Fracture, Fatigue and Failure of Materials
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Lecture 08
Plastic Zone Size

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Hi everyone and welcome back to the eighth lecture of this course fracture, fatigue and failure of materials. In this class, we are going to discuss about the plastic zone size that are forming in presence of a cracked tip that leads to all the changes in the failure mechanism behavior and we will also see that how this plastic zone size formation and its influence on the fracture mechanics of materials are changed for the plane strain or the plane stress condition.

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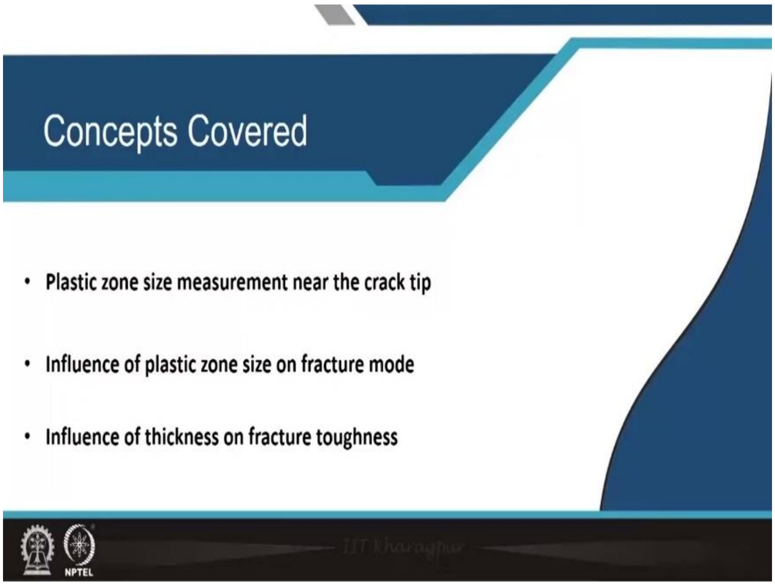

So, let us see what are all the concepts for this lecture. This particularly includes the plastic zone size measurement near the crack tip and how that influences the fracture mode as well as the influence of thickness of the component on fracture toughness of material.

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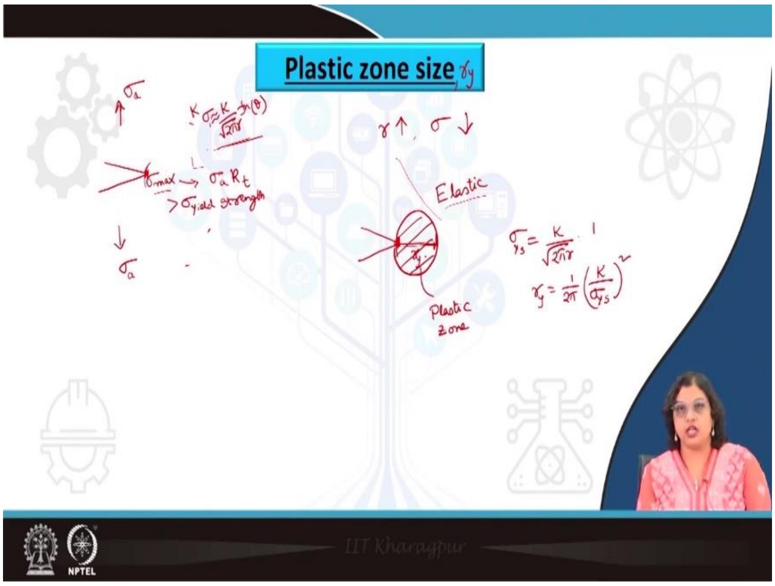

So, moving on to the very first concept about the plastic zone size. So, we have seen that the linear elastic fracture mechanics is based on the basic assumption that cracks are inherent in the material. So, in case there is a component with a crack in it and we are applying stress like this, tensile stress in the mode 1 condition, which is perpendicular to the growth of the crack, then we have also seen that at the tip of the crack, we are getting a maximized stress and that is related to not only the applied stress, but the stress concentration factor, and that is related to the geometry of the crack, the length of the crack, the radius of curvature of the crack, etc.

