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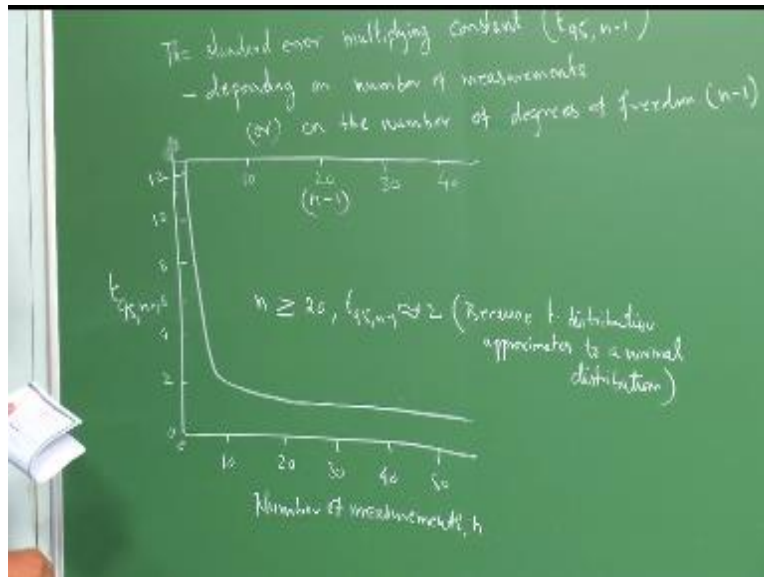
**Tutorial-4  
Materials Characterization  
Quantitative metallography**

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Hello every one welcome to this online material characterization course organized by NPTEL. In last classes we have just looked some of the statically methods to analysis the micrographs. Where we want to calculate the volume fraction of second phase or the point I mean the size of the second phase. How to do that and what the diffracts method available and so on, so I would like to continue that tutorial class we will just recollect what we have seen in the last class. And then I will also talk about the kind of errors standard propagation errors or standard errors. Whatever we have seen in the last class and then I will just take you to the actual problem solving.

Using some of the standard micrographs taken from optical microscope or the standard microscope. So what so if you recall what all we have seen in the last class as a statically methods for propographic analysis. You will recall that we have looked at that volume fraction  $V_v$  and length fraction  $L_l$  and point fraction and so on. So let me continue to,

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So if I recall that I introduced the term as standard error multiplying constant  $T_{95, n-1}$  by 95 is the we are looking at the data. That is random data for 95% of constant level, and then you would like to look at the significant of this though I introduced it I did not talk about it. So let us look at the significant of this and this constant is depending on number of measurements on a random sampling. So how this constant is getting influenced by number of samples as well as degrees of freedom that we will see. So if you look at the graph which we have already seen in the last class also.

The standard error multiplying constant varies with the number of measurements small  $n$  like this. And then you can see that after it reaches beyond 20 the  $T_{95, n-1}$  error constant stabilizes around 2 beyond 20 okay. So you can I have just mentioned in the last class so we will recall this we will write this. So  $N \geq 20$ , the  $T_{95, n-1}$  try to stabilize beyond 2, this is because we can write it in the bracket. Because  $t$  distribution approximates to normal distribution. So when  $N = 20$  or  $\geq 20$  the constant remains 2, because the  $t$  distribution approximates to a normal distribution.

So now what I will do is you can even mention the degrees of freedom this column so this one is 0. This is 10; this is  $n-1$  that is degrees of freedom. That compares with the number of experiments.

