

Fracture, Fatigue and Failure of Materials
Professor Indrani Sen
Department of Metallurgical and Materials Engineering
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Lecture 60
Failure Analysis - Summary

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The slide is a title slide for an NPTEL online certification course. It features a blue and white color scheme with geometric shapes. At the top, there are two logos: the Indian Institute of Technology Kharagpur logo and the NPTEL logo. Below the logos, the text reads "NPTEL ONLINE CERTIFICATION COURSES". The main title is "Fracture, Fatigue and Failure of Materials" by "INDRANI SEN" from the "DEPARTMENT OF METALLURGICAL AND MATERIALS ENGINEERING, IIT KHARAGPUR". The slide also mentions "Module 03: Failure" and "Lecture 60 : Overview of Fracture, Fatigue and Failure of Materials".

Fracture, Fatigue and Failure of Materials
INDRANI SEN
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Module 03: Failure
Lecture 60 : Overview of Fracture, Fatigue and Failure of Materials

Concepts Covered

- Fracture of Materials
- Different Approaches in Fracture Mechanics
- Concept of Fatigue
- Factors affecting fatigue life
- Failure Analysis
- Importance of Designing a component
- Glimpse of case studies

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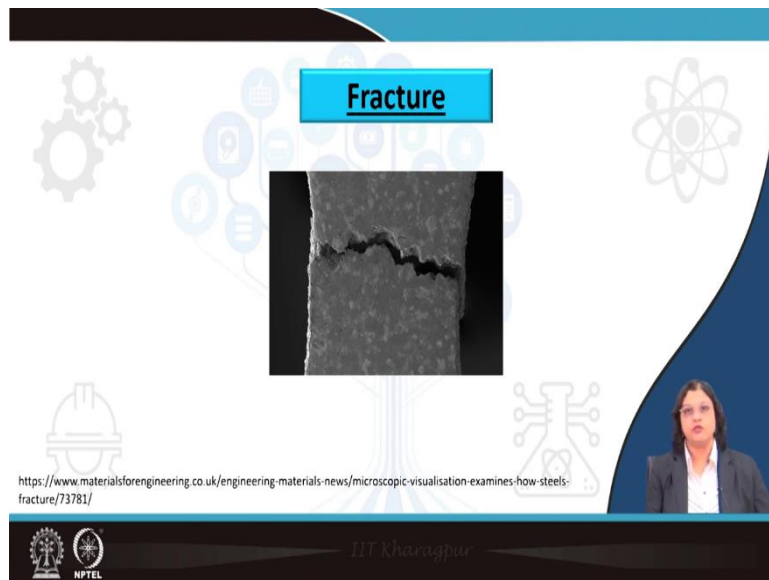
Hello everyone, welcome back, and we are at the last lecture of the course Fracture, Fatigue, and Failure of Materials. And in this lecture, I would like to give an overview of what we have discussed in the previous lecture, so far.

So, coming to the concepts that will be covered in this lecture. We will first start with the fracture of materials and the different approaches that are used in fracture mechanics,

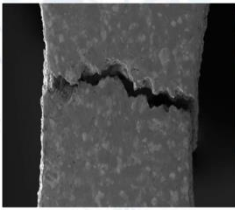
particularly to understand the fracture toughness behaviour of a material, and then the ways by which the toughness can be engineered further.

And from there, we will relate to the concepts of fatigue, different kinds of fatigues that are commonly seen in materials, factors affecting the fatigue life, our main target here is to improve the fatigue performance of a material or a component under certain circumstances, and eventually whatever understanding we have developed from fracture and fatigue, we would like to incorporate that in performing the failure analysis, and in there we will discuss about the importance of proper designing of a component to avoid failure or to delay failure to the maximum extent. And I would be also giving some glimpses of case studies that has been discussed in the various lectures.

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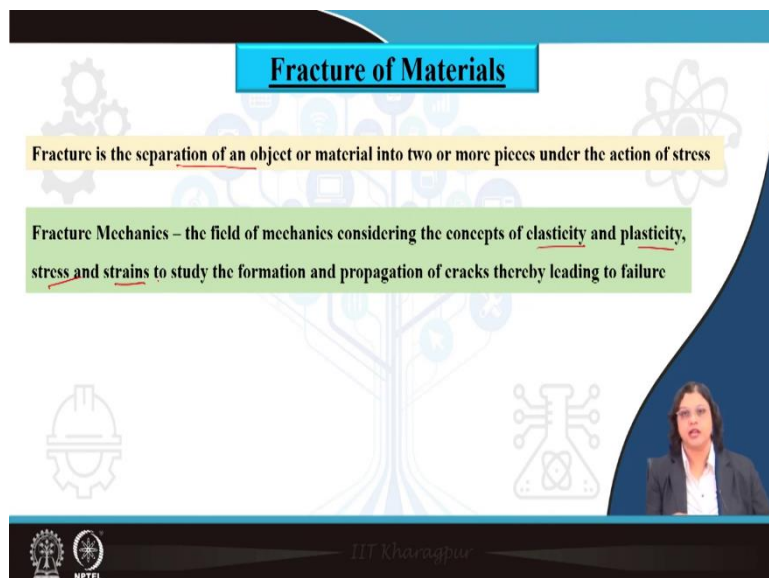
Fracture



<https://www.materialsforengineering.co.uk/engineering-materials-news/microscopic-visualisation-examines-how-steels-fracture/73781/>

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Fracture of Materials

Fracture is the separation of an object or material into two or more pieces under the action of stress

