Metallurgy and Material Science

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Module No. 02

Lecture No. 16

Introduction to Drude Sommerfeld Model

Hello, welcome to this sixteenth class, in this physics of materials lecture series. So, in the past few classes, we have developed the Drude model for materials metallic systems in particular and we looked at how well it was able to explain thermal conductivity, thermal properties, electronic properties and also what it did with respect to the Wiedemann Franz law. So, these are the things that we did. We recognized that it had some limitations that in terms of explaining a material properties. It could only go so far, but was not able to successfully explain all the major facets of material properties that we are able to measure.

As I mentioned in right at the beginning, the way we proceed on this courses that and in fact the way we should proceed and in the search in general is that, the experimental information that we are able to obtain. So far as we are able to obtain it properly, where we have eliminated or significantly reduced or control the errors involved. Once you do all that and we obtain some experimental information that is the supreme piece of information. So, any theory that you put together no matter how suttle it seems, no matter how interesting it seems, when you write it down or when you explain it to somebody. At the end of it or at the end of the whole analysis, it should actually match very well with the experimental data. Your success depends on how well it matches with that experimental data.

So therefore, all the work that we did with respect to the Drude model and the analysis that we did is partially successful. After all it does explain some of the experimental data that we have seen, but we recognize that we have to move forward, we have to look at alternative to it. So that, we can explain the properties more in a more complete manner with less of the kinds of a loopholes or holes that are there in the ability of Drude model to explain the experimental data. So, in this context we digress, we basically said that you know Drude model treats the particles as classical particles and we over the past 100 years or so, we have come to recognize that there are quantum mechanical effects that there are effects that we call, we now recognize as a as or designate a quantum mechanical effects. And that they are very relevant to the kinds of system that we are dealing with, in which, in this case bunch of electrons in a solid.

So, in this context we digressed a bit, we looked at the history of quantum mechanics in the last couple of classes and we did this particularly to understand or to pull together all the major concepts of quantum mechanics that we will use through this course. And also to understand; where they came from? How they related to each other? And perhaps also the kinds of difficulties we face, while utilizing those concept for trying to understand those concepts. And so, that is where we are now and we will proceed forward from here.

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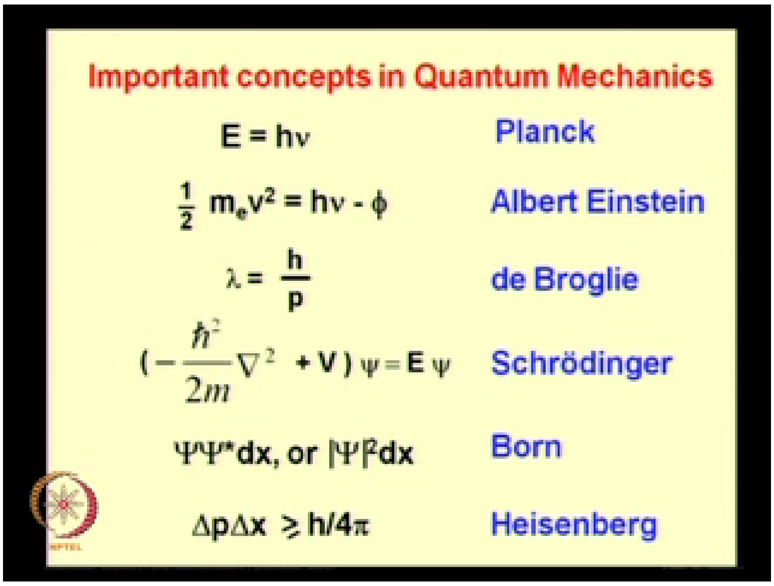

I am right now showing you the in summary the major concepts of quantum mechanics, in the single slide all that we discussed to the last of last couple of classes. So, the very first and foremost equation of quantum mechanics that we will that we saw was that the Planck has given to us, Max Planck. And that is simply that energy is h times nu for for a radiation of frequency given. In other words, it is in that quantum of h nu that you can exchange energy with with a system that is a giving out energy with that frequency nu.

And specifically the whole idea of quantum mechanics comes down to this one equation. In the sense that, if this h had turned out to be 0, then we would be existing in a world where quantum mechanical effects did not actually exist.

So, if h had turned out to be 0, it would be in that a particular source of energy or or a electromagnetic radiation of frequency nu, could give out could exchange energy with any of its surroundings in or in any manner in in any amounts. So, in in extremely small incremental amounts to very large amounts that we know real restriction in how it would go about in this transaction of energy. The fact that the h is actually non 0, even though it is small, implies that now the transaction can occur only in steps of h nu and this is what Planck discovered. I mean he assumed that there are may be step size, which he was not aware of and he treated that to be h nu and he ran through the calculations with respect to black body radiation and came up with the value for h.

And he found that it was non 0. Now that, he has found he found that it was non 0, the implication was that the transaction could occurs could occur only in the in step sizes of h nu. So, it could be h nu, 2 h nu, 3 h nu and so on. Which which of course, meant that if the frequency was very high that h nu value would also be high and it was quite possible that the source of energy did not have the amount of energy required to be equal to that step size. And therefore, if we went to higher and higher frequencies that source of energy would not be able to transmit energy at those higher frequencies or absorb energy at those higher frequencies. So, this is the basic idea that Planck discovered and that has formed the basis of quantum mechanics.

And as I mentioned in those last 2 classes that, he himself was quite uncomfortable with his idea that there was a step size involved in this process and several of the scientists, leading scientist; who worked on quantum mechanics, who have developed all of the theory that we today accept as quantum mechanics have all been uncomfortable at one point or the other with this idea that there was a step size and that nature had a step size. So that is important thing. That that we are finding that nature has a step size and that step size for these transactions is h nu and this is something that lot of people felt was not natural for from an intuitive prospective, but it turns out that if you look at all the examination that people have done over the years seems to indicate that this basic concept is right. I mean that all of the there is so much of data that had not been explainable in about in hundred years ago, out of which seem to fall into place the minute we accepted that there was this h size. So, in even though as a from our own intuitive feel of nature perhaps it comes across as something that is not natural, in that there is a step size apparently nature does have this step size and and therefore, we have to accept is as is.

