Fracture, Fatigue and Failure of Materials
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Lecture 09
Plane Stress and Plane Strain Fracture Toughness

Hello everyone. Here we are with the ninth lecture of the course Fracture, Fatigue and Failure of Materials.

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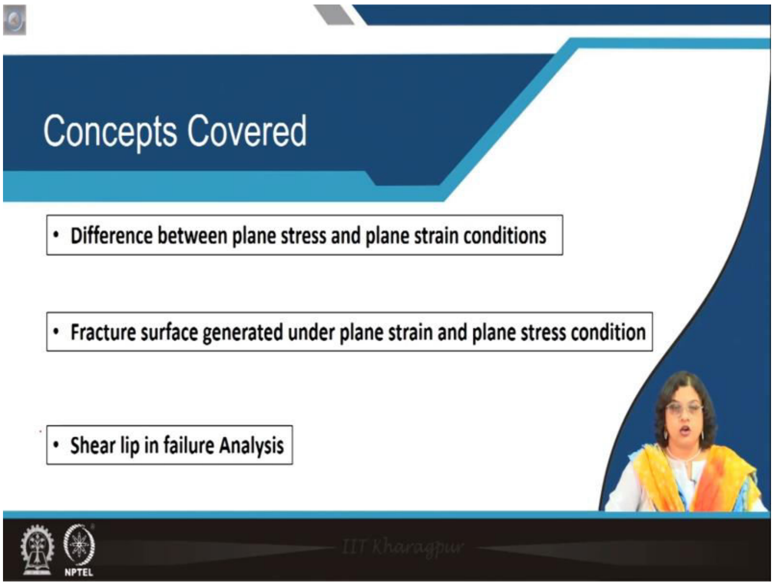

And in this class, we are going to talk about the different concepts which includes primarily the plane stress and plane strain conditions and the difference between these two conditions. We will start from where we left for the last lecture, like the plastic zone size and how does that influence the plane stress or the plane strain conditions and the differences or the similarities between these two.

And next we will be talking about the fracture surfaces that are generated and based on which how we can distinguish between the fracture that has happened in the plane strain condition or plane stress condition. And that will lead us to understand the importance of these two different conditions in failure analysis and how the formation of shear lip can be used to determine the failure condition and the failure analysis of already failed component.

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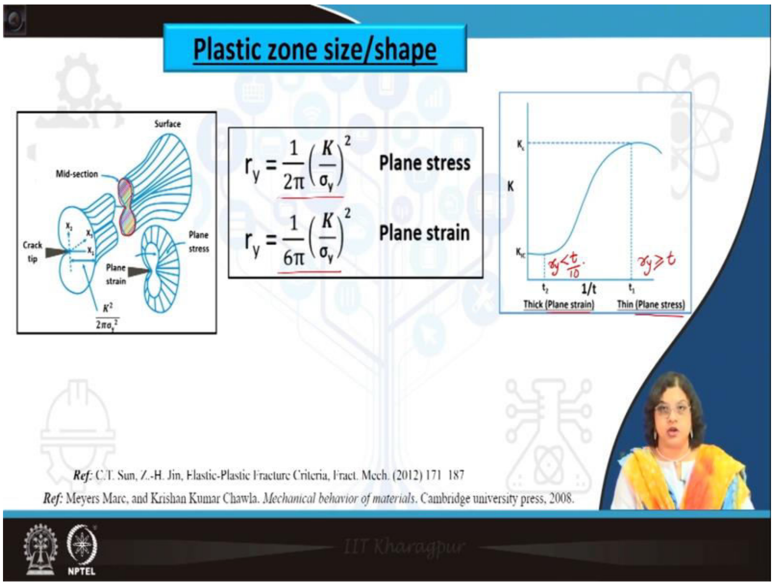

So, let us start with the last lecture where we left and we discussed about the plastic zone size and shape. We have seen that if there is a crack in a component, then near to the surfaces or the edges, the plane stress part develops which includes a higher plastic zone size, something like

$$r_y = \frac{1}{2\pi}\left(\frac{K}{\sigma_{ys}}\right)^2$$

On the other hand, at the interior part, particularly in the middle part, the plane strain condition generates which essentially means a triaxial condition of stress and biaxial strain. So, overall the strain that is generated is being constrained and amount of the strain that is generated is one third of that generated under plane stress condition.

We have also seen that based on the thickness of the component, this plane strain or plane stress can actually generate. And we have seen that for the case of thin component, plane stress condition is active and when we say thin actually we mean that the plastic zone size that is developed is either greater than or equal to the thickness of the component.

On the other hand, when we are talking about a thick component, plane strain condition generates. And that means that the plastic zone size generated is less than one tenth of the thickness or thickness is typically being 10 times the plastic zone size or even more than that. So, based on this, we would like to figure out that how we can determine from the thickness of the component or the additional parameters that whether plane strain or plane stress condition will be active.

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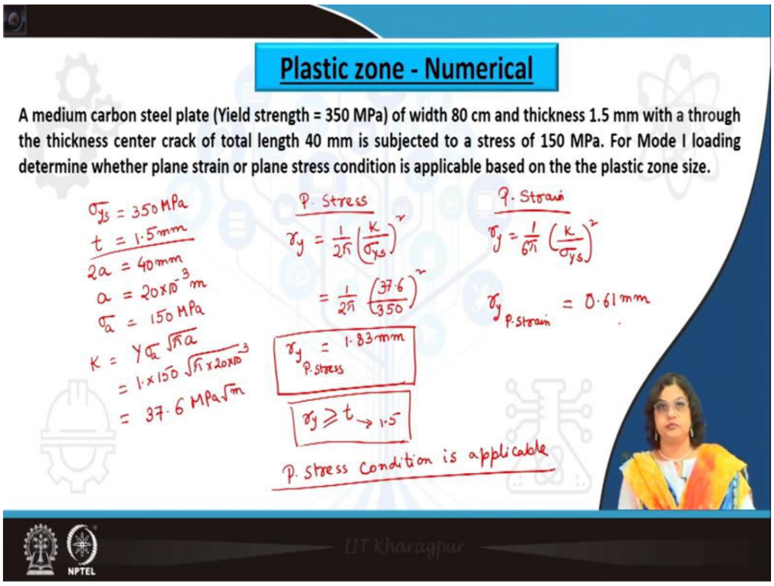

And to understand that we would like to solve a numerical to understand this concept further. So, it says a medium carbon steel plate with yield strength of 350 MPa and width of the plate being 80 cm and thickness being 1.5 mm. It has a through the thickness that means a center crack of total length 40 mm. And it is subjected to a stress of 150 MPa.

