Fracture, Fatigue and Failure of Materials
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Lecture 07
Fracture Toughness and Plane Stress-Plane Strain

Hello everyone, and welcome to the seventh lecture of the course fracture, fatigue and failure of materials. So, in today's lecture, we will be talking about fracture toughness and plane stress, plane strain condition, two very major concepts of fracture mechanics will be discussed starting from today, and the basic concepts that will be covered in this lecture are the following.

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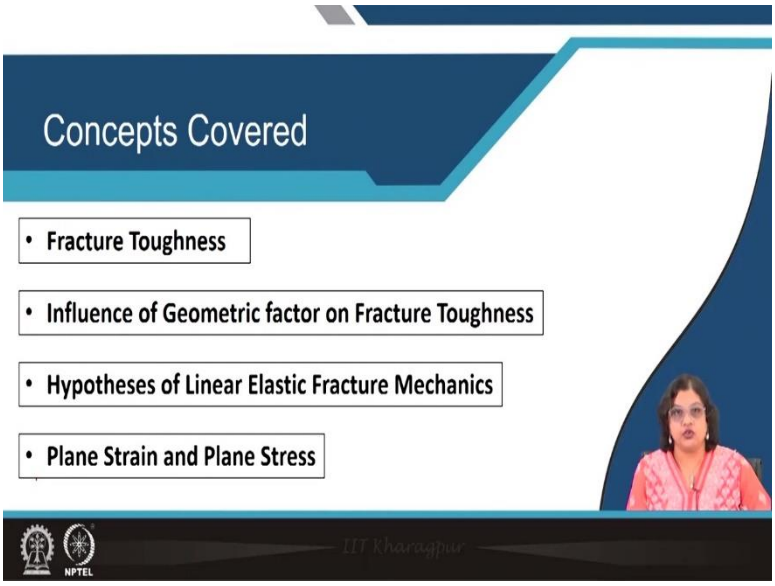

First of all, fracture toughness. So, so far, we have discussed about fracture strength and how this can be generated based on Griffith criterion as well as their modification. In this class, we will see that what is the difference between fracture strength and fracture toughness, what exactly is fracture toughness and why is it so important in case of fracture mechanics.

Following this, we will also look into the different geometric factor that can lead to modification in the fracture toughness of a material. And, further to that, we will discuss about the linear elastic fracture mechanics and the hypothesis associated with that and finally, we will introduce the concept of plane strain and plane stress.

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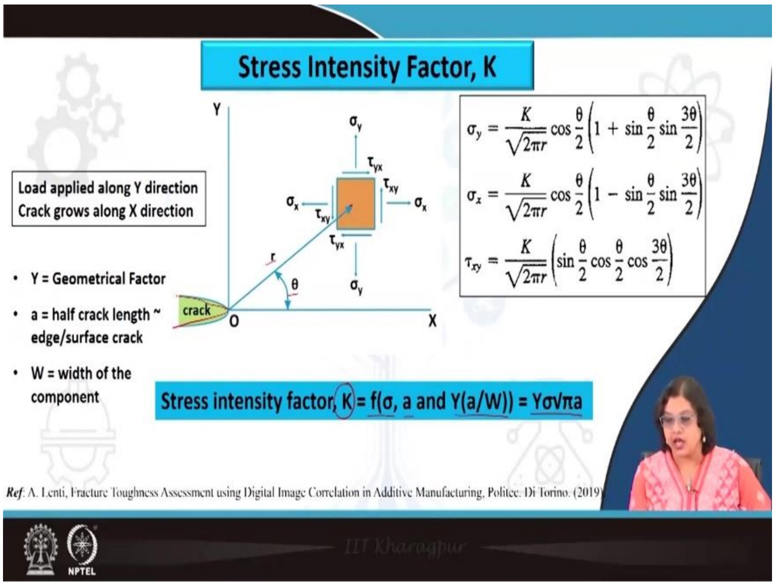

So, let us see, what we have seen in the last lecture is about stress intensity factor and we have seen that if there is a crack present in a material something like this, at any point away from the crack tip, which is at a distance r and at an angle θ, we are able to find out the stress scenario at any point away from the crack tip, and this is given by particularly this factor here denoted by capital K or the stress intensity factor, this is essentially a function of the applied stress, crack length, as well as the geometric factor and it is considered as $Y\sigma\sqrt{\pi a}\ Y$. So, that is how stress intensity factor is generated at any point.