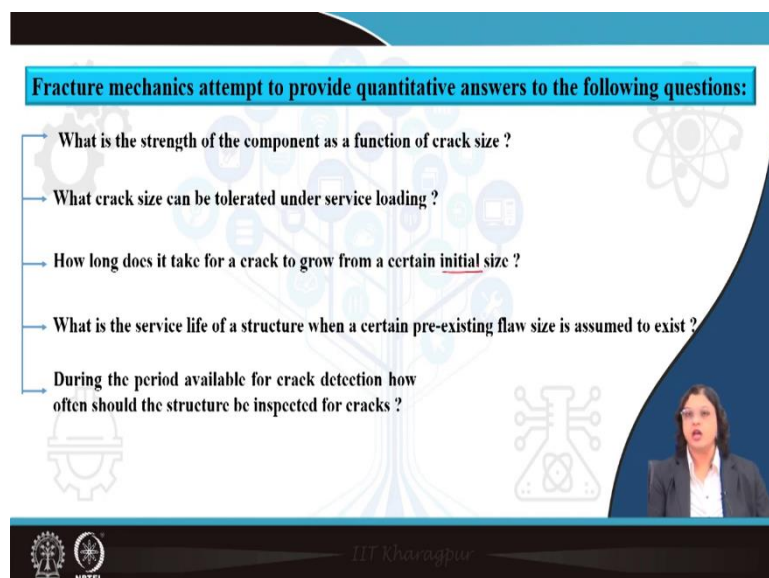
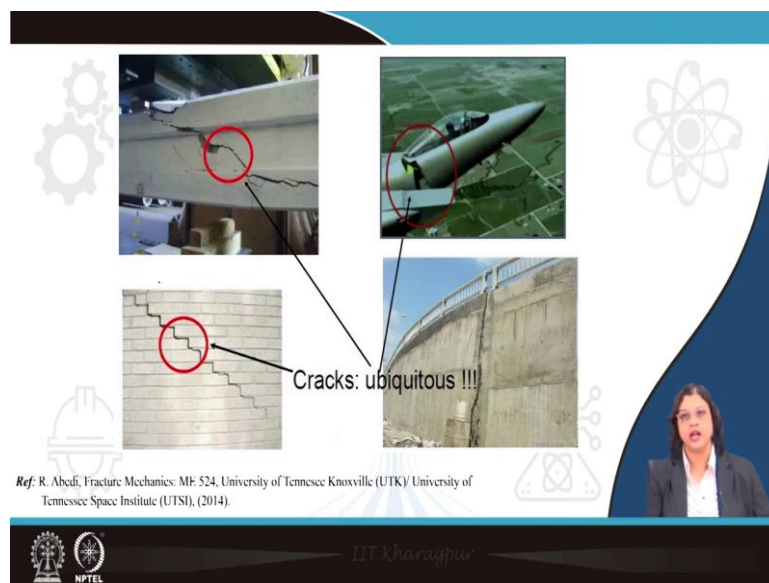
Fracture Mechanics – the field of mechanics considering the concepts of elasticity and plasticity, stress and strains to study the formation and propagation of cracks thereby leading to failure

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So, coming to the first topic or the first module that has been covered in this course, we talked about fracture. Now, if we want to say the very basic definition of fracture, it says the separation of an object or a material into two or more pieces, okay, that is a very generic definition of fracture. And when we are coming to the fracture mechanics, it actually deals with the concepts of elasticity, plasticity, stress, and strain, and eventually on the formation of crack, and finally that will lead to the propagation and the final failure of a material or a system or a component.

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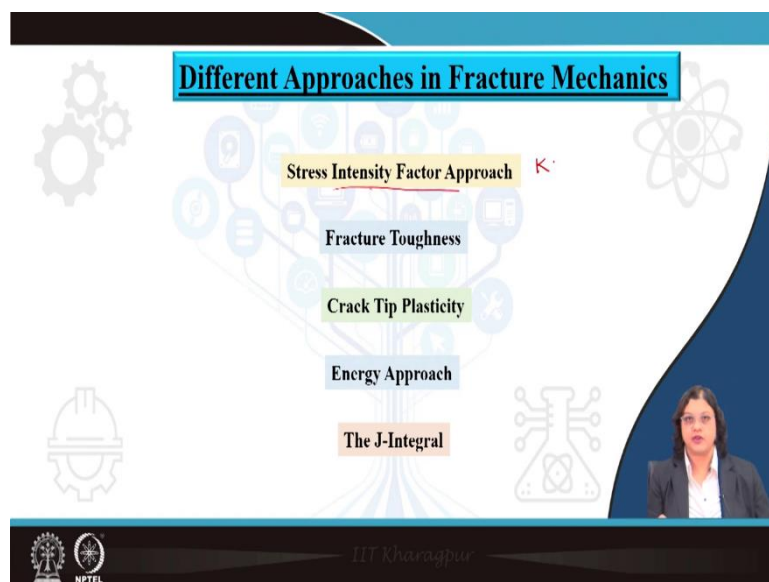
So, as we know defects or cracks are omnipresent, they are almost everywhere we can hardly find any system that are perfect, and that does not have any defect. So, we know of the fact that there are defects in a system and our target is to find out or to answer the following

questions, such as what is the strength of the component as a function of crack size? so if we know that there is an existing crack size how much amount of stress that system or that component can withstand before it undergoes catastrophic failure.

What kind of crack size can be tolerated and the service loading? Okay some if even if we cannot avoid having a crack or a defect in a component our target in that case would be to minimize the quantity or the value of the defect size, so that we can still achieve satisfactory performances. Even if there is a crack of certain size, we would like to know that how long does it take for the crack to grow till a limiting size at which fracture can materialize.

What is the service life of a structure when a certain pre-existing flaw is already present there? So, with that we can actually predict the overall life period of a system. Now, during the period available for crack detection how often should the structure be inspected? Now, this is also very important considering the fact that there are defects or the defects sometimes generate during the course of service but that does not mean that, we have to immediately stop using that system or a component, rather we would like to formulate an investigation procedure after regular period of interval, so that we can monitor the growth of the defect and we can predict the service life of that component.

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There are different approaches of course to assess the fracture, behaviour of a material through fracture mechanics, and the most commonly used one is the stress intensity factor approach. Stress intensity factor is commonly defined by the term K , and from there the critical value of stress intensity factor actually leads to the concept of fracture toughness, and we can determine how much is the toughness of a material in presence of a crack.

