Physics of Material

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Lecture No. # 13

Classical Particles and Quantum Particles

Hello, and welcome to this the 13th class in our physics of material sector series. So, in all our classes so far we have already travelled quite some distance in the general area of the subject. We have successfully built one model for the materials especially trying to explain the electronic properties as well as the thermal properties. And that model was the drude model we were able to build the model, we were able to check weather this model is good in explaining all the properties that we see of the materials. And we found that in fact, it was good in some respects, but it also has some shortcomings specifically with respect to specific heat and the average translational kinetic energy of the electrons. So, we have travelled quite some distance in that sense, in sense of actually gone through one model and examined it is capabilities.

Have not come that far we went even further, we said that to improve upon what we have already done. We need to examine our existing model to see, what is the source of that shortcoming? So, it is not enough that we know, that the model is limited in some ways we would like to have better feel for why it is limited in those aspects. So, in that context we examined the fact that the drude model has effectively used Maxwell-Boltzmann statistics in describing the electrons. And that is itself a piece of information that we have picked up from the fact that the kinetic theory of gases actually uses Maxwell-Boltzmann statistics. And since we have effectively just extended the kinetic theory to metals, the electrons are also effectively believed to be used in Maxwell-Boltzmann statistics within the frame work of drude model.

So, in the last class, we went through the exercise of deriving the Maxwell-Boltzmann statistics at that stage we specifically did this process. One to; of course, understand how this kind of statistical process is employed to handle arrange the particles and kinds of particles and conditions that we are trying to handle. So, that itself a very useful exercise to have because that is the kind of exercise you would have to go through if you are trying to extend this kind of learning process to some other new system that you are trying to examine. So, therefore, in that context we have looked at this derivation of Maxwell-Boltzmann statistics.

But, more specifically we also wanted to see what is it, how is that you handle a set of particles with certain set of characteristics and how you can translate that to statistical information that will that can then be used. The reason we did this is we would also like to use similar process do not exactly identical, but similar process if we were to actually put different characteristics for those particles. So, therefore, understanding how it is done in one case is very useful. So, that we can make the appropriate changes if we were to try and do the same process with slightly different set of rules that we are expecting the particles to behave or obey. So, it is in the context that we have derived the Maxwell-Boltzmann statistics. So, to briefly restate what we did in the Maxwell-Boltzmann statistics, we came up with final expression that looks like this.

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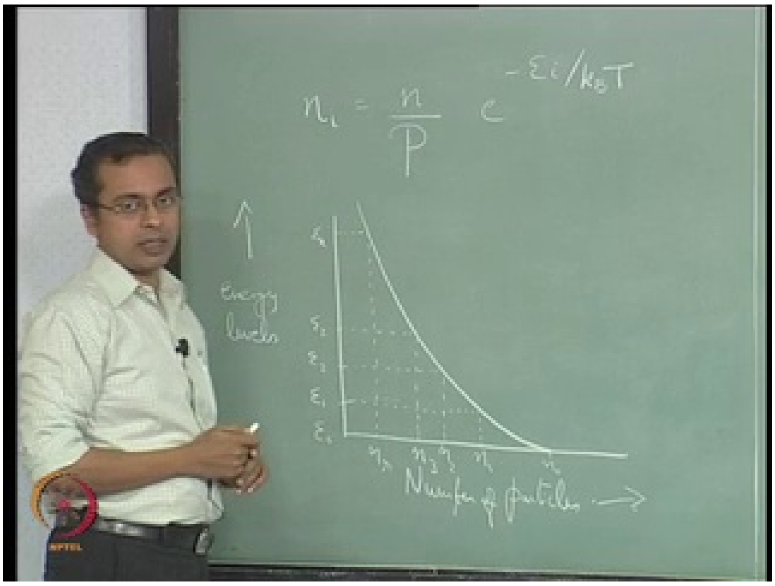

(No Audio From: 03:58 to 04:09) So, where n i is the number of particles which are setting at the energy level e i and n is your total number of particles. P happens to be something that we describe as the partition function, which is essentially the sum of all these terms over all the e i of just this term here term that we see out here. So, that is what this works out. So, if we make a plot of this which we did last class so, we will start with that plot and proceed. So, we will see a plot that looks where we have essentially the energy levels (No Audio From: 04:48 to 04:54) increasing that way and this is the number of particles. (No Audio From: 04:59 to 05:08) So, we have these are the two parameters that we wish to plot against each other.

We see a plot that looks something like this, where the way we read this plot this is energy level e 0 and that has n 0 number of particles. If we take an energy level e 1 (No Audio From: 05:32 to 05:38) it has n 1 level number of particles, e 2 n 2 particles, e 3 n 3 particles and so on. And so, you have some energy level e r, (No Audio From: 06:01 to 06:07) n r number of particles. So, this the way you would read this graph, the way you would read any other graph also, but essentially this is how the two pieces of information that we are trying to co-relate end up, showing up in a graph. And if you; as we can see from this expression that we have put down, as the energy level goes up the number of particles is going to come down because it is minus E i by K B T that we have up there.

So, in general the energy level goes up the number of particles at that energy level, occupying that energy level will come down which is basically what you see here, this is increasing energy here. So, as you go up and up in energy the number of particles occupying that energy level is actually coming down, this is the information that we have. We can also make slightly; this is at one particular temperature. So, when we make this plot in this expression n is a constant and this partition function will be a constant because we have fixed temperature and this particular temperature as a constant. So, now, supposing we were to do this at some other temperature then what is it that we would see, we can; we will make the same plot here.

