Physics of Materials

Prof. Dr. Prathap Haridoss
Department of Metallurgical and Materials Engineering
Indian Institute of Technology, Madras

Lecture No. # 17
Fermi-Dirac Statistics: Part 1

Hello, this is the 17 th class in the physics of material course, and today we are look at the Fermi-Dirac statistics.

(Refer Slide Time: 00:24)

So, the Fermi-Dirac statistics - the Fermi-Dirac statistics describes to as the distribution of Fermions in a system of thermal equilibrium; distribution of Fermions across energy levels for a system in thermal equilibrium. Fermi-Dirac statistics describe the distribution of Fermions across energy levels for a system that is in thermal equilibrium. For our purposes this is of relevant, because the electrons that we are trying to examine electrons with solute that we are trying to examine, electrons qualifier Fermions, they have all characteristics that that going to this terminology called the Fermions. So, they have half into get a spin they obey policy exclusion principle and and therefore, they have qualifiers Fermions.

So, we would like to see this, this of course is useful to as because there it tells as something about the behavior of those electrons, what they can do, what they cannot do, what they will attempt to do, this is the basic idea that we wish to examine in detail, right. So, this is Fermi-Dirac statistics so what are the conditions we would just take them very briefly because we have discuss them in detail earlier. First of all it says that those the Fermions fall obey.

(No audio from 02:30 to 02:42)

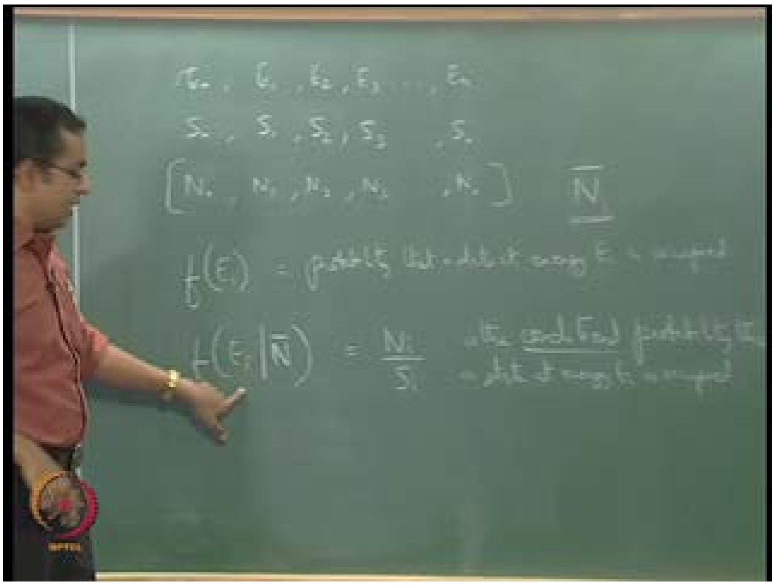
So the particles of the Paulies exclusion principle and therefore, this this discussion applies for them. The particle are identical and indistinguishable. So, this simply means that when you have energy levels and you have certain number of particles in one energy level and another number of particles in another energy level, simply swapping particle between two energy levels is not is not considered as a new arrangement. So, if you have N 1 at energy level m E 1 and N 2 at an energy level E 2, if you simply take one particle from E 1 and exchange it with a particle at E 2 that is not considered as a new arrangement, so that is the important thing you will through it in to the, we will incorporated into the mathematics as we go along.

So, these are the two major thing points. The third thing of course, is that it is in thermal equilibrium plus this is a system that is a that has a fixed extend so in other words the number of particles in it it is a constant. So, therefore, the total number of particles which we have call N total is the total number of particle is constant is fixed it is an thermal equilibrium therefore, the total energy of the system e total is also fixed. So, N total is fixed, E total is fixed, which these are this is the framework that we are operating in and this is the framework with with in which Fermi-Dirac statistics is use to describe the distribution of particles.

So, N total is fixed, E total is fixed and we are basically looking; we are looking for this information, what is the probability (No audio from 04:46 to 04:53) that an energy level that state with energy level with energy E i is occupied. So, we wish to find out what is the probability that is state with energy E i is occupied. This is the piece of information that we wish to find out, this the framework within which we are going do this calculation, right. So, this how we are draw about it so as you proceed you will see how this comes about. I will point out that we would put will put down some equations and proceed step by step and each each put down please carefully, note down what are the conditions under which we are claiming that that equation is operating.

You can always re examine it when you when you done with the class, just to see that you understand what it means when you put down the kind of an equation. Because at each stage we may make some simplification, we will sometimes eliminate some conditions; sometimes focus on a particular condition and so on. So, it is not that as you all the equations are to be just remembered as it is. Please, understand what are the conditions under which we are putting down those equations. So, the system that we are dealing with has some parameters associated with it.