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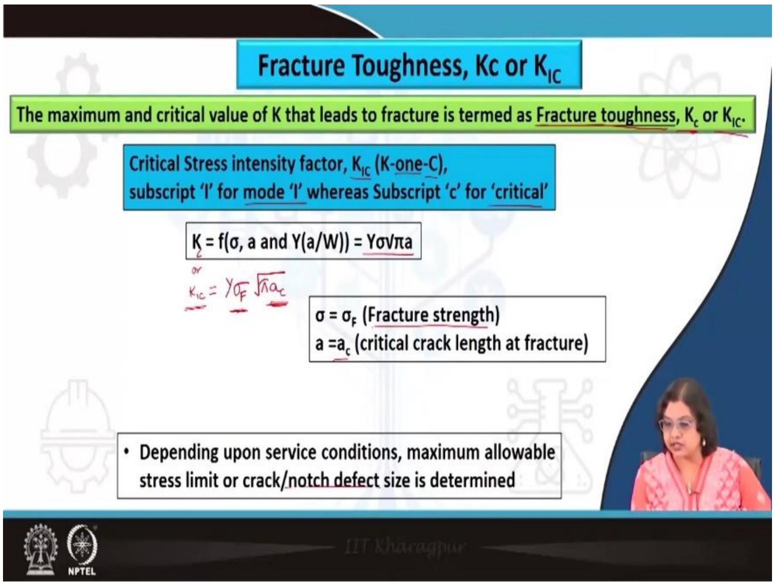

And interestingly, the critical value of this K or the stress intensity factor at which fracture happens that particular value is known as the fracture toughness. So, fracture toughness is nothing but the critical value of K, typically denoted as K_C, C stands for critical or K_{IC}. This one comes from the mode one loading condition, so, this is not I, this is just one capital roman one and C as I said stands for critical, since most of the fracture toughness tests are pursued in the mode one loading condition that is why K_{IC} is very much commonly used as a term to determine or to denote fracture toughness.

Now, at certain point this critical value of K at which fracture will occur that is a fixed one. So, whenever such value is reached, there will be fracture, a fracture will materialize. So, that is how K is very-very important or K_C is actually very important. And if we know the fracture toughness value of a material, we can determine that when the fracture will occur or if we can increase the stress further so, that there will be still no fracture.

So, if we use this relation here, so, K is given by $Y\sigma\sqrt{\pi a}$, at the point of fracture when K turns to K_C or K_{IC}, at that point, we actually have the relation as Yσ$Y\sigma\sqrt{\pi a}$. Now, σ at the point of fracture will be denoted as σ_F which is nothing but the fracture strength of a material and a at such condition at the point of fracture will be the critical value of crack length or denoted as a_c.

So, if we are knowledgeable about the fracture toughness of a material, we can also in turn determine the fracture strength of the material or the critical crack length or vice versa if we are knowledgeable about the fracture strength of a material and the critical crack length, we should be able to figure out the fracture toughness of the material. Now, information about the fracture toughness is very-very important when we are thinking of using this for certain application.

For example, at the service condition, if we know the fracture toughness of let us say titanium alloy, which is used for aerospace application, if we know the fracture toughness of that particular alloy, we should be able to figure out either of these two, either the maximum fracture strength that it can survive or maybe the critical crack length that survived the condition till there will be any fracture. So, this information is very-very helpful while designing something based on the materials properties. So, depending upon the service condition, the maximum allowable stress limit or the defect size can be determined and then we can predict that whether fracture will occur or not.

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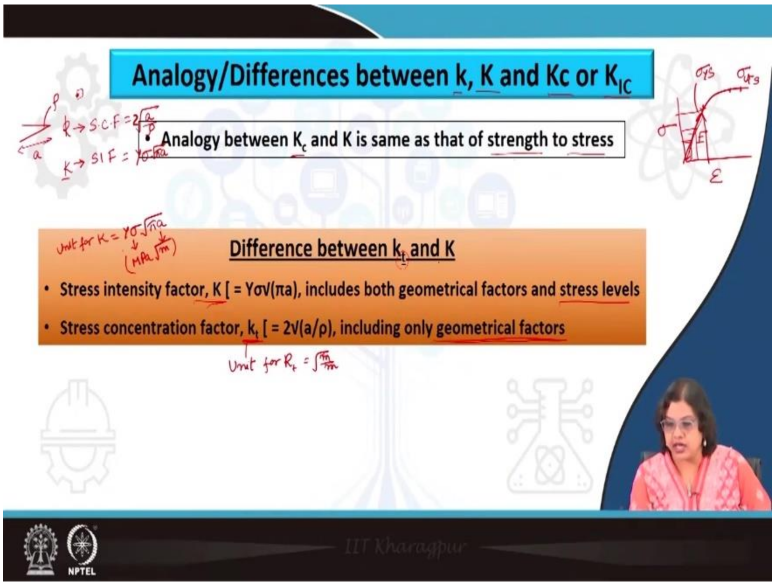

Now, if we try to draw the analogy between the different parameters that we have discussed so, far small k, capital K or the critical value of capital K. Now, small k is nothing but the stress concentration factor, if we can look into our previous lectures, we will see that if there is a crack of half length a and radius of curvature for the crack tip is ρ, then small k or the stress concentration factor, this is given by $\sqrt{\frac{a}{\rho}}$.

