

Physics of Materials
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Lecture No. # 10
Drude Model: Source of Shortcomings

Hello and welcome to this tenth class in this physics of materials lecture series. In the past few classes, we have developed the Drude model for materials and specifically applied it to the metallic systems. And try to see if we can make predictions of the properties of metals and we found that it has a very good capability of predicting some of the properties of metallic systems. And specifically, in the last class we focused on what were the successes and the limitations or failures of the Drude model where it was able to give us a good understanding of why something is occurring and also where it perhaps gave us misleading answers may be even wrong answers.

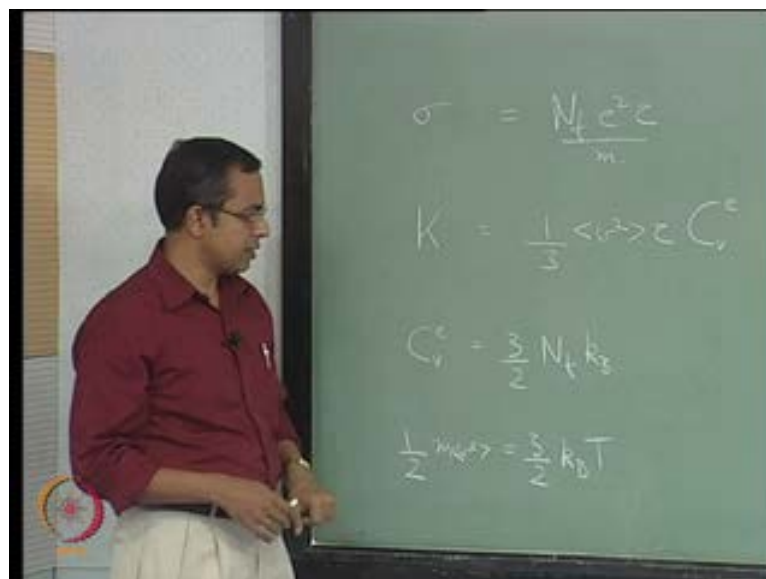
So, this is the journey we have done so far like I said specifically the thermal conductivity, the electronic conductivity are predicted correctly the Wiedemann-Franz law is also predicted correctly. But on further a deeper exploration we find that the prediction for the electronic contribution to specific heat is actually over estimated. In fact, the actual contribution is much less and the mean square velocity that it predicts is actually less than what is actually there in this system. So, this is something that, we have seen or at least we have stated this and come this one.

So, today what we will do and today and in the next class also we will actually focus more on trying to understand why the model has had some issues addressing some of the questions that we find along the way. And what is it that is central to the problem that this model has on (on) a more macroscopic sense we have or an more general sense descriptive sense we (we) already understand that there are certain aspects of the material system that the model ignores. So, for example, it is ignoring the interaction between charged particles. It is ignoring the interaction between those the electrons that are running across the system and the specific positively charged ions that may be present in the system.

At least in (in) the sense that there is no accurate depiction of this interaction there just a generic resistive term that, we throw into the system. So, there are already issues even at a descriptive sense with this model. But still we would like to explore further to see we can identify that specific concept or that specific idea that is very central to this model which and which is not really appropriate for this system that we are applying into. So that is the region of study that we need to look at and see if we can extract this information out of this model.

So, to do this lets step back a little to put down some numbers in (in) some cases I have just told you that. It does it gives (gives) us a value of approximately such and such for specific property we will step back and put those numbers down. And (and) actually done walk our way through those numbers and then see if from somewhere there we are able to get some ideas of what is going wrong and where it is going wrong.

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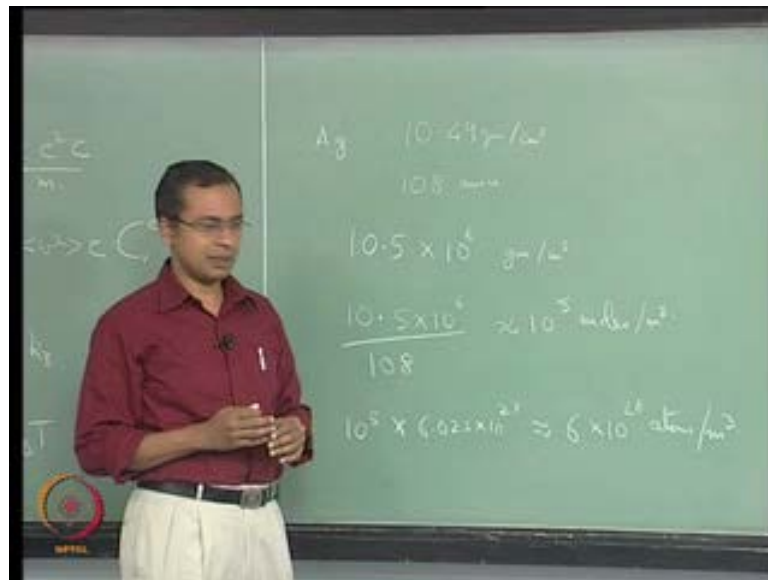


So that is what we will do today? So, if you look at the equations we put down for this model first we had the electronic conductivity sigma and we got that to be $N_f e^2 \tau$ by m . So, this is what we got then we got thermal conductivity K 1 by 3 (No audio from 03:58 to 4:03) 1 by 3 v square average of the v squares $\tau C_v e$ and also this $C_v e$ itself because it comes from the kinetic theory of gases this $C_v e$ is 3 by 2 $N_f k_B$ where k_B is the Boltzmann constant and the average translational kinetic energy (No audio from 04:32 to 04:41) is 3 by 2 $k_B T$ for an electron. So, these are some of the parameters

that we have that are based on this model and all the assumptions that going to the model.