And of course, whenever there is a crack there is also a possibility for developing or generating some amount of plasticity particularly at the tip of the crack and that is very much valid, if we are talking about metallic systems which are by default ductile in nature and have the ability to deform to a certain extent, when there is stress available.

Then there are energy approach, and the J-Integral approach, particularly for elastic plastic materials which undergo significant amount of plastic deformation, ahead of the crack tip and which cannot be ignored anymore and we have to utilize a specialized J-Integral method to determine the toughness of such kind of ductile or plastic materials.

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The slide is titled "Linear Elastic Fracture Mechanics" in a blue box. It contains several text boxes with the following content:

- British Engineer Griffith is known as the father of Fracture mechanics
- Griffith Criteria – 1920
- It states that crack extension (i.e., fracture) occurs when the energy available for crack growth is sufficient to overcome the resistance of the material
- Suitable for estimating fracture strength for brittle materials with very sharp crack
- Modified by Orowan to incorporate the concept of plastic deformation prior to fracture $\rightarrow \gamma_p$
- Further Modified by Irwin in 1950s to incorporate the concept of strain energy release rate in fracture mechanics G

The slide also features a small video inset of a presenter in the bottom right corner. At the bottom, there are logos for IIT Kharagpur and NPTEL.

Okay, now, linear elastic fracture mechanics is the one that is the most developed one and that is actually that forms the basis of fracture mechanics in general. And the British engineer Griffith is actually known as the father of fracture mechanics, and that is particularly because he developed the main rule for fracture mechanics, known as the Griffith criteria, which states that the crack extension occurs when the energy available for crack growth is just sufficient to overcome the resistance of the material.

We have already seen in the various lectures, in the first module, that how Griffith criteria help us to determine the fracture strength of a material as a function of the crack size. And this is particularly suitable for estimating the fracture strength of a material which fails in a brittle fashion, and that too it is having a very very sharp crack, We were also able to determine how sharp the crack needs to be so that Griffith criteria would be valid.

But that is very difficult to determine experimentally, and to that and Griffith criteria has been further modified by Irwin, and he introduced the concept of strain energy release rate commonly known as G , often gives us an impression and idea of the fracture behaviour of the material, fracture toughness of the material.

The Stress Intensity Approach

The stresses in the vicinity of a crack tip take the form as

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta)$$

where r and θ are the radius and angle w.r.t the crack tip

$$\begin{aligned}\sigma_y &= \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\ \sigma_x &= \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\ \tau_{xy} &= \frac{K}{\sqrt{2\pi r}} \left(\sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right)\end{aligned}$$

Stress intensity factor, $K = f(\sigma, a \text{ and } Y(a/W)) = Y\sigma\sqrt{a}$

Ref: C. M. Sewart, Fracture Mechanics: Introduction to Fracture, (2020) <https://doi.org/10.1115/1.3143689>

We have seen that basically the stress can be dissociated into three perpendicular axis as well as there could be shear stress as well, and all these stresses are related directly to this stress intensity factor as well as inversely to the square root of r , r is the distance from the crack tip as well as it is also a function of θ , which represents the angle, at which the point is located with respect to the crack tip.

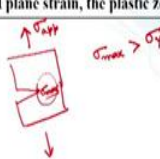
So, overall the stress intensity factor is basically a function of the applied stress σ , the crack length or the half crack length a , and most importantly it also includes a geometrical parameter Y , which is again a function of a / W . We have seen that if we have different kind of crack in a component based on the crack size as well as the total width of the component, we can have different values of one.

If we have an edge crack like this, we have seen that Y is equals to 1.12. On the other hand, if we are having a central crack where this is $2a$, and this length this width is W , then Y will be equal equivalent to 1, but this will be only valid if we have the ratio between a / W is very low.

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Crack Tip Plasticity

- The elastic stress distribution in the vicinity of a crack tip, shows that as r tends to zero the stresses become infinite, i.e. there is a stress singularity at the crack tip.
- Since structural materials deform plastically above the yield stress, there will in reality be a plastic zone surrounding the crack tip.
- For plane stress and plane strain, the plastic zone size would be



$$r_p = \frac{1}{2\pi} \left(\frac{K}{\sigma_y} \right)^2 \quad \text{Plane stress}$$

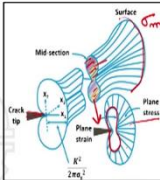
$$r_p = \frac{1}{6\pi} \left(\frac{K}{\sigma_y} \right)^2 \quad \text{Plane strain}$$

Ref: C. M. Stewart, Fracture Mechanics: Introduction to Fracture, (2020). <https://doi.org/10.1115/1.3143689>.

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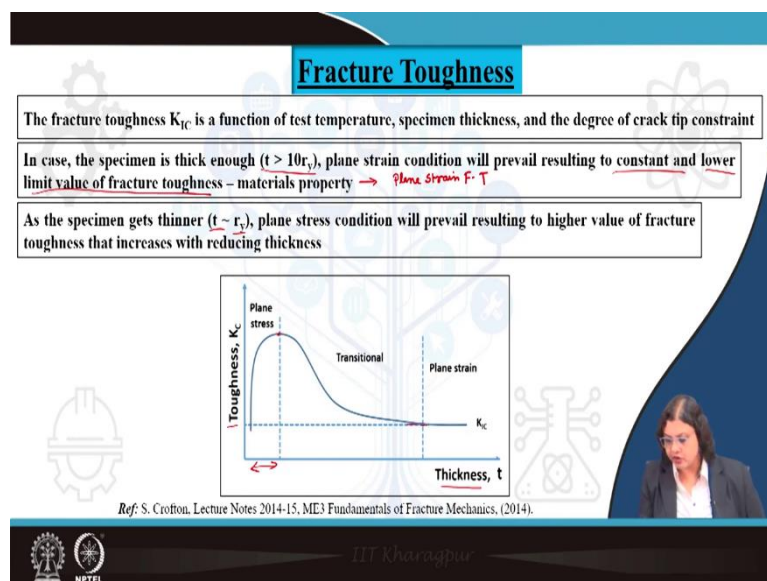
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Now, another thing that we have also understand from this relation between K and σ is that at the tip of the crack. Where r actually achieves a value as small as 0 or very low value, at that point stresses actually achieves an infinite value, which means there that there will be stress singularity at the crack tip, which again is very difficult to determine mathematically, but what it actually indicates us that in structural material, since it deformed plastically, so at some distance, away from the crack tip, when the stress value is pretty high, so that means that although we are having a component like this, and we are applying stress of σ applied, tensile stress, at this point we have already seen that how the stress concentration factor comes into the picture and that maximizes the stress.