On the other hand, if we are talking about the stress value or the stress scenario, the energy at any point away from the crack tip which is at a distance r at an angle theta, then that will be given by the capital K or the stress intensity factor and that we have seen is related to $Y\sigma\sqrt{\pi a}$, at the critical value of K fracture will occur and then it will be the fracture toughness of the material. So, not all the values of K are fracture toughness, but the critical value of K can only be termed as fracture toughness of the material.

Now, at the first point, if we try to draw the analogy between K_C and K, we can think of this as the same as the difference between strength and stress. So, if we look into the stress strain diagram, the typical stress strain diagram of a material, let us say a metallic material, this is what we get stress values varies with strain initially as a linear way. So, this is as per Hooks law and we get the elastic modulus at every point of strain we get the corresponding value of stress, but at certain point, when the deformation mode changes and there is the onset of plastic deformation, that particular point is known as the yield strength of the materials.

Similarly, the maximum value of this is termed as the ultimate tensile strength of the material. So, that is also a stress value, but that critical values are termed as strength either yield strength or ultimate tensile strength, in case of fatigue that can be termed as fatigue strength or fracture strength. So, at that critical value of stress where the deformation mood changes that is known as strength. Similarly, the critical value of K at which fracture occurs that is known as K_C or the fracture toughness. So, that is the basic analogy between K_C and K same as strength to stress.

On the other hand, if we try to find out the difference between small k and capital K, small k it is sometimes denoted as k_t, with the subscript t denoting the stress concentration factor or simply small k as a stress concentration factor. And the basic differences between these two are as follows the capital K or the stress intensity factor includes not only the stress level, but the geometric factors both taken together, so, it includes the stress level and as a result it gives us a wholesome picture in front of the crack tip at any point away from the crack tip. If we know just the crack length as well as the applied stress, we can figure out the critical or the value of K, capital K, stress intensity factor.

On the other hand, small k or stress concentration factor is particularly related to the geometric factors and not the stress level, the applied stress level. So, this in that sense is a kind of conservative 1, the small k one, because it does not include the applied stress levels and so, we cannot predict that what or how it will behave in case of the service when we know that there is certain stress limit that we need to apply we cannot figure that out from the small k or the stress concentration factor, rather stress intensity factor gives us a clear picture.

Also, the unit of stress intensity factor, which is related to unit for K is related to the unit of σ which is MPa and a which is meter, so, the unit is $MPa\sqrt{m}$. On the other hand, the unit for stress concentration factor is just $2\sqrt{\frac{a}{\rho}}$, so the unit of a is also in millimeters, unit of ρ is also in millimeter. So, basically that cancels out, so it is an unitless parameter. So, that is also another difference between capital K and small k.

So, now that we know about the difference and what exactly is fracture toughness and for the case of fracture mechanics or most importantly, if we try to involve this fracture mechanics include this into failure analysis, because our target is to find out that how long a material or a component or a structure can survive and what can we do to enhance the life of this material.

So, we always try to control these parameters, so that we can change and modify the values of fracture toughness or strain in a way that we can achieve higher and higher or better and better performances. And in that way, there this factor here Y, which is what we have not talked much about. So, this Y has a very leading role to play to control the fracture toughness of the material.

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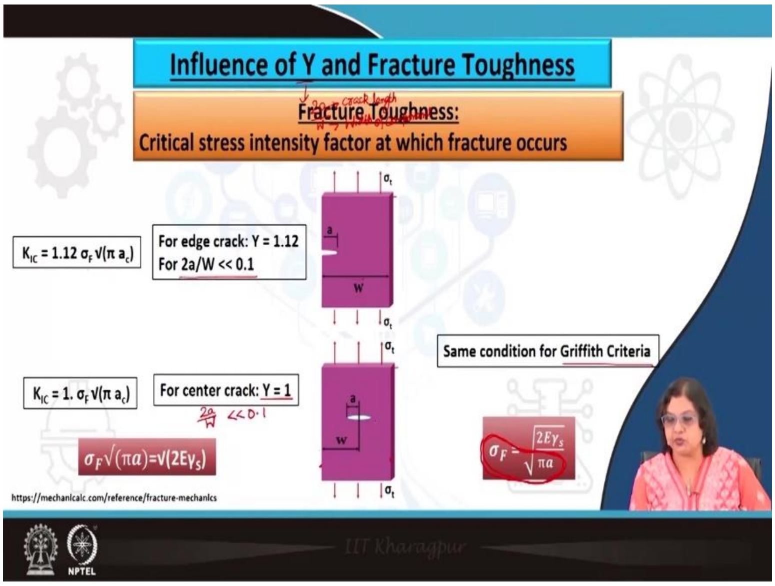

We have seen that how the value of Y changes if we are changing the configuration of the crack and as well as the position of the crack, Y is particularly dependent on $\frac{2a}{W}$, where a is the crack length and W is the width of the component. So, if we are changing this then Y is also changing. So, that means that the component size as well as the crack length is the most important factors, which changes the value of K, the stress intensity factor, and then we can be able to delay or maybe pronounced the fracture behavior or the fracture occurrence if we are varying the parameter Y.