So, this is the first equation, foremost equation that brought us to quantum mechanics and it was put together around in the year 1900, Albert Einstein extended this. And he explained the other phenomenon, which is photoelectric effect. Planck had explained black body radiation, Albert Einstein explained photoelectric effect. Where he took exactly the same idea and see he said that light would now be able to initiate would would then be absorbed from an electromagnetic radiation would be absorbed by a body which could then give out photoelectrons, only in step sizes of h nu. Therefore, he wrote down this equation that we see here, for the photoelectric effect. So, this is the equation he put down. The idea also came about that people began to think that if light could be thought of as particles, which would then, which were then being called as photons and then it was quite possible that may be particles also displayed wave like behavior.

And this was demonstrated using techniques which enabled us to see electron diffraction; for example. And then the de Broglie equation came about which basically said that, if there was a any particle that had a momentum p, then we could associate it with it a wavelength lambda and that they would be related then by lambda equals h by p. So, this was the de Broglie relationship. So, these sort of formulize the idea that all matter could now be associated with a wavelength. So in fact, even macroscopic objects could be associated with a, could be assume to have a wave like behavior and using this equation you could come up with the wavelength corresponding to those objects.

You would find then in fact if you did this analysis, you would find that for macroscopic objects, the kinds of numbers would come up with would put you in a situation, where it is more than enough to treat it simply as a particle and not bother about the wave nature of that object.

So therefore, it is not that the quantum mechanics principle, these principles that have put forth become irrelevant or are inconsistent with the macroscopic life as we see it. It is just that the effects become lesser and lesser significant, as you get a larger and larger objects. And so at at, some point it is very very reasonable for you to assume that we can neglect the effect and you will not know any difference in the calculation in in all that we experience and so on. Therefore, quantum mechanics pervades all aspects of life of of nature from very small scale to larger scales, but largely it is not of enough significance at larger size scales. So, having then discovered that, you know having now reach the situation where we recognize that matter can also show wave like behavior.

All of this information of quantum mechanics was then captured together elegantly by a Schrodinger, when you put together this Schrodinger wave equation. So the Schrodinger wave equation is here. And so, he basically captured the idea that since everything had this wave like behavior, we could now associate with any particle or any system. A wave functions psi and that wave function psi would then capture the most important aspects of that system, the attributes of the system. And then the way function psi could then could actually be obtained by solving the Schrodinger wave equation, which would actually put together the constrains that the system is facing. So, that that is how the Schrodinger wave equation is. The Schrodinger wave equation actually pulls together all the constrains that the system faces and if you pull all those constrains together and place it as the terms of the Schrodinger wave equation, what you will get out of the Schrodinger wave equation is the wave function psi. Once you get the way function psi that then represents the system. It captures all the details of the system. So, he came up with this and it seem to actually, nicely meet the requirements of the quantum mechanical description of a systems, but there was some problem in interpreting what psi was? In I am trying to understand; what the psi represent?

And therefore, the other major contribution was that from Born Max, Born he basically said that psi psi star d x would then represent the probability of an electron existing in the location x and x plus d x. So, that is the contribution of Max Born, which would also be the modulus of psi square. So, that was the contribution there. And then finally, there was Heisenberg, who recognized that when once once you have things, once you have particles and systems being described using way functions, then by the very nature of how the description comes about, there there is an uncertainty principle that is present. So, this is the very again, a very non intuitive kind of an or what should I say? Relationship which does not immediately become convenient for us to accept because of our experience with large scale objects, where we apparently do not have any uncertainty in trying to identify the velocity or position of a ball; for example. So, of a macroscopic ball; for example. So he does something that again, he shows to you that the effects of quantum mechanics are not that significant when you get a large scales, but when you go down to small scales, this is an the impact of that particular kind of a behavior quantum mechanical behavior are becomes very significant. So, this is something that we also discuss, I also indicated you that especially with respect to the Heisenberg uncertainty principle that there is this general based on the descriptions you hear of it. There is a, it is likely that you may get the impression, that it has simply a experimental limitation and I specifically emphasize to you that it is not simply a matter of experimental limitation. It is not simply a case of if you try to observe something you may disturb it and and so on. Although, descriptively that seems to convey in a simple sense, what the uncertainty uncertainty principle is, but more so it is it is something more fundamental than that. It is a fundamental requirement that if you treat particles have has a waves distributed across space, then the more wave functions that you need to add, to get it to get localized would increase the uncertainty, would all enable the particle to have possessive wide range of a momenta; for example. So at a given point in time the more precisely you get its position, the less precisely you can specify it is a momentum and this is got to do with the fact that they are mathematically related each to other as conjugate variables.

So so, these are all major concepts that we saw for quantum mechanics in the past few classes and the reason we saw it was was of course, that we realized that the classical description of a particles was not good enough to serve our purposes. Now, we will move forward from here, we will try to develop our next model or improve model. So, to speak for understanding the behavior of a electrons in a solid. And therefore, explaining the properties of a the solid. So, before we do that I will briefly introduce to 4 personalities, historical personalities, of great importance in the history of science. A couple of whom I have already mentioned, but we will still go over then and then we will see, where it takes us.

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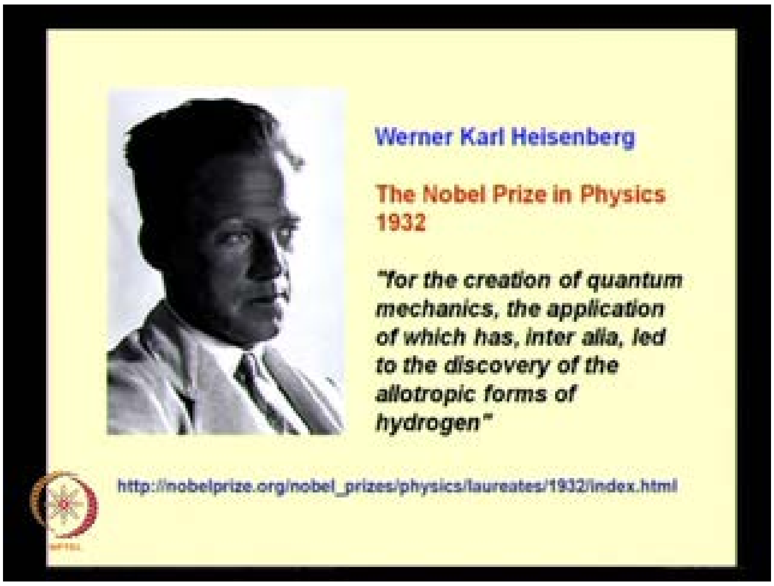

In chronological order in terms of their accomplishments, the first person I will introduce you to is Heisenberg, whom we have discussed in great detail including the very last the very last equation that we saw of the uncertainty principle. So, Heisenberg received his Nobel prize in 1932 and as you can see, it is for the his contribution to quantum mechanics and such. So, he was it was a Nobel prize in physics, in the year 1932 and as I indicated his contributions were very significant to the quantum mechanical description.