So, in case the σ_{max} exceeds the yield strength of the material, suddenly they will be plastic deformation ahead of the crack tip. Now, we have also seen that at any point away from the crack tip, we should be able to figure out the stress scenario. Let say at any point away from the crack tip at different points, we should be able to find out the K value. And not only that, the stress condition is dependent on this K and we have seen that both σ or τ, these are actually related to $\frac{k}{\sqrt{2\pi r}} fn(\theta)$, sin θ cos θ, etc. So, essentially σ is related to $\frac{k}{\sqrt{2\pi r}}$.

Now, in case we have plastic zone ahead of the crack tip, we would like to know the size of the plastic zone, and how that influences the fracture behavior of the material. So, let's assume that the σ_{max} exceeds the yield strength and there is a plastic deformation which is forming at the tip of the crack, but then the target would be to find out that up to what extent this plastic deformation will continue, because we have also seen as per this relation here, that as at the crack tip stress value will be very-very high, but as r increases, that is as we are moving away from the crack tip, K or σ is going to reduce. So, it follows a decreasing trend as we are moving away from the crack tip.

So, accordingly the plastic zone size will be also restricted up to certain levels. So, if we have a crack and there is this plastic zone size, which is starting at the tip of the crack up to certain distance this plastic zone size will be valid over after this there will be only elastic deformation, the stress value will keep on decreasing like that. So, eventually we would like to figure out that up to what radius of curvature, up to what size this plastic zone will be valid.

To find out that we can simply use this kind of relation as we have seen that $\sigma = \frac{k}{\sqrt{2\pi r}} fn(\theta)$. So, if we put θ = 0 so, if we want to figure this out for this straight line here, we would see that by incorporating all the sine θ, cos θ, term that it will come to 1. So, essentially, we can see that, this distance r will be given by $\frac{1}{2\pi}\left(\frac{k}{\sigma_{ys}}\right)^2$. Now, σ at this point of plastic deformation is nothing but the yield strength of the material. So, this should be σ_{ys}.

So, if we know the yield strength of the material and if we know the K value the stress intensity factor or at the point of fracture, we can also use the fracture toughness values then we should be able to find out this plastic zone size. So, this is typically denoted as r_y. So, this r_y is this total diameter over which the plastic zone will be applicable and so, this will be plastic zone and remaining we have the elastic part which is still valid.

So, there will be no or permanent deformation at this location and there will be complete permanent deformation in this zone here, the hatched spacing. Now, for simplicity we consider this as a spherical one, but in reality, at least for metallic systems particularly which is prone to get deformed under permanent deformation that is why I chose the example of metallic systems this is hardly spherical, rather it takes a different shape.

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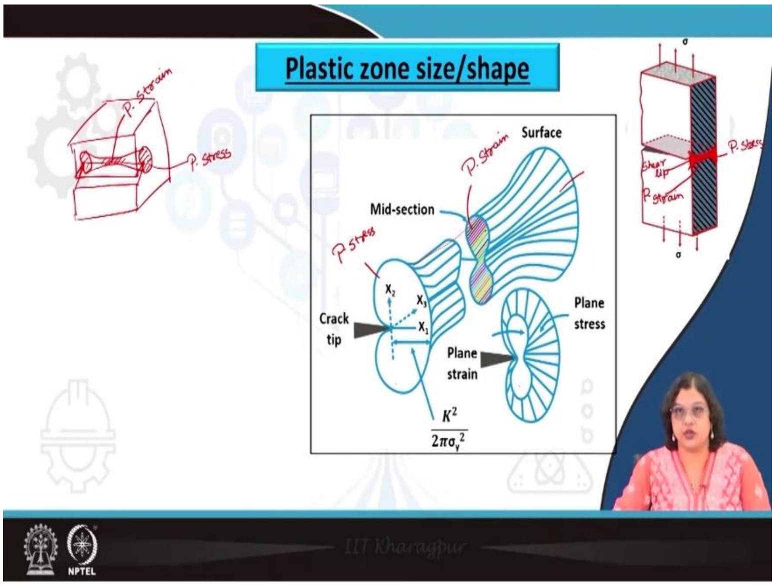