So, these are all the things that we got out of it because we have just used kinetic energy of gases for this process these equations, which comes from the kinetic theory of gases, actually hold in this system. And are also consistent with this model and based on the application of these equations on other aspects of this model. That we did in our last couple of classes we got this expression for the electronic conductivity and this expression for the thermal conductivity. So, with these expressions we will now spend some time trying to put together some numbers for them and then extract some information out of them. So, first let start with N_f we did this little earlier, but we just read do it here so that we can make it complete for this current discussion.

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So, if we take silver as an example, its density is 10.49 grams per cc per centimeter cube and its atomic mass is about 108 atomic mass units. This works out very conveniently for us because the numbers are such that a factor of 10 would approximately take care of (()).

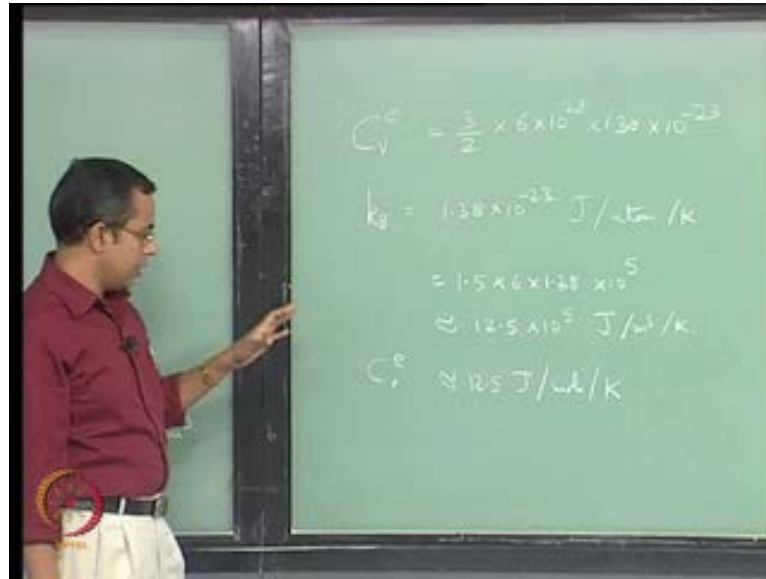
So, let us look at the number of atoms per meter cube. That we are looking at here and assuming that silver behaves in a univalent manner, which it often does then the number of atoms per meter cube is also the number of free electrons per meter cube fine. So, this is what we have so 10.49 grams per centimeter cube or say 10.5 grams per centimeter

cube is the same number (number) of in meter cube it is into 10 power minus (()) into 10 power 6 grams per meter cubes each (each) centimeter would be 10 power minus 2 meter. So, you have 10 powers minus 6 meter cube denominator. So, this is so many grams per meter cube. So, this is the same as in terms of number of moles per meter cube we have 10.5 into 10 power 6 by 108 moles per meter cube and (and) we (we) (()) assume that this is approximately a factor of 10 here 10.5 by 108.

So, this is roughly equal to 10 power 5 moles per meter cube. So, many moles of silver atoms are present per meter cube and therefore, if you assume univalent behavior of silver that is so many moles of we cannot recall it moles. But so, we leave it at that the silver atoms would be then so, many moles per meter cube and this would amount to in terms of number of atoms per meter cube we have 10 power 5 into 6.023 into 10 power 23. So, approximately 6 into 10 power 28 atoms per meter cube. So, the number of atoms per meter cube for (for) silver at room temperature.

So, at silver, which is good conductor of electricity, good conductor of heat and so on? At room temperature has about 6 into 10 power 28 atoms per meter cube and I said that if it is univalent we will have one electron being released per silver atom in the metallic system. And these those electrons would then run around as free electrons if you make that kind of an assumption, then the number of free electrons per meter cube, which is the N f value that we are looking for N f is also same 6 into 10 power 28 electrons per meter cube free electrons per meter cube. So, this is free electrons free electrons per meter cube. So, this is what we got for N f. So, that is one of the quantities that we have there.

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So, if you now look at C_v^e the Boltzmann constant k_B is 1.38×10^{-23} joules per atom per Kelvin this is what we have for the Boltzmann's constant. So, if you look at C_v^e this is $\frac{3}{2} \times 6 \times 10^{23} \times 1.38 \times 10^{-23}$ this is on a per atom. So, if you look at the quantities we have put in here, this is the number of atoms per meter cube or in fact, electrons per meter cube, but since this are coming from the kinetic theory of gases for the movement.

We can even treat it as a atoms per meter cube and later extend it and later just arbitrarily state that there that is the same thing for electrons. So, this is atoms per meter cube this is the Boltzmann's constant in joules per atom per Kelvin. So, if you run through this number this is 1.5×6 so that is $9 \times 1.38 \times 10^5$. So, this is 9×1.38 that support roughly about 12 and half approximately 12.5×10^5 joules per meter cube per Kelvin. So, this the C_v^e that is the electronic contribution to the specific heat on a unit volume basis and per unit (unit) volume we already saw that we have about 10^5 moles per meter cube on a per unit volume basis. So, this is the same as 12.5×10^5 divided by 10^5 joules per mole per Kelvin.

