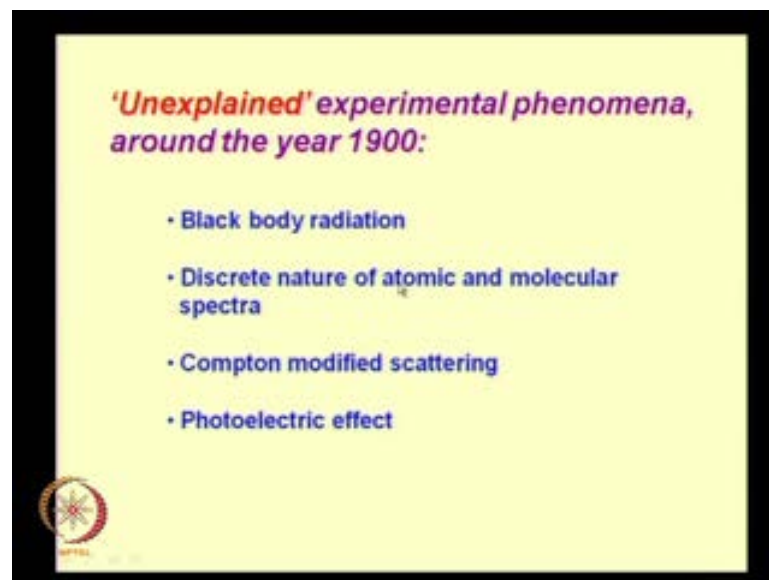


**Physics of Materials**  
**Prof. Dr. Prathap Haridoss**  
**Department of Metallurgical and Materials Engineering**  
**Indian Institute of Technology, Madras**

**Lecture No. # 15**  
**History of Quantum Mechanics – 2**

Hello, this is the fifteenth class in our physics of materials lecture series. In the last class we had a discussion on the history of quantum mechanics, and we will continue with that discussion through this class, and then take it forward.

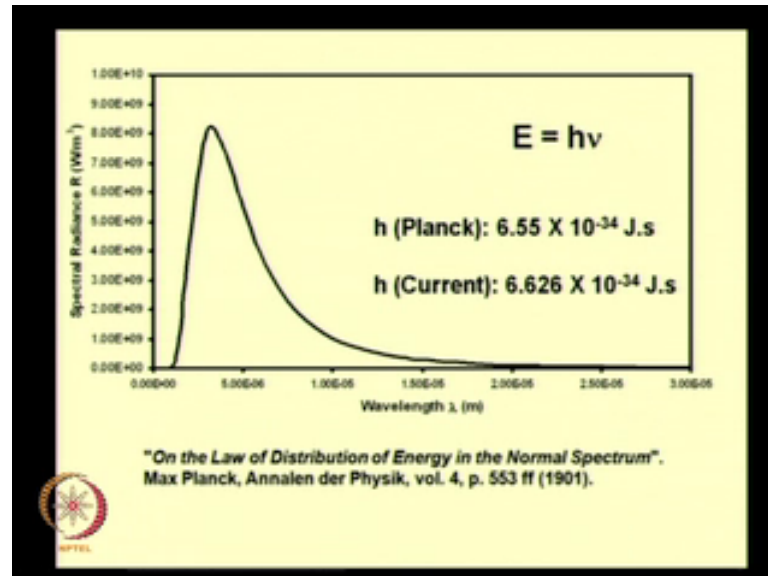
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So, we began last class, I just wanted to summarize a couple of things from there as we proceed forward, we began the last class by looking at some unexplained phenomena at the end of around the year 1900 and amongst them, where the black body radiation, discrete nature of atomic and molecular spectra, Compton modified scattering and photoelectric effect. And we looked in great detail at the black body radiation, it **it** was one of these unexplained phenomena that had been examined, and a theory had emerged for how to explain the data that you obtain from a black body. We looked at what a black body is and how you collect data from it, what kind how the data looks that you get when you get it from a black body. We also briefly discussed the classical way in which people

had try to analyze the data, we looked at the **the** difficulty in explaining the data using the classical approach.

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And, then we looked at the approach that Planck headaker. So, the what you see on your slide is basically the black body data, and the kind of approach that Planck headaker. So Planck head basically to summarize, he had basically put in this equation  $E$  equals  $h \nu$ , where the idea he was trying to explore **he was trying to explore** the idea that, when you have energy being released by a black body at a certain frequency or being observed by a black body at that particular frequency for that to occur, he said that he just assume for the moment that there would be a step size involved in this process. And that step size would then manifest itself in the form of a constant  $h$  and, so he said **excuse me** the energy would be  $h \nu$  and he at that point did not know what the value of  $h$  would be and he basically said that, so it will it some kind of a constant.

So, this  $h \nu$  then represents a step size or a quantum. And, if  $\nu$  the frequency is very low then the step is very low, so the step size is low. So, at low frequencies and therefore, at high wave lengths here, the step size is very low and therefore, even if the overall energy being supplied to the system or being released by the system is small, even if that over all energy is small, since the step size is small those modes are active. So, those modes are able to participate in this those frequencies are able to participate in this process. So, you see the data available from all of those frequencies, when you get to

higher frequencies or therefore, lower wave lengths, the  $h\nu$  value is high and therefore,  $h\nu$  is high. And therefore, the step size involved in the release of energy at that particular frequency is now large, so the  $h\nu$  is therefore large.

And, if the overall energy available to the system is small, then the  $h\nu$  may turn out to be a value that is so high, that you are unable to release energy at that frequency. So, it is a matter of the relative quantity  $h\nu$  relative to the amount of energy available to the system. So, if this is larger than the kind of energy that is available in a shared manner through that system then that particular frequency will not be participating in that process. So using this idea, he try to impose this idea just as a an approach to see if there was a way to cut off the contribution of higher frequencies and therefore lower wave lengths to the black body radiation process, which is what the data show, the data show that at higher frequencies or lower wave lengths.

The contribution of those higher frequencies and lower wave lengths appear to diminish and actually drop to 0 passed a point. So, he wanted to see what is the way in which you could accomplish this in some kind of theoretical fashion and he just made this assumption to see if, that would enable such a process by doing so he was **he was** able to successfully explain this fall of **of** intensity as you go into a half spectral radiance, as you go to higher frequencies or lower wave lengths. And, initially he did not know the value of  $h$ , he just did a curve fit using his theory and assuming some unknown value  $h$  for it and he obtain this value  $6.55 \times 10^{-34}$  joules second with further refinement today the current accepted value for  $h$   $6.626 \times 10^{-34}$  joules second.

So, you can see that, you know even hundred years ago very first stumbled upon this theory, he was able to get a value for this constant, which is very nearly what we use today. We will take a moment to even to examine this a little bit more to understand a certain certle aspect **certle aspect** associated with this entire process that Planck as given us now. The point is this  $h$  the Planck's constant is **is** effectively now we **we** accept that this is a some kind of universal constant, which we are now have, which we are able to use since then for a variety of purposes, but the **the** point is it has as it has this value or in fact this current value which **which** we see here  $6.626 \times 10^{-34}$  joules second.