So, for mode 1 loading, we need to determine that whether plane strain or plane stress condition is applicable. And for that the first thing that we need to determine is the plastic zone size for both the conditions. So, let us see what we have already the information are as follows. We know that the yield strength of the material is 350 MPa and the thickness is given as 1.5 mm.

And most importantly it has a crack of total length that is 2a equals to 40 mm. So, that means a is 2 cm or 20×10^{-3} m. And the applied stress is given as 150 MPa. So, if we need to figure out the plastic zone size under plane stress condition at the first hand or the plane strain condition thereafter, what we need to know, is the value of K. Because r_y is given by

$$r_y = \frac{1}{2\pi}\left(\frac{K}{\sigma_{ys}}\right)^2$$

And in this case r_y will be just one third of that or $r_y = \frac{1}{6\pi}\left(\frac{K}{\sigma_{ys}}\right)^2$

So, what we need to find out is the value of K first. So, again K can be determined by the relation it has with the geometric factor Y

$$K = Y\sigma_a\sqrt{\pi a}$$

And in this case, since this is a center crack, we know that Y equals to 1, σ_a is already known to us and this is 150 MPa. And a is 20×10^{-3} m.

So, if we simply solve this. So, this comes around 37.6 MPa$\sqrt{m}$. So, if we plug this value here for the case of plane stress, we are getting 37.6 divided by 350 whole square. So, that leads us to something like 1.83 mm. So, that is the plastic zone size for the plane stress condition.

And if we do similar calculations for plane strain, since this is just one third of what we are getting for the plane stress condition, so here, we are getting r_y for plane strain is something like 0.61 mm. So, these are the values of the plane strain and plane stress plastic zone size.

Now, we also know that there is a relation between plastic zone size and the thickness of the specimen. So, in this case, thickness is 1.5 mm and this is quite close to what we are getting under plane stress condition. We know that for plane stress condition we need to have this plastic zone size which should be either greater than or equal to the thickness.

And since in this case, the thickness is 1.5 mm, so, this very well satisfies this criteria and then we can say that plane stress condition will be active. So, that is what we can figure out from this. On the other hand, for the case of plane strain to be active, what we have needed is that the thickness should be ten times the plastic zone size.

And that means that the thickness should have been 6 mm at least, which definitely is not being maintained here. So, that means that plane strain condition will not be valid for this particular case. So, this is the way that we need to figure this out beforehand often we need to have the component running under either plane strain or plane stress condition, because of the fact that we know that the value of the fracture toughness does vary if we are talking about plane strain condition or plane stress condition. So, to keep that into account we need to understand this and to appreciate this beforehand.

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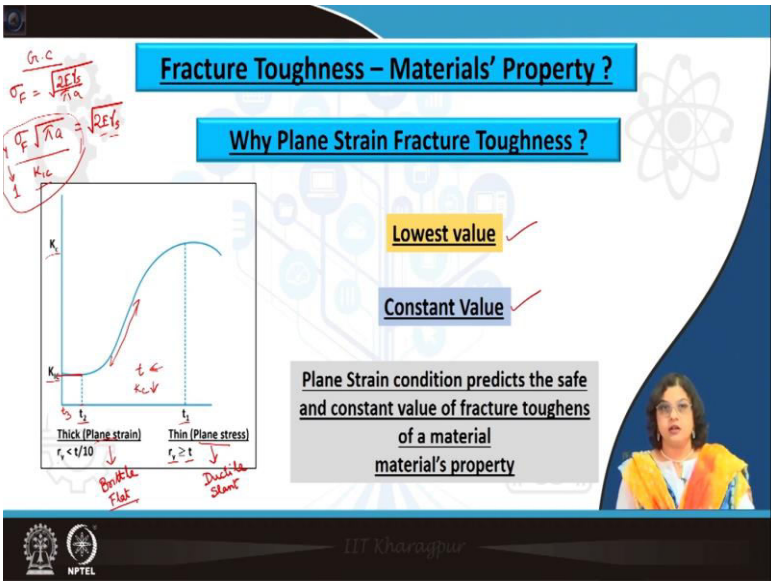

So, then comes to the most important question is that is fracture toughness certainly a materials property? So, as I have just mentioned that fracture toughness values do change for plane strain or plane stress condition. On the other hand, we have also seen right from the Griffith criterion itself, which says that as per Griffith criterion, we have seen that fracture strength is given by

$$\sigma_F = \sqrt{\frac{2E\gamma_s}{\pi a}}$$

and when we just incorporate rearrange this relation, we get $\sigma_F \sqrt{\pi a}$

which is nothing but $K1_c$, because Y in this case is 1, right.

And that leads to $\sigma_F \sqrt{\pi a} = \sqrt{2E\gamma_s}$

We have talked about this that since E and γ_s, the elastic modulus and the surface energy, both are materials property, so, that makes this part on the left hand side of the equation

$\sigma_F \sqrt{\pi a}$ or for that matter $K1_c$ is considered as a materials property.

But we have also seen this diagram which says that if the thickness and the relation between thickness and the plastic zone size is not maintained then plane strain condition will not be maintained or plane stress condition will not be applicable. And then we see a large variation in the fracture toughness values. Both of these Kc or $K1_c$ both of this for that matter is considered or termed as fracture toughness. But in certain cases we consider fracture toughness as the materials property.

Now, this is particularly for the plane strain condition. When we say that fracture toughness is a materials property, we actually mean that plane strain fracture toughness is a materials property. If nothing is mentioned, it automatically means that we are talking about plane strain fracture toughness.

Otherwise, we need to state that this is plane stress fracture toughness and for that we definitely need to mention about the thickness of the component as well. Now, why do we consider plane strain fracture toughness as a materials property? What could be the reason for that?

Because both the cases be it a plane strain or plane stress, both the cases, we need a certain value or a critical value of K for fracture to initiate or to lead to the final catastrophic failure. So, the main important points for plane strain is that it gives us a lowest value that is from the point of view of failure analysis that is very important.

Because unlike strength, for example, if we are talking about yield strength or fatigue strength, we always target to achieve higher and higher strength when we are talking about their structural performances. When we are talking about failure analysis, however, we are a little bit conservative and we mostly try to figure out, what would be the lowest value.

So, that if we can predict the lowest value, we know that in service, if it is having higher than that least expected value, it will survive for definitely that amount of stress. So, it has a lot of significance in case of failure analysis that we want to find out the lower limit and not the upper limit in this case.

But most importantly what is in favor of plane strain fracture toughness being considered as materials property is the fact that it has a constant value. You see for the case of plane stress, when the thickness is increased from this t1. It is still under plane stress condition or in the mixed mode kind of condition but there is a rapid change in the values of fracture toughness. If we are increasing t, t is increasing in this direction and fracture toughness is decreasing, right. And there is a very sharp change in the fracture toughness values.