So, once this point that the $\sigma_{\max}$ exceeds the yield strength of the material, this is bound to deform plastically. So, at some location ahead of the crack tip we are expecting plastic deformation to occur, and that will lead to a plastic zone size, which again will be different under plane stress condition, and plane strain condition. To show that schematically, this is how it will look like, so if we have a crack like this we can see that at the ages, we are having the plane stress condition that is active and at the central location there is the plane strain condition.

So, at the central part from the thickness we are seeing the plane strain condition to be active and at the ages or near to the surface there will be plane stress condition. So, plane stress condition actually has much higher plastic zone size as we can see it from here this appears like a kidney bean shaped size and the plastic zone size under plane strain condition is actually one third of that forms under plane stress condition.

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Now, fracture toughness is actually the value of the critical value of stress intensity fracture at which fracture occurs. This is a function of the test temperature as well as a specimen thickness, and the degree of crack tip constraint. Particularly, thickness plays a very major role in dictating the fracture toughness of the material and also to indicate that whether the fractured after that we are determining whether this is going to change or this is a constant value.

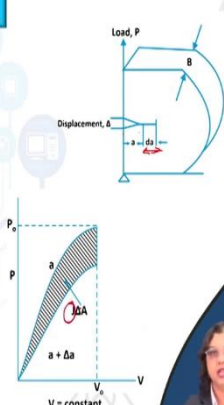
Now, in case the specimen is thick enough such that the thickness is more than 10 times the plastic zone size that is forming at the tip of the crack at that condition, we are going to get a constant and lower limit value of the fracture toughness and that is known as the plane strain fracture toughness, which is considered as a materials property. On the other hand, if the specimen gets thinner, such that the thickness of the specimen is almost equivalent to the plastic zone size, which means the plastic zone almost encompasses the thickness, at that condition plane stress condition will be active and we are getting a much higher value of fracture toughness from there.

However, that fracture toughness value reduces, if we are increasing the thickness, and that is the reason that this is not a constant value and plane stress fracture toughness is not considered as a materials property. So, this is a typical relation that it holds between the toughness as well as the thickness and we can see that for smaller value of thickness for thin plate actually the fracture toughness value increases up to certain level and after that it decreases continuously and very sharply, and finally it achieves more or less a stable value, and that to a lower bound value, which is known as the plane strain fracture toughness.

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The J-Integral

- In 1968, Rice considered the potential energy changes involved in crack growth in non-linear elastic material.
- Rice derived a fracture parameter called J, a contour integral that can be evaluated along any arbitrary path enclosing the crack tip.
- He showed J to be equal to the energy release rate for a crack in non-linear elastic material.
- In spite of this, J has found widespread application as a parameter to predict the onset of crack growth in elastic-plastic materials.



The diagram shows a crack of length a under a load P causing a displacement δ . A small increment da is shown for crack growth. Below it, a graph plots Load P against Displacement δ . The area under the loading curve is shaded and labeled $J dA$, representing the energy release rate. The graph also indicates $V = \text{constant}$ and shows points a and $a + \delta a$ on the displacement axis.



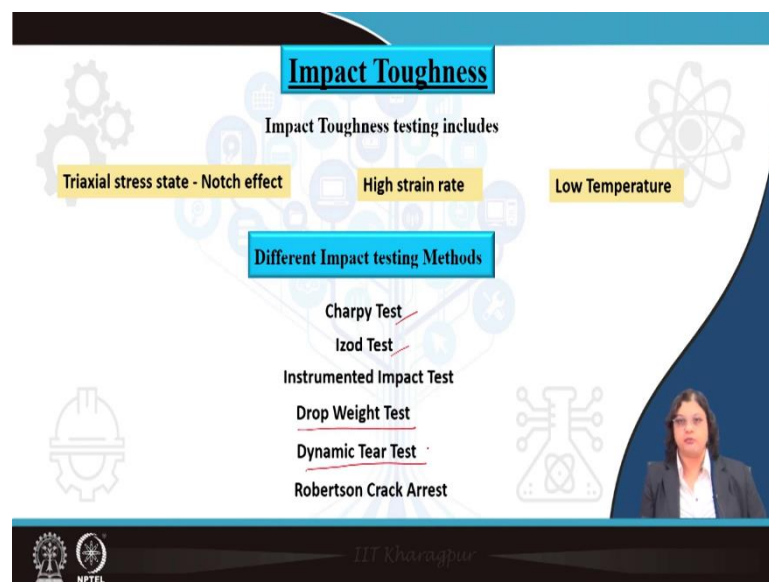

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Now, coming to the material which undergo significant plastic deformation as well along with the elastic deformation, then the K or the stress intensity factor approach is not very much valid, because we understand that the plastic zone size is quite large that encompasses the overall thickness of the component, and the such condition plane stress fracture toughness is applicable, and we have to use the concept of J-Integral.

So, Rice actually discovered or invented or utilized this J-Integral concept to the fracture mechanics system, and he derived a fracture parameter called J which signifies a contour integral that can be evaluated along any arbitrary path, enclosing the crack tip. Rice showed that J is equivalent to the energy release rate for a crack in non-linear elastic material. In case it is an elastic material which undergoes completely brittle fracture, then J is equivalent to G , where G is the strain energy release rate.

It is a little bit difficult to estimate and we often need to simulate the values or use a mathematical expression, but even then, J has been found widespread application as a parameter to predict the onset of crack growth in Elastic Plastic material. So, as you can see if there is an initial half crack of length a , that extends by $B a$, we can determine from here the value of J , based on the energy that is being released under such condition.