To understand that, we need to find out that how this behavior changes and how this plastic zone size changes at the distance away from the crack tip. So, let’s say we have a component something like this in which there is a crack. So, this is a through the thickness crack or this is a surface crack rather, over the entire width and this is the component. So, if there is a crack then at the tip of the crack this plastic zone size will form, both near to the surface. So, these are the two surfaces of this, but this plastic zone size will be restricted in the intermediate section.

So, basically near to the center of the specimen the plastic zone size will be kind of squeezed whereas, this will expand nearer to the edges or near to the free surfaces, why? Because near the free surfaces the stress along this Z direction will be 0. So, that will lead to maximum strain at this condition, the triaxial condition of strain, biaxial condition of stress that will be valid near to the edges. So, this will be nothing but the plane stress condition in the same component itself. On the other hand, in the central part the strain will be restricted along the Z direction and that will lead to a biaxial strain or the plane strain condition.

So, this is how it looks like, you see the shaded part here is the plane stress part and then the central part is plane strain again, and again near to the edges we are getting this enhanced plastic zone size. So, this is plane stress and at the center we have plane strain. So, this plane stress condition whatever this is generated along 45 degrees to the crack direction that is nothing but presence of the shear lip that can be seen on the fracture surface.

Now, if we take this part out, this is how the plastic zone size would look like. So, this is the plane stress condition here and towards the midsection. So, this is simply taken out from the entire component only the plastic zone part and you can see that this is the midsection part this is squeeze and takes a kidney bean kind of appearance. So, this is the plane strain condition, and at the surfaces, both the surfaces, we are having plane stress kind of condition.

So, these are being superimposed on one another and we can see that this is the plane strain the inner one and the outer one is the plane stress. One thing is very much apparent from even this freehand drawing out there, that the plane strain condition has much lower plastic zone size compared to the plane stress condition and we will see that how these two are different. So, overall, this takes this kind of shape and not exactly a spherical one because this has been constrained from the surrounding spaces.

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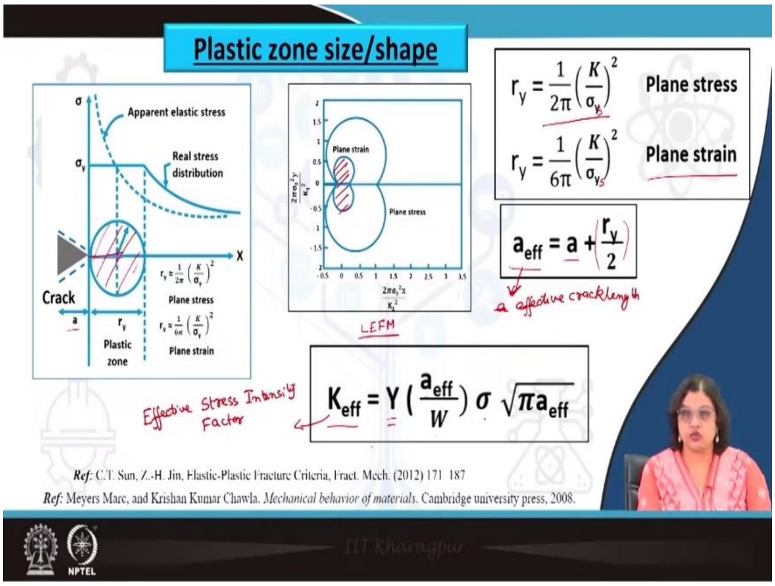