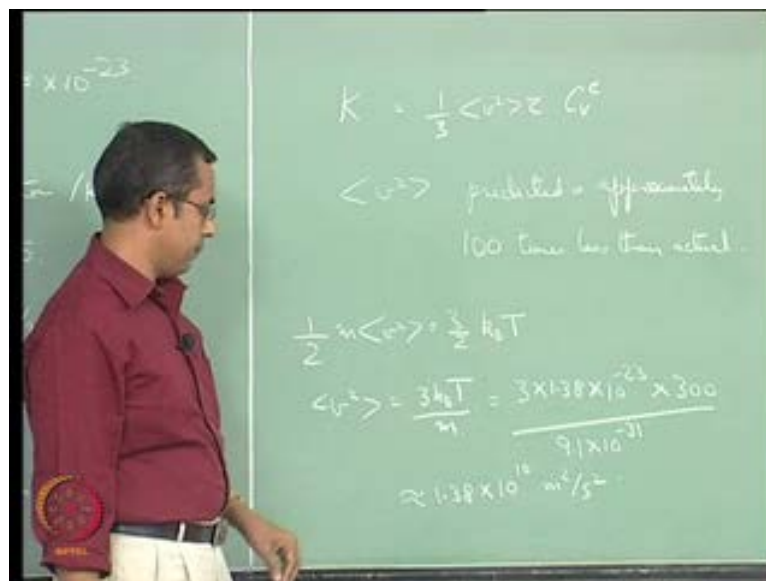
So, this is what we are C_v^e will work out to so at constant volume, electronic contribution to specific heat at constant volume being (()) from the kinetic theory of gases so, this is what we end up getting. Now, if you look at this value. So, this is the value that we get if you actually look at experimental data where people have and there

are lots of studies in the literature. You can actually go and look at if some of these are old literature from the even from the 60's and so on. You can see people have done very careful experiments where they have want to very low temperatures and try to see what is the specific heat and then try to therefore, find out what is the electronic contribution to specific heat.

So, there is a lot of experimental data that enables us to do get this information. What we find is that the actual value is of the order of from experimental data is of the order of 0.2 or even less joules per mole per Kelvin. So, it is of this order it is less than about 0.2 joules per mole Kelvin. So, our theory is predicting about 12.5 joules per mole per Kelvin the or the Drude model is predicting what is what we are currently following up on the experimental values that you will find or about or about 0.2 joules per moles per Kelvin. So, roughly about two orders of magnitude less you can actually come up with exact numbers based on the system you are looking at so on. If you look at it are roughly of the order of two orders of magnitude less.

So, this is so the actual value is two orders of magnitude less then what we predicting or in other words this C_v is being over predicted, we are actually predicting a value two orders of magnitude higher than what is actually present in the system.

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So, on the other hand we find that if you look at the thermal conductivity itself we have $1/3 v^2 \tau C_v$. And so, if you look at this basically what is happening is that if

you assume that the tau is being predicted acceptably, which seems to then we have that the v square value is being (is being) actually, under predicted because this is being over predicted and this is being under predicted by two orders of magnitudes similar magnitudes. So, we will also now so, this is something that we note down predicted is approximately 100 times less than actual predicted by Drude model is 100 times less than actual. So, (()) incidently if you want to see what it is predicted we have we just put down this $\frac{1}{2} m v^2$ is equal to $\frac{3}{2} k_B T$.

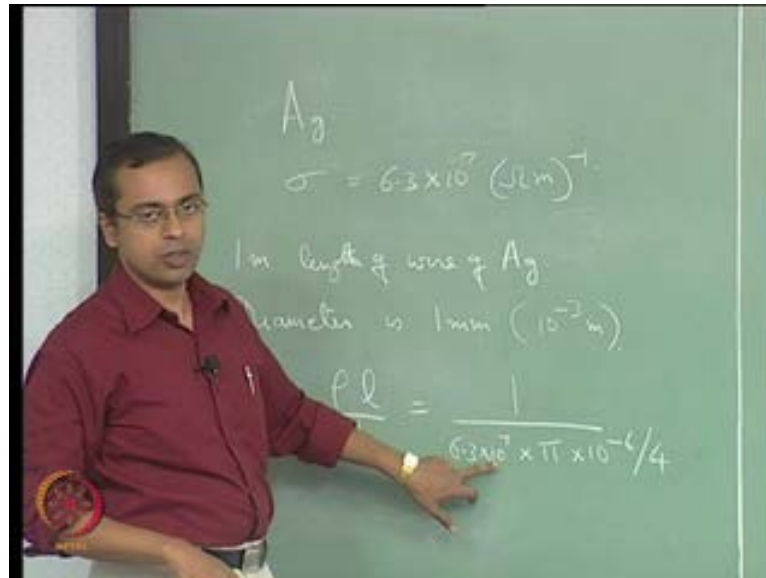
So, at room temperature or there about T would be of the order of 300 Kelvin. So, roughly about 300 Kelvin we can say. So, therefore, v^2 equals $\frac{3 k_B T}{m}$. So, if you put down the values this is 3 into 1.38×10^{-23} into 300 and we divide this by 9.1×10^{-31} which is the mass of an electron 9.1×10^{-31} kilograms per electron is the mass of the electron that we have here. So, we have 3 here, 3 here and a 9 here. So, they will approximately cancel out. So, we have what we have is roughly so, v^2 is approximately equal to 1.38×10^{-23} here and 10^{-21} here.

So, 10^{-21} in the numerator and 10^{-31} in the denominator the rest of them are constants which we just figured out 3, 3 and 9 go. So, this is $10^{-21} / 10^{-31}$ by $10^{-23} / 10^{-31}$. So, that is $10^{-21} / 10^{-31}$ meter square second square per second square. So, we mentioned this in (in) the class without specifically sort of deriving it (()) putting it together. So, we find that the mean square velocity is of this order and based on what we are understanding now this is actually being under predicted by two orders of magnitude. So, this is something that we will look at. So, we will just keep this in keep this information in mind for the moment we will come back to it little later in the class.

So, we just look that mean square velocity. So, I also wanted to I am putting down some values here because it is going to lead us into understanding better what is the problem that we have faced with this model. But also to give you an understanding of what are the different aspects that we are dealing with here and specifically, with respect to velocity we have an we have something that we (we) need to be careful about or at least alert to. So, the other velocity that we mentioned is the drift velocity and that was in the context of the electronic conductivity calculation that we did.