So, the issue is that this is a very small value. So,  $10^{-34}$  as you were aware is a very small number. The key point that we should recognize here, is that if you look at this equation  $E = h\nu$ . If and we accept the idea that  $h\nu$  represents some kind of a step size relative to which the process will occur. If  $h$  had been 0, in a hypothetical case, if Planck had done all this in and in a hypothetical situation, he discovered that  $h$  actually is 0. If  $h$  is 0 then regardless of the frequency, so regardless of the frequency the frequency can participate in the process, because the step size now associated with that frequency is 0.

So whatever little energy is available that frequency can still participate in the process, but since  $h$  is non zero it is a small number, but specifically it is a non zero number. It is a small, but non zero number that is a very specific combination it is a small, but non zero number and since it is non zero, we find that the contribution of specific frequencies especially higher frequencies is now removed from the picture. So, this entire field of quantum mechanics, the entire science of quantum mechanics that has evolved, that was discovered by Planck and then has since evolved itself traces itself back to this one specific idea or specific fact, that we now accept as a fact, that there is this universal constant  $h$  and it is a small number, but it is a non zero number.

So it is that is what makes quantum mechanics possible or rather because it exists that way we are forced to accept quantum mechanics as a manner in which nature exhibits its features. So, so this  $h$  being a non zero number that such a  $h$  existed and that it is a non zero number, is what Planck enabled us to recognize. And by this process, he enabled us to recognize that nature as following this rule, that there is some step size, which is involved in all of these processes which are the quantum as we would call it. And therefore, to understand nature, to understand the various ways in which nature exhibits itself, we have to accept this quantum and that is how quantum mechanics is come to be.

So, to summarize again on this slide, I wish to point out that we have to realize that quantum mechanics is directly related to the fact that there is this universal constant  $h$ , which is a small, but specifically a non zero number. If  $h$  turned out to be 0, we would not really have had quantum mechanics. So, we discovered that this whole process exists, so this is what we did last class we discuss this in great detail and I will just summarize it here on the slide for you. So, we will proceed forward from here, as I mentioned one of the other phenomena that people did not have an explanation

for was the photoelectric effect. The photoelectric effect, the key features of this phenomena, where basically that, when you actually had a surface and you had light incident on it then based on the surface and based on the wave length of light that you had incident on it, some of the surfaces would eject electrons so...

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**Photoelectric effect**

**Key features of the phenomenon:**

- 1) No electrons are ejected regardless of intensity of the incident radiation, unless  $\nu$  exceeds a threshold value
- 2) Kinetic energy of the ejected electrons increases with  $\nu$  of incident radiation and is independent of the intensity of the radiation
- 3) Even at low intensities electrons are ejected if the threshold  $\nu$  is exceeded.

$$\frac{1}{2} m_e v^2 = h\nu - \phi$$

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And the, so this phenomenon was known and it had been studied experimentally and the generalizations that could be made of the phenomena are listed here, first was that no electrons were rejected regardless of the intensity of the incident radiation, unless the frequency exceeded a certain threshold value. So, frequency was a very important parameter here, it was not just the intensity, the frequency was much more important than intensity in initiating the process, it did not matter how much intensity you had if you did not have a certain minimum frequency, no electrons are coming out of that system.

The kinetic energy of the electrons ejected kept increasing with the frequency, once you cross that threshold frequency, but was but the kinetic energy remain independent of the intensity **right**. So, both the release of the electron remains independent of intensity and the kinetic energy of the electron remains independent of intensity, both of them seem related to the frequency. At the same time, even at very low intensities, if the threshold frequency was exceeded there were electrons being released by the system. So, these were the three major features of this phenomenon, which is the photoelectric effect, so

these three features, where studied and Einstein was able to put together an equation which is what is listed down here,  $\frac{1}{2} m_e v^2$ , this  $m_e$  is the mass of the electron,  $v$  is the velocity of that electron, so this is the kinetic energy of that electron.

And as I said, it seems to be increase with the frequency of the incident radiation, so it is proportional to the frequency and there is a certain threshold that you have to cross before this occurs, he basically put in (no audio from 10:41 to 10:54) So, he propose this equation  $\frac{1}{2} m_e v^2$  is  $h\nu - \phi$ , where  $\phi$  work is the work function of that material. And he use this term here, he not only use the idea that the kinetic energy of the electron was proportional to the frequency, he just through in this constant, which was Planck's constant  $h\nu$  into this equation. And again accept, he was basically introducing this idea that once again there some kind of a step size involved here and he **he** push this  $h$  here, trying to see if that was the same process that was involved here. And he came up with this equation and this equation actually fit the data very well.

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$$\frac{1}{2} m_e v^2 = h\nu - \phi$$

**Explanation of Photoelectric effect suggests:**

- 1) Electromagnetic radiation of frequency  $\nu$  cannot possess any arbitrary amount of energy – it can only possess energies  $h\nu, 2h\nu, 3h\nu, \dots, nh\nu$
- 2) Electromagnetic radiation of frequency  $\nu$  behaves as though it consists of 1, 2, 3, ..., n particles each with energy  $h\nu$

*These 'particles' of light came to be known as 'photons'*

Newton believed that light consisted of particles called 'corpuscles', but this idea was abandoned with the discovery of interference and diffraction phenomena which indicated the wave nature of light. (1675)

So, the by so, so what do we have here, we have an equation here, where the kinetic energy of the electron is the related to the frequency a specifically in the form of  $h\nu$  minus some constant value which is the work function of that system **right**. So, given that we are we find that this equation fits the data, what is the significance of it. So, once Einstein found that you know this is the kind of data that the equation seems to explain the data fit the data correctly, what is the significance of this equation, what does it

imply. The implication is something like this, it appears that the electromagnetic radiation of frequency  $\nu$  cannot possess any arbitrary amount of energy, so once you have a specific frequency for the electromagnetic radiant, for one specific frequency  $\nu$  it appears that, it cannot have any arbitrary amount of energies specifically, it can only **possesses** possess energies  $h\nu$ , two  $h\nu$ , three  $h\nu$  etcetera till  $n h\nu$ .

In other words, you could not have  $1.25 h\nu$ , if you if the only frequency available was  $\nu$ , you could not have  $1.25 h\nu$ , you can only have one  $h\nu$  or two  $h\nu$ . So, this is **this is** what the equation seems to imply, so it appears that if you look at electromagnetic radiation of a any frequency  $\nu$ , it seems like it is behaving as though it consists of one, two or three particles or  $n$  particles each with energy  $h\nu$ . So, electromagnetic radiation of frequency  $\nu$  behaves as though it consists of one, two, three or may be  $n$  particles each of frequency  $h\nu$ . So, this is a big change in the way, in which we look at electromagnetic radiation.

So whereas, previously we have normally thought of electromagnetic radiation as waves, we suddenly find that there is one phenomenon. The photoelectric effect, where the manner in which the radiation is interacting with the sample leading to the ejection of electrons, that manner seems to indicate that the radiation is actually arriving at the surface and manifesting itself at the surface, as though it were a particle. It had as they were one, two, three, four particles each with energy  $h\nu$ , all of those particles have a are associated with a frequency  $\nu$  and are associated with an energy  $h\nu$  and it appears that the entire process is occurring that way.

So, suddenly what was always thought of as a wave seems to display particle like behavior and these so called particles of light came to be known as photons. So, Einstein through his equation for the photo electric effect introduced this concept of photons are the equivalent of particles of light. In the sense that, he revealed to us that there are experiments, there are situations, where the manner in which electromagnetic radiation behaves is very similar to that of particles or you could explain the phenomenon, if you consider it as particles and treated them as particles. So, those particles came to be known as photons.