The fracture toughness, as I said is a critical stress intensity factor at which fracture occurs and for the case of an edge crack thing we already have seen that in this case Y is nothing but 1.12 and this is valid only when $\frac{2a}{W}$, is much-much less than 0.1 or in other words in the very basic meaning of this is that the crack length is very-very small compared to the total width of the component. So, this is the width of the component and in that case K_{IC} is given by $1.12\sigma_F\sqrt{\pi a_c}$, at the point of fracture.

So, for the case of center crack on the other hand, we see that the crack length or half crack length here is a and the total width in this case is W. So, that leads us to Y equals to 1, again for this condition that $\frac{2a}{W}$ is much-much less than 0.1, and as a result what we get is K_{IC} equals to $\sigma_F\sqrt{\pi a_c}$. So, this is nothing but what we have seen for the case of Griffith criteria, if you see that this is exactly what we have seen for the case of Griffith criteria.

So, K is nothing but this parameter here, $\sigma_F\sqrt{\pi a}$, and that is a material property that is what Griffith has determined, which is valid only if we have a center crack through the thickness crack and the crack length, which is total crack length is much-much smaller compared to the width of the component.

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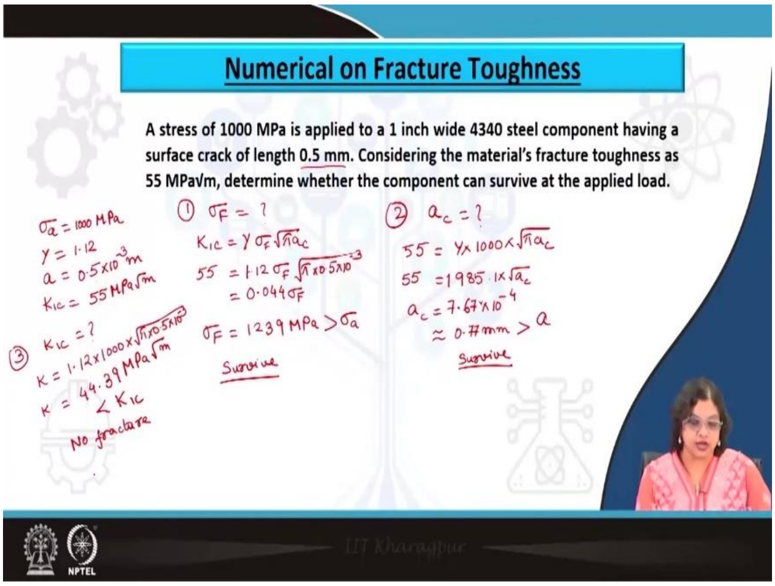

So, based on this understanding, let us solve a problem so that the concept gets more clear. What it says is that a stress of 1000 MPA is applied to a 1 inch wide 4340 that is a grade of steel component, having a surface crack of length 0.5 millimeter. So, that means that σ_a here is 1000 MPa and we have a surface crack so, that means an age crack and of course, for that Y value should be 1.12 and a is given by (0.5×10^{-3})meter. Since, we need to consider everything in the unit of meter for length and MPa for the case of stress.

Now, considering the materials fracture toughness as 55 $MPa\sqrt{m}$, determined whether the component can survive at the applied load. So, we need to figure out that whether the applied stress is sufficient for the material so, that it can survive or whether this will fracture at the point of service. So, K_{IC} given is 55 $MPa\sqrt{m}$. So, now, there are different ways by which we can solve this problem.

Firstly, we can either solve the required fracture strength or the critical crack length that can sustain or the critical value of K which could be lower or higher than the existing or the given fracture toughness value at this condition of σ_a and a that is given. So, let us do this to make it more clear. So, let us say condition one, when we need to figure out σ_F value.

So, let us simply plug this relation we knew that K_{IC} equals to $Y\sigma\sqrt{\pi a}$. So, K_{IC} is given as 55 $MPa\sqrt{m}$. Y we know that this is 1.12, we need to find out that the stress which could be the fracture strength of the material. And we are considering the a value as the critical value, as a fracture occurs at this condition this is the critical length that can lead to fracture, so what is the requirement of stress at such condition. So, let us say (π x 0.5 x 10^{-3}).