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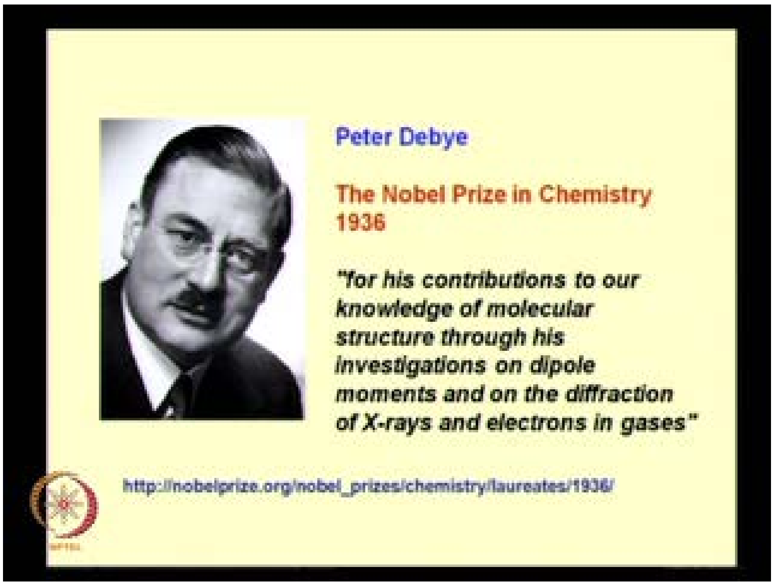

Then we come 4 years down, we we meet Peter Debye, who received a Nobel prize in 1936, in chemistry not in physics in chemistry. And he he had made major contributions to molecular structure. So, molecular structure and through his investigations on dipole moments and also to the diffraction of X rays. So, X ray diffraction was another aspect of a Debye's contributions. So, there are cameras which are Debye Scherer cameras and so on. And so, he had made very significant contributions in all these aspects and for this he received a Nobel Prize in 1936. And Peter Debye is of course, a Debye is a name that a people become very familiar with when you study science.

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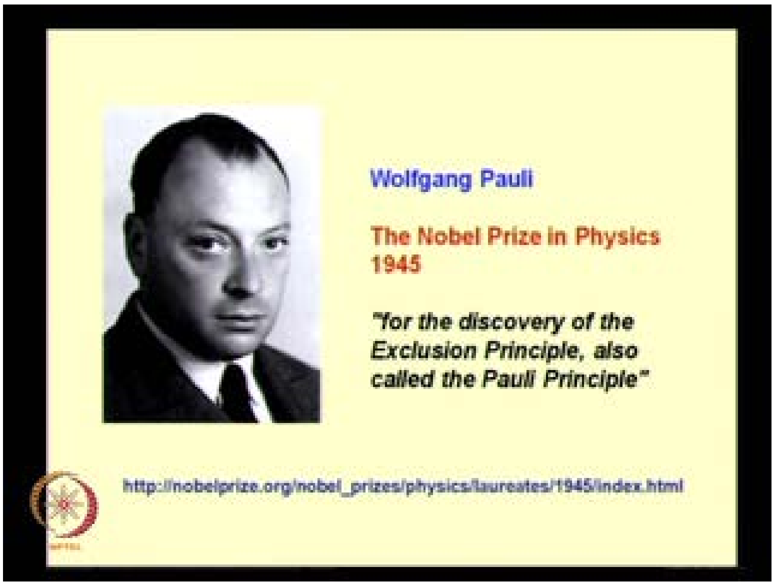

The third person I introduce you to is Wolfgang Pauli, again Nobel laureate, Nobel Prize in physics, in the year 1945. And of course, his contribution is something that we mentioned earlier on which is that he put, he postulated this Pauli's exclusion principle. So, he basically said that you know, if you have this quantum mechanical description of systems existing then you cannot have 2 particles which have exactly the same quantum numbers. They cannot they cannot have all of the quantum numbers being exactly identical. So, that is the Pauli's exclusion principle. It is a very important contribution; in fact we will immediately be using it in this later in this discussion and even in the discussion that will come up in the next class. So, Pauli's exclusion principle is a very integral part of how we are going to look at the behavior of electrons in solids. So he got a Nobel Prize in physics, in the year 1945 so.

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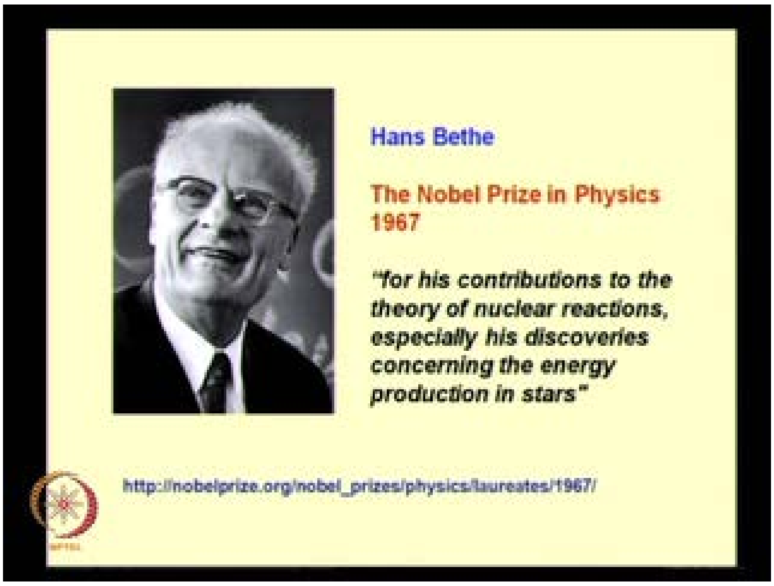

Much later 20 years down the road, we meet Hans Bathe, again a Nobel laureate in physics. So, his contributions are to the theory of nuclear reactions and especially his discoveries concerning energy production in stars. So, we have 4 Nobel laureates spaced out in time over a period of about 35 years. So, Heisenberg for uncertainty principle, Debye for contributions in chemistry and in X ray diffraction. Then Pauli's exclusion principle and then now we have Hans bathe, who has looks at the generation of a energy in stars and such. So, these are 4 personalities, who apparently are you know spread out across the wide areas of science have and not at first glance may be expect for a for Pauli and Heisenberg perhaps not immediately relatable to each other.