Incidentally this is you know past nineteen hundred, but if you go back for more than two hundred years, earlier than that Newton himself had proposed that light consisted of



particles, he called them corpuscles at that point in time. And so that was his idea more than two hundred years before photons came along and he really was examining that idea in great detail, but all of that fell apart when people discovered interference phenomenon phenomena and diffraction phenomena. So, once the people notice that in fact light could display interference and it could display diffraction and that those are phenomena that can be explained, if you treat light as a wave and you can actually calculate all the features that you see in an interference pattern or in a diffraction pattern by treating light as a wave.

Once people discovered that, Newton's original idea of corpuscles was dropped, people basically did not think that this was the worth idea worth pursuing, because there was enough strong experimental evidence that light behaved as waves and did not behave as particles. So, with that that idea ended and it two hundred years later, the idea resurfaced because there was a new experiment, where suddenly light also displayed the phenomenon, displayed characteristics that you would normally associate with particles.

So, the credit for this remains with Einstein because although Newton suggested it he really did not have any specific experimental data to back up his idea or any any theory which would actually be a proven theory, which would actually support his idea, it was just an idea that he was working on and so with that it had to be abandoned, but Einstein actually was able to show, that there was an experimental data against which this idea of particles especially taken in taken along with the idea of Planck that, you can have this quanta of energy associated with a radiation of frequency  $\nu$ . Taking all this together, he was able to show that light can display particle light behavior and therefore, Einstein gets the credit for treating and reviving this idea that light could behave as particles and which are photons.

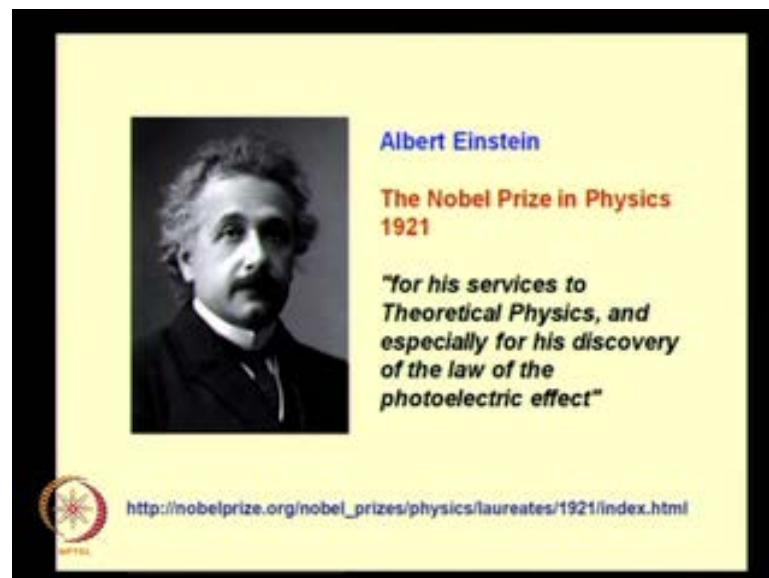
Incidentally, I will also I also wish to point out that while Einstein did this, he basically he extended Planck's idea, he took Planck's idea which he Planck got employed for black body radiation and employed it now took it across to a photoelectric effect, which was another which was the other phenomenon that I am not been really explained. There were a lot of people and we believe including Planck himself, who were quite doubtful whether this was the right way to do it, in the sense they they they faded that, although they got stumbled upon this idea of quanta and and such there was a lot of discomfort on whether or not nature actually behaved this way.



It appeared like they had stumbled upon something, which was working out conveniently and helping **helping** them get good fits for various experimental data, but there was a lot of apprehension, that perhaps somewhere fundamentally they were not right about it. A perhaps there was some further underlying theory, which would not **which would not** require this kind of quantization. So there was a lot of discomfort about this idea of quantization, so when so I **I** think it is particularly important for us to recognize this, because when you and I look at quantization and quantum mechanics. And if, for some reason we experience some discomfort in trying to accept it and utilize it as is.

We can take comfort in the fact that the greater scientist of our **of our** times that we are aware of also had an equal amount of discomfort in accepting these ideas. So, the u state, but somewhere in the back of their mind, they were not completely convince that nature actually behaved this way. So, it is with a lot of such thought process that the science evolved Einstein himself was a quite oppose to the idea of how the basic principal of how quantum mechanics existed, he was quite unconvince, that it was the final ultimate truth of a fair bit of the science that we saw **excuse me**. So (no audio from 18:41 to 18:48) So that is the story of the photoelectric effect.

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And **and** for this of course, Albert Einstein got his Nobel prize in physics, I am sure you are also aware that Albert Einstein is famous for relativity, the theory of relativity, both the general theory of relativity and the special theory of relativity, but he got his Nobel

prize for this equation, which **which** help describe the photoelectric effect. So, we need to recognize that this is a very profound, we have already seen that Planck got a Nobel Prize, we also seen that Einstein got a noble prize for this work.

And, the suggest indications that what they stumbled upon was something phenomenal, it **it** had not been really recognized I mean it **it it** had a not been recognize still that point and they completely change the **land cape of science** landscape of science. So, he got his Nobel Prize in physics in nineteen twenty one. Some of the photos you see here are from the Nobel organizations website, if you go there you can definitely see their lectures, you can see lot more details of all of these famous people and the work they have done **alright**.

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**Louis de Broglie** proposed (1924) the idea that any particle, travelling with a linear momentum 'p', can be thought of as having a wavelength  $\lambda$  given by:

$$\lambda = \frac{h}{p}$$

**'Wave-Particle duality'**

The possibility that particles could be thought of as waves, was successfully explored by **Davisson and Gremer**.

They demonstrated that a beam of electrons could diffract. Electrons from a heated filament, incident on a Ni sample, demonstrated diffraction (1927)

So, Einstein proposed that light could behave as particles, so are **are** demonstrated characteristic of particles in specific experiments, so that was Einstein contribution to the quantum mechanics a picture. Planck initiated it an Einstein brought for this idea that light could actually behave like particles **de Broglie** Louie de Broglie, I believe that's how is name is pronounced, he proposed 1924, that **that** any particle which had which was travelling with some linear momentum p could be thought of as having a wave length lamda, such that the lamda would be given by this equation. So, once again you see h the Planck's constant appearing here and this is p the momentum of that particle

right. So, you have  $h$  the Planck's constant and  $p$  the momentum of the particle and  $\lambda$  is the result of  $h$  by  $p$ .

So, he proposed this idea, that any particle anything that we have conventionally thought of as a particle could have a wave length associated with in and this is what it could work out to. And this idea, when I say any particle in principle, you can extend this idea to a particle that is something, that is, that we can handle. So, when when you look at you know a ball that we have playing with let say even a plastic ball or a like a board like what I showed you couple of classes ago are any other ball that you find are any other object. In principal, you could use this, if we if we if it had a linear momentum associated with it. In principal you can use this equation to see the wave length associated with it.

So, it is it is something that we can extend to life size objects except that we will find that these wave lengths are not of any greater relevance will we talk of life sized object. So that therefore, we do not although in principal this is a concept that we can extend to some larger objects, it is not something that is of any significance in our activity because it is it is of a size scale that would be irrelevant to our activity, we would not be sensing it in a in a physical science at our size scales. So, he came up with this idea and this was actually prove up.