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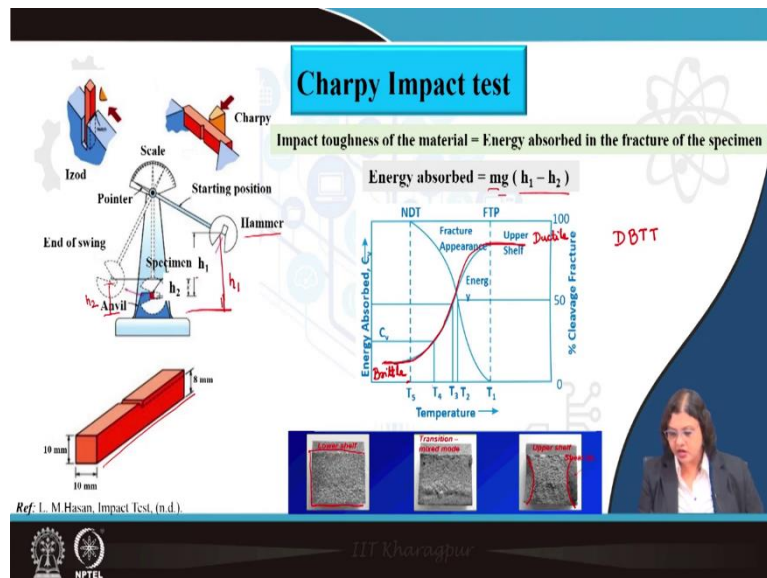


Now, another very important kind of toughness that is often utilized or we need to understand the value or the behaviour is the impact toughness. Impact toughness actually includes application of triaxial stress state in a component which is obtained by having a notch, very prominent big notch in a system as well as when high strain rate is employed during the application of stress, and the tests are performed at very low temperature. So, all these

conditions we have seen that increases the strength of the material and actually decreases the ductility or decreases the toughness of a material.

So, we apply all these three conditions all together to find out the fracture behaviour under the worst possible scenario. There are different kinds of impact toughness testing methods the most commonly used one is the Charpy Test, and then there is also Izod Test or Instrumented impact Test, which gives us a much clearer picture and much more understanding and information about the impact toughness testing. Then, there are Drop Weight Test and Dynamic Tear Test, utilizing very big specimen, which are often utilized in the practical application condition. And then, there is also Robertson Crack Arrest Test, where we can determine the crack arrest temperature.

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So, Charpy Impact Test is the most widely used one. It actually uses a specimen of rectangular configuration having a v-notch of particular dimension as per the ASTM standard. And we put the specimen somewhere here and then it is hit by a pendulum having hammer of certain weight, we determine the height of the pendulum at the initial condition, so h_1 and then after it hits the sample and breaks the sample, it stops at quite some lower height, and we determine that height as h_2 , and from there we can determine that how much energy is being absorbed by the fracture of the specimen which is given by mass of the specimen G , as acceleration, due to gravity and the difference in the height of the specimen $h_1 - h_2$.

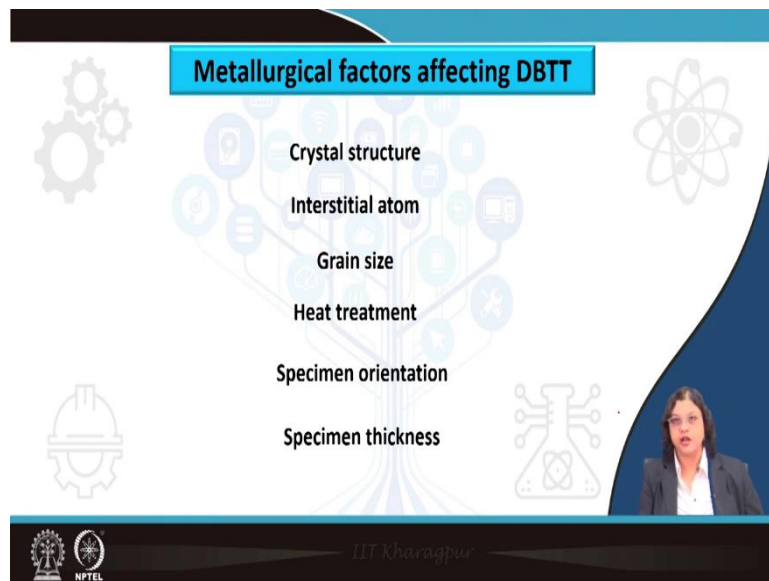
From there, we actually determine the energy absorbed during fracture, and when we are plotting this, when we are performing this at different reducing temperatures, and we plot this

energy versus time curve, we get this kind of S shaped curve having, a distinct upper shelf and a lower shelf. So, here the material behaviour is mostly ductile or it fails under ductile mode, whereas in the lower shelf, the material fails under brittle fashion.

And there are different ways by which the ductile to brittle transition temperature, commonly known as DBTT are determined. There are actually five different ways by which we can predict either the maximum temperature at which it is completely ductile or the minimum temperature, at which it undergoes completely brittle failure, and then the most common one is however T_4 , when it uses a particular amount of energy such as 20 J.

And we also simultaneously investigate the fracture surface where you can see when it fails under upper shelf we have a distinct shear lip signifying certain amount of plasticity. On the other hand, at the lower shelf, there is a completely flat fracture and no signature of any plastic deformation is evident.

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Now, there are various metallurgical factors that affect the DBTT so our target of course is to understand the concept of any kind of fracture toughness behaviour and then, our next target is to control that. We would like to have a material that will have higher and higher fracture toughness. And, so in case of impact toughness there are several factors such as the crystal structure of the material, the presence of interstitial atom, grain size, sometimes we will prefer finer grain size that will lead to lower and lower DBTT. And different kind of heat treatments ensures that we reach, we attain the particular microstructure that is beneficial to have lower value of DBTT.

And specimen orientation is also another factor that is very important to generate a good amount of impact toughness. Specimen thickness is as well for any kind of fracture toughness plays a very big role in determining the ability of the material to resist fracture.