(Refer Slide Time: 06:20)



So, we will just list those down we will just say that it has energy levels E 0, E 1, E 2, E 3 and so on, E r. At each energy level this is the system we are we are basically saying Paulies exclusion principle applies and and for it to have any meaning they has to be a certain fixed number of states. If they are infinite state there is no meaning in saying Paulies; even then you would say you know if Paulies exclusion principle applies into electrons for not have exactly the same state, will not occupy exactly the same state, where the state incorporate for of the quantum number information.

So, we basically say that these are the energy levels. At each energy level there is a certain finite number of states so you just call that S 0, this is the number of state available at energy level E 0, S 1 number of energy level available at with energy E 1, S

2, S 3 and S r, which is the number of states available at the energy level r. so, we will just say that this is the steam that we have. Now, what we; so, if you take our system given that is in a thermal equilibrium and given that other constraint to the system around fixed, the energy levels it is a fixed volume system it is a closed system so the energy levels are all fixed.

So, in our calculation, in all the calculation that we will do, up front these numbers are all fixed. We are not specifically giving a energy at value in terms of joule or in terms of joule specifically we are not explicitly saying so but regardless the point is for our calculation purposes this are certain fixed numbers, we have to be we have to recognize that. So, this is not a variable in our calculation it is fixed all of this numbers are fixed. Similarly, the number of states that are available at each energy level are is also fixed. So, what they are fixed at is is another calculation which we will do later on so that is not something that immediately concerns us. For our system this is fixed, this is fixed we do not have any choice, it is fixed.

What is not fixed, what is variable is the actual number of particle number of electrons in this case that are happen to be sitting at E 0, E 1, E 2, E 3 and E 4 and so on. So, those are N 0, N 1, N 2, N 3, N r for the state r and so on. So, this is not something that is fixed, at is as it start out the calculation, as we start out the calculation we do not know what this values are. In other words, we can consider a verity of situations in each of the situation the difference will be the actual value of N 0 was is the actual value of N 1, N 2, N 3 and so on, those numbers can change. You could add some particles here, you could remove some particles here you could add some more particles here all of this possibilities are allowed.

Where as you cannot do that with the the number states available at the energy levels or the energy level themselves, this is all fixed. You cannot just increase E 0 you cannot increase S 0 and so on. When you say E 0 it is fixed E 0 there is no other value you can have, S 0is also the number states that you cannot change that anywhere. Whereas, the number of particles that are sitting there, subject to the fact that it is not greater than S 0 you can actually increase the number of particles or decrease the number of particles so this is the system that we have. Now, we wish to know; so let us what we will do, we will we will starting some limited fashion then we will in our in our calculation if narrow down to a certain starting point from there we will start.

We will then write a more general equation then we will find so this is the layout of what we are going to do. We are going to start with narrow our focus of one particular situation, then we will write a more general equation which covers the entire arrangement that we can think of. And then we were find that even though there is a larger equation that is of interest in that larger equation there is only one term that is of real interest. If you focus on that one term we will get the information that we want. So, we will, so even though you are write a much more elaborate equation, most of that equation we can set aside. We can say that most of that equation is not is not necessary for as to solve or analyze that entire equation, that one term in the equation is good enough.

It it is the most important term that we need to analyze so we were focus on that one term and our calculations will be based on that term, so that is what we will do. So, what is this narrow starting point that we will start with? So, our narrow starting point is that we will first say that I told you that these numbers can change, right, N 0, N 1, N 2, N 3, N 5 and so on can change. Now, I will start with a situation where and we were call this a distribution. So, this this entire information that we have here I will call this N bar. So, as a particular N bar or I call this n bar I for example, so our particular N bar implies that a particular set of N 0, N 1, N 2, N 3, N 4 and a particular arrangement has been chosen.

A particular combination of N 0, N 1 a set a particular set of N 0, N 1, N 2, N 3 has been chosen as N i. If you change this it will be some other N, N bar so that is that is what we are referring to. Now, I put something down here and then you will, I will explain what it means this is the probability this f (E) is the is the probability function which tells as the probability of that an f (E i) let say, this is the probability that the energy of the this is the probability that state at energy level E i is occupied. So, this is (No audio from: 12:42 to 12:51) some E i is occupied so that is the probability that it is occupied. So, this is so this is for a particular n bar will will remove the side for it does not confusing at the movement we leave it like that n bar is a particular set that we having.