It was successfully explore by Davisson and Gremer they did an experiment, where they actually heated a filament which would then emit electrons and those electrons, where then incident on a nickel sample. And, it they were able to see in a in a reflection mode, the diffraction pattern for that nickel sample, so and as I mention before that, when you see diffraction or interference that gives you an indication that you are seeing wave like behavior. So that is the because the theory that explains diffraction or interference comes from wave like behavior. So, the fact that they were able to see diffraction from a source which was actually giving them electrons, then implied that electron was displaying a wave like a behavior in that particular experiment, so that is what, it implied right.

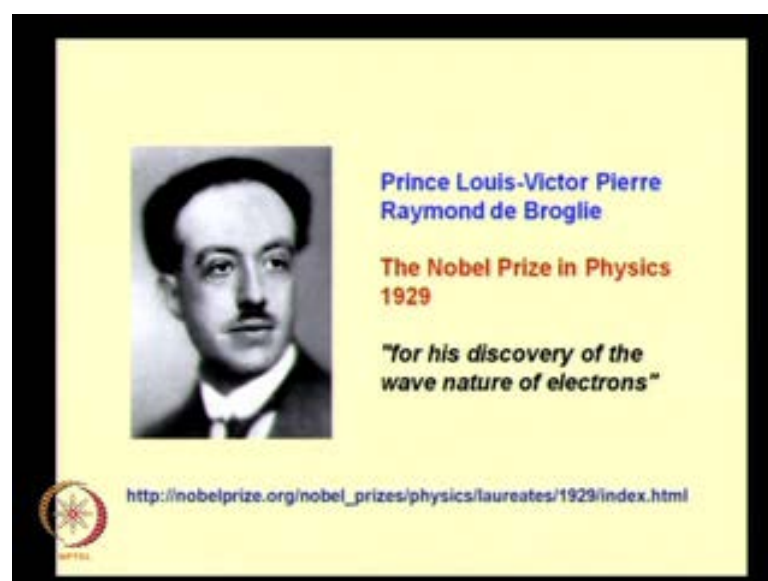
So, what they were able to show is that, what we conventionally thought of as a particle an electron was actually also be capable of showing as wave like behavior. And, it so happen that the, an they were also able to say that you know we we are able to say with with some level of certainty, what is the wave length we can associate with that particle.

So they were able to prove this idea of the Louie de Broglie and **and** able to establish that his idea of particles also showing that any **party could** particle could also show wave like behavior was actually correct. In fact incidentally, I mean as is true with a most of you know path breaking science there is some element of luck involved in all of this.

The kinds of voltages they used then it really know exactly what they were dealing with at that point in time. The kind of voltages they used somehow enable them to get the diffraction pattern in a manner that they could actually absorb and that really help them out. And they are five needs in that sense and they were able to show that particles can show wave like behavior. So Einstein was able to show that light are what us conventionally thought of us waves, could show you particle like behavior and Louie de Broglie and Davisson Gremer showed as that what was conventionally thought of as particles an electron, could show as wave like behavior.

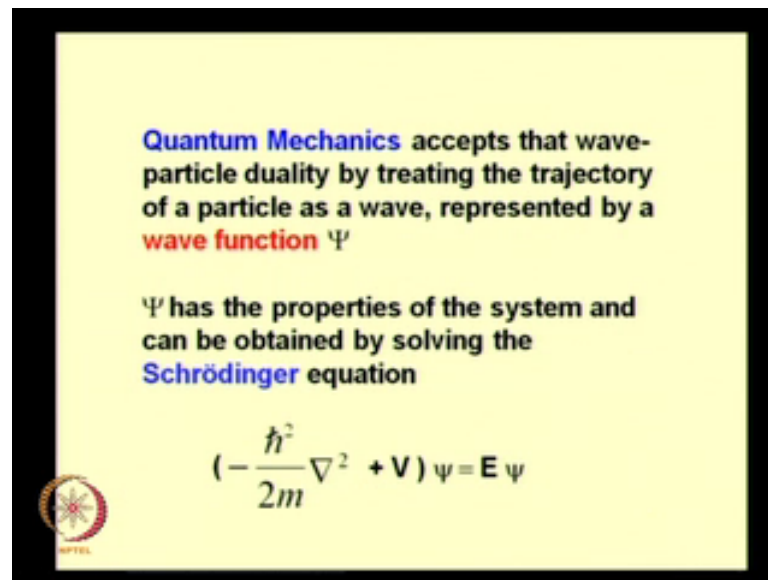
So, between the two of them, they are main they are **they are** contribution can be summarized by this phrase here wave particle duality. So between Einstein and Louie de Broglie they have successfully show in the world, that quantum mechanics and it is rules are enabling us to recognize that all waves can show as particle like behavior, all particles can show as wave like behavior. So, these are major contributions Planck showed as that, there is the concept of a quantum, Einstein showed as that light can behave like a particle.

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Louie de Broglie and then Davisson Gremer through their experiment showed as that particles can be behave like waves, sure enough Louie de Broglie got himself Nobel prize for us discovery of the wave nature of electrons. And again, you can look up this information and greater detail in the Nobel organizations information.

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**Quantum Mechanics** accepts that wave-particle duality by treating the trajectory of a particle as a wave, represented by a **wave function**  $\Psi$

$\Psi$  has the properties of the system and can be obtained by solving the **Schrödinger** equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = E\psi$$

So, we now see that you know we have this people in the about nineteen hundred so and above in later. People had surely slowly, but surely stumbled upon more and more aspects of quantum mechanics. So, it then turned out that to proceed forward people recognize that, the quantum mechanics was actually accepting within it is frame work, it was accepting this wave particle duality and what was previously being thought of as trajectories of **of** a particle, where now could also be considered as a as something that belong to a wave. And so particle trajectories could now be represented by something called a wave function psi. So, quantum mechanics was encapsulating this idea of a wave particle duality by associating trajectories of particles with a wave function, what is it called wave function and this represented by this character psi.

Now, psi it turns out has the properties of the system, so a particle and whatever else you want associate with it, that entire system has all it is properties are encapsulated by this wave function psi and it can be obtained by solving this Schrödinger equation, which is what is listed here. So, this is minus h bar square by 2m, this is del square, so h bar is actually h by two phi, which where h is Planck's constant, so h bar is simply h by two

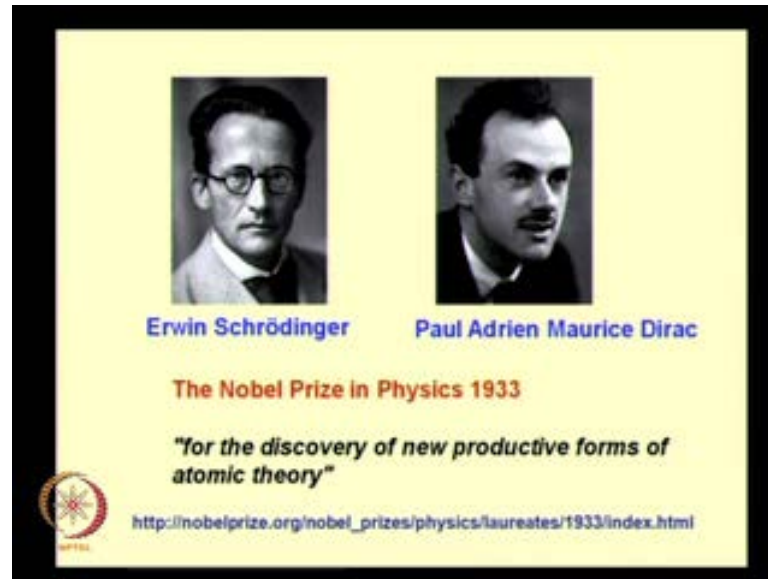
$\psi$ , so  $\hat{H} \psi = E \psi$  is the operator. This is the potential energy of the system  $V$ , this is the wave function  $\psi$  and this is equal to  $E \psi$ . So, this is an equation incidentally this is **this is** the very famous equation attributed to Schrödinger, where he attempted to see, how best you can capture this idea of wave **wave** in the quantum rel and see what **what** all carry you obtain from it and that is how he examine this  $\psi$  and came up with this equation.