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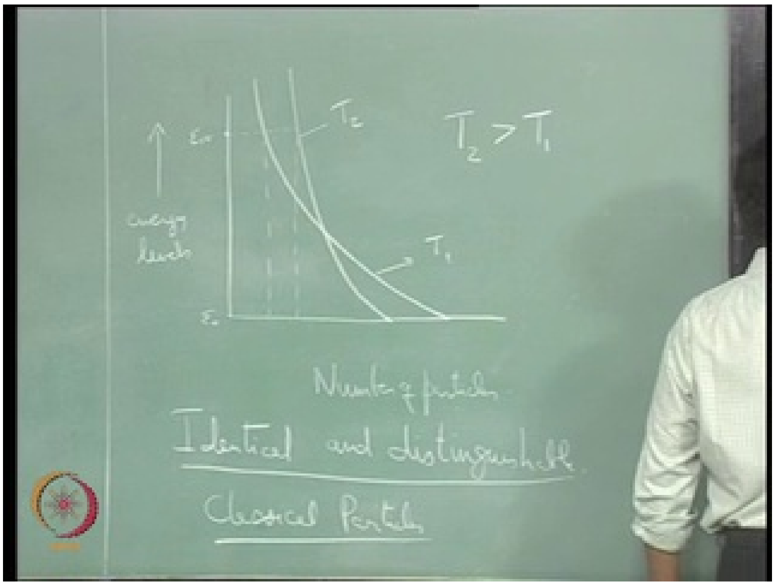

So, this number of particles (No Audio From: 07:22 to 07:28) and this is energy levels. So, without going into much detail of all the particles that we just saw, we would just have to curves that took like this. This is at T 1 and this is at T 2, where T 2 is greater than T 1. So, what this basically means is that, as the temperature goes up in this case T 2 is greater than T 1 the higher energy levels here, will have more parts at given any energy level. So, let us whatever, I will just arbitrarily say that as e 10 and this is still e 0. This is just arbitrarily chosen as e 10, at e 10 which is high energy level originally we had certain number of particles, now we actually had a larger number of particles. So, the higher energy level, some arbitrarily level e 10 originally we had some number of particles, now we actually have a larger number of particles.

So, the higher energy level is actually getting more populated, is more heavily populated at this; in this temperature condition. If you go to low; at lower energy level such as in fact, even e 0, at e 0 originally it was this many number of particles, now at higher temperature it is lower number of particles. So, what is happening is, as you raise the temperature lower energy levels are getting less populated and the higher energy level are getting more populated and this make sense overall because first of the number of particles is fixed. So, you cannot arbitrarily raise the number of particles at some energy level without making any difference to the number of particles at the other energy level so, in general number of particles is concerned.

So, therefore, if it at all goes up at some energy level, at some other energy level the number of particles has to come down, so that part is correct, that part is consistent with what we are dealing with. But at the same time the overall energy level of the system is going up, when we say temperature is going up that basically what we need, when temperature is gone the energy level associated with that entire system has gone. So, which means, the total energy of the system has to go up which means the sum of energies of all the individual particles that sum has to go up. For that sum to go up, the number of particles at lower energy level comes down so, the contribution of lower energy level decreases, but the contribution of higher energy level increases. And therefore, actually sum goes up and that is why; that is how the overall energy of the system goes up.

So, this is the picture of this system that is following Maxwell-Boltzmann statistics and how that system behaves as you change the temperature, specifically in this case we are raising it from T 1 to T 2. Now, when we spoke about the limitation of the Drude model, one of the things that we said is that it is; it does an incorrect prediction of the electronic contribution to the specific heat. So, in fact it is over predicting the electronic contribution to specific heat. Now, if you see here this diagram that I have drawn here, is actually showing you the difference in energy, the difference in layout of energy in the system as a function of temperature. Now, when we talk of specific heat that is essentially what we are talking about.

At specific heat simply that if you have a some temperature for the system, you would like to know how much energy you have to provide to that system to raise the temperature of that system by small delta amount, small delta T. So, I have arbitrarily chosen T 2 which is may not be small delta T higher than T 1, but it still (it still) encapsulate this information. This information is in some ways related (( )) is in fact, directly related to the specific heat of the system. Because that (( )); that is also capturing the same information, except it is giving it to you in some specific unit that we have put it down in a graph right. So, when we say that the electronic contribution of specific heat is not being accurately predicted by the drude model and therefore, by the Maxwell-Boltzmann statistics which is hidden within the drude model, or in fact, is the basis of the drude model.

What we are actually saying is that, this kind of change in distribution of energy level and number of particles of those energy levels. This kind of change in distribution as predicted by Maxwell-Boltzmann statistics is not valid or does not appear to be valid for electrons which are present in a solid. So, that is the basic information that we are gathering or the kind of idea that we are forced to consider because this is what the informations and this is where the errors right. So, we now see that you know, we have; by coming up with Maxwell's by looking at Maxwell-Boltzmann statistics in greater detail. We have actually been able to get very close to where is it that we are having any issue with this derivation and as a result of which this model is having problem with making some predictions right. So, this is the curbs of the problem that we face.

Now, having come this far, lets probe this issue a little further and see that this graph has not come arbitrally right. This graph has come because we made, we assumed that the particle has certain characteristics that they followed certain rules. Only because we assumed; only because we made that assumption, we were actually able to develop the rest of statistics that statistics is based entirely on those assumption right. So, the; and in this case the assumption; assumption that is of significance, is the assumption that impact the manner in which we count the states. The manner in which we count the states, the manner in which we figure out what is sitting in which state, whether to count it as a one state two state and so on. So, that is very integral to what we are talking of when we say Maxwell-Boltzmann statistical distribution.

