-Dirac distribution expression expression for Fermi-Dirac distribution and to summarize. This is an expression that holds for bunch of Fermion's which are particles that are identical and in disguisable and they follow poly exclusion principles which usually means and Fermion's all therefore, essentially means that it has to have a half integer display

So, electrons in a solid qualify as Fermion's electrons qualify as fermion's and the system we are dealing with or is a whole bunch of electrons that are existing with in a solid which have which happened to move that feel through that solid. So, the Fermi-Dirac distribution that we have derived will apply perfectly fine perfectly well for the system that we have choosing and this is expression that we have arrived at for this system. So, what will do in our next class is we started this be saying that we know, we have trued model which know modifies trued summer figure model the trued summer figure model is heating the electrons as quantum mechanical particles which are following policy exclusion principle.

And therefore, so much field essentially took Fermi-Dirac statistics and employed it and ensure and applied it to the distributions of electrons in the system. Therefore, to the trued model and to understand the process we have now derive the Fermi-Dirac distribution. So, what we will do in the next class is we will look at we have known come come up with expression mathematical expression for the Fermi-Dirac distribution we will make a plot of this distribution and we will compare it with the Maxwell Boltzmann distribution. Then therefore, we will see that by assuming this is the distribution that holds with in this system what is the actual fundamental distribution we have made with respect to the model that exist for the electrons in the system since where now saying the new model thus.

So, more fields modification of trued model and hence the trued summer field model is employing this distribution what is the implication of it is something we have not it consider we have just mathematically arrived at that particular distribution. When once, you put it down in the form of plot, we will get a better understanding for what is the implication of assuming a disc distribution holds for the electrons in the system. What is the implication of distribution in terms of the of the prediction that we can make of the properties of the electrons. And therefore, the properties of the entire solid which are largely determined by those field electrons in a metallic system. So, we will do all of that in the next class and we will fault this distribution today. Thank you.