This is, this equation incidentally cannot be derived, it cannot be derived from any fundamental principles. In fact, it is often described as a not as a derived equation, but as an inspired equation. If we want to call it that, that there is no way of formally defining a deriving this from more fundamental principles, because this equation itself is the fundamental principle, meaning all it is telling you is that the total energy is the sum of the kinetic and the potential energies, so kinetic and potential energy contributions, contribute to the total energy.

It is just put in this form, in this format that helps us now look at a wave function and see how the wave function is also satisfying this overall situation side. So this is how, this Schrödinger wave equation is and in, many in most of a quantum mechanics, we start by first considering, what are the constraints that the system is facing and using those constraints we solve this Schrödinger wave equation based on the constraints that the system typically a particle or collection of particles whatever it is that, is facing those constraints.

We take that system and we solve this, we put those constraint into this equation typically in the form of the potential, which could be a function of position and **and** then we solve the Schrödinger wave equation, when we solve the Schrödinger wave equation will get a wave function  $\psi$ , that wave function now belongs to that system. And we captures all the properties of the system, so that is how we utilize the Schrödinger wave equation right now we I am just stating it, when we use it we will see how it is used and how any other features that we need to become familiar with above Schrödinger wave equation, we will see at little later stage.

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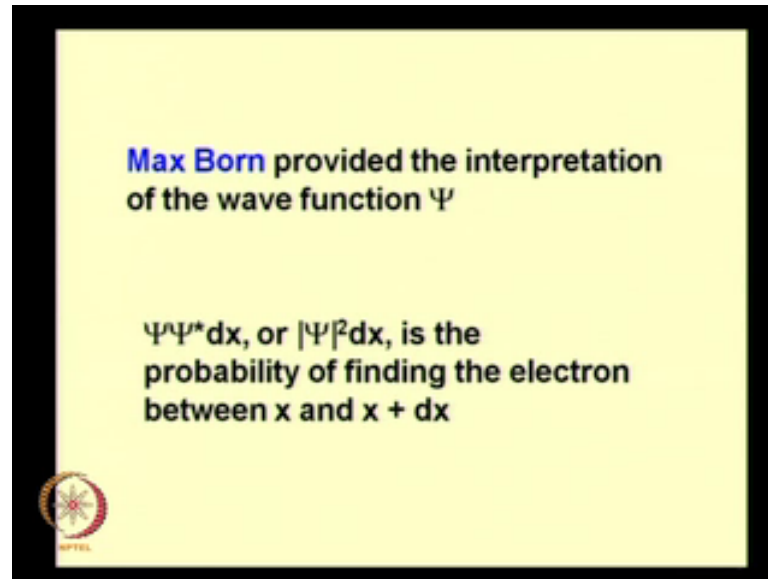


The Schrödinger wave equation turned out to be a very major contribution, because it will it **it** really gave a some theoretical frame work within way from which we could actually extract a lot of understanding of the system within the rules of the quantum mechanics. So, it **it** really moved the idea of how you handle quantum mechanical systems distinct step forward and for this of course, you got as got a Nobel prize in physics in nineteen thirty three. He shared that he was noble prize with Paul Dirac who also made significant contributions to quantum mechanics and we will see those contribution this little later and so they standard it for this **this**, what is described as for the discovery of new productive forms of atomic theory, so this is Erwin Schrödinger.

And I was point out that you know, the Schrödinger equation actually what you see here is a is one version of the Schrödinger equation, so this is called the time independent version of Schrödinger equation. There is another version called the time dependent version of the Schrödinger equation. And **and** you can see that you know, when you look at a time dependent form of Schrödinger equation, it **it** has conflicts with relativity and theory of relativity, so there is **there is** still room to understand what all of that means, but basically this time independent version of the Schrödinger wave equation is, what we use for understanding quantum mechanics are for utilizing quantum mechanics. I will also tell you a little bit more about it in a **in a** few moments in relation to the contribution of yet an another person to the **the** field of quantum mechanics. And, at that point perhaps the significance of Schrödinger's contribution will be more clear process **right**



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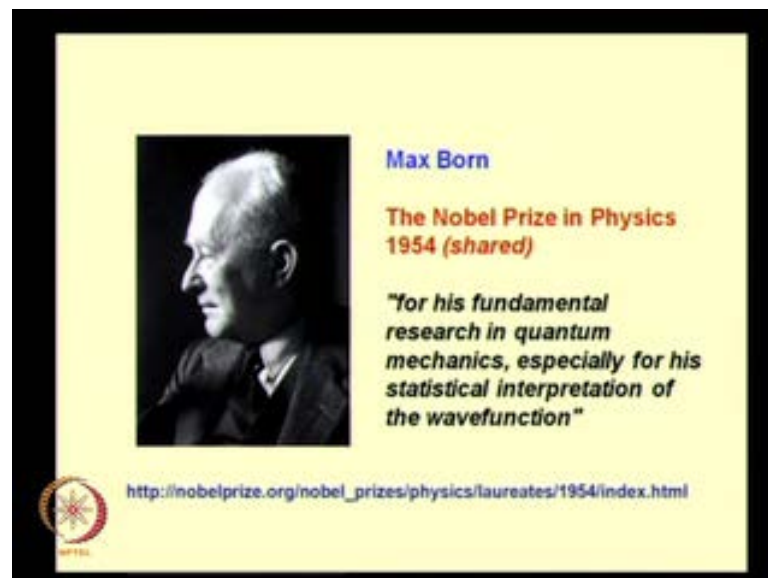
So, Schrödinger wave equation gave us the wave function  $\psi$ , but there was a lot of confusion on what that  $\psi$  represented. So, people found out that there was  $\psi$  and it **it** seem to capture a lot of the details of the system, so people were unable to actually tell what is this  $\psi$ , what is the  $\psi$  and what is the how can you sort of physically understand what  $\psi$  is, it was **it was** a very elusive concept. Although  $\psi$  was there and mathematically you could find it and then you could get information out of it, what exactly was  $\psi$  was something that people were unable to understand, although it seem to work were for **for** a variety of purposes.

Max Born is credited with helping as come up with an interpretation for  $\psi$ , he basically said that  $\psi$  is a wave function, so you can actually have complex conjugate of it. And  $\psi^*\psi dx$  or in other words the modulus of  $\psi$  square of the modulus of the  $\psi$  times  $dx$  is the probability of finding the electron between  $x$  and  $x + dx$ , if electron is the system that have we are working with. So, what if what basically said is that, when you have a wave function, if you calculate, if you use that wave function and you actually do this calculation for  $\psi^*\psi dx$ . Then depending on  $\psi^*\psi dx$  across the universe, you can **you can** calculated through entire all the space that you have available to you, you will get some **some** picture to some plot you will get, the value of the  $\psi^*\psi dx$  is then an indication of the probability of existence of the electron at that is part.