So, if we solve this let us see what it gives. So, (π x 0.5 x 10^{-3}) and root over of that into 1.12. So, that leads us to 0.044 into σ_F, and that leads to σ_F equals to 55 divided by this term, which is 1239 MPa. Now, this value is greater than the applied stress, which means that the applied stress that the stress that we are applying is less than the fracture strength that is required for fracture to happen. Obviously, the material is going to survive under such condition. Now, this is one of the way by which we can solve this.

The second way as I said is by finding out the critical crack length. So, similarly, we use the same relation, but in this case, we can use the value of sigma a as σ_F we are considering that the applied stress is sufficient for the fracture to occur and we need to find out the value of a_c. So, Y will be 1.12 once again so, that makes it $1985.1\ x\ \sqrt{a_c}$ that is equivalent to 55, so that makes root a_c as 0.02 and a_c will be something like (7.67 x 10^{-4})or we can say that this is 0.77 millimeter.

Now, the applied the crack existing crack length is only 0.5 millimeters, so, that makes that this is greater than a, given a. Of course, that means that if we have a crack of 0.7 millimeter and if we are applying the stress of 1000 MPa only then fracture can happen, since the crack length is less than this value, so this means that fracture will not materialize under such condition. So, this also gives us the same understanding that it will survive.

And thirdly, we can also solve the value of K_{IC}. At this condition when we are applying a stress of 1000 MPa and crack length of 0.5 millimeter, let us say what is the value of K. So, K equals to in this case will be given by $1.12\ x\ 1000\ x\ \sqrt{(\pi\ x\ 0.5\ x\ 10^{-3})}$, whole under root. So, that will be something like this, this comes to 44.39 $MPa\sqrt{m}$. So, this value of K is obviously less than K_{IC}, that is specified here 55 MPa root meter. So, of course, there will be no fracture once again or it will survive the condition.

Either of these methods are correct and you can use anything for your convenience. But I just wanted to specify the different ways that by which we can use this and of course, depending on the service condition, we often need to solve it accordingly, so that in some cases we need to figure out the maximum stress level or the maximum crack length that can be sustained or maybe the critical value of K.

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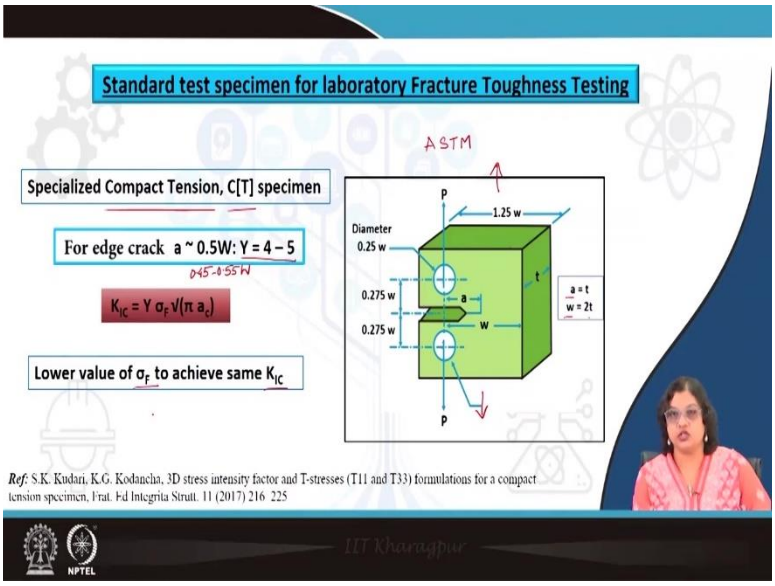

Now, moving on now, that we have seen the importance of Y on the fracture toughness of the material or the estimation of K, actually, now, fracture toughness of a material under certain condition is a fixed value. So, it is a constant value and if that value of K is achieved, there will be fracture as simple as that and this is valid only under certain condition known as the plane strain condition we will discuss in details in the next lectures. But for now, we all know that fracture toughness is a certain constant value that needs to be achieved for fracture to happen.

Now, when we are figuring out the fracture toughness of a material in the lab scale, like when we are testing the material, we need to select the kind of machine that will be used, the kind of specimen that will be used. So, the typical way, the specimen that we use for fracture toughness testing is known as a compact tension specimen or CT specimen.

So, this is as per the ASTM standard American Society for Testing of materials standard that a CT specimen is designed, and you can see all the dimensions are dependent or related to one another particularly a and W are fixed and then the thickness is as per this relation and the distance between these holes, these holes are used for loading it through pins and stresses are applied like this.

Now, there are some advantages of using this kind of specimen dimension, the major advantage is that it so, this is a edge crack. So, this is an edge crack and for this a, the total length of the a is something like almost like 0.5 W, 0.45 to 0.55 to be very specific, this is like this.