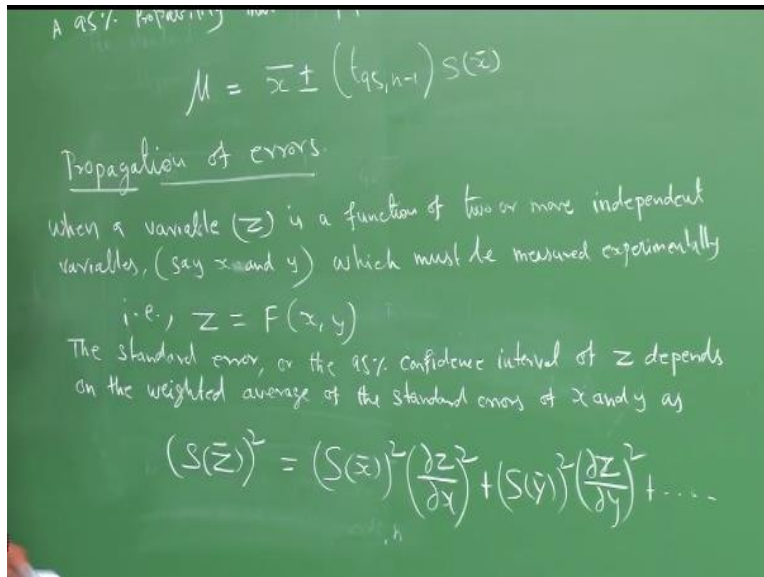
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$n-1$	$T$
1	12.706
2	4.303
3	3.182
4	2.776
5	2.571
6	2.447
7	2.365
8	2.306
9	2.262
10	2.228

So what I will do now is I will just give you some values in a form of a table for 95% constant limits as a function of degrees of freedom or  $\sim$  which is nothing but  $n-1$ . So that you can have the ready reference for this how this constant varies with the number of experiments. So let us spot  $\sim$  versus  $T$ , so you have the value of  $\sim$  that is degrees of freedom with  $T$  up to 10 so I will give you few more for the reference  $\sim$  and  $T$  may be 15 I will give you for example, 2.131 and 20.086.

So you may find these values in standard textbooks also but this is just for the regular reference for you to calculate the radians for freedom for the  $T$  and then you can use these values for the statistical method which we are going to use for calculating various micro structure parameters or stereological parameters and now what I will do is, I will barfly discuss the propagation of errors and before that let us recall the.

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A 95% probability that so this is the formula which we see in the last class just to recollect because we have now tabulated the T for different degrees of freedom where you have the 95% of probability that the propagation mean value lies between this range, so all the stereological parameters whatever you measure should be represented or should be presented in this form if you really want to take care of the accuracy or the most meaningful values of your sequence analysis.

Then this is considered useful so let us now look at the propagation of errors because it is very important that one should know what is the error bar you have for your data and also you should know how the propagation of the error goes along with the number of measurements of the kind of samples you do a random sampling and so on. So what I have written here is, when a variable Z is a function of two or more independent variables say X and Y.

Which must be measured experimentally like your one of the stereological parameters like volume fraction or something like that, that is  $Z = F(x, y)$  the standard error or the 95% confidence level of Z depends on the weighted average of the standard errors of X and Y as  $S(\bar{Z})^2 = S(\bar{X})^2 (\frac{\partial z}{\partial x})^2 + S(\bar{Y})^2 (\frac{\partial z}{\partial y})^2 + \dots$  like that it will propagate depending upon the

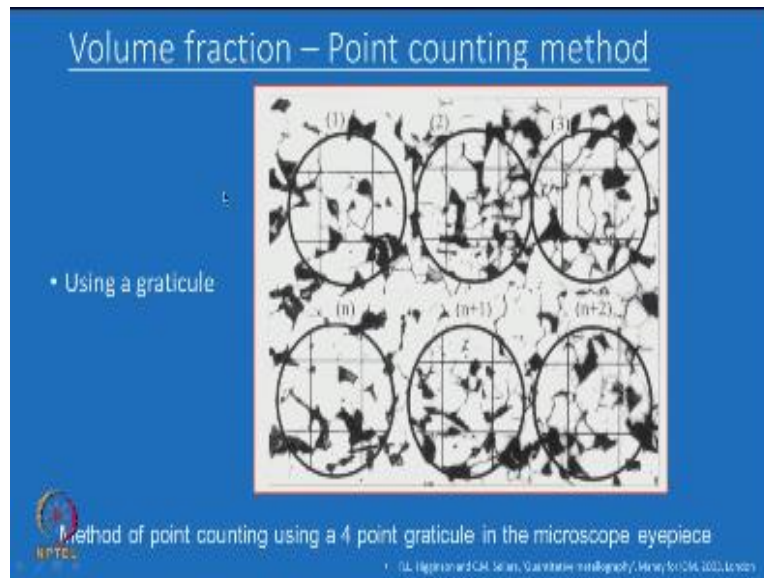
number of variables if you have z then the z term will again x, y, z term also will come we can write the same thing in another form.