And spread out in time, spread out in areas and science, but perhaps the thing that is common to them, the immediate obvious thing that is common to them is that there are Nobel laureates. And so, their highly accomplished in in their areas of work. What is of interest to us, is to is to recognize that there is something more common to this 4 people. Which which is more relevant to our immediate discussion. What is common to this 4 people and relevant to our discussion, is the fact that all of them did their P H D or doctoral thesis under the same person. There was one teacher for who was common to all of these people, they are all they are all students for the same person and who is this person?

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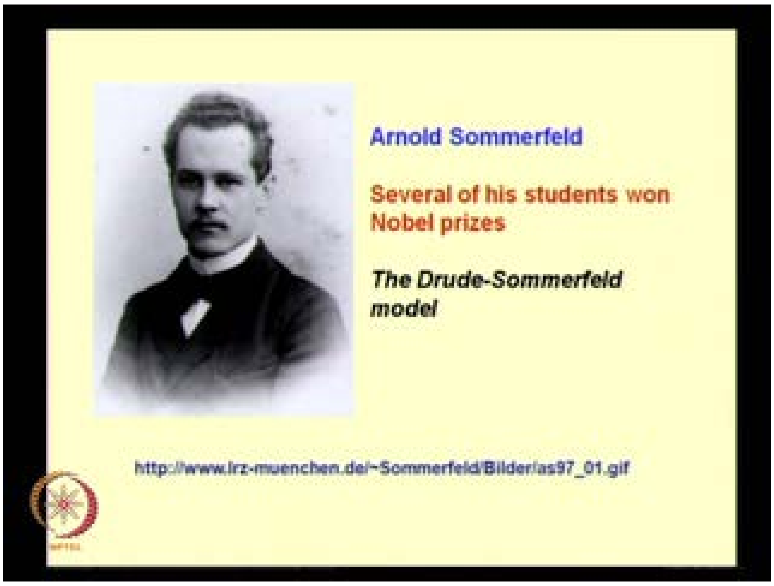

This person is goes by the name; I have the name Arnold Sommerfeld. So, he is considered a very eminent scientist and several of his students, you saw now 4 of his students won Nobel prizes. Which is perhaps perhaps quite unique, I am not aware of any other persons, whose whom who had 4 students earning up us Nobel laureates. So, he has this distinction that he had 4 students who became Nobel laureates. He had several students, who who went on to win several awards themselves and are or also extremely distinguish. The specifically the 4 that we discussed went on to win the Nobel Prize. Sommerfeld, himself was nominated for a Nobel Prize several times, although he did not win it.

And he has made major contributions in the fields of in the fields of mathematics, in the fields of physics and so on. So, he is an highly accomplished person, who did not who of course, who apparently did not win the Nobel Prize though, but has won a number of other awards and was the doctoral adviser for 4 people, who won the Nobel Prize. So, his contribution is many fold as I mentioned, the specific contribution of a Sommerfeld, which is interest to us is the Drude Sommerfeld model. So, what and that is the model that we are going to examine in some detail, the in the immediate time that we are going to discuss this.

And what he has actually done is he looked at the Drude model and looked at what he said that was good about the model, what was said that apparently seem to be lacking in the model and then try to make improvements on it. So, he came up with a new model which is a modified Drude model, as you made you may want to call it and in fact as called a Drude Sommerfeld model or this is considered very important contribution. Because it really helped move forward the theory of the solids, to understand why he said this solids have the properties that they have and what can we do, what can we say about the fundamental properties of the constituents of the solids and how they add up to give us the property of the solid. So, the Drude Sommerfeld model is what we will look at in the immediate.

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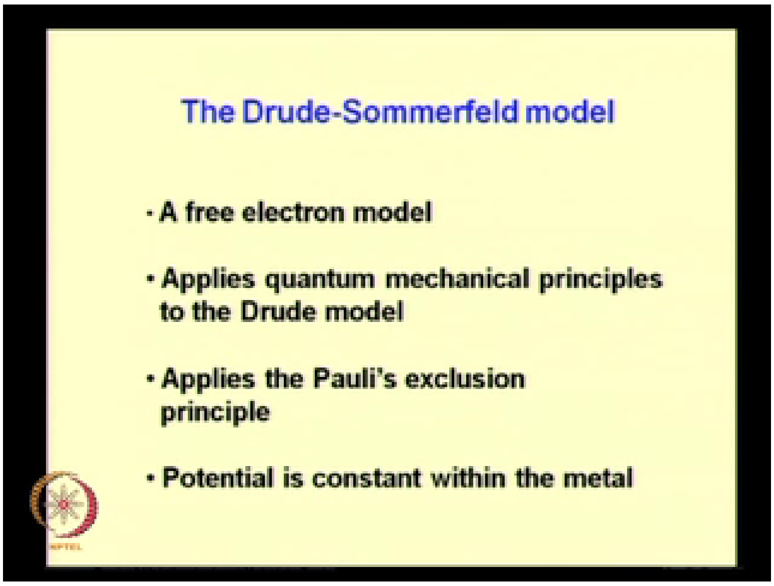

So, the Drude Sommerfeld model is what we will see, it is actually in continuation with the Drude model as I said, it is going to take some features of the Drude model and it is going to improve on that. So, we will see the features that it takes up and features that it improves in our discussion right now.