So, that means half of the width is the crack length. Of course, this violates the basic concepts by which we can determine Y equals to 1.12 for edge crack, rather Y increases to a much higher value, which is around 4 to 5. Not only that, the application of load from this pin positions here, so just add the weight of the crack that is also advantageous in the sense that fracture can happen much early.

So, the main advantage of using this in the lab scale testing is that we can still achieve fracture very quickly so, that means certain load. Of course, the typical value or the critical value of K at which fracture will happen that is a constant value, so that is not going to change, but, the load content that is necessary to reach that value is much less.

So, the advantage of this is that, in lab scale testing then we can use a much lower load capacity machine that can consume lesser electricity will make it less expensive, specimen, material wise, also it will require less amount of material, so that will conserve the total amount of material and we can do much more number of tests to verify that. So, that is particularly the reason that we use the compact tension specimen for lab scale fracture toughness testing. So, that we can achieve the K_{IC} condition with lower load of the stress conditions so, that we can use low load capacity machines.

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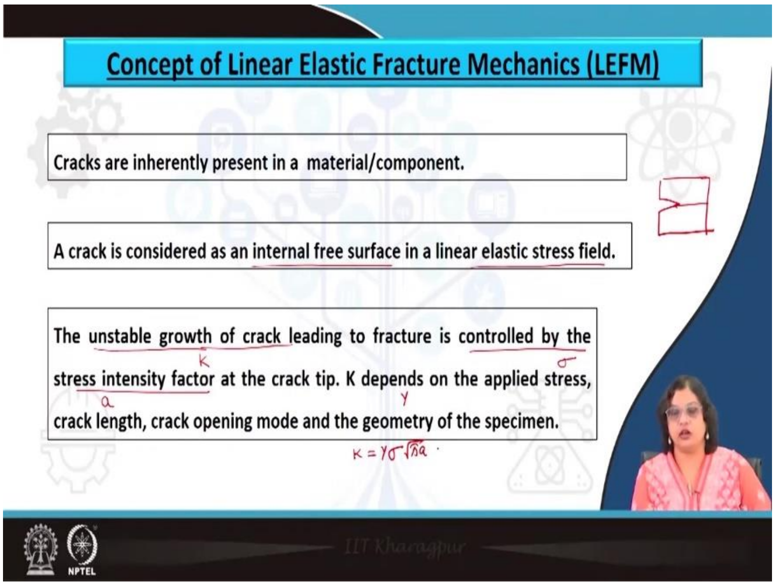

So, with this the concept of linear elastic fracture mechanics are considered as follows. So, first of all it is assumed that cracks are inherently present in a material or a component, it could be any form of defect that can lead to fracture and we should consider all the calculations based on this assumption that crack is present in a material.

Now, what is crack? Crack is nothing but an internal free surface. So, this is a free surface formed by the breakage of the bond and that is present in a linear elastic stress field. So, whatever stress we are applying that is converted to strain as per the very basic linear elastic model and it follows that, so, that we can use this for the calculations.

Also, another important parameter for linear elastic fracture mechanics is that the growth of the crack is unstable, which means that once the crack starts growing, it hardly takes any time to lead to fracture. So, let us say we have a compact tension specimen as we have seen, so, once the crack starts growing, there is already a notch here, this notch is machined. We can determine the length, the tip radius, etc. But once this notch starts growing then it will immediately lead to fracture. So, this pattern of growing is known as the unstable growth because everything is according to the linear elastic model, which means that it fractures as per the brittle concept. This is another thing which is very important for the case of fracture mechanics.

Here, when we say something is plastic means there is a permanent deformation, when something is elastic that means, that it is like a brittle one. So, this unstable growth of the crack is controlled by the stress intensity factor, we have seen that K value and K depends on the applied stress crack length, crack opening mode, whether it is mode 1 or not and the geometry of the specimen. So, basically K is given by $Y\sigma\sqrt{\pi a}$. So, that is how the entire concept of linear elastic fracture mechanics stands upon.