Now, whatever change is, that is something very difficult to predict. So, we always look for values which are constant, or which are not bound to change even if we are making some minor changes. Otherwise, if someone is determining the plane stress fracture toughness with a thickness of one millimeter of the component and if someone else is determining the plane stress fracture toughness with a thickness of let us say 1.1 mm, there can be some differences in the fracture toughness values. So, obviously, that cannot be used as a standard one.

Rather, if we achieve a thickness of t2 that is the minimum thickness that is required for plane strain condition to be valid, that is equivalent to r_y equals to t/10. And if we are having even a thicker component t3, like t3 could be twice the value of t2 or even more, then also this value is constant. It is not changing any further.

So, we are getting the lowest value and we are getting a constant value and that makes plane strain fracture toughness a materials property. So, typically when we say that fracture toughness of titanium alloy is something like 50 MPa root meter or some category of steel is let us say 35 MPa√m, we definitely mean about the plane strain fracture toughness.

And if we look into it more carefully, we will also understand that plane strain fracture toughness actually means the failure is in brittle mode. We have a flat fracture surface compared to a thin component which leads to a ductile mode of failure and that leads to some amount of plastic deformation and that leads to a slant fracture surface. So, that means this lowest value of fracture toughness has something to do with the lack of plastic deformation.

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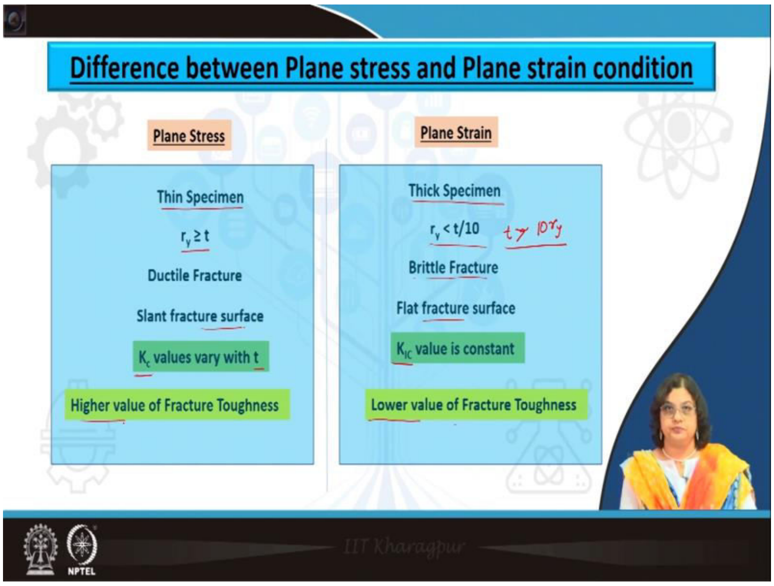

So, let us look into the differences between plane stress and the plane strain condition one more time. And what it says is for the case of plane stress can be achieved in case of thin specimen whereas we need a thicker specimen if we want to look for the plane strain fracture toughness, typically, this thickness has something in relation with the plastic zone size.

And the plastic zone size should be greater than or at least equals to the thickness for the case of plane stress to be valid. And for plane strain the thickness should be 10 times the plastic zone size.

So, 10 times of r_y at least or even greater than. And as I mentioned the plane stress will lead to a ductile fracture and that will have a slant fracture surface. Because typically, any kind of ductile deformation will occur along the 45° shear plane and that leads to a slant fracture surface.

On the other hand, plane strain is giving us a brittle fracture and that obviously is related to a flat fracture surface. Most importantly, this plane stress fracture toughness value which is typically denoted as Kc that varies with any change in t or the thickness. On the other hand, plane strain fracture toughness which is typically termed as $K1_c$, is a constant one and it does not vary even if we are increasing the thickness value.

And essentially, we get higher value of fracture toughness under plane stress condition and lower value of fracture toughness under plane strain condition. So, if we are doing the laboratory testing, we have to be careful to design the component in such a manner that it satisfies this criteria of thickness being greater than 10 times the plastic zone size, so that we get a lower value of the fracture toughness. And in practice, even if it goes for the plane stress condition, we know that it will require higher amount of K value to fracture under plane stress condition.

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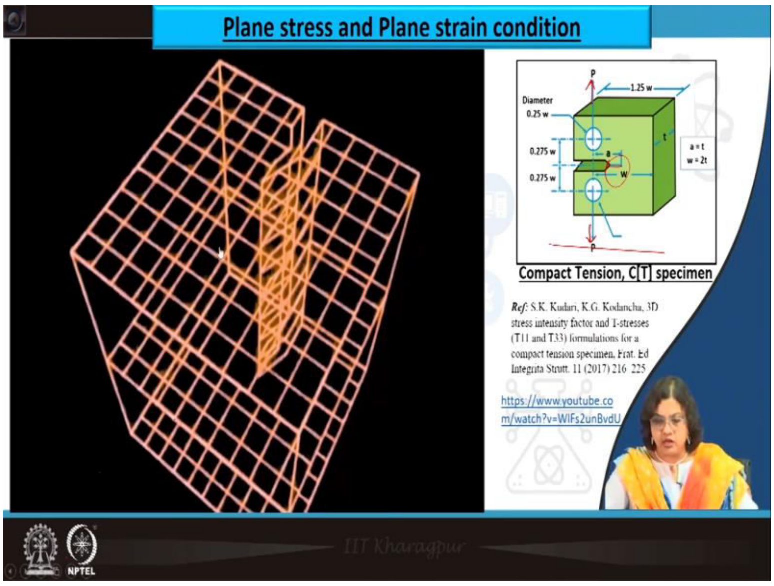

So, here is a video which explains how the plane strain and the plane stress mechanisms are active. So, this is particularly done on a compact tension specimen which is typically used for fracture toughness testing. So, this is three-dimensional specimen that you can look for. This one is quite thick here and when we are loading in along this direction, the crack which is perpendicular to this loading direction, expands along this direction.

And all the activities are happening in this zone ahead of the crack tip. So, let us see how this works in this video. So, we have seen that in actual practice when we have a component like this, how we can see the the crack propagation behavior. So, this is a three dimensional component which is having a crack in it. And we can see that how the crack grows.