So, this is the probability that are state at energy level E i is occupied, now what are it say is this is about general probability we will narrow down and I put the equation down here then then we will discuss it. (No audio from 13:26 to 13:38) What I mean is this is called the conditional probability that state at energy level E i is occupied so conditional probability.

(No audio from 13:49 to 14:19)

So, this is the conditional probability that state at energy level E i is occupied. So, what this means is, you have various different N bars available to you, I will picking one particular N bar. So, one particular arrangement of N bar I am picking, which means a particular set of N 0, N 1, N 2, N 3, N 4 and so on. So, at this point I have frozen all three numbers, right.

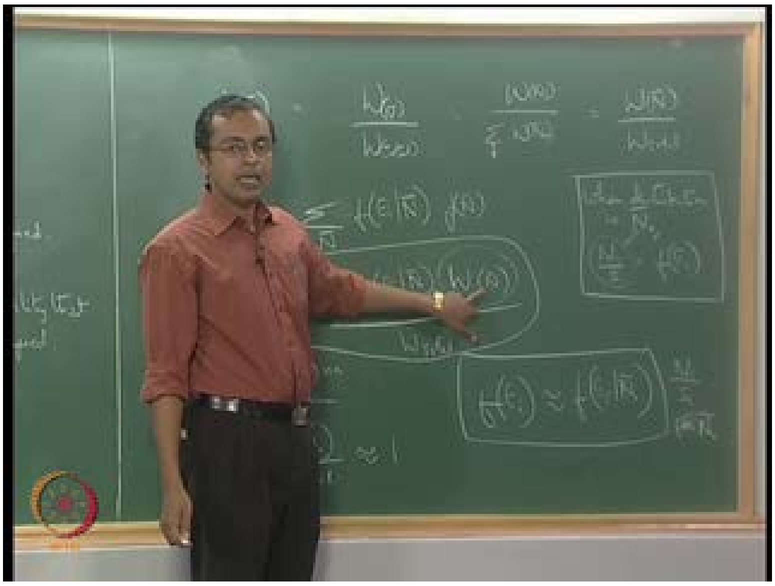
All three numbers are anyway these two are frozen I also frozen this number for this particular N bar because I happen took deliberately choose to do so. Given that I have frozen it, what is the probability that are state at energy level E 3 is occupied, it is imply the ratio of the number of particles occupying those state to the number of state that are available. So, there are S 3 states available, there are N 3 particles occupying those states. So, any one of the states if you take, in if there are let us have we are 10 states here and there are 4 particles here so the chance that a particular state at S 3 is occupied is simply 40 percent.

Because there are 4 particles occupying one of ten states, I mean 4 of 10 states so that is what you have is 40 percent. So, but that is for a particular N bar, if I change that N bar that this number would change at at the next N bar that I select, this N 3 might change to instead of 4 it might become 7, in which case for that new N bar the probability of occupancy is 70 percent, right. So, this when when I say N i by S i that is for a particular n bar so this is called conditional probability. This is just a general probability that is state at energy level E i is occupied and we are going to write a expression for that. This is the conditional probability that the particular state is occupy and the condition is that this the distribution that currently exist so that is the condition we have placed that is why it is called the conditional probability.

So, this conditional is have to do with this choice of this N bar, fine. So, this is the conditional probability that it exist state of at energy level E i is occupied. So, what do we now need? So, if you want to look at if you you want now, what we are going to do now is to find what we need to do to this term, so that we can end up on this term so that is the more general statement that we wish to make. What are we have to do using this term to come up with this term.

So, what is it that we have to do? We have to see what is the probability that; now, we have different variety of arrangements possible each each arrangements corresponds to a set like this so we need one information which is what is that probability that we are at this particular N bar, right. So, I have 1000 particles out of which I have put 10 here and I have put 100 here, I have put something here, something here, something here. In another arrangement, I have put 500 here I have put 50 here and so on. So, all this possible arrangement might exist each of which corresponds to an N bar. So, for every N bar there is a there is a probability you will be at that N bar I mean as suppose to any of the number of other N bar set are possible. So, you will write an another equation and then we will see what it means; again I will put down the equation then I describe it to you.

(Refer Slide Time: 17:37)



We will say first of all that p of N bar equals W N bar by W total. What does this what does we, what do we mean by this? This is the probability that the system actually has that specific distribution N bar that specific distribution N bar. What is that probability it is the number of ways in which you can create that N bar divided by the total number of ways in which you can create any N bar. Or in other words this is simply W N bar by a sum of all the N bars if the let say if you put a j across all j. So, N bar so, you have N 1 N 2 N 3 and so on, you have you can come up with N bar j you can say, we call this j for example, and this is j.