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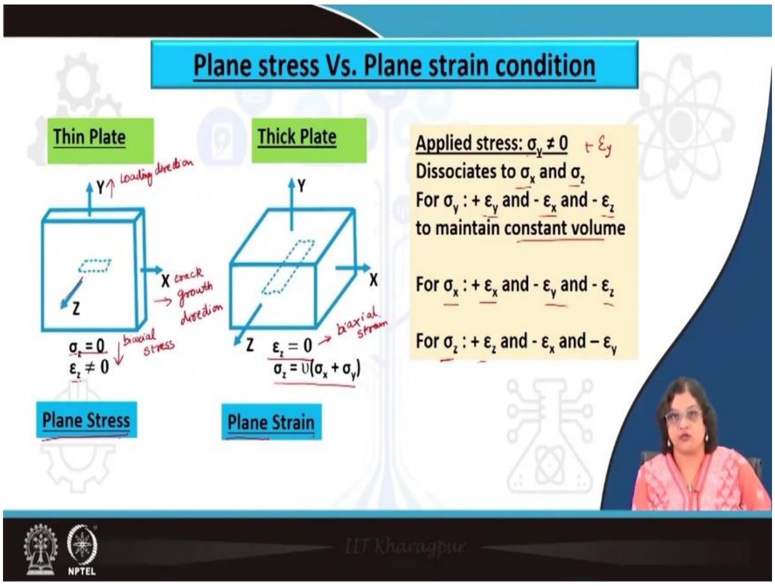

Now, coming to the concept of plane stress versus plane strain condition what we need to see is that, even if the component have similar pattern, if we have different thickness. Now, so, far even for the Griffith criterion, we have not talked much about the thickness, we have mentioned that the width or the length of the specimen or the component is infinite, but there is a finite thickness, why, because this thickness has a very important or very big role to play to control the fracture toughness of a material or the critical value of K for fracture.

In case of thin plate what happens is that whenever we are applying stress, let us say we are applying stress along this direction along the direction of Y and the crack is growing along the direction of X, so this is the loading direction and X is the crack growth direction of course, the crack is growing perpendicular to the loading direction. Now, at any point, as we are applying the load, the load is supposed to dissociate to X and Z direction also. So, even if we are applying the stress along Y, this is dissociating into σ_x and σ_z counterpart and for each of these stress values we are getting the corresponding strain values also.

Since, we are applying the stress along the Y direction there will be a positive ε_y, strain along the X direction will be counterbalanced by the growth of the crack, but the strain along the Z one is a critical one, but for each condition, we are getting the strain also which are getting dissociated in the perpendicular direction.

If we are getting a positive strain along the Y direction, then there will be a negative counterpart of strain along the X and Z direction to conserve the volume, to maintain the constant volume condition. So, for each of these conditions, we can determine the plus values, the positive values of strains and the negative values for X and Z, as I said for X, this is a direction for the crack growth, so this will be counterbalanced there, but it is a Z one which is important.

Now, for the case of thin plate, if we have a very thin plate, actually, the stress along the Z direction, σ_z is nothing but 0. So, that makes the stress acting only along the two directions. On the other hand, strain is triaxial in that case, because strains are not zero, we are getting this ε_z component from the other parts of stress also, other counterparts of stress. So, that makes it a condition when we have stress as biaxial but strain as triaxial.

On the other hand, if we are having a thick plate, at that circumstances, σ_z cannot be considered at 0, so σ_z has a finite value. But on the other hand, since we are generating plus ε_z and minus ε_z from the different counterparts of stress those will get nullified and finally, we end up having $\varepsilon_z = 0$. So, that leads to biaxial strain in that case.

So, we have biaxial strain for the case of thick plate and we have biaxial stress for the case of thin plate and based on this biaxial thing, it is named as plane stress or plane strain. Since, biaxial means that it is acting along a plane, along two directions, that is why this is known as plane stress. On the other hand strain is acting along two direction X and Y and that is why this is known as plane strain.

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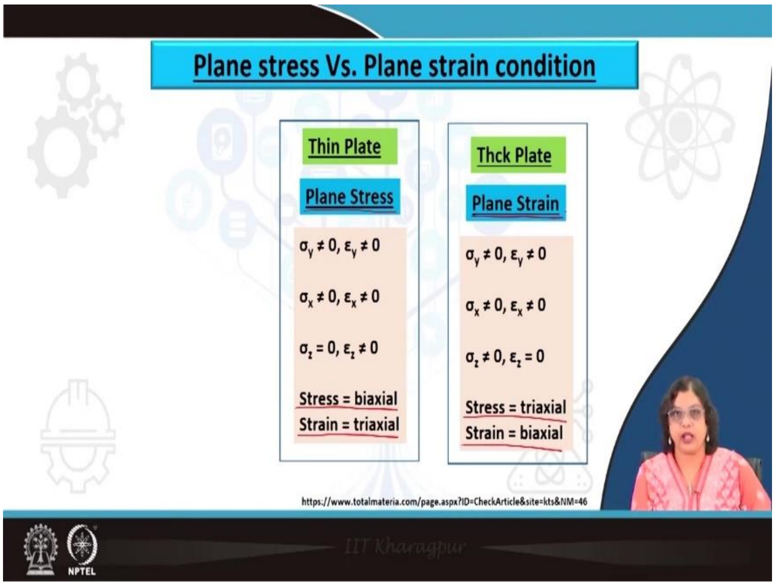