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For example, if  
 $z = x^n y^m$   
 $(S(\bar{z}))^2 = (S(\bar{x}))^2 (n x^{n-1} y^m)^2 + (S(\bar{y}))^2 (m x^n y^{m-1})^2$   
 or dividing above, the relative standard error  
 $\left(\frac{S(\bar{z})}{\bar{z}}\right)^2 = \left(\frac{S(\bar{x})}{\bar{x}}\right)^2 n^2 + \left(\frac{S(\bar{y})}{\bar{y}}\right)^2 m^2$

So when you have the variables X and Y the standard error propagation settle like this for example you can have variables of this form then the standard error has written in this fashion or you can also write this equation divided by the irrelative standard error then you have the  $S(\bar{Z}) / \bar{Z}$  in terms of  $S(\bar{x} / \bar{x})^2$  times  $N^2 + S(\bar{y} / \bar{y})^2$  times  $M^2$  so for example if you are interested in the use measuring the grain size or colony size and etc we should use the error distribution and then represent you are the results, so these are the brief introduction of all the statistical methods what I have been talking about, now what I will do is I will just take out some practical example how this statistical methods are actually evaluated from the micro graph which is taken through optical microscopy or scanning electron microscopy. So in that first exercise what I will do is I have few slides to show you.

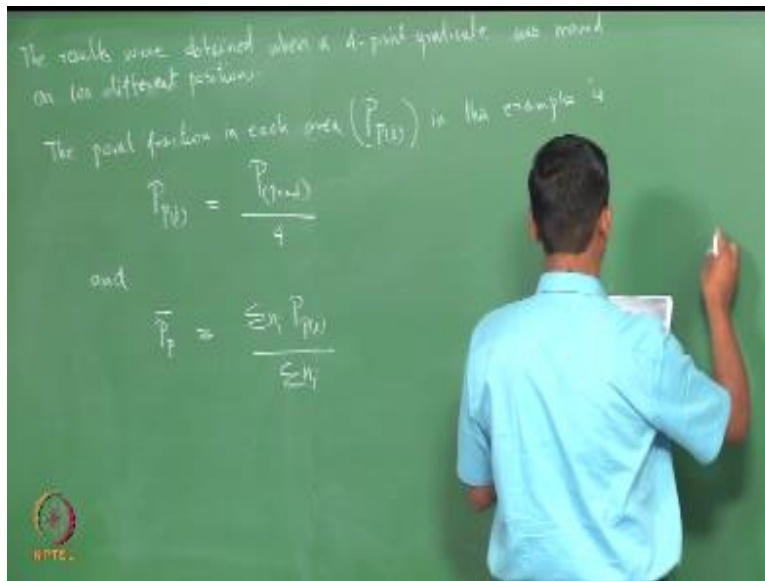
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If you look at this slide this is the micrograph taken from a steel sample you have you are seeing the bright white background and a dark patches this is a microstructure those who are not familiar with this steel microstructure extra you can also consider this as a two phase microstructure where you have the white as a matrix and the dark patches or a second phase and for those who are familiar with the metal energy of steel this is a frayed pyrolite microstructure.

So what I want you to concentrate on this how to use a graticule in a microscope optical microscope to find out a volume fraction using a point counting method. So that is the idea, so you have to just use some of the graticule in the eyepiece of the microscope that is why it is shown in a circle, suppose you take this micrograph at the magnification of 100X then if you use graticule.

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So what I have written here the results which I am going to present on the black board is obtained when a 4 point graticule which I showed on the slide was moved on 100 different positions on the using the microscope, so our interest is to find out the volume fraction here so but here we are kind time to find out the point fraction in each area that is  $P_{Pi}$  in this example is  $P_{Pi}=P$  here it is written that black phase is called pyrolite phase it could be anything in your case, so just for a clarity I have written it is a  $P/4$ .

So we can write, before I write the other formula so what you have to understand here is using this probe 4 point graticule you can get 3.5 or 4 or even 0 so those probabilities are there let us see what kind of values we get for measuring this values so the same thing can be written for the average values for n number of counting so you get.