So, if you sum it across all the N bars then whatever you will get is called total so, we will we are just interested in one them so, we just say N bar by W total. And this is the total number of ways in which you can arrange the particles across all possible combinations, all possible N bars that you can get. So, all specific ways in which you can specify N 1 N 2 N 3 N 4 and so on you take all of that together you can come up with one W total of which for one of them whatever you get is appear. So, this is what it is, this is the probability that the system is at particular state N bar. So, therefore, f of E is now simply I will write it down.

So, (No audio from 19:34 to 19:51) what it means is, the general probability that an state at energy level E i is occupied, which is this term that is there, is this is the term here which is the conditional probability that that particular state is occupied. The condition begin that the state is at the overall set is at N bar times the probability that it is at N bar right and summed over all possible N bars. So, this states in to account all possibilities so, we will I will so I just write it down and then we will examine this little so that (No audio from 20:19 to 20:44) so, what we mean is so, this is f (E i), so we have all the information here, I will just restate what I have just put together here, so that you follow what is being done right.

This is the probability that an energy level at that state at energy level E i is occupy so, this is the probability right so this is the term that we are interested in. So, this is an in general regardless of all the conditions in all conditions taken into account in all possible ways that the system could exist all information put together this is the probability that state at energy level at energy level E i is occupied right. Now, if you go back to what we put down here we had this three possibilities our energy levels are fixed the number of states at those energy levels are fixed. But we have freedom in selecting how many particles happen to be sitting at that energy level.

We had this freedom, we can keep we do have this freedom we can select different number of particles and then distribute them at all this energy levels. So, all these values can change so, we since they can change and we have many many possibilities of those changes, we started out with one such possibility which we we have designated as N bar. And when it we say N bar that means, for that instant we have froze on the values of N 0 N 1 N 2 N 3 N 4 and so on. So, if you want to consider the entire gamete of possibilities available to you, you have to look at all possible N bar arrangements right N bar meaning this collection. So, I this one specification of N bar another way you could specify it you will called it another N bar and so on.

So, you have lot of possibilities of N bar of which only one is put down on the board here right. So, what we are basically saying is that the probability that state is occupied here of energy level some energy level is occupied, is it is equal to the probability that it is occupied, when you have specified a particular N bar. Times that total number of the probability that it is the; it is at that N bar and you have to sum that up right. To stated again N bar is this collection here and since you have many such N bars, there is even associated with N bar; there is a probability that you are there at that N bar right.

Because you have you know several several possibilities of N that you can several changes, you can make here so, for every given change there is a new N bar so, you have a huge number of N bars available. Given how easy or how many times you can make it that way you can have a certain probability that you are at that particular N bar, when you put down the terms you will see that. So, all all I am saying is that you have a variety of values available for N bar and if you look at those values you will find that given that the system. This hole analysis is also based on the assumption that all the possibilities that we can think of or all equally or individually equally probable all right.

So, in other words if I come with a particular arrangement of the system it is just as as probable as another arrangement right. So, what we are interested in is therefore, to see that if you take a particular arrangement, in how many ways you can attain that arrangement. Each of those rows is equally probable and therefore, if one arrangement can occur in ten in ten different ways another arrangement can occur in 100 different ways. Individually the first arrangement is equally probable as this each of those ten arrangements is as equally probable as each of those 100 arrangements. But because one can occur in ten different ways, the other can occur in 100 different ways; the second arrangement is ten times more probable than the first arrangement this is the basic idea alright.

So, you have ten ways to ten ways to attain N bar in a particular manner and 100 ways to attain another another N bar, each of those ten ways is just as probable as each of those 100 ways. And therefore, if you look at it taken together this 100 ways will occur ten times more likely than this ten ways will occur so, this is the basic idea. So, so, we have here the probability that are state at energy level E i is occupied, that is equal to the conditional probability that state E i is occupied the condition being that it is at N bar. Times the number of ways in which, you can accomplish this N bar by the total number of ways in which you can accomplish everything.

So, so this is the information now what we will find is, which is the basic statistical mechanics tool that we employ; is that, if you look at the number of ways in which you can accomplish N bar this term right. You have for every arrangement there is certain number ways in which you can accomplish it, if you look at if you focus on this term you will find that for every arrangement you can come up with a value of W N bar. And you will find one arrangement, one particular N bar which corresponds to a particular value of N 0 N 1 N 2 N 3 N 4 and so on. That particular arrangement will be the most probable arrangement, meaning the number of ways in which, you can obtain that arrangement will be huge.