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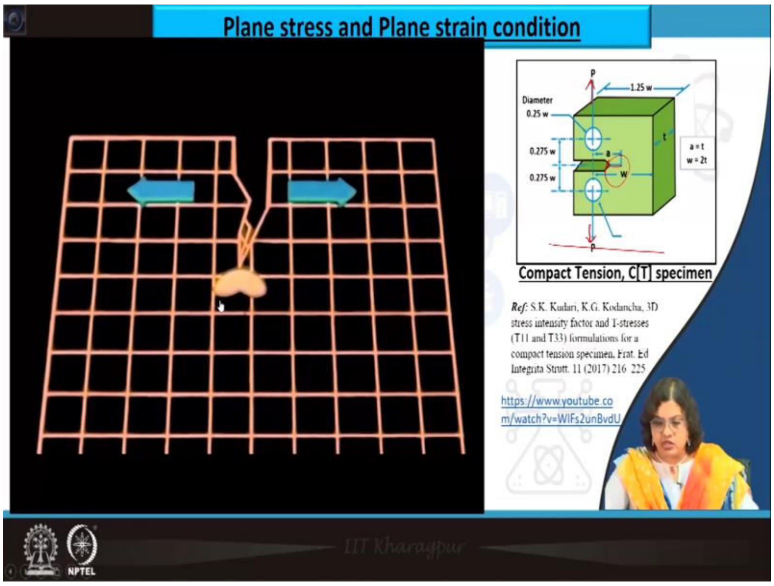

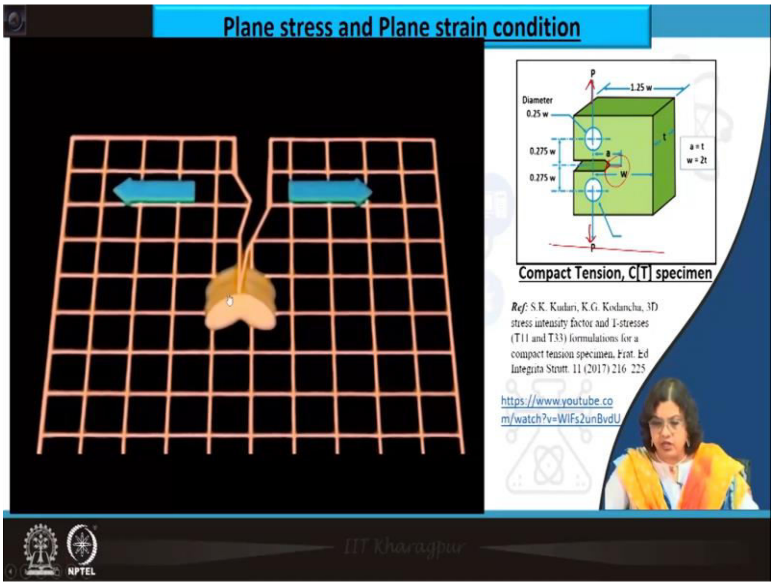

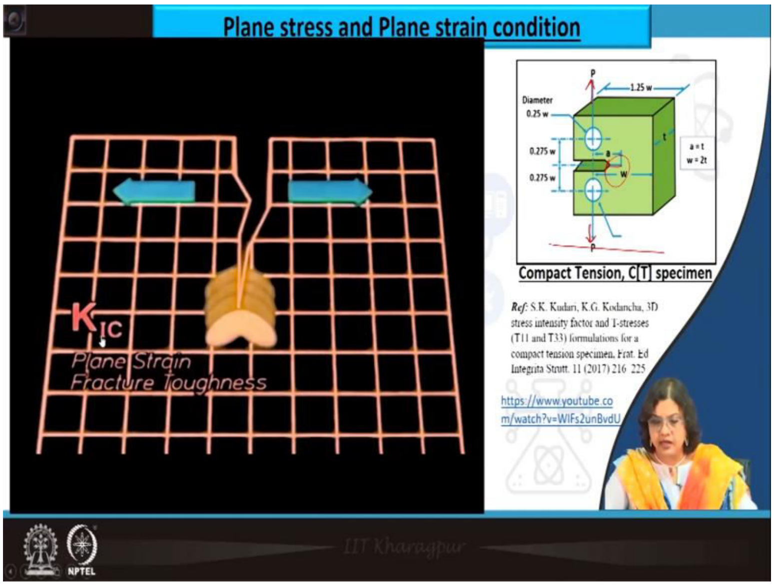

So, essentially, when we are having the stress along this direction, the crack grows on the perpendicular direction. And it forms a plastic zone size which eventually grows as we are applying higher and higher stress values. And when it grows it actually leaves a wake of plastic zone on the crack size. And that is dependent on the plane strain fracture toughness of a material.

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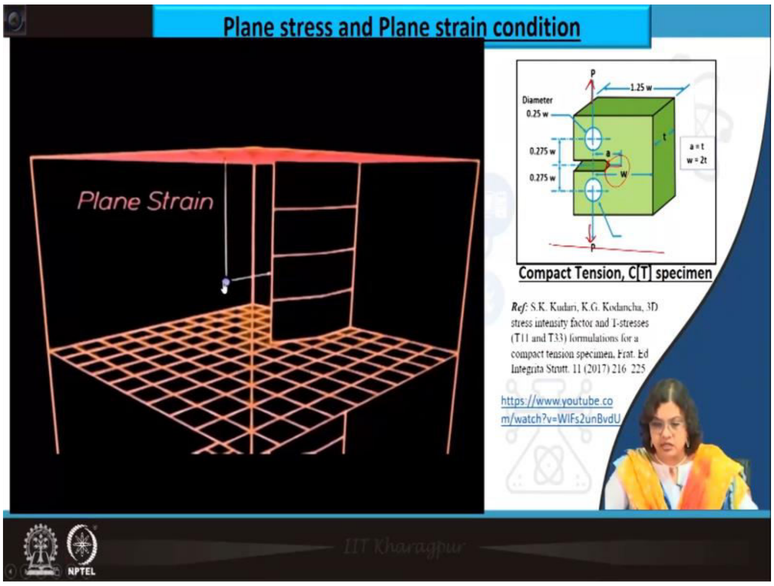

Now, plane strain fracture toughness of a material is varied for one material to another. When we are talking about a three-dimensional space, let us talk about a point which is at certain depth from the surface and at some distance from the crack tip. And it is in the plane strain condition because it is in the constrained stress.