So, somewhere there (somewhere there) in the rules that we made saying that electron in the solid follow those rules, somewhere there we have made an error so, that is where the issues. Now, the specific aspect of the Maxwell-Boltzmann statistics that stands out, which is very fundamental to the; to why the graph ends up in that way, is that we have said that those particles are identical and distinguishable. (No Audio From: 14:06 to 14:14) We have said that the particles, that are classical particles we have used all those terms we have said classical particles is the term we have used, we have said identical particles, we have said distinguishable particles these are all terms that we have used. And any physics book you take or any solid state physics book you take, you will find some mention of these words classical is the word you will see, identical is the word you will see and distinguish able is the word you will see.

So, we have assumed this and we have also like I said mentioned these are classical. (No Audio From: 14:44 to 14:52) So, fundamental to Maxwell-Boltzmann distribution is the fact that we assume that particles are classical and that therefore, they are identical and distinguishable so, this is the combination. So, in today’s class, we will actually spent we will try and examine this combination very carefully. What we mean by saying classical and what do we mean by saying classical means identical and distinguishable and so on. I will also; I also want to sate this that for longest time; one of the way reason why we use this kind of terminology especially with respect to classical, is that for the longest time this is all we knew. For the longest time all the particle that we examine at all levels including large particle which I mean and large object and so on.

Where all consider classical we did not know any different, we were not aware that there was possibilities that we could consider them in any other manner. Only around the year 1900, did the idea come up that actually consider them in somewhat different manner and that is possible that they may be behaving in a different manner. And that there are specific circumstances under which we get closer to; when it is necessary to in fact, think that they do not behave this way, all those things came up only around the year 1900. So, all of the; sort of the old physics that if you wish to see, you know the way when you say Newton’s law of motion and so on, those were all described assuming that particles are classical.

So, all those things are true and as we examine this further I would like you to keep that in mind. So, that when you go back and look at any old notes you have on Newton’s laws of motion and all the other things that you have done in physics. You can reacess that you have learnt or reexamine everything that you have learnt. From the perspective of this idea that much of what was described, was pertaining to something that is classical and therefore, even though physics is then called as classical physics so, this is the ethic. So, now we look at the identical and distinguishable so, these are the terms we look at and we will see and what we will mean by it.

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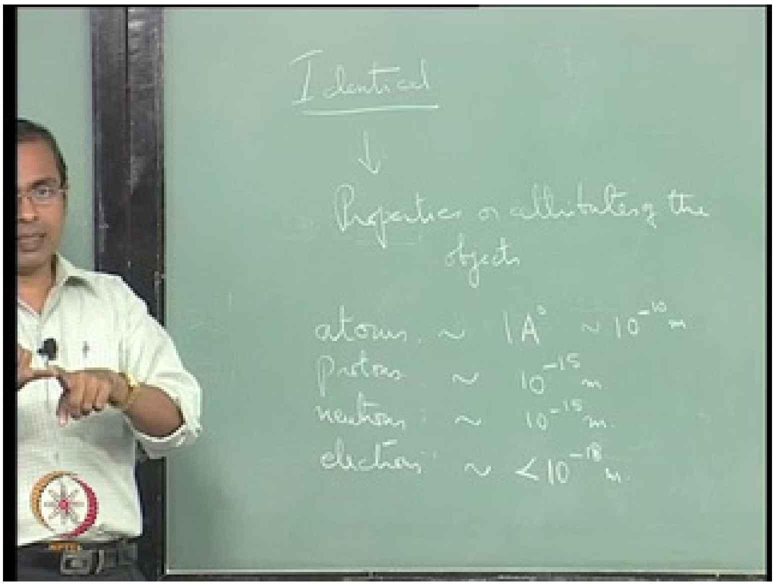

So, first of all we say identical, now when do we say something is identical or when do we say this is not identical this is the first question we will ask ourselves. The simplest way, I mean when we say something is not identical, when we say two objects are not identical to each other it means, we are able to clearly demarcate which is one object and which is other object. The simplest way we do that is simply look at the properties of that properties or attributes (No Audio From: 17:43 to 17:57) properties or attributes of the object. So, you just look at the properties or attributes of the object if they are not the same, they are obviously two different object and you are able to say so convincingly. So, to assist as in this process, I am simply; I am just going to show you I just brought along few plastics balls, toys that you can easily find anywhere else.

And I am going to pick up some of them and show you use them as examples to assist as in this discussion. So, I randomly pick up a couple of them and then we will see what we can see about them. So, we now have; I now have two balls, two plastic balls which you could get anywhere in my hand and as you can see they are essentially more or less the same size. In fact, in this case they happen to be the same size within experimental error and but clearly they are not identical right. So, in this case the color is the distinguishing character, it is very easy for us to distinguish between these two. They are large scale objects, they are not small objects we have two different ones very easy to handle in your hand and they are not the same so, this is definitely not identical.

So, when we say that we are able to distinguish between them there is no issue here, these are not identical and they are distinguishable so, this is what we have here. We could also consider another combination here. So, in that case the sizes were the same, but the colors were not the same, in this case neither the sizes the same nor is the color is the same right. I have one pink ball in the left hand and a green small ball in the right hand. So, it is easy for us now differentiate between these two because neither the color is the same nor the size is same right. So, in a; what is called the classical physics, this is the manner in which we go about things, these are anyway not; there is enough that is different about these two that we have no issues demarcating between them right. So, this is how it goes on, I will take two more here, now (these are) these are exactly the same size and again within the experimental error, these are exactly the same size and they are also the same color.