So, we calculated something called drift velocity. So, now, let us try and see if we can come up with a number for this drift velocity fine. So, again we will take the example of silver because it is a very good conductor and on that basis we will see if we can come up with some value for drift velocity.

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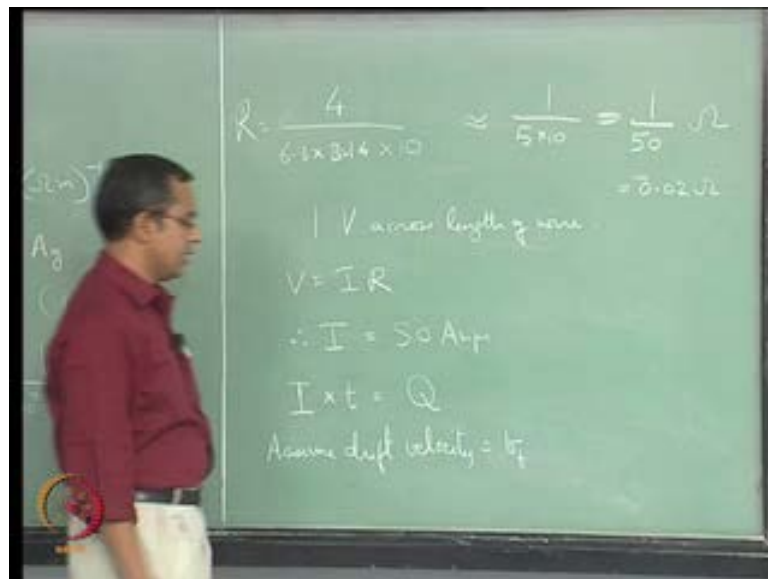
So, what we see is so if you take silver the parameter that is now of interest to us for our immediate discussion is its resistivity. So, I will just let me start (()) putting down its conductivity value sigma is about 6.3 this you can into 10 power 7 I just look this up in (in) a database any book many books will give a value. So, you can easily look this up the other values have put down are also value you (you) can find in any standard book. So, the atomic mass of silver the density of silver, the conductivity of silver resistivity of silver and of course, the other constants Boltzmann constant and so on. So, these are all things you can look up and so, I have looked this up somewhere so, this is the value that you have 6.3 into 10 power 7 ohm meter inverse and of course, it is one of the best conductor that you can find (()) incidentally.

So, this is one of the best values that you will find for conductivity in metallic systems so, this is what we have. So, we will assume that we have a one meter length of wire of silver and let see it is diameter is one millimeter. So, we have and this is not farfetched you can have a one meter long wire we just once one m diameter of course, if it is silver that is an expensive wire, but this is just for our discussion and you can (()) buy such a

wire it does not so, expensive that you cannot (()). But any way the point is you could use such a wire and you could easily certainly find one such a wire and it serves for our purpose of discussion so, we will (()) with it.

So, let us start by see what is the resistance of this wire. So, resistance R is rho l by A l is one meter we have got a we have to get for this diameter and rho is simply the inverse of (()) conductivity, conductivity is whatever it is this value here the rho is inverse of this. So, this is actually 1 divided by the conductivity which is 6.3 into 10 power 7 area is pi d square by 4 so, we will have into pi d square is this is in millimeter so, this is 10 powers minus 3 meters. So, d square is 10 powers minus 6 so, (so) that is pi d square divided by 4 so, this is what we have here.

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So, let us just work this number out so, this is 4 divided by 6.3 into pi or 3.14 and we have 10 power 7 10 power minus 6 so, we have 10 in the denominator effectively into 10. So, this is roughly point eight times this will go roughly 0.8 times and if you multiply 6.3 by 0.8 you will roughly get 5. So, (so) approximately we are we are only looking at approximate values here 1 by 5 1 by 5 into 10 so, that is roughly 1 by 50. So, resistance is 1 by 50 ohms so, this is what we are looking at R is equal to 1 by 50 ohms fine. So, we this is the resistance of a silver wire that is one meters that is one meter in length and has a diameter of 1 millimeter this is the resistance. So, we will we will just apply a one volt potential across it so, these are all numbers that are very reasonable numbers.

So, these are all I mean very (()) I mean you go and buy a battery in a shop it is one point five volt battery so, we could easily arranged to have one volt put across this silver wire. So, we will just put 1 volt one volt across wire across length of wire so, then we have v equals $I R$ ohms law R we know is 1 by 50 ohms so, if you want you can put it down specifically it is 0.02 ohms so, 1 by 50 ohms is 0.02 ohms. So, v equals $I R$ therefore, the current through the system I is simply v by R which is equal to 50 amps.