And statistical mechanics which operates for these kinds of system basically says that, when you identify that particular arrangement, which has the maximum number of ways, in which you can accomplish it. You will find that it is way more probable, significantly more probable than any other arrangement possible with any other N bar that is possible right. Therefore, at that point what you will find is that we will put that down here, I will call that most probable arrangement as N bar zero. So, N bar zero is the most probable arrangement that you can get for which implies (No audio from 26:33 to 26:44) N bar zero has maximized this W N bar zero alright.

When W N bar zero is a maximum you will find that its value will be much higher than all the other N bars that are present in the system, all the W N bars that are present in the system, which will imply W N bar (No audio from 27:08 to 27:16) will be approximately equal to one, when this happens and we say that in our system that given the kind of system that we deal with, the most probable arrangement will will be will be much more probable than even the next most probable arrangement. So, it will be not just much more; we are talking of you know we did this example right at the beginning, we did when we consider the maximum Bolts man statistics, we did this example where we found that as you increase the number of particles in the system.

The most probable state in the system starts becoming more and more probable relative to all the other possible arrangements in the system. The most probable micro state becomes more and more probable than the other micro states combined and we did that for you know two three and four particles and I basically highlighted the idea. That when you go to larger and larger systems and we know when you talking of one mole of a solid of a material which has ten power 23 particles and not just two three four particles, ten power twenty three particles. When you go to that kind of a system the statistical mechanics indicates that in the; in that kind of a system that most probable arrangement will essentially be 99.999 percent probable, something like almost like 100 percent probable.

All the other arrangements combine will constitute a very mini squall number of arrangements related to the most probable arrangement. So, therefore, statistical mechanics basically operates on the idea that the most probable arrangement has a probability of occurrence of almost equal to one. So, when we do a statistical mechanical approach, what we are essentially doing is; even though the system has all the possible arrangements available to it and this case we need all the possible N bars available to it. Because each N bar is one arrangement, one micro state, so; one micro state is one N bar, even though you have so many different possible micro states available.

The most probable micro state the most probable micro state which is N bar zero that micro state will occur in so many ways, that it will it will be way more likely to occur, than all the other micro states in that system combined. Not just the second one, not just the third one, second plus, third plus, four plus whatever, whatever it is all the possible micro states that you can think of of the system. You you add them all up the number of ways in which they can occur as a sum will be a very tiny fraction of the number of ways in which the most probable micro state can occur. So, we have not put down the equations for the most for the manner in which we will calculate this number of ways in which the most probable micro state will occur, but this is the idea all right.

So, therefore, if you keep that in mind this is when I say N i N i N bar zero, we are talking of the most occurring micro state. So, therefore further most occurring micro state that is the idea that I just mention, the probability that it exist will be almost equal to one. So, the number of ways in which you will accomplish the most probable micro state, which is N bar zero divided by the total number of ways in which you can attain any micro state. This this term in the denominator also includes W N bar zero which includes W N bar zero, which is the most probable micro state plus second most probable micro state plus the third most probable micro state and so on.

It is just that from the second the term if you set aside this term all the other terms are a very negligible contribution to this term. So, the denominator is actually the same as the numerator plus a very very very tiny at extra term. So, all so, the denominator w total is n bar zero plus a very very tiny amount over and above that so therefore, this ratio works out to be one in the most probable micro state. So, we are saying that if you have manage to identify that particular micro state which which is the most probable micro state then the of course, that it it directly means; that the number of ways in which you can accomplish it is a maximum. So, therefore, we are basically saying that when you have done that.

(No audio from 31:31 to 31:53)

So, this is the frame work so this is the equation that that is that helps us narrow down the calculation that we wish to do, what which is the basically directly take taking this equation which is the general case. The general case is that the probability of occupancy at energy level E I, in general; is the probability of occupancy at energy E i given a particular arrangement, times the probability that that arrangements exists. When it becomes the most probable arrangement this ratio becomes equal to one, in the most probable arrangement we are designate the designating this as N bar zero and therefore, this is also N bar zero.

When this ratio becomes equal to one essentially we are basically saying that the probability of occupancy is simply the occupancy at at the at this at this particular value N bar zero. So, where you have already reach the maximum so that then the sum becomes meaningless so, then you will basically just directly become this probability of occupancy at E i is the probability of occupancy at E i times the at this particular condition. In principle the sum would include other terms, but all those other terms are approaching zero, relative to this term. So therefore, the conditional probability that a particular energy level E I, a particular a particular state at energy level E i is occupied.