So, as I mentioned that this is the zone for the plastic deformation to occur and the stress value is continuously decreasing and that leads to a condition similar to this which are the experimental values that has been shown here from the references. And the typical plastic zone size under plane stress condition is what has been shown as $\frac{1}{2\pi}\left(\frac{k}{\sigma_{ys}}\right)^2$ and on the plane strain condition as we have seen that the plastic zone size is much smaller in this case and this is just one third of that under plane stress condition. So, in this case we have $r_y = \frac{1}{6\pi}\left(\frac{k}{\sigma_{ys}}\right)^2$ So, that is how we can figure out the plastic zone size and based on the size we can use the kind of relations that will be valid to get the fracture toughness or the fracture behavior of the material, and this has a very prominent role on the fracture behavior of the material whether a material is prone to undergo permanent deformation or not that dictates its mode of fracture, that dictates it the fracture surface, that will be formed and just by looking on the fracture surface also, we should be able to figure out that whether this the loading condition has been under plane strain or plane stress condition.

Now, this is not the end of it, rather what we are concerned now is that, so far we have considered crack length like this as ‘a’, as you have seen and we have used this ‘a’, for all the calculations and we assume that once the crack grows, as I have mentioned in the last lecture, the basic assumption of linear elastic fracture mechanics is that, once the crack starts growing near the tip of the crack, it will lead to the propagation or final fracture in no time. So, this propagation is unstable, the growth of the crack is unstable and that is considered based on the fact that this entire deformation is elastic or brittle.

So, that means that whatever is the crack length if we use this in the relation such as K which is given by $Y\sigma\sqrt{\pi a}$ or even Griffith criteria, which says σ_F which is related to $\sqrt{\frac{2E\gamma_s}{\pi a}}$. So, everywhere we consider crack length as the most important term and based on that we can figure out the changes in the fracture strength or the stress intensity factor, considering that there is no permanent deformation and everything is brittle or elastic.

Now, what happens if there is a plastic zone in front of the crack tip? The growth of the crack is not be following the same manner or same trend as it was for the unstable growth in case of elastic deformation. So, if there is a plastic deformation that acts as a hindrance to the growth of the crack. Obviously, more time and energy will be spent on overcoming this plastic barrier, higher the plastic zone size, longer time will be required, the fracture will be delayed or the energy that is required for fracture will be increased.

So, one thing is clear that we cannot use the term ‘a’ anymore in case there is significant plastic deformation, if the plastic deformation is negligible, no problem, we can still consider ‘a’ as a prime factor prime parameter and we can use it for all the calculations, all the quantification, but in case there is a significant amount of plastic deformation, then we need to consider the plastic zone size also and in that case, the crack length is considered as ‘a’ effective.

So, this is effective crack length and this includes not only the crack length, but also half of this plastic zone size. So, that is how we consider the effective crack length, then in that case all the calculations will be done based on this ‘a’ effective, so, in that case the K effective instead of K simple stress intensity factor or the effective stress intensity factor. So, this is effective stress intensity factor that is equals to Y depending on the center crack or edge crack, we can determine Y and then but that is related to ‘a’ effective and W.

So, instead of a by W this one is also related to a effective and that may also vary and then sigma and pi ‘a’ effective. So, everywhere we have to use this ‘a’ effective term in case there is a significant amount of plastic deformation. Now, how significant the plastic deformation is or when can we consider the plastic deformation as significant and when is it okay to ignore that or consider that as negligible that also is not very arbitrary or and there are rules for that also and that we are going to explain.

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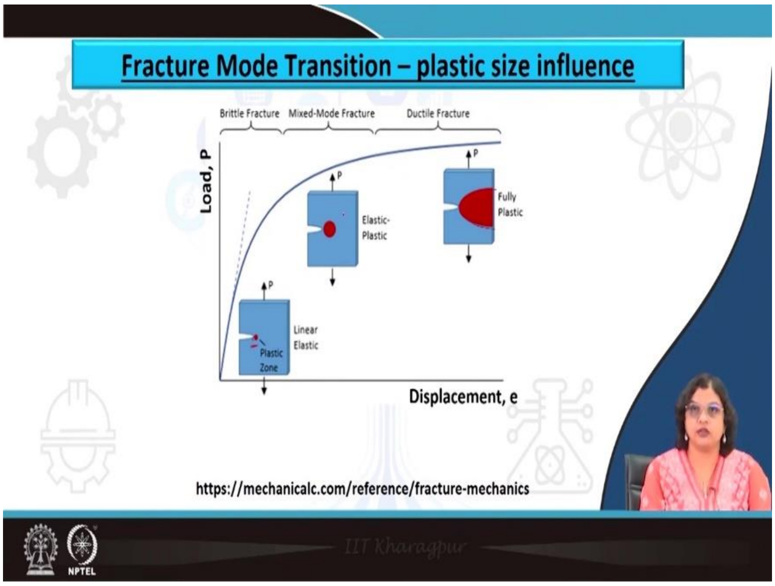