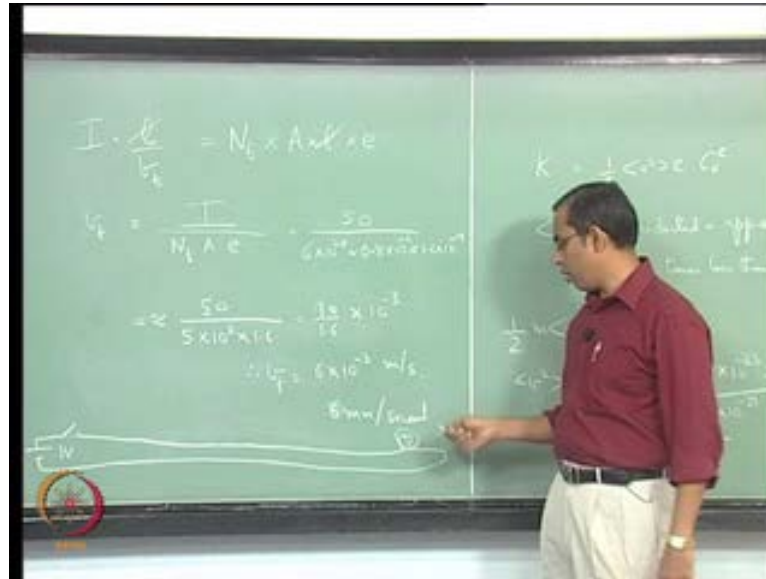
So, we now have so, effectively if you take a 1 meter long silver wire with 1 m m diameter and we just put one volt across it you will get the you will support a current of 50 amps and 50 amps by the way is a lot of current. In fact, it is a lot of current so, you may have a lot of heat from it and so on, but this is the basic calculation the calculation shows you that it try to support 50 amps it is electronically it is capable of supporting 50 amps at this point, because of the voltage that you have placed across it right.

So, now 50 amps so, when you talk of current that is basically transport of charge so, current times time the charge per unit time is current so, therefore, if you look at if you write you can write if this way I times t equals q . So, the charge transported per unit time would be or per (()) charge by time is the current or similarly, therefore, current into time is the charge right. So, this is the situation we have now, the time that it takes for (()) assume that the electrons are moving with this drift velocity which we do not know at this point in at this point what that value is we will assume that the electrons have a drift velocity (()) so, that is the drift velocity fine. So, the time it takes to go across a length one of that conductor time taken.

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So, time taken to travel distance l is simply l by v so, that is the time taken to travel the distance. So, (so) we can now substitute that (()) which we will do now right.

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So, we have the current I into l by v_d this is equal to the total charge that is getting transported across as a result of the passage of the current now, what is the total charge? That total charge is the number of free electrons (N_f) they are all getting transported as a result of this (v_d) is number of free electrons per unit volume of that sample which is the cross sectional area of the sample into the length of that sample. So, this is the volume of the sample this is the number of free electrons in that sample per unit volume times their charge so, this is the total charge that is present in that system in the form of the free electrons.

The free electrons possess this much amount of charge and when you say current is passing through then this is what is getting transported so, that is the total charge and because it has to maintain charge neutrality so, whatever is getting transported has to be transported back (N_f) so, this is what (N_f) . So, if you simplify this we have these two l will go so, we have the drift velocity v_d is simply I divided by N_f the cross sectional area A and the electronic charge e **alright**. So, now, let us put the numbers back into this equation we had the current being 50 amps **right** N_f we said is about we just came up with six into 10 power 28 electrons per meter cube.

The cross sectional area is $\pi d^2/4$ so, this is about 0.8×10^{-6} $\pi d^2/4$ so, $\pi/4$ will be 0.8 and the d^2 will give you the 10^{-6} into the electronic charge 1.6×10^{-19} coulombs **right**. So, now, let us just

simplify this so, this is basically you will have actually this is 6 times 0.8 is about is roughly about 5 you can say and so, that will become ten on top. So, we will have we just put it down this is approximately 50 divided by this 6 and 0.8 (()) become roughly about 5, we have 10 power 28 10 power minus 6 and 10 power minus 19. So, these two become 10 power minus 25 so, 10 power minus 25 so, and ten power 28 we have so, 10 power 3 we will have minus 25 and 28 it give you 10 power 3 and you still have 1.6.

So, this is roughly 50 by 1.6, I am sorry 10 by 1.6 so, 10 by 1.6 into 10 power minus 3 and so, this is actually about 5 no it is about 6 sorry so, 6 into 10 power minus 3 meters per second. So, 6 so, this is 6 into so, this is $v = f \cdot \lambda$ 6 into 10 power minus 3 meters per second so, this is actually what when, if you look at this in millimeters per second this is only 0.6 millimeters per second, I am sorry 0.6 centimeters per second so, 6 millimeters per second. So, we see that the drift velocity is 6 millimeters per second so, this is not a large number this is a very small number fine so, in fact, if you look at it let say you know if you look at this board the length of the length approximate length here is about one meter so, roughly about one meter we can say.

So, if you think of some simple circuit that we can draw so, let just say we have a battery here and we have a wire a silver wire at that and a bulb the wire comes all the way back and we have a switch somewhere here. So, let say we have this hypothetical circuit which we could very easily step up and let say we have access to some (()) expensive silver wire one millimeter in diameter one meter in length. Actually we would need two meters we would need one meter to get to the bulb one meter to come back. So, now, let say we close the switch so, now, the electrons all over silver wire it is charged neutral so, we are not really I mean we are not losing electrons anywhere we just close the switch.

If you see given that and let say we are just put one volt here so, one volt is what we have put let say and so, we have now, we have a circuit we just consistent with all the calculations that we have done. If you close the circuit you see that you know one meter length, if it has a drift velocity of 6 millimeters per second. So, it is basically one meter by 6 into 10 power minus 3 meters so, the time that it will take is about 100 and 50 seconds or more little more than 100 and 50 seconds for the electrons (()) travel this distance and reach the bulb. So, 100 and 50 seconds is a lot of time it is two and half minute's right.

I have actually walked across the board here (()) second or two to come to this side of the board it will take two and half minutes from the time close that circuit for the electrons. Which are close to that switch or close to that battery to come all the way up to this a bulb, if you if you say that this is the drift velocity. So, this is something that we have to really be I mean we should become aware of when we close the switch the light comes on instantaneously. So, we do not see any difference, I mean we switch it we close the switch and we (we) just do not know the difference almost immediately the light is on.