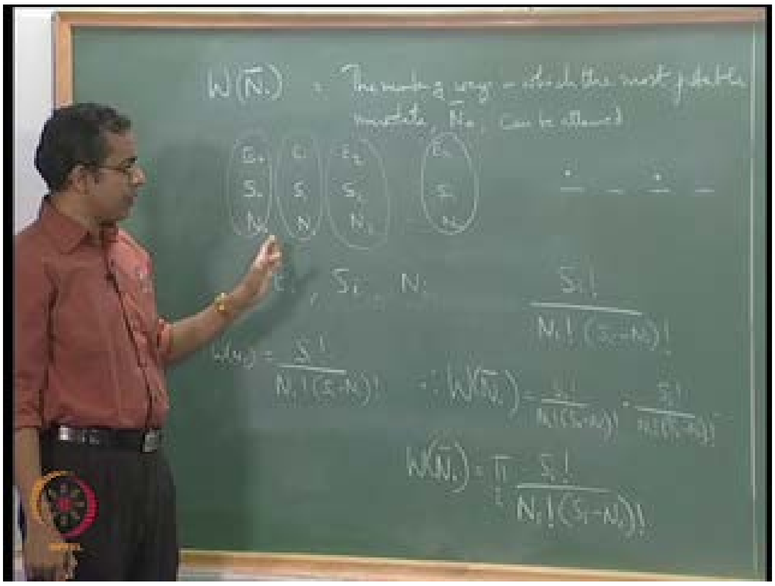
When you have attained the most probable distribution is therefore, the actual probability that state is occupy regardless of all condition. So, this is the simplification we wish to them or we find is valid for the system right. Now, our problem now changes to like I said we started with a narrow condition we widened it, our narrow was that we only looked at in general; what is the conditional probability that at particular state at energy level E i is occupied given a particular distribution N bar. That is the that is the narrow point or the focus point where we started our calculation, we widen that to a more general equation here, which is the, which gives as a more complete picture of a what is that probability that state at energy level E i is occupied.

Then we find that even though with this is the even though this complete term is the more more complete whole picture of the system. Most of the term in it are negligible so, we can eliminate in fact all of those other terms, it is enough if you find out, what is the probability that are state at energy level E i is occupy in the most probable distribution that exist in the system. If you if you find that out you already automatically found this out so, we have started some of focused point, come up with large larger expression out here and from that again we find that it is enough if you if you just focus on a particular term. What is this? This is simply if you compare with what we have here, it is simply N i by S i subject to at N bar N i.

So, or I put it over here it is simply (No audio from 35:09 to 35:30) so, this is the final generalization that we make or final simplification that we make. So, when the distribution is that most probable distribution N i N bar naught, for that distribution when if you know the ratio N i by S i that is equal to the probability that state is occupied so, that is the direct result of this equation here right. So, what I have written here, when distribution is N i N bar naught N i by S i that is simply this term here so, this term is equal to this term which is what which is what has out here so that is. So, for the most probable distribution if you know N i by S i you automatically know the probability of occupancy of an state at energy level E i subject to this conditions that we have spoken of.

So therefore, what we know come down to is we need to find out N i by S i for N i naught for this particular distribution N bar naught. So, if you we only need to focus on this distribution N bar naught and in that distribution if you know N i by S i we have got the answer that we are looking for. So, to do that what we are going to do is, we are going to look at the number of ways in which you can accomplish any one given N bar any one given N bar and then maximize it, to see in under what conditions you get the maximum value for this. In the calculation that we do, you will at some point you will get a value for N i by S i and that is that is basically all the information that we will get all right. So, this is the approach we are going to take all right.

(Refer Slide Time: 37:05)



So, we are looking for an expression for W N bar naught

(No audio from 37:12 to 37:48)

So, this is the number of ways in which the most probable micro state N bar naught can be attained this is what we are looking for and that is what that is the expression that we are interested in. So, what do we have? We have E 0 energy level, S 0 is the number of state available at that energy level and N 0 is the number of particles in that particular state, then we have E 1 S 1 N 1 E 2 S 2 N 2 and so on so, we just say E r S r N r some r we have all of this. Now, this is not some arbitrary number, we have some at this point we do not know what these numbers are, we want to see the condition under which the probability, the number of ways in which this arrangement can be attain it is a maximum.

So, we will start with so, what we are saying in general is; we will start like this we will say energy level E i has S i states and in this S i states there are N i particles. We want an expression now to see the number of ways in which we can accomplish this. So, number of ways in which we can do this arrangement even that we have an particular collection of N 1 N 0 N 1 N 2 N 3 N 4 and so on. So, to do that first we will take any one particular energy value S i for example here, we will see the number of ways in which this can be accomplished right, times the number of ways in which this can be accomplished.