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Intrinsic and Extrinsic Fracture Toughness

Toughening mechanism in engineering solids: (1) Intrinsic toughness
(2) Extrinsic toughness

Intrinsic Toughness provides the inherent resistance to crack nucleation and propagation

Extrinsic Toughness occurs by reducing the crack driving force.

Ref: R.W. Hertzberg, R.P. Vinci, J.L. Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials*, 5th ed., John Wiley & Sons, Inc, 1982.

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The slide features a blue header with the title 'Intrinsic and Extrinsic Fracture Toughness'. Below the title, it lists two toughening mechanisms: (1) Intrinsic toughness and (2) Extrinsic toughness. It then explains that intrinsic toughness provides inherent resistance to crack nucleation and propagation, while extrinsic toughness occurs by reducing the crack driving force. A reference to 'Deformation and Fracture Mechanics of Engineering Materials' by R.W. Hertzberg et al. is provided. The slide also includes the IIT Kharagpur and NPTEL logos, and a small video inset of a presenter in the bottom right corner.

Now, coming to the different ways by which fracture toughness can be enhanced is by intrinsic toughening or extrinsic toughening. And by intrinsic toughness what we mean is the inherent resistance of the material to crack nucleation and propagation, so that depends on several factors. Whereas extrinsic toughness is by which we are reducing the ability of the crack to grow further, so it involves reducing the cracked driving force itself.

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Intrinsic and Extrinsic Fracture Toughness

Intrinsic Toughness Mechanism

- Electron Bond
- Crystal structure
- Degree of order in materials

Extrinsic Toughness Mechanism

- Crack deflection and Meandering
- Zone Shielding
- Contact Shielding
- Combined Zone and Contact Shielding

Ref: R.W. Hertzberg, R.P. Vinci, J.L. Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials*, 5th ed., John Wiley & Sons, Inc, 1982.

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This slide continues the discussion on fracture toughness mechanisms. It is titled 'Intrinsic and Extrinsic Fracture Toughness'. It is divided into two main sections: 'Intrinsic Toughness Mechanism' and 'Extrinsic Toughness Mechanism'. The intrinsic section lists three factors: Electron Bond, Crystal structure, and Degree of order in materials. The extrinsic section lists four factors: Crack deflection and Meandering, Zone Shielding, Contact Shielding, and Combined Zone and Contact Shielding. A reference to the same textbook as the previous slide is included. The slide also features the IIT Kharagpur and NPTEL logos, and a small video inset of a presenter in the bottom right corner.

Intrinsic toughness is dependent on the electron bond, the nature of the electron bond of a material, crystal structure, as well as degree of order in material, so that those kind of factors dictate how much ability the material have by default to resist fracture. On the other hand, extrinsic toughness is when we incorporate certain situations to control the fracture toughness, and this can be obtained by ways by which we can make the crack to deflect its regular path by metering or the zone shielding by which the crack will achieve some kind of hindrance for its growth.

Contact shielding when the two phases of the crack come in contact or there could be combined zone and contact shielding, all this can actually arrest the growth of the crack and that can delay the fracture procedure, which means that it can enhance the fracture toughness or resist the fracture behaviour to the maximum extent.

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Toughening of Material

- Toughness of metallic systems can be enhanced by influencing any or both the Intrinsic and Extrinsic Toughening mechanisms
- Both Ceramic and Glass materials have low intrinsic toughness.
- Toughness of Ceramic materials can be increased by Extrinsic Toughening mechanism such as geometrical toughening, crack tip shielding, crack wake shielding etc.
- Toughness of glass can be enhanced by no break scenario and safe breakage scenario.
- Crystalline polymer has higher toughness than amorphous polymer.
- Toughness of brittle polymer can be enhanced by particle toughened mechanism.

The slide features a blue header with the title 'Toughening of Material'. Below the title, six yellow and green boxes contain text about material toughness. The background includes faint icons of gears, an atom, and a hard hat. A video inset in the bottom right corner shows a woman speaking. The footer contains the IIT Kharagpur and NPTEL logos.

Now, there are different materials for which fracture toughness enhancement by intrinsic or extrinsic toughening is very very important, particularly for metallic systems toughness can be influenced or enhanced by changing any or both of the intrinsic and the extrinsic toughness mechanism. On the other hand, both ceramic and glass materials have very low intrinsic toughness, and for those case particularly extrinsic toughness mechanism works such as, for the case of ceramic materials toughness can be increased by extrinsic toughening mechanisms, such as geometrical toughening proactive shielding, crack wake shielding, etc.

On the other hand, toughness of glass can be enhanced by employing a no break scenario or a safe bracket scenario. Crystalline polymer on the other hand has comparatively higher

toughness, compared to the amorphous polymer, so this kind of brittle polymer toughness can be enhanced by particle toughening mechanism.

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Fatigue

Stress
Tension +
0
- Compression
Cycles →
 $\sigma_{max} = \sigma_{min}$

Concept of Fatigue

- Load is applied cyclically to material/component
- Leads to progressive, localized and permanent microstructural change
- Initiation and propagation of cracks due to fluctuating stress
- Once a fatigue crack initiates, it grows, often un-noticably with each cycle producing striations
- Crack continues to grow until the point of catastrophic failure

Now, coming to the next module where we have discussed about fatigue. So, the word fatigue means when we apply cyclic loading and a material fracture at a stress level much lower than the expected tensile strength of the material or the yield strength of the material. So, for the concept of fatigue we actually need a cyclic load or fluctuating load being applied on a material and that leads to progressive, localized and permanent microstructural change, which keeps on adding up in every cycle and finally lead to initiation of crack, and once the crack initiates it leads to propagation of it until the final catastrophic fracture in occurs.

So, one of the major disadvantage or what makes fatigue of concern is the fact that, once a fatigue crack initiate it grows often unnoticeably, and with each cycle it generates a striation which cannot be seen or observed without doing a thorough fractographic investigation, and suddenly it fails. So, that catastrophic failure comes without any significant alarm and that makes studying fatigue very very important for practical application.