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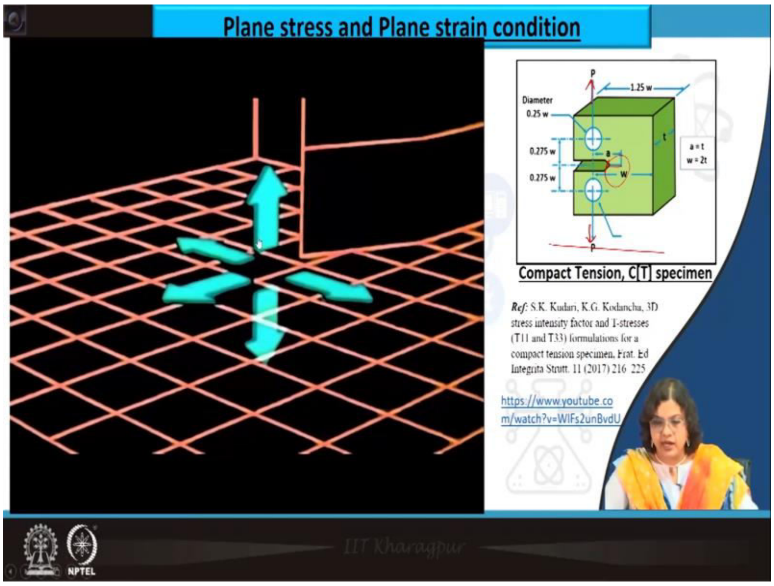

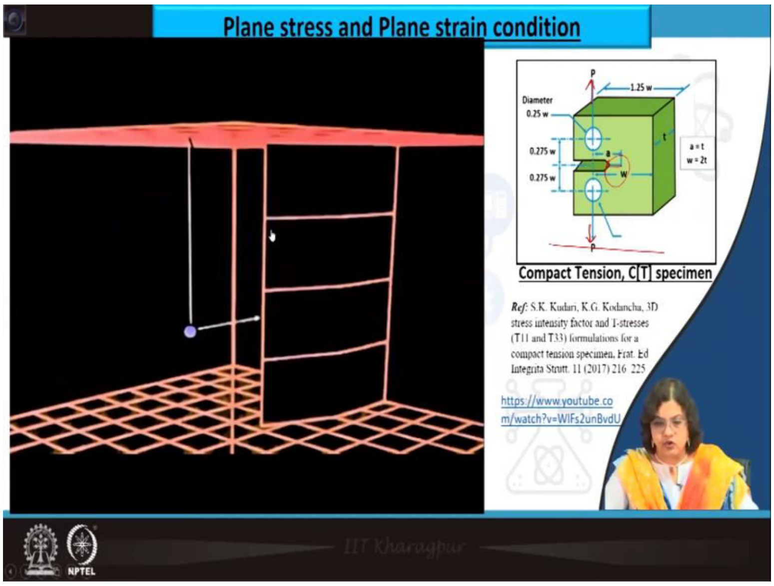

So, let us see how it is. We are applying the stress, the maximum along this direction and the minimum along the perpendicular to it, but in this direction we are getting a compressive stresses, which somehow is being constrained by the surrounding elastic space and to overcome that there is a tensile stress already developed which leads to a triaxial strain.

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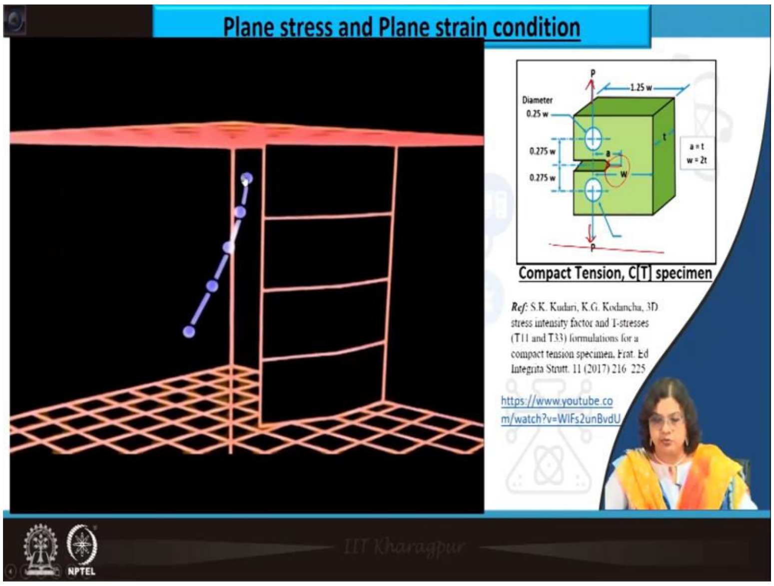

To obtain similar kind of values, if we are shifting the point upward, we need to shift it towards the crack also, to maintain this plane strain condition. And eventually, it has to be very very near to the crack tip to achieve this plane strain condition.

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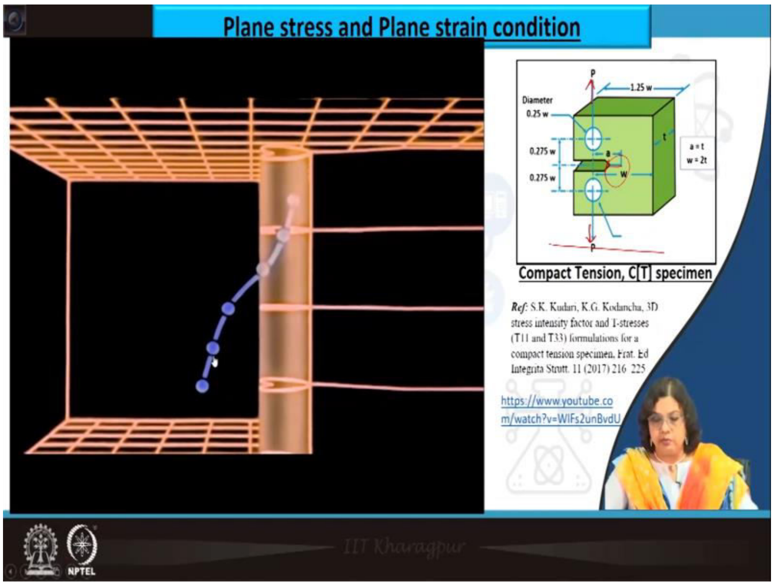

But, so far, we have not talked about the plastic zone and its influence. So, in this case in presence of this plastic zone, these points are actually moving near to the surface and near to the crack tip to achieve the plane strain condition. And eventually, it will be just embedded within the plastic zone itself, leading to the plane stress condition being active towards the surface.