So, if  $\psi^* \psi$  happens to a very large value somewhere in space, that means we can say with greater certainty that the electrons actually exist at that particular location. If it is virtually 0 at some other location, it means the electron will most likely not be available at that at those parts. So, when you look at this is very important, because if you **if you** want to understand how electrons exist say with in atoms or within the solid and so on. This is the kind of analysis that help us understand, where is the electrons going to be, is it a bound electron, is it a free electron, is it if it is bound electrons then you will find a very high concentration some location you will not find it in other places and it will **it will** drop to 0 and so on.

So, understanding what this  $\psi^* \psi$  gives us is a very useful step in understanding what is the electron actually doing in **in in** the given the constraints that it has been placed in side. So, Max Borns interpretation of the wave function is considered a very significant contribution to quantum mechanics, because it helped as, why people were groping in that arc on what is this wave function even though it seems to work, it helped us put some kind of physical picture to that wave function.

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So, this is what his contribution was and he got a Nobel prize for this I am just pointing out that all this people got this Nobel prizes, because I want to highlight the fact although you may already be aware of it I wish to underline and highlight the fact that the subject matter that we are dealing with is considered very profound, I mean although it has been

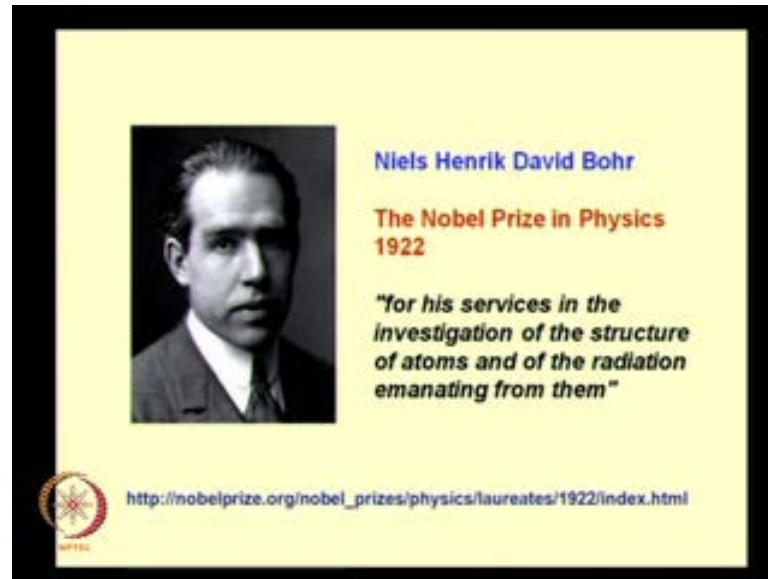
establish now over a hundred years and people use it and people take many things of it many of it, much of it, for granted. The point is it **it** all of these were considered a very profound additions to the body of science and requiring I mean consider the highest contribution to the body of science through the years that have gone by.

And therefore, those people were recognized for this contribution and it also tells us the kinds of delay mass that they faced in trying to understand this idea of quantum mechanics and to see to grapple with this special, whether or not nature was really actually following this rules of quantum mechanics are was there something else that we needed to consider and so on. So, this would help us get a idea of the complexity of the **the** concept of quantum mechanics, at is with respect to how easily or difficult or with how much difficulty, we end up having to accept this sides as something that is valid.

So, other the greatest minds of this of the past hundred years are so had lot of difficulties trying to accept this as a as the basis of with which nature a **seem to** seems to be display it is characters character and therefore, it is not surprising that when we also go through this process, we also at various stages have difficulty in understanding some of this. I would also point out in **in** some of these cases, the some of the theories, where you know they are best guess that, this is probably what was happening in the system and then check to see if in fact that was happening based on all their knowledge that they had.

So in some cases, you know it is **it is** difficult for us to actually pin it down to a very specific mathematical process by which the **the** person who came up with idea actually I was able to pin it down to that. So, there is some level of abstraction in these and some amount of intuetably for faith, so to speak in trying to reach many of these principles of quantum mechanics, even from the **from the** perspective of the people who actually originated this idea, I it I will always take you back to this idea that  $E = h\nu$  is itself something that Planck himself had lot of apprehension on whether or not in fact that was the way nature behave and that is the basic equation that makes quantum mechanics, what it is.

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So Max Born got a Nobel prize for this, the other person who contributed very significantly to the overall a feel that we are looking at is **is** Niels Bohr, he is the one who actually attempted to look at those atomic and molecular spectra, I mention that was another one of those not so well understood phenomena, people did know, why you could not have a continuous spectrum coming out from a atom, why there were discrete lines, why is it that the intensity of the atomic and molecular spectra was very intense at specific locations and virtually 0 at other locations, so I with respect to wavelength.

So Niels Bohr suggested this model that, you would have a nucleus and you would have a electrons was going on going around in a fixed orbits and then, when an electron jumps from one orbit to another orbit, there is an energy difference associated with that process. And therefore, you would have very specific those are fixed energies and therefore, the difference in that energy would show up as a very specific at a very specific frequency. So, this was the idea that here put for although the eventual understanding of the atoms and the structure of the atoms, did not support this idea that you had this fixed orbits.

The planetary model was not really the best way of describing electrons present with in a solid, but he was still successful in a explaining the his **his** approach still successfully explained the spectra that people saw. Incidentally, when we I **I** mention that Schrödinger actually had this wave equation, so what he did was looked at the hydrogen

atom, which is what Bohr had actually described, so Schrödinger looked at the hydrogen atom, he used the constraints of the electron placed that the hydrogen atom placed on the electron, he tried to capture those constraints in a **in a** form that would fit his wave equation, that would be useful for us equation Schrödinger wave equation and he solved for the wave function corresponding to the hydrogen atom with respect to the hydrogen atom.

And, the result that he got from his analysis was consistent with what Niels Bohr had put out for us theory. So that was considered as a big success for Schrödinger, because he was able to show not only show that he had this theory in place which worked, which he had good reason to be work, but he was able to actually compare both with some experimental data, which is the hydrogen spectrum and also compare well against Niels Bohr's theory for the hydrogen atom. And therefore, there is some amount of linkage of these ideas in validating each other so to speak and **and** therefore the Schrödinger wave equation then became an even more accepted and agreeable idea so to speak. I will also point out the Schrödinger wave equation, while it works I mean while where able to solve it for the hydrogen atom.

If you make the system more and more complicated it becomes more and more difficult to solve the equation, the kinds of mathematical methods and tools you will have to use get **((C))** sophisticated. So, it is not a very easy thing to use even though the equation itself is very elegantly put it is not a very trivial thing to actually utilize it for a variety of circumstances. So, you will often find in many books the Schrödinger wave equation being solved for simple cases, in fact for most of these for our course, we will look at specific conditions and we will solve the Schrödinger wave equation for those conditions, but you will find that you can easily think of conditions, which would make the entire calculation process significantly more complicated. So, it is not an easy and straightforward thing to do and so that is something that you should keep in mind **right**.