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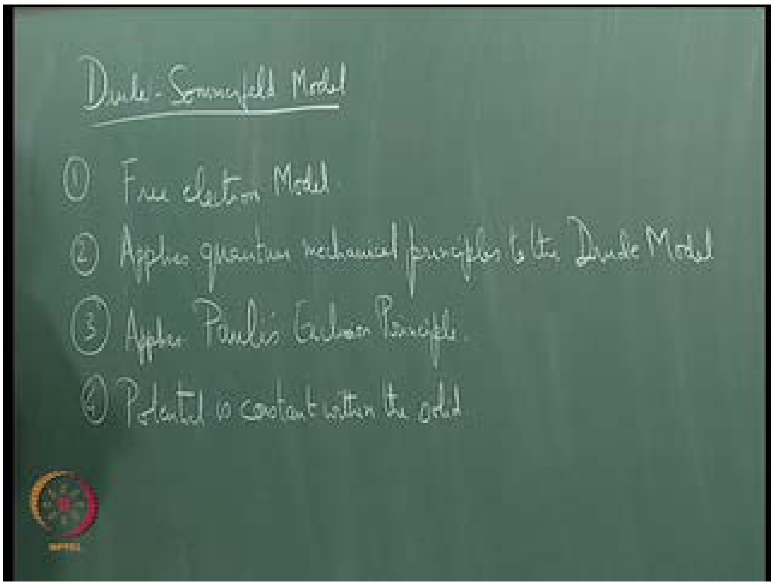

So, the Drude Sommerfeld model, I said the Drude Sommerfeld model is first of all a free electron model. So in this sense, in the sense that we call it a free electron model. In this sense it is borrowing the same idea or it is starting out with the same idea that the original Drude model did.

So, in in the sense that, when we say it is a free electron model, what we mean is that you would think of a solid as containing those ionic cores, which are present within the solid. And that there are electrons, which are free to run across the extent of that solid and that largely those electrons are not really impacted by those ionic cores. So, they are they are free, that is what we mean. There is no specific preference for them to be at any one location, they can freely run across, all of those electrons can freely run across the extent to the solid. This was a primary assumption and requirement in the original Drude model and it is it remains an assumption and a requirement in the Drude Sommerfeld model.

So therefore, in this sense it is the same as the original Drude model. Then, the immediate thing that, the Sommerfeld model, the Drude Sommerfeld model does is that it applies, it takes quantum mechanical principles. And we have discussed all the major quantum mechanical principles that we will atheist immediately use and it takes quantum apply mechanical principles and it applies them to the Drude model. So, what that means, I mean it is put down as a sentence here, that it applies quantum mechanical principles to the Drude model, exactly what that means we will we will see in in just a few moments.

So, but the idea is that that in the original Drude model, the fundamental idea that the electrons are classical particles is being utilized. And we have discussed that, we will again touch up on it at least, but here we specifically move away from that idea. We recognize that, we cannot merely treat electrons as classical particles; there is something more to it. So, we recognize this idea and in the Drude Sommerfeld model that is basically done. It is formally incorporated into the model, formally we incorporate the fact that the electrons are not classical particles, they are quantum mechanical particles, they show quantum mechanical behavior.

And therefore, the way in which we handle the electrons in our analysis has to change, to accommodate for the fact that they are showing displaying quantum mechanical behavior. Specifically, the model also incorporates. So, specifically the model also incorporates the Pauli's exclusion principle. So, we already have the quantum mechanical behavior that is being incorporated, in that specifically we also incorporate the Pauli's exclusion principle. So, that is what the Drude Sommerfeld model is doing. Incorporating the Pauli's exclusion principle into the Drude model, the Drude Sommerfeld model does that additional thing.

It also makes the assumption that; so, this is an assumption that again existed already in the Drude model. And so, this is simply continuing with that assumption that potential is constant within the solid, this ties with this idea that it is a free electron model. So the fact that the potential is constant within the solid, simply emphasizes the idea that the electrons have no specific preference, that they need to be at one that all the electrons need to be in any one preferred location, all the free electrons. So, it recognizes that there are electrons, which are tightly bound, closely bound to the ionic cores and neglects them. And then treats the whole of the solid as being of some kind of a uniform potential. So that, the electrons that have escaped from each of those electron atomic or ionic cores. And therefore, are the free electrons that fill this solid, those electrons are actually free to run through the entire extent to the solid, there is no preferred location in terms of potential that they would have to gather or where they would have, which they would they have to avoid. So, this is being assumed in this model. So, as these are the major ideas that the Drude Sommerfeld model employs and the manner in which we the significance of these ideas we will explore a little more.

And then we will see that, there is a formal way in which we can incorporate these ideas into the model from a mathematical prospective. So, these are the major things for the Drude Sommerfeld model.

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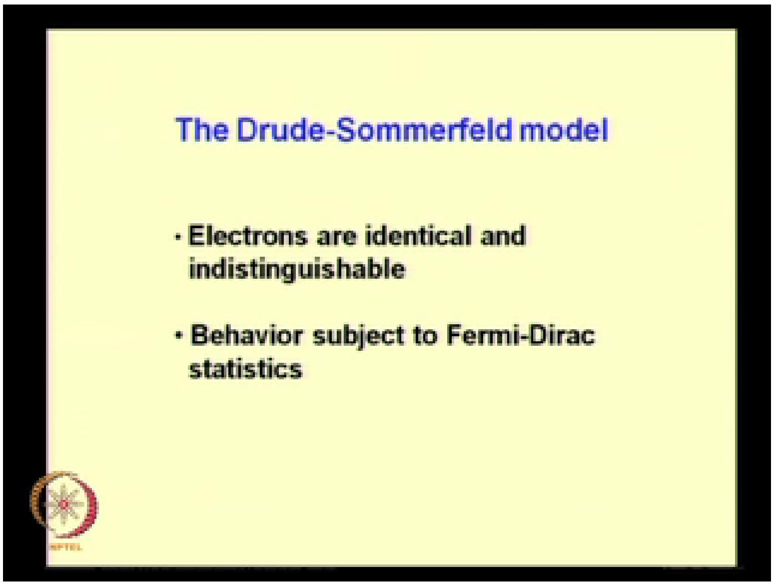