But if you go into the details of the system we find that electrons at one end of that system may take this long of the order of two and half minutes under the circumstances that we have just describe for them to actually complete to reach the bulb and actually participate in this process fine. So, in fact, what is actually happening is because of continuity the electrons which are very close to the bulb start burning the bulb to begin with and then the other bulbs the other electrons arrive at the bulb so, this is basically the process that is occurring.

But I want you to get a sense for the fact this number is rather small so, even if you are actually in fact, to draw an (()) from daily life, if you are actually watering plants with (with) who was send to the water coming through the holes the velocity of water inside that is going to be higher than those. So, (so) when we think of you know electronics when we think how instantaneously things occur and (and) so on, if you going to some details you find numbers like this which (which) are very surprising in this context in the context of what is normally associate with electronic process. So, this is a very small velocity the drift velocity, that we have come up with an incidentally if this is the drift velocity we can also work out a value.

And I would also like to draw your attention to the fact this drift velocity is actually just to the to the extent that it is a small number it is very therefore, drastically different from the mean square velocity that we previously discussed. In the mean square velocity calculation we came up with a number of 10 power 10 meter square per second square and even that we said is actually under underestimate of the velocity. So, what is the difference here please remember when we talk of thermal conductivity.

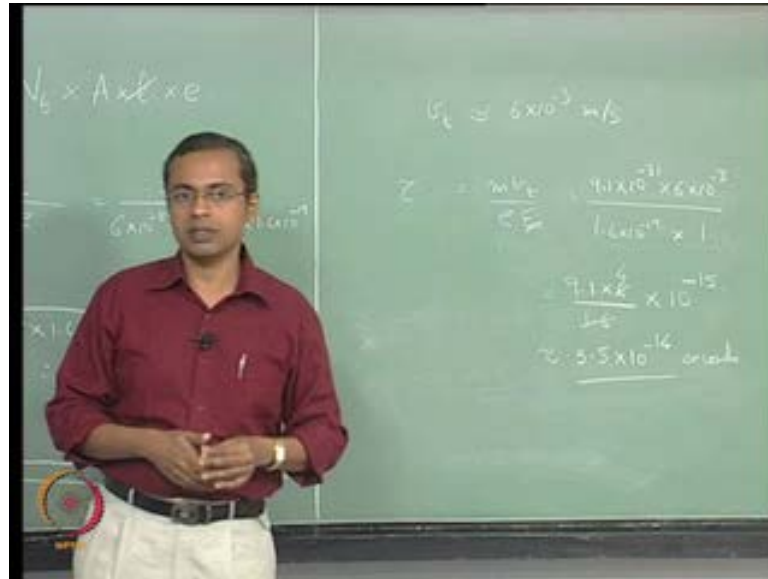
When we talk of thermal conductivity there is no the electrons are not going anywhere so to speak only heat is being transferred from one side to the other side so, and that is occurring by the collisions of those various electrons with each other. On the other hand when we talk of electronic conductivity you apply a potential and there is a continuous movement of electrons, electrons are entering one (one) end of that wire and exiting the other end of the wire and (and) there is a loop when they come back, but the process is there is a movement there is a net movement of electrons. So, if there is some way in which you could follow them you find there is a net movement of electrons.

So, there are two things that are occurring given the temperature of the system let say room temperature or any other temperature, the electrons are all randomly moving. So, they are randomly moving about very rapidly they move about very rapidly they bounce half of each other and so on. But their net velocity is zero they are not (()) anywhere overall (overall) on average their net velocity is going to be zero, because they are largely going to stay within the system they are not headed anywhere. But if you actually look at the average of their speeds or average of and therefore, the average of the square of the velocities that is going to be a very large number because at a given instant it has a very large speed in some particular direction.

So, if you take that number into account it is large number, but overall they are not going anywhere so, overall if you look at it (it) is essentially stationary. When you apply a field in addition to this high speed random motion that is going on within this system in addition to that we have an overall drift of the electrons. So, that overall drift of electrons is this drift velocity so, therefore, it is now not now that we recognize this it is no longer that surprising, that within the system we have a very high speed random motion going on which results in this high average v square value.

But the average drift of the electrons over and above this random movement is actually a very small number so, therefore, these are two different numbers so, (()) in this derivations we have walk through them at different (()) I just wanted to highlight that that there is a difference. And now, having come up with this drift velocity we can also put down a number for that mean time between collisions, which also we previously just randomly arbitrarily put it down and gave some value for it we can actually given this value we can actually come up with a number more appropriately.

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So, we will do that, we see that the v_f is 6×10^3 meters per second and τ the mean free time between collisions is $m v_f$ by $e E$ this is what this is how we defined it this is how we saw it in the equation that we derived couple of classes above. So, this is what we have so, if you look at our system here this is m is the electron mass so, that is again 9.1×10^{-31} kilograms v_f is the value that we have here into 6×10^3 meters per second. The electronic charge is again known 1.6×10^{-19} coulombs per electron, this field that we have put we have put one volt across a length of one meter.