So, in our kind of a discussion, it is this scenario that is of significant to us that we have actually two objects which have all their attribute being the same. In terms of, in this case color and the size and if presumably the weight and so on because it is the same material and so we have the same two objects. So, in classical physics what we say is that even when we have two objects like this, you can actually distinguish between them. Meaning I can say this is in my right hand and I can say this in my left hand and I can say with confidence that they remain in this way right. As long as I keep them this way, as long as I do not consciously interchange them, this is the one this is the ball that is in the right hand and this is the one that is on in my left hand.

So, I am able to confidently distinguish between the two of them. This may not immediately sound, like a very significant statement, but we look at a alternate condition and then we will find what is the significant of the statement that I may. So, we will just hold this for a moment and (get back to) get back to this just a moment. Now, if you look at the atomic particle so, to speak atomic and subatomic particles, what do we have? We have atoms, we have protons, we have neutrons and we have electrons. Now, if you look at the size scale that we associate with some of these, this is of the order of an Armstrong. So, that is roughly 10 power minus 10 meters. So, an atom is roughly about 10 power minus 10 meters.

In fact, incidentally the typical inter atomic space is about twice this of the order of two Armstrong. This typical inter atomic distance that you can talk of in a; so, want a generic number that you want to work with inter atomic distance, this is the number you would work with two Armstrong of which each atom is about an Armstrong. So, that is what we are looking at protons and neutrons are believed to be of the order of 10 power minus 15 meters. (No Audio From: 22:28 to 22:34) So, they are estimated at 10 power minus 15 meters is the size scale that associate with proton or neutron. So, that is already five order of less than magnitude associate with the entire atom so, that is very small number. So, in fact, if you look at; this is something that we are looking at 10 power minus 15 meters, we elaborate on that a little bit.

If we look at electrons, the estimate are that is less than 10 power minus 18 meters. That is very small number 10 power minus 18 is very small number it less than that estimate of what it is exact number are; I believe very difficult to get. But that is the kind of estimate that we have for the general sizes of the atomic and sub atomic particles that we conventionally we deal with. If you look at you know the best materialize characterization techniques, they are getting into one Armstrong and sub Armstrong size scales. But you are; you would be hard pressed to actually look at individual electron and so on. So, this is going to be; this is that; I mean when we are talking of size scale this is a significant drop, when you go form 10 power minus 10 or to 10 power minus 15 or 10 power minus 18 this is like a huge drop in size scale that we are talking of.

So, we are actually looking at object of this size scale. Now, in real life when I took, when I picked up two orange balls here and showed it to you, this is you know completely different size scale, we are talking of order of the say five centimeters. So, 10 power minus 2 meters, the kind of size scale that will 10 power minus one to 10 power minus two meters is what we are talking of. So, that is actually about almost 10, 8 to 9 order of magnitude higher than this and about you know virtually you know 10 power is of the orders of 16 orders of magnitude higher than what you have here. So, this is one of the issues that we have, that we have to become aware of when we are talking of object of different size scales. First of all we must be conscious of the fact that we are actually dealing with objects of different size scale.

The reason I point this out is that some of the things that we talk of conventionally in physics in fact, if you talk of when people says Newtonian physics and so on. The reason many of the things, many of the theories of the past are held well with all of that has been described with Newtonian physics. Is because typically all of the objects that where described or where all large scale objects so, large scale in this heads. So, when we talk of when we look at any problem I mean, if you really look at you know physics text book that talks of billet ball hitting another billet ball such as such velocity or arriving at such and such speed or projectile being thrown you know at some particular velocity and some particular angle that is described to you and so on, they are all large scale objects.

So, in that sense and it turns out that many of the things that we say of large scale objects is therefore, works out to be very valid for those objects. As you go to smaller and smaller size scales and this is the size scale that we are talking of. It turns out some of the things that we are saying is not; it is not possible for us to say the exact same thing with the same level of certainty when we go down to smaller and smaller objects, this is one issue that we used to crop up. So, this is something that we need to be aware of, but we will examine it further as we try and discuss this little be greater detail. Now, clearly when I spoke about these various example of you know a green color a small green ball versus a big pink ball and so on.

Obviously, it is possible for us to distinguish between an electron and proton an electron and neutron an electron and an atom, similarly proton versus an atom and so on simply because they are of completely different size scale right. Being able to distinguish between these various particles is quite straightforward because their size scales are very different except of course, the proton and neutron combination. Here the size scale is the same, but charge is different, it is the same as I saying that I had a pink color ball and orange color ball. The size scale is exactly the same, but the color is different there. So, another attribute of the ball is different and therefore, you are able to distinguish between the two of them. In this case the attribute of a particle that is different happens to be the charge.

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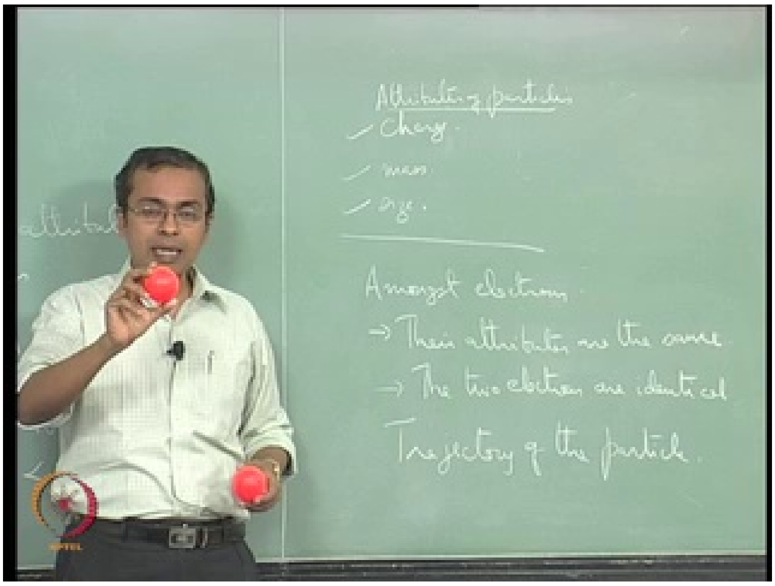