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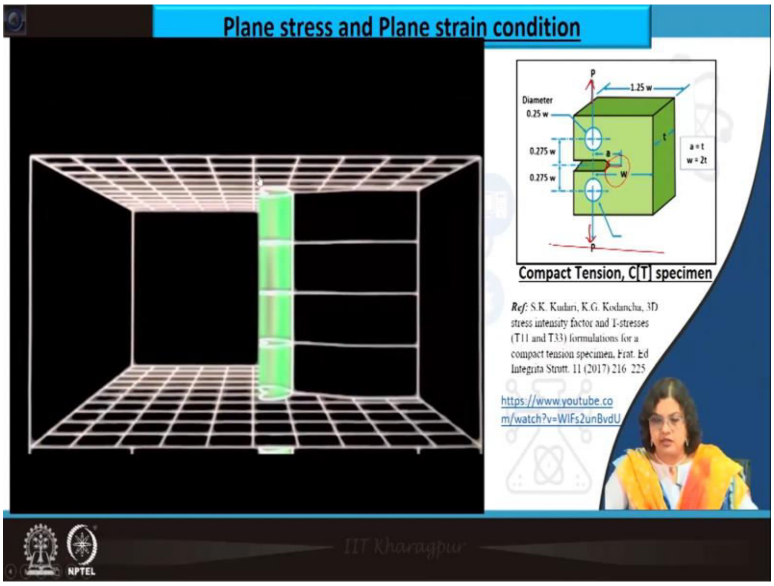

And that leads to a plane strain in the middle, but plane stress near to the surface for a very thick component.

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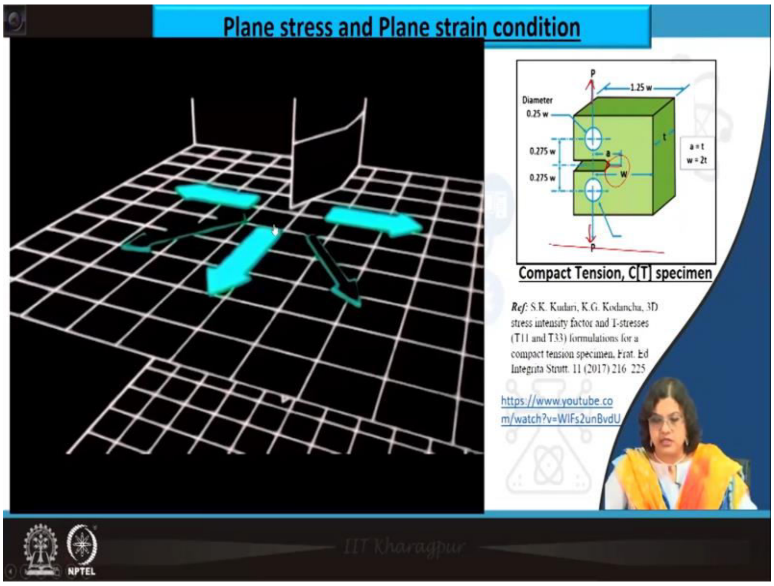

So, if you are talking about the fracture surface, these are the stresses that is being applied tensile stresses and there are the shear stresses along 45°.

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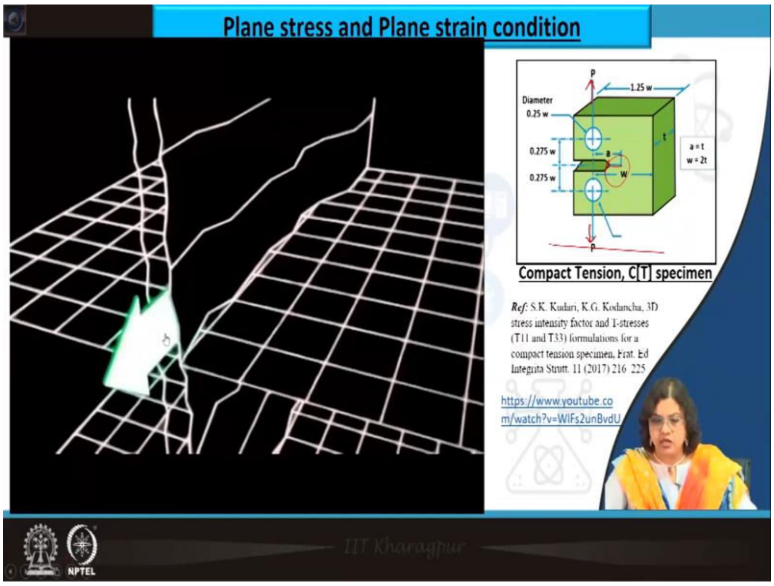

But since, this tensile stresses are mostly active, so, fracture is happening along this way, which is leading us to flat fracture.

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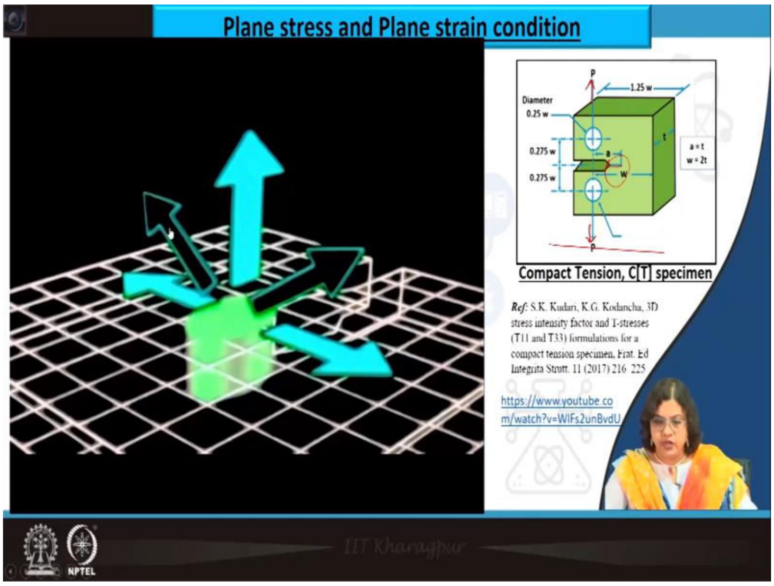

Now, when we are talking about the thin component, you see the plastic zone here is pretty big that is encompassing the entire thickness and we are applying the tensile stress along this direction which leads to a stress perpendicular to it, which however is 0.

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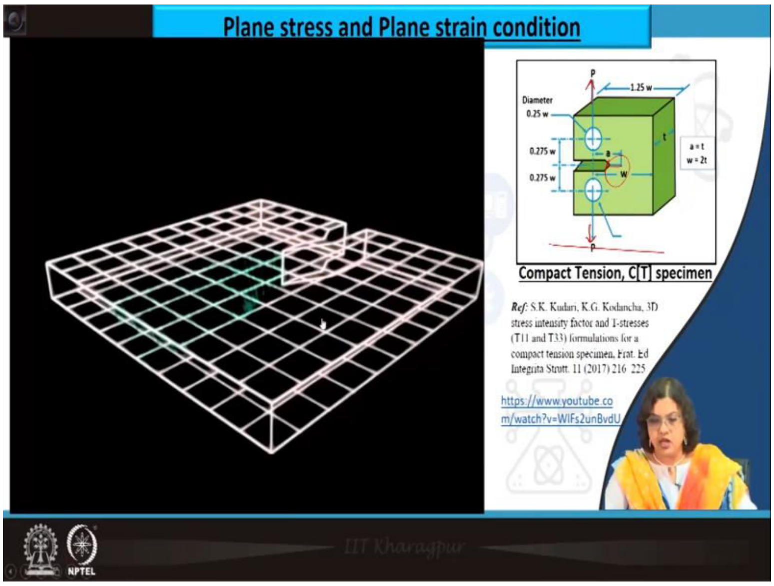