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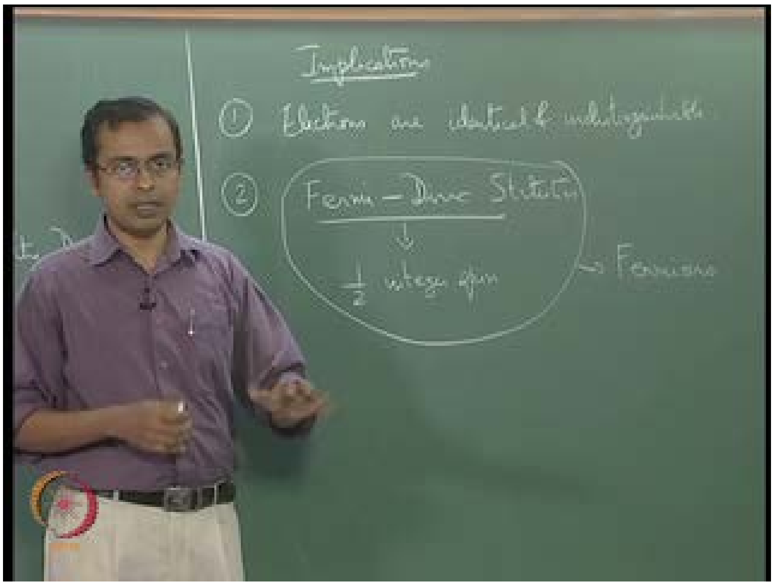

So when we say that the the ideas that I have specifically indicated as the Drude Sommerfeld ideas. When those ideas are now incorporated into the model, there is specific implication to incorporating those ideas. So, what we will look at briefly or the the implications of those ideas. Excuse me.

The first is that a electrons are identical and indistinguishable. The first is that electrons are identical and indistinguishable. So, this is the basic idea that is being that this is the basic point, where in we are incorporating the fact that the Drude Sommerfeld model is employing quantum mechanical principles in the model, where originally the Drude model did not look at quantum mechanical principles treat at the electrons is classical particles. So, the electrons were treated as identical, but distinguishable particles. Now because we are using quantum mechanical description, they are giving treated as identical and indistinguishable particles. So, this is where the quantum mechanical principle comes in.

So so, we will see the significance of that in in just a moment. And therefore, the fact that they are identical and indistinguishable impacts directly impacts the way in which we do the statistics of the set of particles. So, once again as we did with the original Drude model, we will now have to develop a statistical distribution for how the electrons behave within the solid and except that that will now incorporate the fact that the particles are identical and indistinguishable. And therefore, that impacts the mathematics the way in which we put in, put the equations together to develop that statics. And therefore, the result will will also change.

So, when you do that the original work that that actually did this. That looked at identical and indistinguishable particles. And therefore, particles that were demonstrating quantum mechanical behavior and also incorporated the fact that they were obeying Pauli's exclusion principle. So, those that combination of identical and indistinguishable particles obeying Pauli's exclusion principle, that combination was a a examined and the statistic corresponding to that combination was first put together and demonstrated or indicated by Fermi and Dirac. And so, it is named after them.

So so, there is this statistics, which are referred to as Fermi Dirac statistics or Fermi Dirac statistical behavior. So, Fermi Dirac statistics actually examines the, excuse me, the behavior of a set of particles, which are showing us the quantum mechanical behavior in that they are identical, but indistinguishable, but also that they are actually following Pauli's exclusion principle. Now the thing is the for it to follow Fermi Dirac statistics, the we are saying that Pauli's exclusion principle is followed and which normally means that it applies to basically particles that have half integer spin. So, one of the criteria is that you should have half integer spin.

And and therefore, the particles that are having this half integer spin and are identical and indistinguishable and follow Fermi Dirac statistics. So, this combination is then, this a particle that does all of this is referred to as a Fermions. So Fermions, these particles that follow the Fermi Dirac statistics and therefore, actually also have a half integer spin. They are referred to as Fermions. This is to be distinguished from our original set of particles that we explore explore explore or investigated under the original Drude model. So, the Drude Sommerfeld model uses the Fermi Dirac statistics to describe electrons in a solid.

And therefore, incorporates all of those ideas that we are seeing, that the Drude Sommerfeld model is trying to put into the picture, which is that the electrons or quantum mechanical particles and they are following Pauli's exclusion principle. So, the Drude Sommerfeld model treats the electrons as Fermions. Now that you understand what we mean by Fermions, I can simply say this that the Drude Sommerfeld model treats the electrons in a solid as Fermions. And therefore, they meet all these criteria. They also; therefore, we we will put that down the original Drude model, treats the particles as classical particles.

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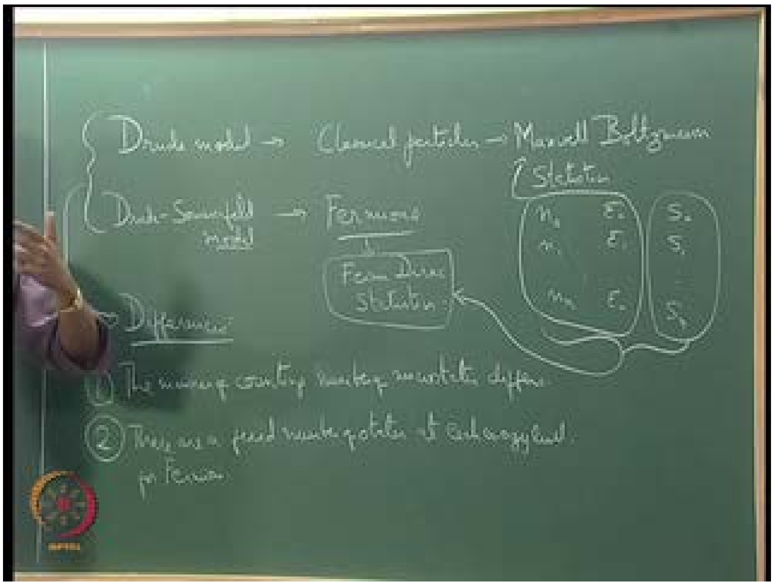

Drude Sommerfeld model treats the electrons as Fermions. So, this is the difference. Drude model treats the particles as classical particles; the Drude Sommerfeld model treats the particles as Fermions. So, this is the difference between the Drude model and the Drude sommerfeld model. So, and I mentioned that we are talking of, when we say classical particle versus a quantum mechanical particle. We are basically saying that in classical particles, the particles are identical, but distinguishable and in quantum mechanical particles, we are talking of identical, but indistinguishable particles. So, when we build this statistic, we were we are actually going to build this Fermi Dirac statistic. We are actually going through, going to do this calculation, which is the Fermi Dirac statistic.