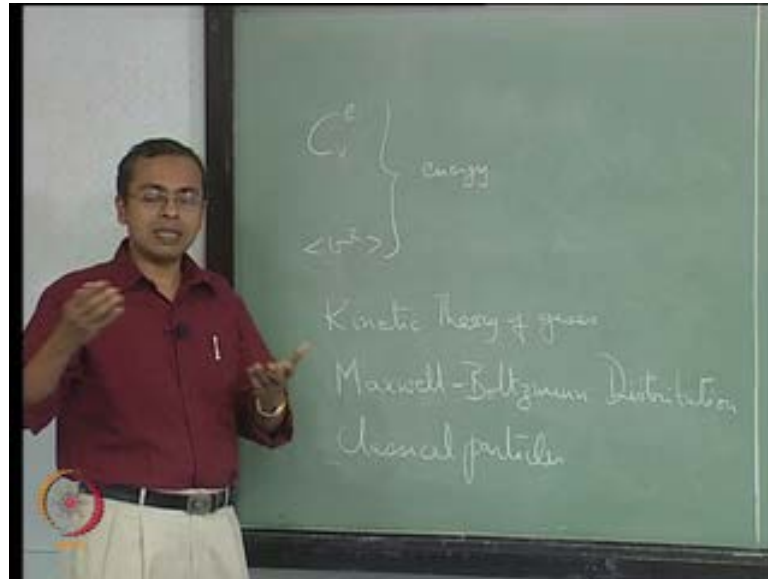
So, it is one volt per meter (()) field that we have so, into one so, this is all we have in this system. So, now, we can run through this calculation this is 10^3 power minus this is 10^3 power minus 31 10^3 power minus 3 so, this is ten power minus 24 and we have 10^3 power minus 19 down here so, if you bring this up here it will become 10^3 power plus 19. So, (so) this is equal to 9.1×6 by 1.6 into so, we have 34 so 15×10^3 power minus 15. So, 1.6 so, 10^3 power minus 19 become plus 19 so, this become plus 16 so, we have minus 15 there so this is what we have. So, if you work this out this is roughly 4 times roughly so, this is roughly about 49 so, about 35 or 3.5×10^3 power minus 14 seconds. So, mean free time between collisions is 10^3 power minus 14 per seconds 10^3 power minus 14 seconds and that is the order of magnitude that we mentioned earlier when we are doing our discussions.

And incidentally also wanted to point out that you know, if you put a larger field larger field across the system then accordingly things will change you may have high currents and so on. So, some numbers may change here in there, but these are numbers that we can say or the right order of magnitude this is the kind of number that we mentioned earlier also. So, all these number we mentioned earlier we did not really put them down and I will show you how they come about so, today I have taken the (the) time to show specifically how these numbers come (come) together so, that we can relate to them much better.

So, that is the way these numbers come or at least one way in which we can arrive at these numbers, and we also have been discussing through the day about how reasonable they are what the error is. So, as far as the drude model is concerned we have looked at all the major results of the drude model come up with numbers in (in) one case looked at the fact that experimental values are distinctly different from the theoretical prediction and (and) that is one thing that we noted to orders of magnitude of and so on. So, with this feel for the drude model and the expressions it gives us the values it predicts and so on.

We will exanimate a little more and to see with (with) also this understanding that the drude model actually succeeds in some places and fails in some others. With the help of these equations and these numbers we will exanimate a little bit more to see, if we can get a better sense of what is the particular aspect of the drude model where the error lies. So, that we can then examine that aspects some more and find alternate ways in which we can address this situation so, we will do that now.

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So, if you put it down now we have two places in the drude model where we specifically have issues with what it predicts one is the electronic contribution to specific heat at constant volume, and the v square value. So, this v square value then I mean we talk of you know $\frac{1}{2} m v^2$ has being the translational kinetic energy of an on per electron basis this is also energy and how (how) that energy changes as you change the temperature so this is. So, we find both of these are related to energy so, they are somehow related to energy of the system, and this velocity that we are talking of we have actually put this down as the mean of the squares of the velocities, because we said that is more representative of a system where you have so, many you know ten power twenty three order of magnitude kind of electrons.

So, what we need to understand what we need to recognize from this situation is that somewhere, along the line we are using some process by which we are coming up with this average for this v^2 , which is possibly where there is an error. So, because that then gives us the energy then and essentially that all relates together here, because it is all related to energy (fine). So, this gives us a clue that first of all the issue is with how we are estimating energy how we are estimating the behavior of the energy of that system behavior in the sense of this how much more energy would you require to rise that the energy of the system by the temperature of the system by one degree and so on.

So, all that relates to how is energy present in that system so, that is the key question that we are that (()) out of this look at what the drude model has done and what it is predicted. So, somewhere along the line in our description of this model, we have made some assumptions about how energy is held within this system and that assumption is affecting us later on in the predictions that this model is making fine. And as I said before this model starts using the kinetic theory of gases as its basis. So, the (the) manner in which energy is held within the system is the same manner in which energy is held within the system as it is indicated in the kinetic theory of gases so, that is the link that we have right.

So, in the kinetic theory of gases there is a statistical distribution statistical description a description of the manner in which energy is stored within the system. In (in) the kinetic theory of gases as well as in the drude model where we are dealing with a very large number of particles (very large number of particles) it is not feasible it is highly infeasible for us to actually know the every of every single individual particle. So, even when we talk of a v^2 value here average of the squares of the velocities it is not actually calculated by somehow having a detector which actually now measures the velocity of every single electron or every single molecule present within the system and then somehow you (you) know that add them all up and divide by the number.

So, this is not something that we can actually physically do instead what is actually done is a statistical method is adopted, where we make some assumption on how the energy is distributed amongst particles, that assumption is not I mean it is based on what (what) what we understand of the system based on how what we understand of the system. We actually come up with this description of how energy will be stored how energy will be distributed amongst the particles, you have a very large collection of particles you have some energy all the particles do not attain exactly the same energy at exactly the same time so, there is a distribution.

So, some assumption is made on how that energy is distributed or a theory is built to tell us how that energy is distributed amongst those particles. So, that theory in the original kinetic theory of gases has been built for this large collection of ideal gas molecules, non interacting ideal gas molecules is that system that has been taken up for description or description. And in that system given that manner in which those ideal gas molecules behave given that understanding (given that understanding) of how ideal gas molecules

behave based on that understanding a description has been developed on how energy would be stored amongst those particles. So, that description is referred to as the Maxwell Boltzmann distribution.