So, overall if we are looking into this load versus displacement curve for specimen, we can see that initially when there is a brittle mode of fracture in such cases, there is a very negligible amount of plastic deformation, you can see this red circular zone is very-very small. On the other hand, if we are talking about the ductile mode of fracture, there is a significant amount of plastic deformation we can see that the plastic zone has reached to the edges and it is a very a pronounced plastic deformation certainly this cannot be ignored.

And in case we have a mixed mode fracture, we have a comparatively pronounced plastic zone, but not as high as the fully plastic one. So, depending on that we can figure out and we have to impose the relations that whether this is an elastic fracture or brittle fracture, ductile fracture or a mixed mode kind of fracture. In reality, most of the cases the failure occurs in the mixed mode condition and we often need to use such kind of relations, but for lab scale testing or for the very basic concepts about fracture toughness and all we still prefer to use the linear elastic fracture toughness.

Now, when this linear elastic fracture toughness would be valid, and when it is not, that can be understood by its relation with the plastic zone size and as well as the thickness of the specimen, we will elaborate more on that. So, thickness of a specimen has a big role to play on the fracture toughness, this sounds maybe not familiar to all of you, but if we think about the influence of thickness on the plane stress, plane strain condition, we should be able to understand this.

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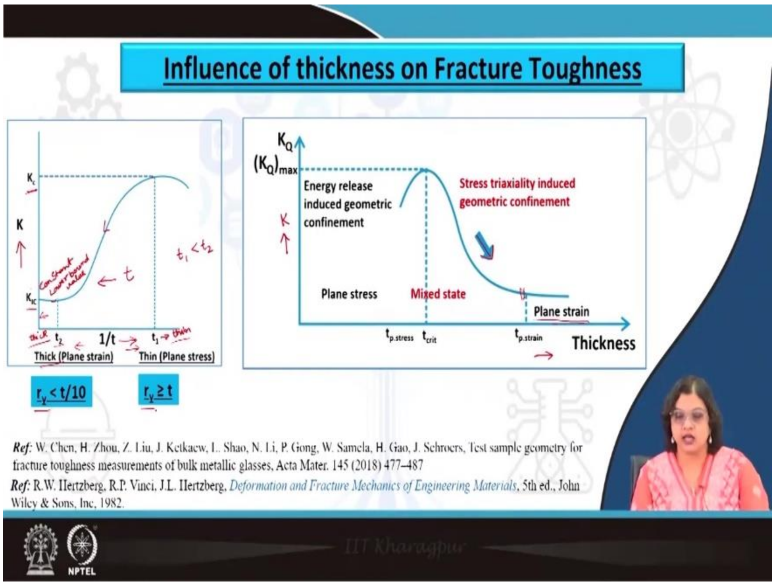

So, let us see how it varies. So, this is a plot for K on the y axis and 1/t, inverse of t on the x axis. So, that means that 1/ t is increasing along this direction or in other words, t is decreasing along this direction. So, t is increasing along this direction. So, that means, t_1 is actually less than t_2. So, this is a thin plate, a thin sheet, and this is a thick plate, fine. This one y here shows the variation in the stress intensity factor and we can clearly see that for a thin plate K value is quite high, the critical value of K that leads to fracture that is the K_C is quite higher value compared to the thick plate, which shows the K_{IC} value is quite less.