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Fatigue testing

- Load/Stress controlled - Un-notched specimen - Crack Initiation - HCF $N_f \sim 10^6 - 10^8$
- Displacement/Strain controlled - Un-notched specimen - Crack Initiation - LCF $N_f \sim 10^3 - 10^5$
- Stress intensity factor controlled - Notched specimen - Crack Propagation - FCGR

Handwritten notes on the slide include: "gauge section" with a drawing of a specimen, a stress-strain hysteresis loop, and the equation $\frac{da}{dN}$.

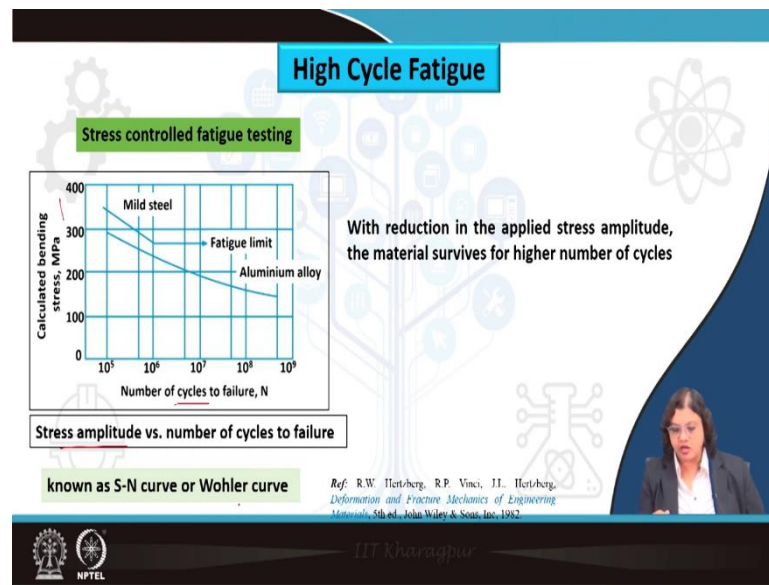
The slide footer includes the IIT Kharagpur and NPTEL logos.

Now, there are different ways by which fatigue can be tested in lab scale, so that we can understand the fatigue behaviour of material, this could be done under load or stress controlled condition or under strain controlled condition or displacement control condition, for both of this actually we use an un-notched specimen, a cylindrical specimen which looks something like this, where there could be a pronounced gauge section or not, but it is actually an un-notched specimen, for both of this we need a specimen which has a perfectly smooth gauge section, which means that there is no prominent or dominant crack or not present there.

So, for that kind of fatigue stress controlled or strain controlled, high cycle or low cycle fatigue, we are mostly concerned to find out the crack initiation. We once the fatigue crack initiates it hardly takes a few numbers of cycles to propagate and lead to catastrophic failure. On the other hand, there could be also situation when there is an already pre-existing flaw or a sharp corner or some kind of stress concentration already existing, and for those cases we perform the test under stress intensity factor control, and in this case, we used a notched component the one that is commonly used in the lab scale for metallic system is known as the compact tension specimen or in short CT specimen.

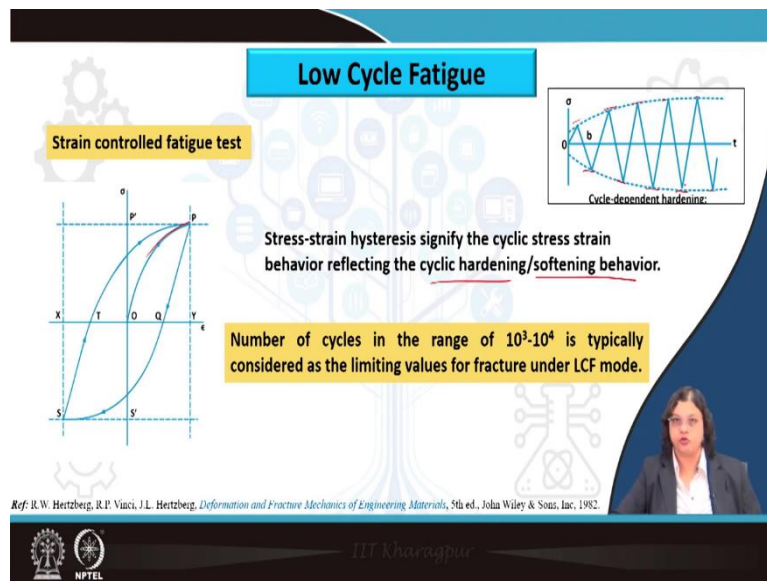
In this case cracked propagation is of our concern, and what we generate in turn is the fatigue crack growth rate curve. On the other hand, under stress control we get the high cycle fatigue, where the number of cycles for failure could be something like $10^6 - 10^8$ cycles, and for the case of LCF N_f varies between $10^3 - 10^5$, and on the other hand for the case of fatigue crack growth rate curve we determine the da / dN versus ΔK curve.

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So, let us look at each of this different kind of testing very quickly. So, this is the stress control fatigue testing for high cycle fatigue, and what we get here the result has the stress amplitude on the y axis and the number of cycles to failure at the x axis, and what we get is a SN curve or the stress amplitude versus number of cycles to failure, and from there we can actually understand how many number of cycles a specimen can survive, at a particular stress amplitude or if we know that a particular amount of stress amplitude will be used in service, we can determine the number of cycles that the specimen can survive, at that particular stress amplitude.