So, but it both the Niels Bohr and Schrödinger checked out the hydrogen atom and the kind of spectrum that it had and so on. And they found that they were able to I mean independently they were able to see their results that were comparable with each other. So, Niels Bohr is also credited with having made significant contributions into the structure of atoms and of the radiation as it stated here for his services in the investigation of the structure of atoms and of the radiation emanating from them. Incidentally, you can see that

all of these people actually are I mean have made significant contribution since say the first two or three decades, after the year nineteen hundred **right**. So, they were all contemporaries of each other and we will talk of one more contemporary in a next slide well will talk about that in moment.

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The **wave function** can be considered as a combination of more than one wave function, each having a momentum. In such a case some measurements will indicate one value of momentum and other measurements will result in other values of the momentum.

'**Localizing**' a particle will require that the probability of finding the particle in a very small region should be nearly 1. Therefore its wave function should look like this:

$\Delta p \Delta x \geq h/4\pi$

The graph shows a vertical axis labeled  $\psi$  and a horizontal axis labeled  $x$ . A single, very narrow and tall red peak is centered on the  $x$  axis, representing a localized wave function.

This wave function so any way, they **they** were contemporaries and we will **will** discuss what this, what they manage to do as contemporaries of each other and some of the nice things that they were able to work with each other and contribute to each other's work and how that came about. We briefly look at the wave function a little bit more, the wave function can be considered as a combination of more than one wave function, each having a momentum. So, this was an idea that as people became to accept this wave function, they basically said that the wave function would have a momentum associated with that and there would have a **and** therefore, wave length and so on. In such cases, in **in** and if **if** you had a wave function, you can actually the Schrödinger wave equation may give your complicated wave function, but we can treated as a sum of a bunch of other way of functions.

So then, it helps us a interpret the wave function and the features that we see more conveniently **right**. So therefore, the meaning of saying that wave function can then we thought of as a combination of other wave functions is simply that, if each of those wave functions as a momentum associated with it, then when you make measurement of the

system. In some cases, you will get one value of momentum, in some cases you will get the other value of momentum, so to repeat, if you have a wave function in which you can then write as a some of other wave functions then you can, when you take a random measurement of that system.

You will get any one of those momenta associated with any of those function, so this the what happens, when you look at accombination of wave function leading to the wave function of the system. So, a very important contribution of quantum mechanics is based can be thou, can be understood using this approach. So, let say we would like to localize a particle, localizing a particle is simply this idea that you now, can say with confidence at this particle in this very narrow region or very narrow location, like I said the electron could exist anywhere in the universe right.

So, on other hand we will say that this electron is now struck to this very small region, which is only say five Armstrong across just to give you an example or even less may be two Armstrong across, one Armstrong across and so on. So this means the probability of finding the electron in that region should be very nearly one and it should be 0 virtually every where around it right. So,  $\psi \psi^* dx$ , which is which I said is the bonds interpretation on what is the probability of finding an electron at a given region that  $\psi \psi^* dx$  should reach a maximum only in that region and should reach virtually 0 on either side of it.

So, if you have such a situation the  $\psi$ , the wave function on  $\psi$  of such a situation, where did you have a low electron that is really localize to once what would look something like this. It it would be virtually 0 everywhere at a very small region corresponding to some particular value of  $x$ , you would have a sharp  $p$  and then it would drop to 0 on a on the other side also. If, you have such a curve then the  $\psi \psi^* dx$  of this would also would looks very similar to this and that would be consistent with this pack picture that we have now got the electron localized, so all of this is consistent with the basic concepts of quantum mechanics right.

So we have put this thing together, so what is the consequence of it, to get yourself a wave function like this, what you can do is you can think of it as a some of series of waves of various different wave lengths, such that they all subtract out on either side of this location right. So, they distractively interfere an either side of this location and it so



happens that all of them constructively are in some way interfere to give you some positive value only in this region and again destructively interfere on this region **right**. So, you have a huge number of waves, it will turn out that if you actually do this analysis, the sharper you want to make this peak, the sharper you try to make this peak, you will have to add more and more waves of different wave lengths to enable you to do that.

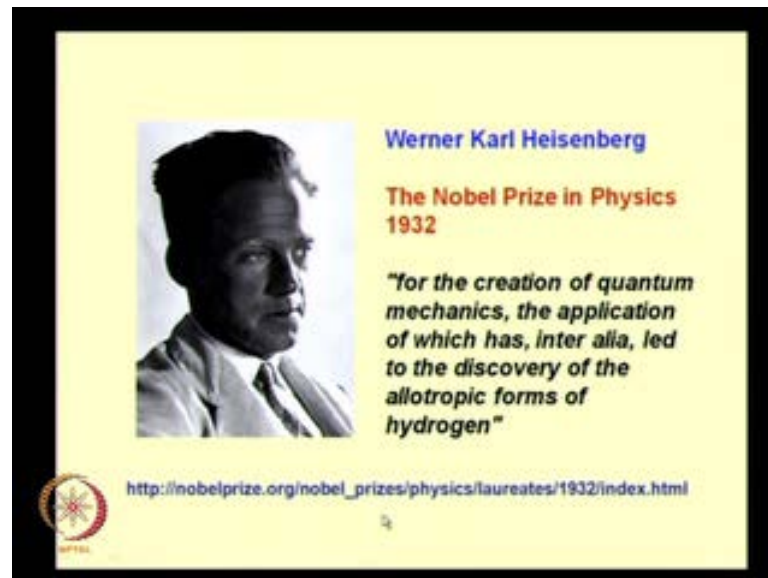
So, what do we mean by localizing **by localizing** we mean, if **if** you what is the error in finding that particles **right**. So by localizing we mean that, if you look at a width of this location, it will be some small  $\Delta x$ , localizing means the  $\Delta x$  is as small as you can make it. If you make it smaller and smaller and smaller, it means if you are saying with greater certainty that the particles is sitting only in that location **right**. So, making  $\Delta x$  smaller and smaller is the same as the **is the** idea that is captured is the idea that exist, when you say that your localizing a particle, but by doing so, you end up having to have wave functions, which are of our variety of different wave lengths and therefore, a variety, if you use the de Broglie hypothesis, it comes amounts to variety of momentum or momenta.

So, this is what happens, each of those waves, wave functions would corresponding to a particular momentum. So therefore, if you now look at the range of momenta that the particle could have **right** that we can again represent just the way range of positions, we represent as  $\Delta x$ , we can represent the range of momentum that you can have as  $\Delta p$ . So, if **if** it can have only one momentum the  $\Delta p$  is 0, if it can exist only in one position exactly precisely at one position  $\Delta x$  is 0. So that is, but we say it has a small range over which you can oscillate, so there we it has a finite  $\Delta x$ , for it have a finite  $\Delta x$ , we have to a you have a very large number of waves with a very large number of momenta associated with them, different momenta associated with them, which will have to add up such that they all subtract out in all other locations except this one location.

So therefore, making  $\Delta x$  small requires as to make  $\Delta p$  large **alright**. So  $\Delta x$  for  $\Delta x$  to be small,  $\Delta p$  will have to be large, if you make  $\Delta p$  0, which means it now only has one particular momentum, then you can find that a you would  $\psi$  will be such that it will **it will** exist the particle can exist it will have a non zero value from minus infinity to plus infinity. And having a non zero value from minus infinity to plus infinity simply means the **the** electron can exist anywhere from minus infinity to plus

infinity. So, minimizing  $\Delta x$  will require you to maximize  $\Delta p$ , minimizing  $\Delta p$  will require you to maximize  $\Delta x$ , this idea is captured by this inequality here,  $\Delta p \Delta x \geq \frac{h}{4\pi}$ .