(No audio from 44:40 to 45:58)

So, the Maxwell Boltzmann distribution so, we just attributed to these two people Maxwell and Boltzmann. Maxwell Boltzmann distribution is fundamental aspect of the kinetic theory of gases, in the kinetic theory of gases where we come up with v square mean square velocity that mean square velocity comes from $\langle v^2 \rangle$ as a direct result of the Maxwell Boltzmann distribution. And the Maxwell Boltzmann distribution merely looks at this specific case that you have this large collection of particles of large collection of non interacting particles and then tries to explain how energy would be distributed in that system. In other words at given energy how many particles should be there at some other energy how many particles should be there and so on.

And what is the logic based on which is distribution is occurring so, all of that is put together in this Maxwell Boltzmann distribution, if you look at the Maxwell Boltzmann distribution in detail from Maxwell Boltzmann distribution you can come up with a velocity distribution. So, there two things here a Maxwell Boltzmann distribution which shows how the energy is distributed from that you can $\langle v^2 \rangle$ and get how the velocities and distributed or therefore, how the squares of the velocities are distributed. And so, from there we get a value for v square this average of the squares of the velocities.

So, this is therefore, the energy of the system how things are distributed how energy is distributed within the system and therefore, something that affects your $C v e$ calculation how $\langle v^2 \rangle$ specifically how it affects your v square calculation mean square mean of the squares of the velocities are directly related to how the Maxwell Boltzmann distribution describes the distribution of energy within that system. So, this is very fundamental to the kinetic theory of gases and to the extent, that we have extended the kinetic theory of gases in pose the rules of kinetic theory of gases directly on to the free electrons that are present within the wires the metallic systems that we are looking at to that extent.

In the description even though we so, far did not $\langle v^2 \rangle$ as stated what we are effectively stating is that electrons within a solid the free electrons within the solid are actually following Maxwell Boltzmann distribution in terms of the how they are distributed

amongst the various energy levels that are available to that. At this movement I am just indicating it you as energy levels and so, on in our next class we will discuss that (()) detail so; it will become more clear to you. But, the point is the point I wish to highlight is that there is such a thing called Maxwell Boltzmann distribution, which looks at how energy is distributed within the system from which you can get lot of things out of that system in terms of for example, the velocity distributions and so on.

And the Maxwell Boltzmann distribution is valid for a set of non interacting particles, but there is some other specifications too which we will get too. And therefore, it is perfectly valid for this ideal gas system which then and therefore, it is perfectly valid for this ideal gas system which then and therefore, perfectly valid for the kinetic theory of gases. We have taken this distribution and (()) it without necessarily stating it upfront we have (()) on the system of electrons present within the solid, which and therefore, ensured that you know or at least therefore, use this idea that their energy is and therefore, their velocity is are also similarly, distributed.

So, we have impose this idea on to those electrons and as a result of (()) the numbers that we have come up with we came up with some numbers for $C v e$ and v square and so on and it turned out, that those numbers wrong. So, somewhere along the line the use of this Maxwell Boltzmann distribution for the electrons has turned out to be incorrect. So, that is the underlying message that we are now able to extract from all the understanding that we have now come up within the system. So, something that was valid for non interacting particles we imposed on particles which can actually interact with each other electrons can get attracted to the ionic cores.

So, we impose this on those particles and (()) that is not really correct it is giving us two orders of magnitude error here and two orders of magnitude error here so, therefore, that is the so, there is an error imposing this. So, what we will do in the next classes we will look at this distribution in greater detail and we will actually derive this distribution, we will look at all the assumptions of this distribution and derive this distribution in the next couple of classes. And we will see that a and we will see what that prediction is and then what we will find is that, I kept mentioning that these (these) are non interacting particles and so on.

There are specific assumptions made on the behavior of a particle which (which) allows us to use Maxwell Boltzmann distribution to describe its energy we (we) cannot use this description to describe the energy of just about anything we wish. The kinds of particles that only particles that have certain attributes can (can) be reasonably made to for, I mean can be reasonably expected to follow the Maxwell Boltzmann distribution fine. So, what we will see is the kinds of particles that actually follow Maxwell Boltzmann distribution are referred to as classical particles.

So, I have now given you two terms to think of Maxwell Boltzmann distribution and classical particles this is something that you will here mention in many places many books will mention classical particles and so on. What we will do specifically in the next few classes is we will look at this we will derive this Maxwell Boltzmann distribution and understand our sources of error. So, therefore, that is itself and immediate task that we have a (()) and (and) by doing the derivation and understanding the sources of error we will also see if there is a we will see if there is there are pointers on where we should (()) see if there is a better way of describing the system that we are dealing with.

And that search for a better description for the manner in which the particles in a in the system that we are dealing with the electrons solid in that search for that a better description for those particles. We will actually focus on this definition of what is a classical particle. So, we will look at this distribution and as a result of having of what we understand from this distribution we will come down to this specific term classical particle, which is often in many books is just mentioned. We will actually analyze this description the definition of a classical particle very carefully we will see what is the kind of a particle that gets called as a classical particle and what is a non classical particle, what is that specific feature that distinguishes between a classical particle and the and an electron.

For example, it specifically in our case (()) dealing with electrons what is it that is different between a classical particle and an electron. And therefore, if Maxwell Boltzmann statistics can be used for classical particles what is the other description that we could possibly use for electrons. So, this is the direction in which we will head once we identify this other description that can be used for the electrons, we can then go back and redo our equations and see if we are now in a position to better make predictions and specifically if we can correct for this errors specifically the errors in the specific heat

capacity at constant volume. So, this is the direction in which we head by look at this distribution and look at the definitions and come up with a $((\))$ distribution.

So, with that we will halt in this class and we will take up these issues in the upcoming classes. Thank you.