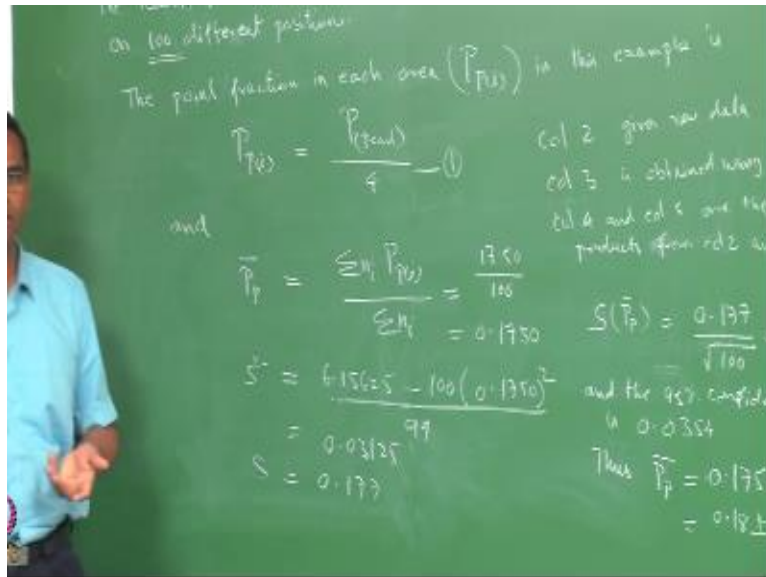
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(2) Number of areas counted $n_i$	(3) Point fraction in per light $P_i$	(4) $n_i P_i$	(5) $n_i (P_i)^2$
40	0	0	0
8	0.125	1.00	0.125
38	0.20	4.50	2.375
	0.375	1.50	0.5625
	0.50	3.50	1.75
	0.625	1.25	0.78125
	0.750	0.75	0.5625
	0.875	0	0
	1.00	0	0
		$\sum n_i P_i =$	$(\sum n_i P_i)^2 =$

So what I have done is here is the big table but still work to go through it one by one so that you will be able to appreciate what we are going to see in the falling fraction counting method results so you have the column one has the number of points in for make the black phase what I have shown the micro graph this is number of areas counted so which is 100 because I said here 100 different positions the practical is mode so  $\sum n_i = 100$  and the point fraction in per light  $P_i$  and  $n$  times the  $P_i$  and then this is  $n$  times square of  $P_i$  so now I will also write how do we get this each of this.



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So the first one the point fractions is taken from this and the average  $P_p$  is taken from this so you have this  $\sum n_i$  is 100  $\sum n_i P_{Pi}$  is also available in the table so you can write just for the reference column two just gives the raw data column three is obtained using this formula for example you say one using points column four and five for the products from column two and column three so you have some so this is a raw data according to this micrograph column three is obtained using this equation and column four and five.

Or the products from column two and three so two and three you get this and then of course this the simply a square of this so what you will see here is so you can just let us write here value is about 17.50 and here so 6.15 625 and if you can substitute same thing here so this is what you get as the average  $P_p$  that is a point fraction if you use the 4. Radical more or different position on the micro graph itself what I have shown so this is the final value but now we have to calculate the standard error how to do that.

So simply we know that the previous class I will show how to calculate the standard deviation so you can directly take that formula and substitute these values, so we can calculate the standard error from this standard error is PPS of S which is nothing but 0.177 we have use this formula

then so which is nothing but we can write it so on the 95% confidence limit is 0.0354 and thus your PP bar is equal to  $0.175 \pm 0.035$  or you may round it off to  $0.18 \pm 0.04$  so the point fraction lies between these limits.

And that is how the data is represented finally with the 95% of confidence limit for this simple point counting method so what is important here is you have to remember that the number of samplings is important the more the measurement the more accurate will be and the you generate all this statistical data carefully and then calculate this standard deviation and the standard error and present the final data to the 95% of confidence limit like this so we can look at the similar analysis in the calculation of the falling fraction using a linear analysis also like mean intercept length also we can do that I will just demonstrate that in the next class thank you.

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