The total number of ways in which this this arrangement can be accomplished, the total number of ways in which this arrangement that have been put on the board can be accomplished is is the product of the number of ways in which you can attain this, times the number of ways in which you can attain this, times the number of ways in which you can attain the other one and so on right. So, the product of all the ways in which you can attain each of these combinations, given that you have selected a combination so this is the way we have go about it. So, we will just take a particular case E i energy level E i which has S i states and N i particles available, the number of ways in which you can do this is simply S i factorial by N i factorial times S i minus N i factorial.

Why I said this? We it is essentially the same as aim that you have N i particles and S i minus N i empty spaces. Because you have S i states right you have some number of states here state state state state you have, some number of particles you have, some particle here, some particle here it is same as saying you have two particles, and two empty states. In how many ways can you jumble them up, that the number of ways in which you can jumble them jumble them up at the number of unique ways in which you can attain them, and therefore that represents the number of ways in which you can accomplish that condition right. So, that is same as so, you have this is just a particular example S i is the number of states we have so, the total number of ways.

In which you can arrange those S i states is S i factorial or select those states, in which N i factorial is the number of particles that happens to be there or or comes from the number of particles there and the number of vacant states that are available. So, if you do this that gives you the total number of ways in which you can attain a particular a combination of N i particles sitting in S i states right. And I said that the for the entire so, this we will call a small w so, small w for a particular N i is this one. (No audio from 41:46 to 41:56) So, the total so, this is for any one of this combinations right which are put down here any one of those combinations can be attained for once you specify the value of N i you can attain it in these many number of ways.

So, for the overall system given that you have N 0 at E 0 N 1 at E 1 N 2 at E 2 and so on. The total number of ways in which you can accomplish this is simply the product of the number of ways you can accomplish each one of them. So therefore, W of N 0 bar is simply the product S 0 factorial by N i N 0 factorial times S i sorry S 0 minus N 0 factorial product times S 1 factorial by N 1 factorial times S 1 minus N 1 factorial and so on. So, for the for energy level E 0 this is the number of ways you can accomplish the arrangement, energy level E 1 this is the number of ways you can accomplish this arrangement, product similar term will come for energy level E 2 E 3 E 4 E 5 and so on so, this is a product, a product of terms that put like this right.

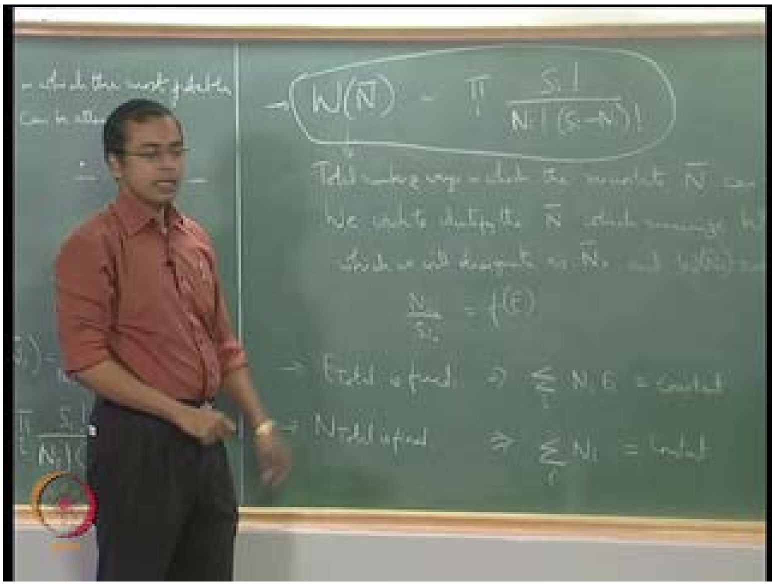
So, the way of writing it is just the way we and we describe discuss this earlier, just the way we write sigma for a sum, we write pi for a product so, we write W N bar is pi over i of.

(No audio from 43:50 to 44:09)

So, W N bar naught is the total number of ways in which you can attain this particular arrangement, all things considered so, all things consider meaning; we have already said that this is one particular arrangement. So, what is a total number of ways in which, we should not designate this yet as the N N bar naught, we we do not yet know that this is the N bar naught, we are going to find the condition under which this becomes N bar naught so, this is N bar in general. So, up given N bar is there a given arrangement is there a given set of N 0 N 1 N 2 N 3 N 4 and so on up to N r is there. So, given arrangement exist for which this is the number of ways in which you can attain that arrangement.

So, number of ways in which you can attain the arrangement is the number of ways in which you can attain the arrangement for energy level E 0 times the number of ways you can attain it for E 1 and so on. And so that is pi over i this, when you maximize this, when this becomes maximum it becomes N bar 0 all right. So, our calculation is in fact our interest in fact is to maximize this so, when you maximize this you will get N bar zero all right so, that is what we are going to do. (No audio from 45:20 to 45:42)

(Refer Slide Time: 45:40)



So, we just rewrite that here this is pi over i (No audio from 45:47 to 45:58) pi over i of S i factorial by N i factorial minus times S i minus N i factorial. So, this is the total number of ways.