This is a normal nomenclature that we use that for the plane strain condition we use the fracture toughness as K_{IC} and for the case of plane stress condition fracture toughness or the critical value of stress intensity factor for fracture is considered or termed as K_C instead of K_{IC}. But this essentially means the critical value of K that leads to fracture. So, what we are seeing here is that K is continuously decreasing or K_C is continuously decreasing as we are increasing the thickness, so, t increases along this direction.

So, as we are increasing the thickness, K is decreasing. But, most importantly what we are seeing is that after it reaches a certain value of t_2, even if we are increasing the t value further, there is no change in the K_{IC} value, it achieves a constant value and not only that, it achieves a lower bound value.

And this can be understood in this relation here, where the y axis once again it denotes a variation in K, stress intensity factor, on the other hand x axis here is the thickness not inverse of thickness but rather the thickness. And we can see that as the thickness increases, fracture toughness or the critical value of K decreases continuously till we achieved the plane strain condition when once again the fracture toughness values that we are achieving here is constant one and with the lower limit values.

So, in that case again we would like to know that, if we are telling that the thin plate, we are seeing higher fracture toughness and thick plate we are seeing lower fracture toughness so, how thin or how thick can we consider is valid for a plane stress or plane strain condition. Thickness, thin or thick these are all qualitative terms, so we need to quantify that if we want to control something, we need to have the exact values for those things. So, how thin or how thick, are something which are important.

Now, this is related, this is once again not arbitrarily dictated as thin or thick, but this is considered based on the plastic zone size. So, in case the plastic zone size is less than the one tenth of the thickness that means thickness is 10 times the plastic zone size. We have seen that plastic zone size is very-very small for the case of plane strain condition, but if that plastic zone size is one tenth of the thickness, then only plane strain condition will be valid and then only whatever fracture toughness or the critical value of K that leads to fracture can be obtained that is a lowest value with the constant nature of it.

On the other hand, in case the thickness is almost comparable or slightly lesser than the plastic zone size or in other words plastic zone size is either same or more than the thickness that means that the entire thickness is being encompassed with the plastic zone size, in that case the plane stress condition will be valid.

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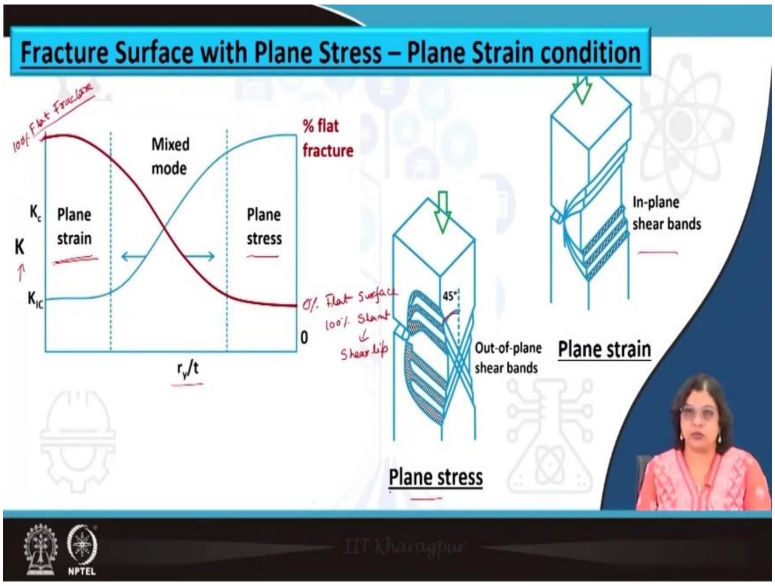

So, that leads us also to another very important feature based on which we can differentiate between plane strain or plane stress condition being active right from the fracture surface itself. So, the blue curve here signifies the variation in K as we are changing the r_y / t. So, we have normalized the plastic zone size with the thickness and this follows a similar kind of trend that for lower value of this r_y / t, that is for a thick plate, we are seeing the plane strain condition to be active which gives us low value of K_{IC} and again which is constant. On the other hand, if we are considering thinner plate, then we are having higher value of K and that is under plane stress condition.