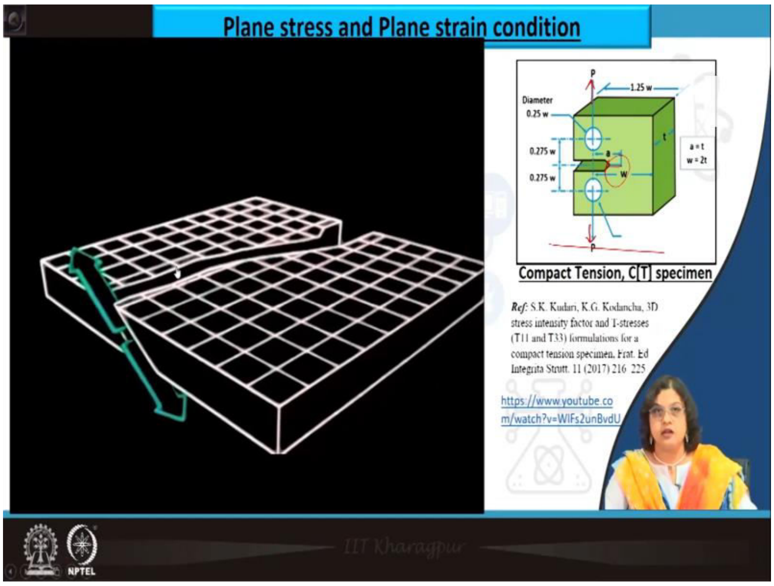

And then there is a shear stress also 45°. And accordingly the crack will move along the shear planes and that will lead to a slant fracture surface. So, that is what we essentially see in case of plane stress or plane strain condition.

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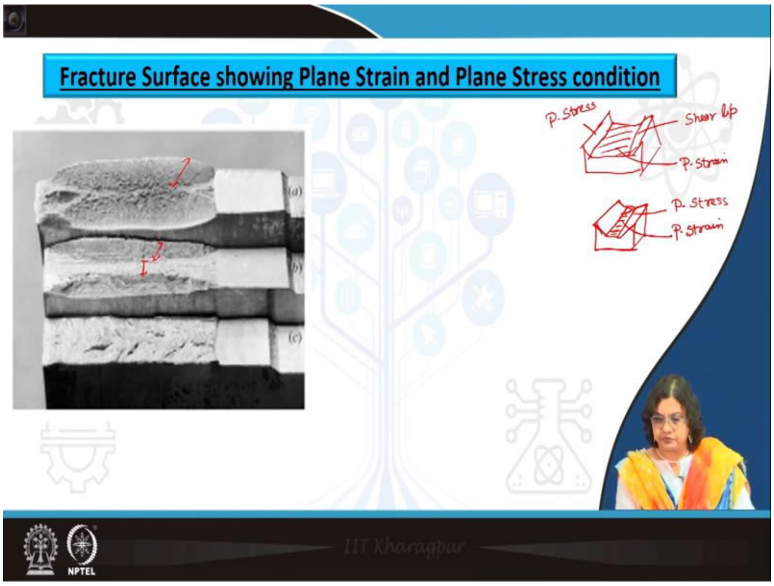

And if we try to see how the fracture surface looks like, we can see here that in case we have a thick component something like this, where we are getting the plane strain condition in the interior which is giving us a flat fracture surface as well as shear lip on the edges in which it is deviating to 45°.

So, here essentially, we are having plane stress condition. Now, if we are taking a thinner specimen then the shear of this plane strain part will be lesser and lesser. So, now, we have the plane strain also near to the middle but you can see that the shear or the ratio of the plane strain to the plane stress part is being much reduced for the case of thinner specimen.

And we can draw thinner and thinner specimen to see how the plane strain part is being squeezed and the entire part is taken over by the plane stress or the shear lip part. This is just some of the pictures of the broken specimen which shows how this shear leave has been changed. So, in this case you can see that the shear lip is quite pronounced, whereas, here we can see that the portion of the plane strain part is little bit more extended and the shear lip part is being squeezed.

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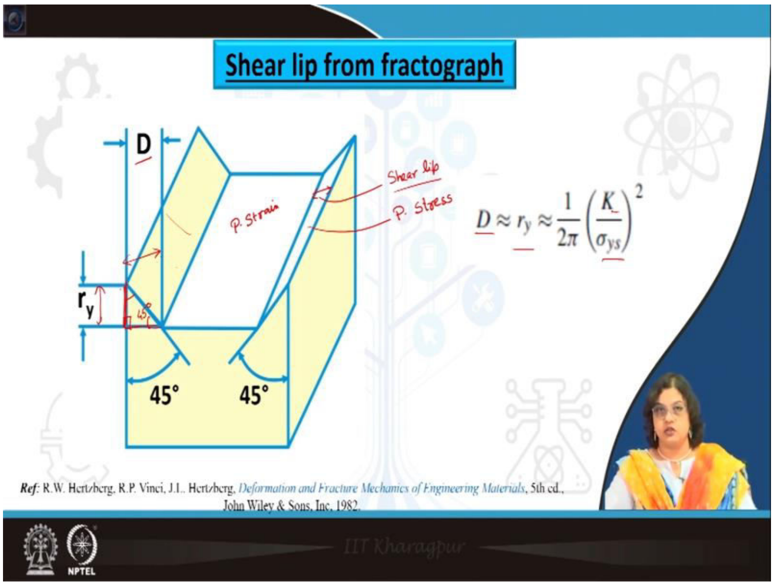

So, based on this, we actually can use this for the practical estimation of the K value or the fracture toughness values or the stress intensity factor value at that point of fracture just from this shear lip or from the fractograph itself. So, let us see how this can be done. So, this is a schematic of a fracture surface as we have seen.

So, essentially, this is the part where we are having the shear lip or this is the part where we are having plain stress, right. And this is the part where we are having plane strain. So, we can very well measure this shear lip width or shear lip depth. So, this capital D here is the shear lip depth that we can measure and what we can see is that if we are simply extrapolating this, we can see that this is nothing but an isosceles triangle. Isosceles triangle means one in which the two of the axes are of same length. Why?