So, the Drude model, which applies classical particle behavior is what we saw and for this we actually went ahead and we we explore this in greater detail. We try to understand, what we said that, when we say classical particle, what is the statistical behavior that we need to impose on the system or or what is that behavior that we are assuming about that system. When we did that the statistical distribution that we found was of relevant to this kind of behavior was the Maxwell Boltzmann statistics. So, when we used classical behavior for the particles, when we assume that the electrons behave like classical particles, effectively we had impose the Maxwell Boltzmann statistics on those particles. So, all the results that we got were consistent with the fact that those particles were assumed to obey the Maxwell Boltzmann statistics. So, an and we went ahead and derive this statistics. We we looked at the mathematical process by which, if you mathematically incorporate all those descriptive things that we say about those particles, we will end up with set set with a certain set of equations, it would then give us how the particles are distributed across all the energy levels available in that system. And so that, is how we came up with the Maxwell Boltzmann statistics.

Now, we are saying that, we have modified the model and it is now the Drude Sommerfeld model, the Drude Sommerfeld model has changed the kinds of assumptions we have made about the particles. And therefore, as I mentioned of the beginning that set of assumptions then represents the model that we are talking of for the particle. So, the Drude Sommerfeld model changes those assumptions, it is basically says that now the particles are identical and indistinguishable. Therefore, they and therefore, they are no longer classical particles and and therefore, the statistical distibution way in which they are distributed changes. Or they are following Pauli's exclusion principle. So, all of that is captured in the set of particles that are called Fermions using the Fermi Dirac statistics.

So, this Fermi Dirac statistical behavior or the Fermi Dirac statistical distribution is the is the thing that we are going to derive. So, we we will derive this Fermi Dirac statistics because it is of immediate relevant to what we are doing, in particular it will also tell us specifically what is it? We can say about electrons present within the solid, when we understand, when we pull this derivation together, we will understand in in the the results of the derivation will show us how we have changed the picture of the electrons in the solid.

When when I say we have changed, all I am saying is we have, how we have changed from a theoretical prospective, the electrons in the solid already already have whatever it is that they have. So we in that sense, in the in that fundamental sense, we are not altering anything about the solid. We are only trying to look at our ability to describe it and so our ability to describe it using Fermi Dirac statistics changes the kinds of things we could, we will say about those electrons. And and on the basis of those change statements are we are now able to make of those electrons.

We will see, if some of the properties that we have a difficulty explaining; for example, this specific heat. That we have a difficulty explaining using Maxwell Boltzmann statistics, we will see if by using Fermi Dirac statistics that anomaly that existed on the on the a prediction of the specific heat of the electronic contribution to specific heat. That anomaly that we face, we will see if by simply incorporating Fermi Dirac statistics are we have been able to overcome that problem. Therefore, that is very important contribution, here we we over estimated the electronic contribution to specific heat by a factor of 100. So, that is quite a significant over estimate. So, two orders of magnitude we we over estimated, we will see if by using Fermi Dirac statistics we are able to actually make make the correction.

So so, these are, this is the direction in which we will proceed, that we will look at the contributions in that way. Even as so, as we go ahead and make the development, we will I will just high light here, some of the key aspects in which these 2 statistics are going to differ. So, and that will set the base that will enable us to then properly relate whatever it is that we are deriving at this point with what we have derived earlier. So, I will high light those specific aspects of those derivations that are different, that are going to be different. Now relative to what we have derived earlier on for the Maxwell Boltzmann statistics.

So, the first thing that we first manner in which, these 2 differ differences. The first thing is of course, as I said descriptively Fermions are identical and indistinguishable. Whereas, Maxwell Boltzmann classical particles are identical and distinguishable, what is the difference from a mathematical prospective? For a mathematical prospective, what it means is that, when we try and write this equations for the manner in which the particles are distributed at various energy levels. If you consider a situation, where you have a certain number of particles in one energy level and so let us say, n 1 particle sitting in one energy level and n 2 particle sitting in a other energy level. If you have this situation, if you simply swap 2 particles, if you move one particle from the higher energy level to the lower energy level and at the same time move another particle from the lower energy level to higher energy level. What have you done? You have not changed the number of particles at each of those energy level. So, you still have n 1 particle sitting at the higher energy level, you still have a n 2 particles sitting at the lower energy level. So, in terms of the number of particles those 2 energy levels, you are not changed anything. Now if you have this situation, but they are classical particles, since the each particle is distinguishable from the other, simply swapping these particles would now be treated as another arrangement.

So, even though the number of particles at the higher energy level remains the same, number of particles in the lower energy level remains the same and everything else about the system remain the same. The fact that you swapped 2 particles, you moved 1 particle up and simultaneously you moved another particle down. This situation, this step, it will now result in the system being treated as though it had attained a new state. So, it would be counted as a new state, in a classical system, in a in a classical way of counting the statistics of the state of the system.

So, when we looking at micro states of the system, macro state of the system and so on, this would be one another way in which the same micro state is being attained. In in the quantum mechanical description of the system, when you when you say that the particles are identical, but indistinguishable. When they are identical and indistinguishable, if you swap a particle you have n 1 sitting at higher energy level, n 1 particle sitting at a higher energy level, n 2 particle sitting at a lower energy level, when you just take 1 particle from there move it down and take 1 particle from below and move it up in energy level. Since the particles are anyway indistinguishable, what this means is that this swap cannot be treated as a new state, it cannot be treated as a as a new implementation of that micro state. It cannot be treated as that and why is it, why is this issue coming? It is coming because of what I already described for you regarding this idea of indistinguishability. It is simply that, when you say it is a quantum mechanical particle, it we no longer uniquely think of it is a hard object, we think of it has being distributed in space, as something that has being distributed in space there is a probability of it is existence, which is distributed in space. So, when when when you have 2 particles have with certain, I mean to identical particles 2 electrons for this in this case, having specific attributes, the fact that they are in this indistinguishable and the fact that they are actually just probability distributions across a particular regional space. Simply, implies that there is always an inherent chance that, they may swap with each other.