Now, if you look on the fracture surface, there is another trend that we are going to see. For the case of plane strain condition which means the thick plate which fails in a brittle fashion because there is not enough plastic deformation, the plastic zone is very-very restricted. So, that means there is the fracture mode is elastic or it follows the linear elastic fracture mechanics in that case if we have a brittle fracture, we are going to see a flat fracture surface. So, we are seeing 100 percent of flat fracture surface here. So, this is 100 percent flat fracture.

If you recall in the very first lecture, we have discussed about the change in the fracture surface under ductile and the brittle mode, and ductile mode is supposed to show us cup and cone or shear lip kind of fracture, whereas, the brittle one is going to show us a flat fracture surface.

So, in case of plane strain, we are going to see a flat fracture surface and in case of plane stress we are going to see almost zero or negligible flat fracture surface. So, rather what we are seeing here is slant fracture surface. So, flat surface rather what we are seeing here, so, if we just plot the opposite way, so, this should be either 100 percent slant or at least there will be some slant fracture surface and this slant part is nothing but the shear lip which we have seen that it deviates along 45 degrees.

So, just by looking on the fracture surface itself we can figure out that whether plane strain or plane stress condition has been active. So, you have seen that this is deviated along 45 degrees and in this case, we are seeing continuously flat fracture surface. So, that leads to the plane strain condition or the plane stress condition. So, just by looking on the fracture surface itself, we should be able to figure out the loading condition as plane strain or the plane stress condition. So, this would be very-very helpful if we are talking about the failure analysis mode and if we are doing a post mortem analysis on the basis of the fracture surface.

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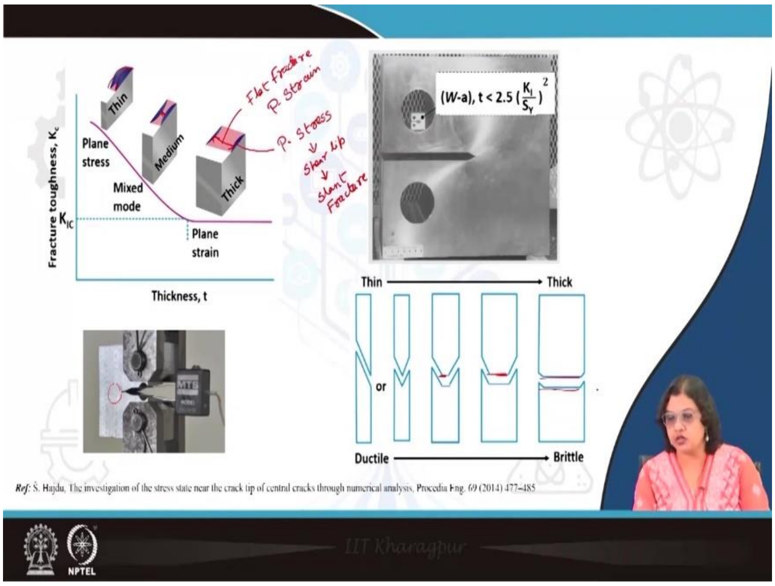

So, this is how it looks like in case of thin specimen. We are seeing that there is a pronounced shear lip, pronounced plastic zone which you can see in this blue zone. In case of mixed mode this shear lip width is decreasing and the shear of this flat fracture is keep on increasing. On the other hand, if we are looking for a very thick component, we see that there is a pronounced part which is flat fracture, so that means that this is nothing but plane strain.

So, now, if you recall the lecture, we have already explained that near to the center part there will be plane strain always active and near to the edges there will be always plane stress and this plane stress will be reflected as shear lip or slant fracture. So, we can figure out that how much is the shear lip width and based on that we can try to find out the fracture surface. So, here is one example of a CT specimen loaded and if we look into this carefully, we can see that there is a plastic zone which is forming in this location.