So, you could have charge of the particle, you could have the mass of the particle and we just put those down so, primarily the charge and mass. (No Audio From: 27:16 to 27:27) So, for our purpose this, these are the two we will look at charge and mass. And charge and mass to the extent that and therefore, the size scale if you want this size of that particle, by looking at protons, electrons, neutrons and the atom and a single atom. We find that you know between them either the charge is different or the size is different or the mass is different or more then one of them is different and therefore, you can distinguish between them right. So, therefore, in this discussion that we had, we are able to extend to an atomic level and look at ways in which we are able to identify independent non identical particles, is very easy to differentiate between non identical particles because they have enough of attribute that is different.

Now, the question of importance to us, we are; this entire discussion is at some level eventually going to applied to a bunch of electrons in a solid so, that is the direction which we are headed. Now, when we talk of bunch of electrons in a solid, the issue with respect to this attribute of the particles that immediately stands out. Is the fact that amongst electrons their attributes are the same? (No Audio From: 28:46 to 28:58) So, when I take two electrons and compare them then you find that their charge is the same, their mass is the same, their size is the same right. So, those three are; the three attributes at we are looking at this case the charge, mass and size are same that is why we are able to you know specifically ask you look up a reference book it will give you a mass of an electron 10 power minus 31 kilograms.

So, that is you get as a mass of an electron, you can get 1.6 into 10 power minus 19 coulombs for the charge of an electron. So, these are some standards so, you can find these standards. So, therefore, their attributes are now the same. So, the question is if you have two electrons can you distinguish between them so, that is what the question is. So, we have, in other words the attribute being the same of the two particles being the same, when you have such situation we describe that by saying that those two particles are identical. (No Audio From: 29:57 to 30:08) So, the word identical in our general discussion of identical versus distinguishable and so on. The word identical in our context is being used to capture the information that the attribute of the two particles are the same, the mass is the same, the charge is the same, the size scale of the particle is the same. So, therefore, those two particles are identical.

Now, the question is given that you have two identical particles, can you distinguish between them. So, this goes back to this idea that I started off with which is that I have two orange balls, where essentially their attributes are the same, their mass is the same. In this case if you want color is the same and to the extent that you know their charge is neutral at this point. Their charge is also the same, their size is the same everything is same about these two plastic balls that I have in my hand. So, therefore, they are identical, attribute wise they are identical and I wish to know I can distinguish between them. In this case at this size scale (at this size scale) which is the order of 10 power minus 2 to 10 power minus 1 centimeter sorry, 10 power minus 1 meter.

In thus; in this size scale I am able to distinguish them, simply because I know for a fact that I am holding one in my right hand, one in my left hand and they remain that way. I have no confusion on this aspect I have no reason to believe that they have in any manner changed their positions. I have control on where they are and how they are position. So, therefore, I am able to distinguish between them at this size scale. So, these are identical, but distinguishable at this size scale, at this size scale that we are dealing with. Now, we wish to know that if you have two electrons which are now having all the attributes being the same, in just the manner that we have these two plastic balls and I know essentially talking of very small size scale less than 10 power minus 18 meters.

If we are getting down to that kind of size scale and you have two electrons are they identical and distinguishable. Can I say for a fact that I have control on exactly which is electron, the first electron that came into the system and whereas second electron so, this is the issue that we have. What happens is that upfront we do not know the answer to this question alright. So, the question has been post, I post the question that these are two identical particles can you distinguish between them. So, upfront we do not have an answer to the question, the way we go about it is we assume an answer and we check whether it is correct or not alright. So, when we look at classical mechanics (which is the) which is what I have mentioned all along. The assumption that we make for two electrons is the same assumption that we make for two plastic balls that I showed you.

So, the two plastic balls which are at the size scale of 10 power minus one meters. I am able to make say with confidence that even though identical, I am able to distinguish between the two of them. In other words if I keep one ball up and one ball down and I swap them around, I can say that the second ball only has gone up. So, I am able confidently say that which ball has gone up, which ball has gone down I am able to say confidently. In relevance with respect to the kind of a system and kind of problems that we are trying to discuss, the concept of interest for us is the trajectory. (No Audio From: 32:28 to 33:38) So, the concept of our interest was the trajectory of the particle, I just mention to you another concept of interest which is the position of the particle.

So, I just said that you could have two different orange balls and I could keep one up and one down right. So, I have two different positions for these orange balls, I am able to say that if I swap them around and the ball in the left hand has gone up, the ball in the right hand has come down. I am able to say that I know this for a fact that whatever was in my left hand has only gone up and whatever was in my right hand has only come down. So, this is a new position with the different ball on the top and different ball at the bottom. So, this is what we mean by the fact that they are identical, but distinguishable. In with respect to the particles and kind of situation in which we are examining them, which is that these particles may be electrons or atoms. If it were a kinetic theory of gases they would be gas atoms or gas molecules and if it is an electron in solids they are electrons, they are all randomly moving around alright, this is something that we have discussed already.