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And this is an equation, that we now recognize as the Heisenberg uncertainty principle, so Heisenberg's uncertainty principle is captured in this kind of this equation that you see here, and in fact it is not just this, there are **are** serious conjugate variables, when you do Fourier transforms, you see these conjugate variables. Any conjugate variable will create this kind of a situation, where you will have the variation in one and the possible variation of the other and the product of them will have to be equal to or greater than a particular value. And therefore, if you try to minimize one the others will go up, it turns out that, so in a sense this is actually now a mathematical requirement.

So, that is what I wish to highlight here, that Heisenberg's uncertainty principle is now consistent with all the features of quantum mechanics and it is also in a **in a in a** very fundamental sense it is a mathematical requirement. In many of the high school books, we find this description that you know, if you have an electron and you try to locate it is **locate it is** position you shine a light and that disturbs the electron and therefore, you cannot locate its position very accurately. The more accurately you try to locate its position you will disturb it and its velocity will change therefore, momentum will change, that description is a simplified description given just to get us going on this

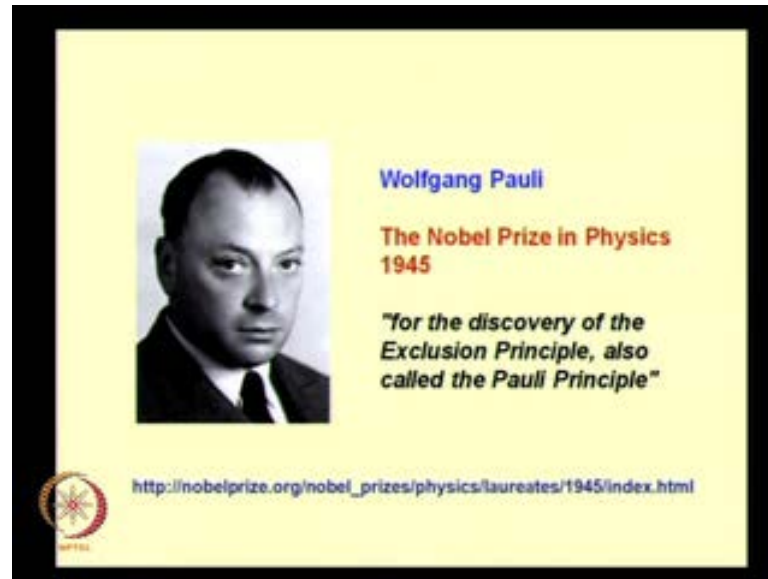
concept of uncertainty principal, but it is not an accurate **description** description in the sense, it **it** is a misleading description.

Because it somehow conveys to you, the idea that uncertainty principal is simply an experimental limitation, that is the idea that is somehow being conveyed with this kind of a description that you shine a light and something happens. It is not an experiment, it is not simply an experimental limitation, it is a reality, it is a theoretical reality, it is a mathematical requirement, it is a theoretical reality, because as I said these are conjugate variables and the way they relate to each other will force you to have this inequality. If it were an experimental if it **if it** was simply an experimental short coming then over the last hundred years the uncertainty should have been decreasing **right**, there is no decrease in uncertainty.

We have had you know so much of advancement and experimental techniques, you would have said that in a uncertainty was high in the 1920's and it is actually very negligible in 2010, that is not the case uncertainty is uncertainty and it remains that way **right**. Uncertainty principal has not it does not fundamentally change that itself is an indication that this is not simply some experimental limitation that we have to put up with. This is the reality that the variables the way they behave or such that you have this inequality that if you try to reduce a  $\Delta x$ ,  $\Delta p$  will go up the error **error** in the momentum estimation and vice versa.

So, Heisenberg's uncertainty principal is very profound in that sense **right**, it captures all the essence of quantum mechanics plus it also tells us what are the limitations and how you can use it. So, for this Heisenberg got a Nobel prize for the creation of quantum mechanics and he this is the way it is described that, but his primary contribution was in this uncertainty principal that he had put forth for us.

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And the other major person that we **we** recognize for quantum mechanics is poly, he of course is credited with this Pauli's exclusion principal, which is simply that the all the quantum numbers cannot be the same for two electrons and that is something that we use when we understand some of the features of electrons in various solids, I just to I **I** briefly mention that one idea little while earlier and I just wanted to spend a moment telling you little bit more about it. All of this people, where contemporaries of each other in the **in the** first fifty years are so after nineteen after the year nineteen hundred.

And they had a lot of discussion in on the quantum mechanics they had a regular meetings grappling with this idea of a quantum mechanics, because many of them for example, Einstein himself was unconvinced that even though they were proparence of this theory, they were very unconvinced that nature behave this way. They always some of them felt that there was something hidden, that they had not yet is discovered and then that they would soon discover that nature **(( ))** really follow the rules of quantum mechanics, but that has not happen, the more they delta on it, the more they analyzed it, they more they were convinced that quantum mechanics is the reality that the world is following in a more in a fundamental manner in all of the aspects that it manifest itself and so we have to accept as accept it as it is.

So, if you **you** can read lot of literature available there, which tells you of all the discussion that people have had the kinds of meetings that they had, early meetings that

they had trying to grapple with this. And the kinds of experiments they challenged each other with thought experiments that the challenged each other with. To try and explain this to try an get an understanding better understanding of the either the scope or the limitation of quantum mechanics.

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Summary	
$E = h\nu$	Planck
$\lambda = \frac{h}{p}$	de Broglie
$(-\frac{\hbar^2}{2m}\nabla^2 + V)\psi = E\psi$	Schrödinger
$\Psi\Psi^*dx$ , or $ \Psi ^2dx$	Born
$\Delta p\Delta x \geq h/4\pi$	Heisenberg

So I will finish by this with this summary, these are the major equation that are the essence of quantum mechanics E equals h nu attributed to Planck, which **which** plot forth the idea of a quantization of energy and therefore, the **the** fact that we now had something called quantum mechanics that we had to deal with and this was then extended by **...** So, if this idea was then extended by Einstein to tell us that light could behave as particles and then de Broglie then showed gave as this idea that particles could be treated as waves. So, all of that comes here e with this de Broglie relationship then Schrödinger gave us this idea that you know, now that your struck with this concept that there is, this wave like behavior and so on. There is a wave function that you can associate with the system, from which you can extract all of this properties and so then therefore, we have the Schrödinger wave equation.

Born gave us this interpretation for this psi, which is that psi psi star dx are this modulus of psi square dx is the probability of finding the electron at x between x and x plus dx. And then Heisenberg gave as uncertainty principal, these are all the major equations associated with a quantum mechanics and also of course, Einstein’s contribution. So,

these are the major equations and for the purposes that we will use through rest of this course, we will use this equations as necessary at various stages to help us get better theories. And better understanding of how electrons behave in a solid and what properties of the solid, we can now have a much more solid understanding of much **much** better understanding of based on the understanding that nature uses these rules. So, we will do this in the subsequent class, we that we will finish for today, **thank you**.