Now, depending on the inherent nature of the material, this is a metallic system if we are talking about very ductile materials which are by default prone to fail under permanent deformation mode, we should be able to see this permanent deformation plane. So, this is the schematically shown if we have a thin plate there is a complete fully slant kind of fracture and or a cup and cone kind of fracture in this as we are getting mixed mode kind of fracture, then the shear of this plane strain component is going to increase and finally, if we have a very thick plate, we are going to see the most pronounced mode of fracture there is under plane strain condition which is supposed to show us a flat fracture surface.

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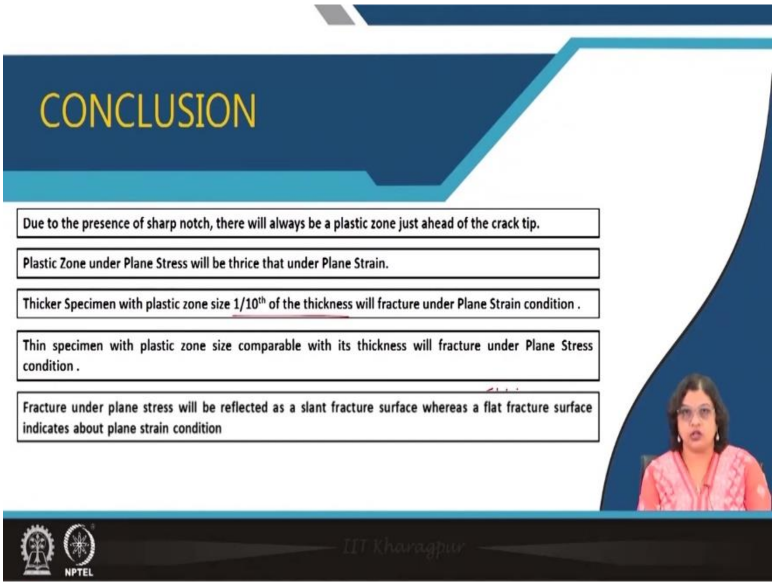

Based on this let us conclude today's lecture that what we have seen is that due to the presence of sharp notch there will always be a plastic zone ahead of the crack tip and this plastic zone under plane stress condition will be bigger and it is almost three times bigger than that under plane strain condition. Now, thicker specimen will have this plastic zone size, which is equivalent to one tenth of the thickness. So, that means that the plastic zone size is very-very restricted.

On the other hand, thin specimen will have this plastic zone size which is almost comparable with the thickness and that will lead to change in the fracture mode. So, fracture under plane stress will be reflected as a slant fracture, whereas, flat fracture surface indicates about plane strain condition. So, looking on the fracture surface itself we can figure out the actual loading condition.

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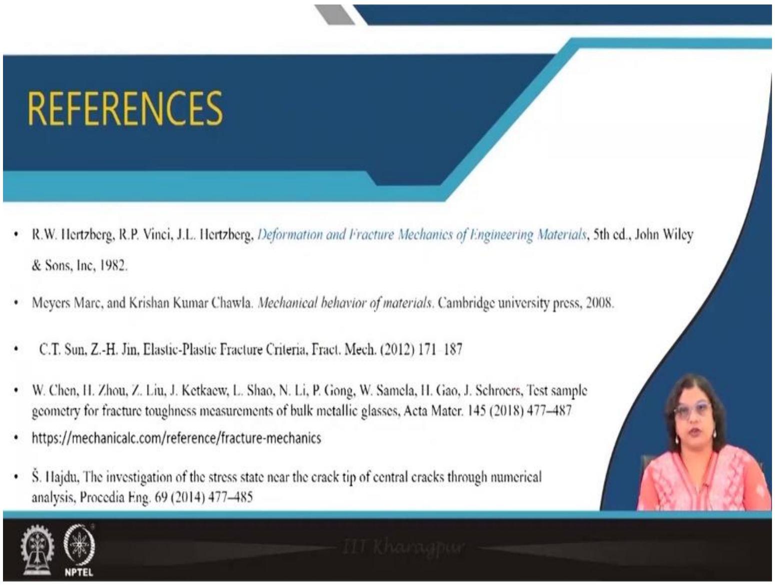

And these are some of the references that has been used for this lecture. I hope you enjoyed the lecture. Thank you very much.