So, more specifically we use the example that when they colloid and they move up apart. If they were classical the fact that, you they collided and moved apart, would still enable you to say what was the ball that started on the what does the particle that started on the left side and where it ended up? What was the particle that started on the right side and where did it end up? The minute it is quantum mechanical and these are only probability distributions, when you go through this process, there is as a chance that they would have swapped, there is a non 0 chance that they would have swapped. And therefore, you cannot say for a fact, that the particle does started with on the left side is the is the particle that is sitting here, the particle that started on the is the particle that is sitting here. They might have anyway switched. So, when you have such a situation, when you have 2 energy levels and you switch, you do everything else is the same and you simply switch the particles, you cannot with confidence call it as a new state. Because for all you know it might have occur at anyway and even if you had not intended for it occur.

So, between the Drude model using Maxwell Boltzmann statistics, so more specifically between the Maxwell Boltzmann statistics and the Fermi Dirac statistics. The first and most important thing that changes is the manner in which you count the number of micro states. And that is very integral, I mean, when I say that I have change the manner in which I am counting the number of micro states or the number of ways in which that micro state can exist, if the the minute I change that manner in which I do that. At that very significantly alters the result that I am going to end up with. Because that is the basic idea that is there in that whole process.

It is the it is the most it is the core of that entire statistical distribution process. So, whatever result we get, very critically depends on the manner in which we count the number of ways and which the micro states can exist. So, a fundamentally they differ and I and I just mentioned, they differ simply because of the character of the particle that we have made an assumption, the assumption that we making about the character of the particle. So, that is the fundamental manner in which they differ. So, this is one very important difference between Maxwell Boltzmann statistics and Fermi Dirac statistics.

There is a second important difference and that is got to do with the fact that as Fermions, the particles are assumed to obey the Pauli's exclusion principle. So, the Pauli's exclusion principle basically tells us that when you have quantum numbers assigned to all the particles, then you have a situation where all the quantum numbers including the spin quantum number, all of the quantum numbers cannot be exactly the same for 2 electrons. So, at least one there has to be a change, in at least one quantum number, so at least one quantum number has to differ. Therefore, when you look at it that way, the fact that there are quantum numbers and so on, it also means that at a given energy level, we will have now have to specify the number of states that are available. So, if so in for each energy level, we will now have to say that quantum mechanics allows us to have so many energy levels and then we will look and see how we are going to fill those numbers of states using particles.

So, the Pauli's exclusion principle creates situation, with respect to differences where you are talking of Fermions, we are basically saying that there is a fixed number of states at each energy level. And that is where; we are able to say that with respect to the states, we have to ensure that you cannot have more than (( )), if those states also incorporate all the quantum numbers. And that is how you are actually indicating the number of states at that particular energy level. We ensure that that puts a limit on the number of particles you can place that energy level. So, this is a very important difference, when we did the Maxwell Boltzmann statistics, when we derived the Maxwell Boltzmann statistics, at at at one of the early stages of the derivation, we simply said that we will have let there be n 0 particles at E 0 energy level, n 1 particles at E 1 energy level and so on. So, that is how we did it, we came down and said we have n r particles at energy level E r. We when we did this description for the Maxwell Boltzmann statistics, at no stage did we place any restriction or what is the upper limit for n 0, we did not place any restriction for the upper limit of n 0, no restriction for n 1, no restriction for n 2, n 3, n 4 and so on. Up to n r they was no restriction at all and that is fundamental to the idea that these are classical particles and and there is no question of no 2 particles being in in exactly the same state, all those issues are do not arise if you are talking treating them as classical particles. And therefore, we simply had some number at some at a given energy level; we did not care of what that number was, now the minute we instead of treating them as classical particles, we now start it treating them as Fermions.

Which are quantum mechanical particles also following the Pauli's exclusion principle? Once we do that, we cannot just in addition for the system itself, in addition to the energy levels that are available in the system, we are also up front placing some restriction that at a given energy level. There are only so many states available. So, we are saying that in in our system now in the description for our system, once we talk of Fermi Dirac statistics, so this much would be valid for Maxwell Boltzmann statistics. In addition we would now incorporate as 0 states, S 1 states, S r. So, only if if you have only n 0 at E 0, n 0 particles at E 0, n 1 particles at E 1 and so on. And n r particles at E r and this is all the restriction you place on the system and your entire description for the system, your mathematical description that you build for the system is based only on this much information, which is present within the first box, then that would then that would lead us to the Maxwell Boltzmann statistics. If on top of it we also place the restriction that there is at E 0, you cannot an arbitrary number of particles because there is a fixed number of states available at E 0. And therefore, there is a certain upper limit on the number of particles you can place in those this number of states because Pauli's exclusion principle exists or is valid for our system.

So then n 0 is not some arbitrary number, it is in some way restricted by S 0, n 1 is restricted by S 1 and so on. n r is restricted by S r, if you take this entire body of information that there is certain number of energy levels, there are the which are indicated here. There are certain number of states at each of those energy levels, which are indicated here and the fact that there are the particles will now have to populate the states within those restrictions. The fact that there are so many limited states at each energy level and Pauli's exclusion principle prevents them prevents you from putting any number of particles within a limited number of states because if you if you cross some number at you will be forced to ensure that you would be forced to put 2 particles into exactly the same state.

And Pauli's exclusion principle prevents you from doing so. If you assume Pauli's exclusion principle is valid, we can we cannot do that. So, that that places an upper limit on the number of particles at each energy level. So, this combined picture now. The energy levels the number of states at those energy levels and then the number of particles that you can place in those states. That combined picture will now lead us to the Fermi Dirac statics. So, if you step back here, the 2 major differences are that the manner in which we count the states simply because we are they are identical and indistinguishable under Fermions, but they are identical and distinguishable as classical particles. In that fundamental way, the the 2 statistics will differ plus the fact that at each energy level, we now have, we are also specifying controlling the number of states available at each energy level. And therefore, you cannot put an arbitrary number of particles at at a given state, at a given energy level.

So, what we will do in our next class is, we will actually derive the statistics and all of these ideas that I have now described to you and where I have shown you the difference between these 2 systems between what is that we have done? And where it is that we are going, we are headed? All of these ideas will be incorporated in our derivation of the Fermi Dirac statistics. And we will come up with the actual final result for the Fermi Dirac statistics; we will then see that having got that result, what does that imply in terms of material properties. And how successful is this new description in taking care of the Anomaly's that the Drude model had a problem dealing with and so in in other words we will see in in what ways is the Drude Sommerfeld superior to the original Drude model and then we will see if there is if there is room for even further improvement before and beyond that. So, with this we will halt for today, we will pick it up in the next class. Thank you.