So, that they are randomly moving around they are bouncing off of each other, they bounce off of the walls of the container they are in; and so on. So, at that point the additional attribute of the particle is the, it is trajectory. So, one particle moving in some direction and another particle moving in some direction it may be that their trajectories intersects so, they hit each other and they bounce off right. So, all these things could occur in system consisting of particles kept at some energy level, where they are moving around randomly they could hit each other, they could hit the walls of the container and so on. So, this is the ongoing process in that kind of a system. So, now, our question of interest is that with respect to the trajectory of those particles can you distinguish between the particles just the way I said with respect to the position, I was able to distinguish between these two particles.

I could move them up and down and I could say with the confidence which is the particle that is up and which is the particle that is down, even though all their attributes are exactly the same right. So, now, with respect to trajectory what we need is that you know, if let us say the two balls come they collide and they move away right. For this kind of a problem you can take the Newtonian approach, you can say energy is conserved and so on. And you can write equations which show you that you have an initial path for this ball, you have an initial path for the other ball. You can predict exactly when they will collide and at the end of collision you can say that you know energy is conserved and so on and you can say in which direction, will the ball on right side move and which direction will the ball on left side move right. So, you can clearly predict the approach of the two balls predict the fact that they collide, predict the fact that they move away and on top of it you can predict clearly. Not just the position of the two balls, but I could have marked one as a and other as b, I can tell you exactly where a will be at the end of collision, exactly where b will be at the end of collision and there will be no confusion in it right.

So, we would like to know that with the bunch of electrons can we extend the same kind of argument. In other words if you know the trajectory of the electrons and if you have a bunch of electrons and you take two of those electrons and you know which is the path in which one of those electrons is moving, which is the path in which other electron is moving. And let us say for example, that they collide just for the purpose of our discussion, let us say they collide and move on. We would like to know for the fact whether are not we can say that the electron that was let us say on the left side has gone to particular position and electron that was on right side has gone to particular position.

So, that is what we mean by saying the fact that they are identical because their attribute are identical, but they are distinguishable because I can keep track of individual electron with confident that it is that very same electron right. So, now, when we talk of classical physics and extend the principle of classical of classical physics to electrons in a solid. We are extending this idea that the two balls can distinguished from each other which is valid at this size scale, to two electrons which is present in a solid. So, we are saying that two electrons are identical because their attributes are the same, but by keeping track of their trajectory you can uniquely identify a given electron at all points and time. That is what we mean by saying that the electron is identical, but distinguishable.

So, classical physics uses that line, it says all electrons or any particle that is classical particle is identical, even if it is identical is still distinguishable based on it is trajectory. So, if you know the trajectory of the particle, you can say for a fact that is the specific electron that you were following throughout your discussion so, that is the thing. When we talk of classical particles this is all this is true and we are also saying that as long as you are keeping track of the trajectory of the particle, you can uniquely identify the particle fine. So, now, till about 1900 this is all that we were aware of till the year 1900. And all the discussions that people had about various particles on various objects and so on and calculations that people made on systems, were all based on this basic assumptions.

And therefore, were all based on Maxwell-Boltzmann statistics because the Maxwell-Boltzmann statistics actually say effectively assumes this right. By in what form does the Maxwell-Boltzmann statistics assume it. It basically says that you know if you have for example, two energy levels right. Two energy levels again if you look at it in this case the up down analogy is useful for us, two energy levels I have (( )) a higher energy level and a lower energy level. I have a particle at a higher energy level and a particle at the lower energy level and these two particles are identical particles alright, they are identical particles. Supposing I swap so, I count this as one state, the system is in one particular state when this particle is up here so, the other particle is down here, supposing I switch them around.

Based on classical understanding of the particles the classical theory of particles, since I am able to distinguish between the particles. Since I can say with confidence that this is a different particle and this is a different particle. Since I can say that with confidence this new arrangement is a new arrangement it is not same as the old arrangement, I have swapped the particles right and this counts as the second arrangement. So, in classical physics this is one arrangement and this is another arrangement so, even though you have two particles having the same energy levels that is how to speak occupying similar energy levels. Now, around the year 1900 and beyond we (we) stumbled upon thanks to famous people whom we will; like max Planck and this is something that we would discuss in great detail in the next couple of classes. Thanks to a bunch of people who did a lot of interesting work in the early from the year 1900 to about 1920s also.

We came up with new set of rules that we believe that the particles followed and an entire new area of physics came out which was called quantum mechanics. So, now, in quantum mechanics the one of the concepts that is hidden that is fundamental to quantum mechanics is that when you talk of a particle. What you are actually talking of is the probability that a particle exist at a particular point. So, when you point; when you make some diagram and say a particle is at a particular point, what you are actually saying is that there is a high probability that the particle is at that point. It is very important to understand that saying something is highly probable is very different from saying that it is certain so, that is the difference.

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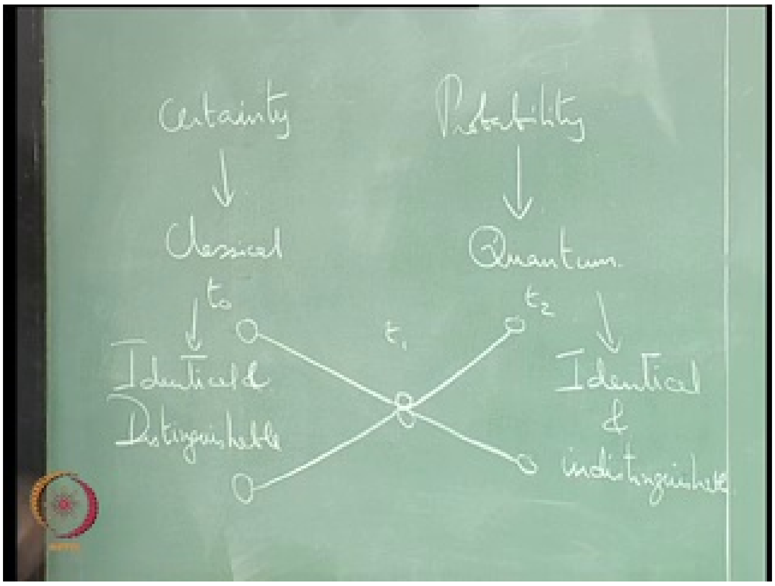

So, we have the concept of certainty and probability.