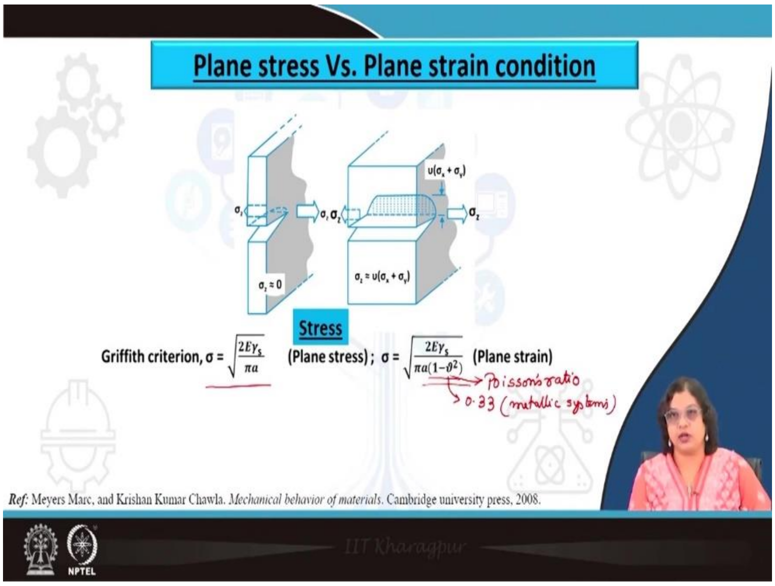

So, eventually, what we mean is for the case of thin plate plane stress condition is valid, which leads to biaxial stress, but triaxial strain. On the other hand, for the case of thick plate, we have triaxial stress but biaxial strain and that leads to plane strain. We will look into more details of this plane stress and plane strain concept, but for now, we can see that the Griffith criteria can also be modified and the σ value that we have generated is particularly for the plane stress condition.

On the other hand, for the plane strain condition, since, we have the stress also acting along the Z direction, we have to take care of this poisson's ratio term. So, there will be compression on the perpendicular direction. So, that leads to the importance of poisson's ratio to be considered, while calculating the fracture strength of the material.

So, to this Griffith criterion in the denominator, we have to add this $\sqrt{(1-\vartheta^2)}$. For the case of metallic systems usually, nu or the value of poisson's ratio is something like around 0.33 for metallic system and this could have different values. So, for metallic system, there is not much of a change and we can calculate these values under plane stress and plane strain conditions.

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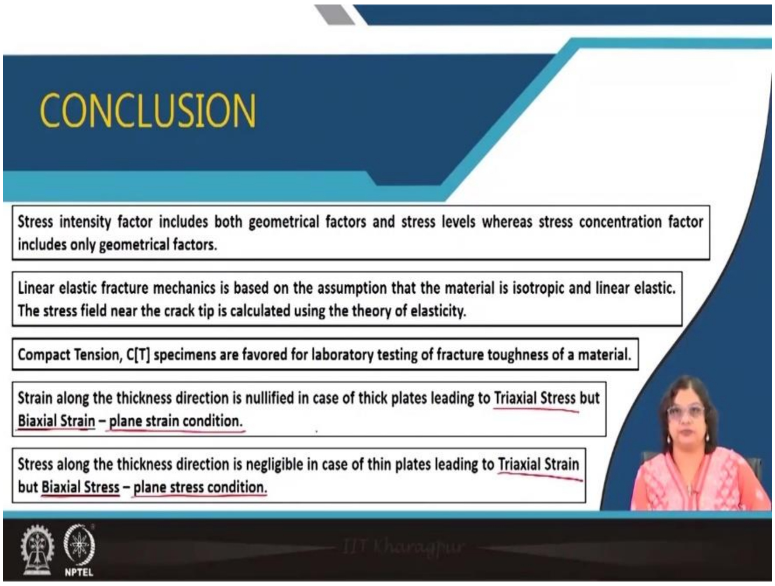

So, the conclusion for this lecture is that the stress intensity factor includes both the geometrical factors and stress level. On the other hand, stress concentration factor relies only on the geometrical factors. Linear elastic fracture mechanics is based on the primary assumption that material is isotropic it behaves same along all the directions and it is linear elastic, which means the stress and strain are followed by this elastic relation and based on this theory of elasticity the stress will is calculated.

Compact tension we have seen and that such kind of specimens are favored for laboratory testing, because we require less amount of stress to achieve the same value of fracture toughness. And in case of plane strain condition, we need to remember or understand that the strain in this case is biaxial. That is why this is termed as plane strain, on the other hand, stress is triaxial. On the other hand, in case of plane stress condition, the stress is biaxial whereas the strain is triaxial.

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So, these are some of the references used in this lecture, and I hope you have learned something new in this lecture. Thank you very much.