(No audio from 46:09 to 46:35)

This is the total number of ways in which the micro state N bar can be attained all right so, this is the this is what we are looking at and.

We wish to (No audio from 46:46 to 47:11)

We wish to identify that particular N bar which maximize the value of this term here so, that is all we wish to do. And when we do that the we will designate that which we will designate (No audio from 47:25 to 47:45)

So, we will designate that as N bar and W N bar zero respectively so, that is what we will do so, our calculation will proceed on this phases. And in fact I will also tell you that as we precede through the calculation our purpose even though this is the framework within which we are actually operating. And this is this is the important idea that we are following; our intent actually is in this process of identifying this N bar zero and therefore, W N bar 0. In this process at an appropriate step we will find the ratio N i by S i and we will corresponding to this W to this particular N bar zero, corresponding to this particular arrangement.

So, we will designate that as N i zero and S i zero so, this is the probability that as energy level that is state at energy level E i is occupied in the most probable distribution that exist within the system. So, that is important then this is the probability that an state at energy level E i is occupy in the most probable N the distribution that is available in the system. Then we do this we have got the this is equal to f (E), since it is most probable distribution all the other terms from the calculations that we have done so far can be neglected and we will be able to say that this is the f (E), so this is all that we have all interested in.

Now, in our system so, this is what we are looking at there are two more pieces of information that are relevant in our system which we will use for our calculation. The first is that we said right at the beginning that the system is at thermal equilibrium which means the total energy available to the system is fixed. So, E total is fixed. (No audio from 49:35 to 49:42) So, what is E total this is simply implies the sum of (No audio from 49:53 to 50:02) the sum of all the the number of particles times the energy level that they are at sum over i so, you have N i particles sitting at energy level E i so, N i E i is the total energy of those particles at that energy, because of that energy level, because of those particles at that energy level.

So, if you sum it across all energy levels that is the total energy of the system right. You have different number of particles at each energy level at each of that different energy levels so, within that energy level, the energy available is the number of particles times that energy level. You sum it across all energy levels that is a total energy available to the system, since this system is at equilibrium that total energy is fixed, we do not change the total energy. So therefore, this term is a constant all right, so that is how this is going to be and we also said the total number of particles in the system is fixed. (No audio from 51:52 to 51:58) N total which is the total number of particles in the available in the system it is also fixed. So, this is what is it, in terms of what we have designated within the system in terms of the number of, in terms of the way we have distributed the particles across the system and so on or the state of our system.

It this simply means the sum over i of N i (No audio from 51:20 to 512:26) the two constrains that we have here is that the total energy of the system is fixed. Therefore, the sum of the number of particle at each energy level times the energy level; times that energy level that sum is a constant, because that sum is equal to the total energy of the system. So, that is fixed so that is a constant so, this is one constraint that we are placing on the system, the other constraint we are placing on the system is; that the total number of particles in the system is fixed. So, any any time you do an rearrangement you you have to remove some particles from some state to put it in to another state right.

So, you cannot just on arbitrarily add on, you cannot arbitrarily increase the number of particles in any given energy level. They have to come from one of the other energy levels or some of those other energy levels so therefore, the total number of particles is fixed N total is fixed. That means; the sum of the number of particles at all the energy levels which then therefore, totals to the total number of particles present in the system that is fixed that is a constant. So, our job in fact our immediate task for example, is basically to see how we can maximize this term, if you maximize this term we are able to identify the most probable state of the system.

Our task is to maximize this term here subject to the constraints that the energy is fixed therefore, this term is fixed and this number of particles is fixed therefore, this term is fixed. So, we have two sums here and you have product here, so we want to maximize this subject to these two constraints. When we do that, we will get under the maximum conditions, under the conditions that give as the maximum number of ways we can attain a particular micro state. We will automatically find that the corresponding N i naught by S i naught is the info piece of information that we are looking for which is simply at the end of it all is the probability that has state at energy level E i is occupy.

So, that is all finally, it comes down to this we want that one particular probability we have a lot of equations available to us from which we narrow down to this three equations. If you look at focus at these three equations we will get this term, when we have got this term, we have got the answer we are looking for and at that point you will look at the implications of that term. So, with this we will halt this in this class we will continue with this derivation in the next class, where we will start from this point here and we will move forward with our calculations and see how we can arrive at this by focusing on this equation here subject to these two concepts. So, we will see that in the next class. Thank you.