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What we are saying is in classical approach of dealing with mater, there is a certainty that something is somewhere. So, in any particle that I am talking of no matter what size scale whatever, the moment I say that it is a classical particle. I am saying I can say with certainty where the particle is I am able to follow the particle and do lot of things with specific particle. The moment I am; I say that you know In fact, the particle is behaving more like a quantum mechanical particle then what we are talking of is that we unable to say with certainty that the particle is at a particular location. We only; we are only able to say there is a high probability that the particle is at that location. So, why is this important to us? This is important because now when you have two trajectories.

So, I have a particle here and a particle here, it moves this way this moves that way they strike here right. So, I just say there is one here and one here now, at this point and they bounce off. So, let us say we have this situation this is T 0, this is T 1 this is T 2 time, time scale. So, at T 0 there are at two positions, they are moving towards each other they strike each other at T 1 and go to T 2 right. So, classical mechanics says that we can say with certainty that this particle is here, we can say with certainty it is up here, we can say with certainty it is up there. We can say with certainty the second particle is here, it second particle is here and second particle is here, quantum mechanics says that we are only able to say that there is high probability that this particle is here.

In fact, we in quantum mechanics we use wave function and so on, wave function eventually represent probabilities that will leads us to probabilities of existence of a particle at a location. So, there is a high probability the particle is here, there is a high probability the particle is here, there is a high probability that they are here and similarly there is high probability that they are here. But the fact that it is not a certainty, actually creates a situation that when you say that there is high probability particle is here and particle is here, there is a definite non zero probability that they actually got inter changed. In classical physics that is not allowed so, when you have, when we look at our example that we have two. When we go back to this example that we have two orange balls they come and they strike and they go out. In classical physics there is no question of confusion of which ball ended up where.

So, they started here they hit each other they went out, there is no confusion whichever ball in my right hand remains there. In quantum physics we are saying that the particle only has a certain probability that it will be there. So, when you go through this process there is a non-zero probability that somewhere in this process they actually interchanged and you have no idea when that had happened, you had no control on that process. So, you would still have two particles arrived like this. You will still have two particles sitting at those two positions, but you have no way of saying that what stared out in right hand has shown up on this side. You can; you have no way of saying this is the actually correct situation as supposed to this.

What you will see is the two particles at those two positions that is all you will see. You have no way of knowing that in fact, specifically the one that was on your right hand ended up on the right side and one that was on your left hand ended up on your left side. So, your inability to; so, what we find is that they; when you apply the rules of quantum mechanics, since there is a certain probability that particle could be somewhere and therefore, there is always a probability that if you have two different particles. You have two particles which are actually identical, in the sense their charge is the same, their mass is the same, their size is the same and everything else is the same. There is always a non-zero probability at any given instance we swapped position.

So, there is non-zero probability that they swapping could have occurred therefore, in addition to being identical particles if you say set of particles follow quantum mechanical rules, we also additionally say that they are indistinguishable. So, when we apply the rules of quantum mechanics, we are saying that particles are especially applied to a set of electrons which are inherently identical particles. We are additionally saying that not only these particles are identical we are unable to distinguish between them because at any given point they could anyway be swapping with respect to each other. In classical mechanics we are saying that even a bunch of identical particles, the possibility of swapping does not exist because there is a certainty that it is here and there is a certainty that it is here.

So, they are always distinguishable so, this is the difference. So, quantum mechanics talks of (No Audio From: 46:11 to 46:18) identical and indistinguishable. (No Audio From: 46:24 to 46:29) These are identical, but distinguishable. (No Audio From: 46:32 to 46:45) So, classical particles are identical and distinguishable, quantum mechanical particle simply they talk of probability of particle being present somewhere are identical and indistinguishable.

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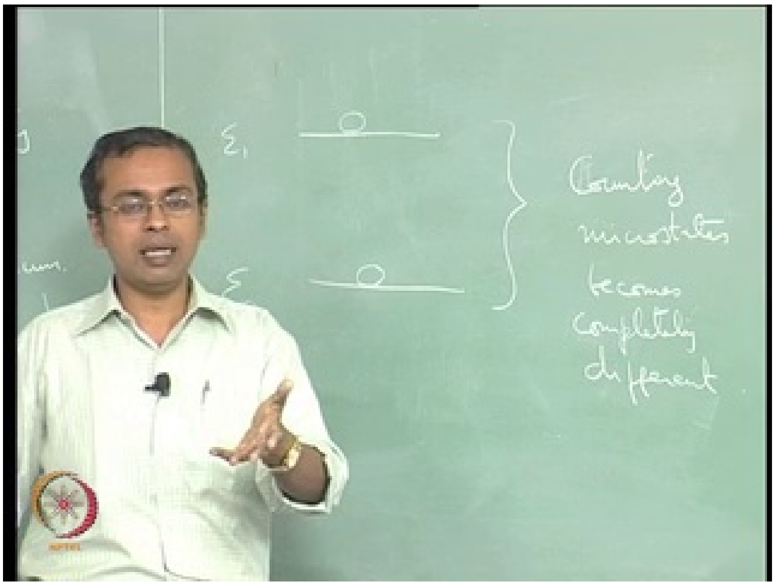

Why is this important? So, it is important because if you have two energy levels e 1 and let us say e 0 and I have a particle here and I have a particle here. In the classical way of viewing it, if I swap these particles that is a (news) new possibility that is a new arrangement. In quantum mechanical way of looking at it, if we swap these particles we do not get a new arrangement because they might have swapped any way, in the sense you have no control on this. Even all you can say is they happens to be a particle up there and a particle down here, we have no idea which particle is down here, you have no idea which particle is up there. And the reason this is important because this totally changes the way in which you count the number of.