Because this is happening at 90°. So, that means that this two angle are 45°. So, this shear lip depth that we are measuring here, this is equivalent to this one and this is nothing but the plastic zone size under plane stress condition. Because we know that this shear lift develops because of the plane stress condition.

So, essentially, this shear lip width or depth that we are determining, that gives us the plastic zone size. So, we can simply use this relation which says that

$$r_y = \frac{1}{2\pi}\left(\frac{K}{\sigma_{ys}}\right)^2$$

as per the plane stress condition. And from there, we can measure this D value and we can get this yield strength value, either from further studies or from the literature and if we know that then we should be able to find out the K value. Now, at the point of fracture this K could be the fracture toughness of the material.

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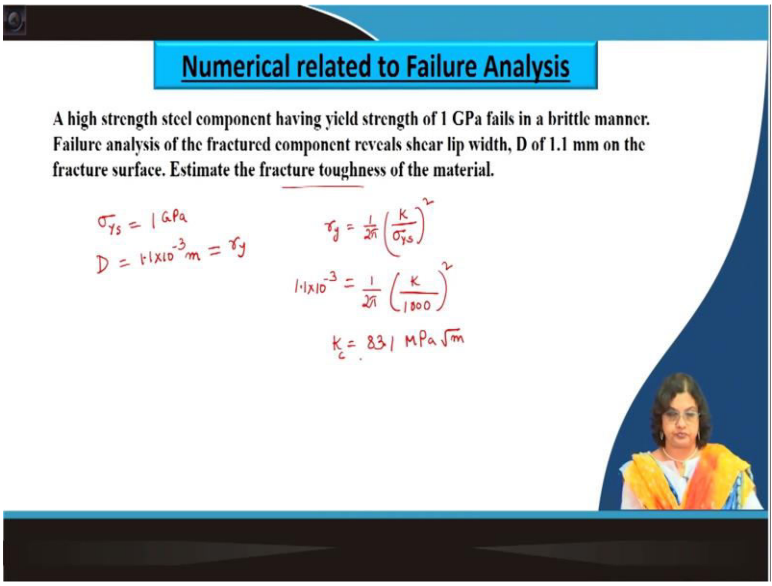

So, based on this there is a simple numerical to solve this and and to also appreciate that how fractographs and the information as well as the understanding can be used to pursue the failure analysis. So, it says high strength steel component which is having an yield strength of 1 GPa that fails in a brittle manner and failure analysis of the fractured component reveals shear lip width.

So, D equals to 1.1×10^{-3} m on the fracture surface. We need to estimate the fracture toughness. Now, this D is nothing but the plastic zone size, we have seen that. So, if we simply use this relation

$$r_y = \frac{1}{2\pi}\left(\frac{K}{\sigma_{ys}}\right)^2$$

we can get these values very easily. So, this is nothing but 1.1×10^{-3} and this is K by 1000 MPa. So, that makes K equals to 83.1 MPa√m. So, that could be the critical value of K for fracture and that could be termed as the fracture toughness under such condition.

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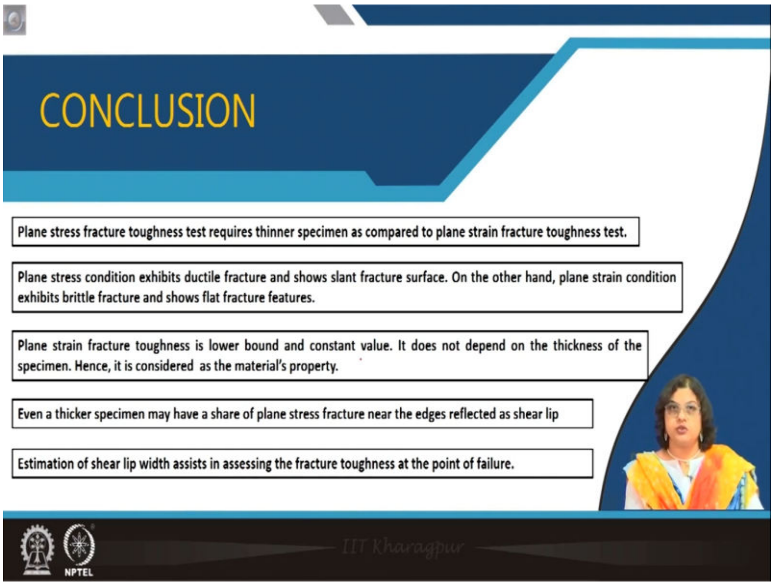

So, with this let us conclude this lecture what we have seen in this lecture is that the plane stress fracture toughness test requires a thinner specimen. On the other hand, plane strain fracture toughness, if we want to do we need a thicker specimen. Plane stress condition exhibits typically ductile fracture and that leads to a slant fracture surface.

And on the other hand, plane strain condition exhibits a brittle fracture and with a flat fracture surface. Of course, in case of plane strain fracture toughness, the values are lower bound the lower limit values and the constant value and that is why it is considered as the materials property.

We have also seen that in case of thicker specimen, there is still a share of plane stress fracture toughness near the edges or near the surfaces, whereas, in the central part it is mostly the plane strain condition. Also, if we can estimate the asses the shear lip width or shear lip depth, we should be able to find out the fracture toughness at the point of failure we can determine that value.

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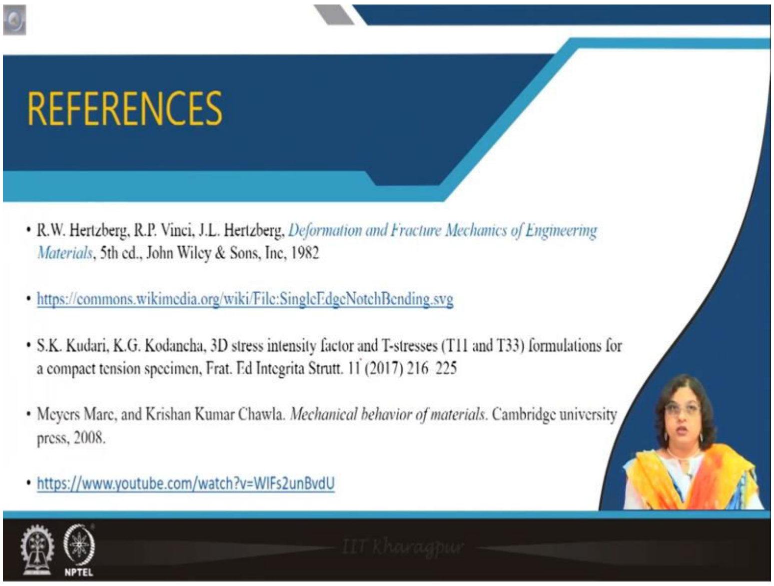

So, these are the references that has been used for this lecture and thank you very much for your attention.