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So, the number the way in which you count the number of micro states becomes completely different based on your assumption whether or not classical or quantum. So, this is the critical step at which the derivation of say the Maxwell-Boltzmann statistics will have to be changed, when you try and create a new statistics which is valid for quantum mechanical particles. So, what I wish to highlight is that you know, this will change the way in which you; the assumption of an identical and distinguishable particle which is valid for classical particles, creates a certain way in which you count your microstates and therefore, has a direct impact in the statistics that comes out of it and therefore, direct impact in how particles are distributed at various energy levels and therefore, how the specific heat of that system behaves.

Whereas, when you talk of particles which are obeying quantum mechanical rules, they are identical, but indistinguishable particles and as a result the way in which you count those microstates becomes completely different. And because that becomes different the final distribution that you get, corresponding to quantum mechanical particles becomes completely different with respect to classical particles. And again that has direct impact on the manner in which that system behaves with respect to energy and therefore, the manner in which it behaves with respect to specific heat. So, this is where these distinguish characteristics begin to appear and we have only briefly mention that there is such a thing quantum mechanics and it came up in the year around the year 1900.

And in next couple of class we will spend great about of time specifically examining the idea of quantum mechanics. How it come about, what are the fundamental ideas of quantum mechanics? Why is that people have so much difficulty in trying to understand quantum mechanics, and get the grasp of what it is quantum mechanics is trying to tell us. I must also point out that you know, all of these the idea goes back to something that I have stated all along. Upfront we do not know whether a particle is a; whether it is appropriate to say that certain particle is a quantum mechanical particle or it is a classical particle. All we can do is we can only assume that something happens to be behaving in a quantum mechanical way or happens to be behaving in a classical way. Based on the assumption we derive a certain expectation of how that system should behave.

If in fact, the experimental data matches that prediction then our assumption is right. So, for example, when we talk of particles which are atoms of an ideal gas so, the size scale is 10 power minus 10 meters. In that size scale those atoms are also a whole set of large collection of particles in a container. It turns out that if you make the assumption that they are classical particles, if you make the assumption that they are identical, but distinguishable. And therefore, employ the Maxwell-Boltzmann statistics then the behavior you get for those particles exactly matches the behavior that we can measure experimentally. Therefore, it is fair and reasonable to say that those particles behave like classical particles in fact, every all the large scale objects we deal with in our day to day life will all fall in that pattern. If you go to the size scale of electrons, we find that the classical mechanical rules and Maxwell-Boltzmann statistics are not predicting their behavior correctly.

So, whatever property we are measuring of the material which we attribute to electrons is not getting accurately measured is coming out to be a number very different from what the theory is predicting. On the other hand if you just assume upfront without any prejudice, we just assume up front that this may be a quantum mechanical particle that may be identical and also indistinguishable. If you make such an assumption the kind of statistics that comes out of it, the kind of equations that come out of it will predict a certain behavior for a collection of electrons. It turns out when you do it that way the prediction actually matches the experimental values that we get. As a result we can now in Hein sight we can say that therefore, it is correct that our, we have assumed our assumption that it is a certain kind of behavior for electrons is actually correct.

So, it is only in this process that we are able to say that you know, certain such particles behaves in a quantum mechanical way or the rather the quantum mechanical effects are very significant for such a particle and insignificant for atoms. They are very significant for electrons and very insignificant for atoms and such particles. And so, that is the manner in which we utilize this. So, what we will do in the next; so, that is the thing, we do not know upfront we make the assumption and then we see whether are not we are right and in this case this is the difference between the two systems that we might follow. So, what we will do in the next couple of classes is we will look at the history of quantum mechanics in great detail and try to examine how it has evolved. And then in subsequent classes we will derive, I have just shown you that microstates can be counted in very differently if you put the rules in slightly differently right.

So, we will actually derive another set of; another statistical distribution which applies exclusively to; which applies very well to electrons in a solid and that would employ all the quantum mechanical rules. So, what we will discuss in next couple of classes, you will all get, in addition to this fundamental idea that those particles are now identical and indistinguishable. All of those ideas will put together in the next statistical distribution that we will derive which incidentally would be; would will be called Fermi Dirac statistics, acknowledging the people who contributed to the creation of that statistical distribution.

So, we will put all that together three classes from now and create and look at the derivation of the Fermi Dirac statistics and that would then be employed to the electrons in a solid. And we will find that, we will see at that stage and examine, again and see how well the statistical distribution is predicting the properties with respect to what we get experimentally. And we would also examine in what ways even if it has in addition to the fact of identifying in what places it has succeeded. We will try to compare it back with Maxwell-Boltzmann statistics and see in what form it has accommodated for the errors of Maxwell-Boltzmann statistics. So, this is what we will do in the next three classes so, with that we will halt for today. Thank you.