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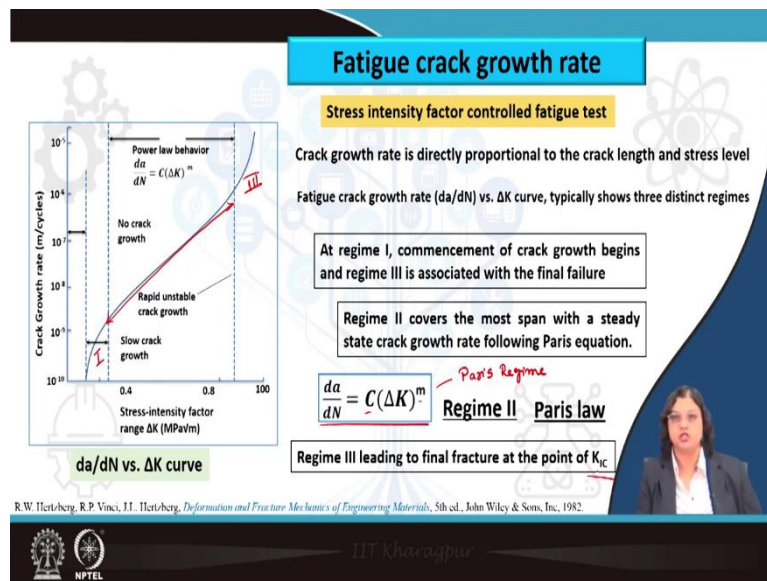


Coming to the low cycle fatigue, we typically use a strain control fatigue test and what we can generate from there is a stress versus strain curve, actually it is a hysteresis as you can see that there is an initial elastic loading and then certain amount of plastic loading, also and we keep on repeating this hysteresis until it fails. There are typically different kind of behaviour that are seen under low cycle fatigue testing, particularly cyclic, hardening or softening behaviour.

One of the examples is shown here, where we are doing the test under strain control conditions, that means that we are applying particular values of strain for each cycle, but even then we are getting a continuously increasing value of stress both in the tension and the compression side and that signifies that the material is being hardened, when we are applying cyclic loading. This continuous for the first initial few cycles or few blocks, and after what it stabilizes till it fractures.

In this case, the number of cycles are quite low of the range of around $10^3 - 10^4$ or 10^5 , and that is related to the amount of stress or the strain values that we are applying during testing.

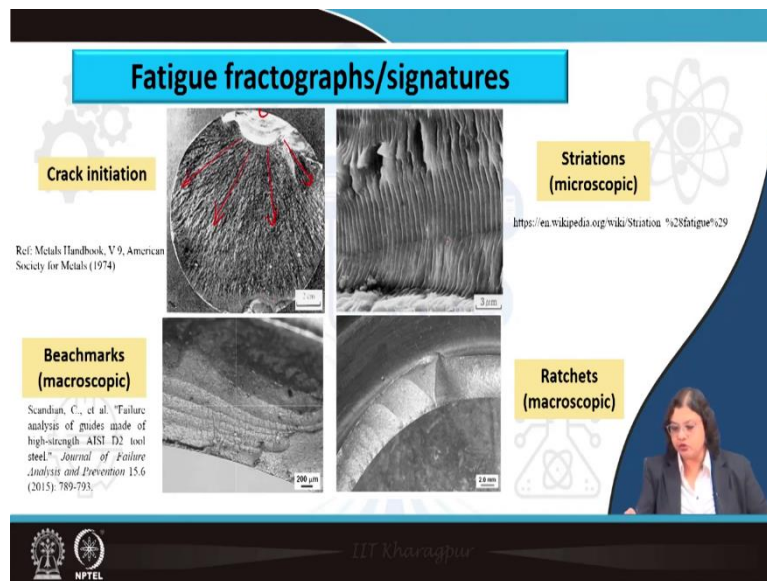
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Now, fatigue crack growth rate curve, as I mentioned it includes a stress intensity factor-controlled fatigue test and what we have here is the crack growth rate is directly proportional to the crack length and the stress level, and eventually this can be related by the stress intensity factor. what we get here, is the da / dN versus ΔK curve, so it has basically three different regime, regime 1 occurs when the commencement of crack growth begins, on the other hand regime 3 is when the final fracture occurs.

Most important part is this prolonged regime which is regime 2 at which the crack grows following a relation known as the Paris equation or Paris relation, also this regime is known as the Paris regime because it follows this relation which says that $da / dN = C (\Delta K)^m$ where the C and m are the constant. Regime 3 is the final fracture regime at the point when the K_{max} reaches the value of K_{IC} or the fracture toughness value at which fracture occurs finally.

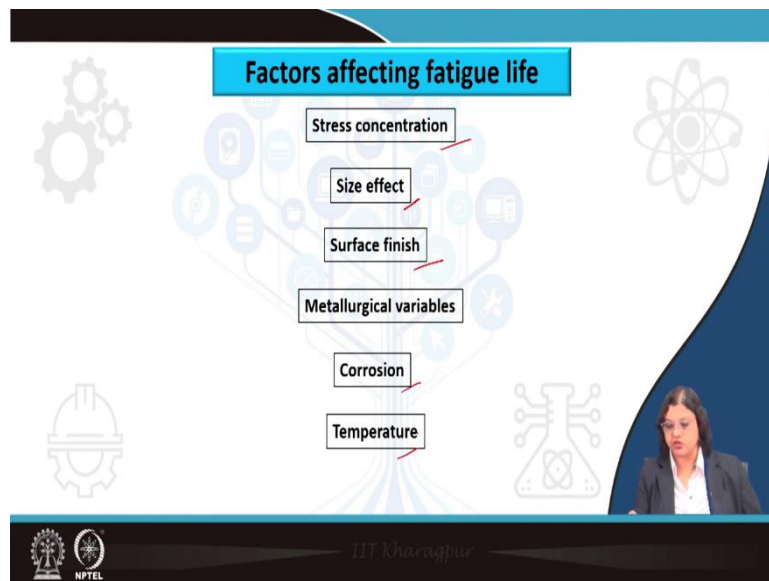
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Now, fatigue is very important to determine, particularly when we do some post-mortem analysis which means the failure has already happened, and we wanted to figure out the reason for failure one of the indications that it has undergone cyclic loading or it has failed under fatigue condition is if we look into the microstructure. If we look in into the microstructure or the fractography that means the microstructure of the fracture surface, at a very low magnification we can very well determine the crack initiation side, which in most of the case is near to the surface from which it propagates.

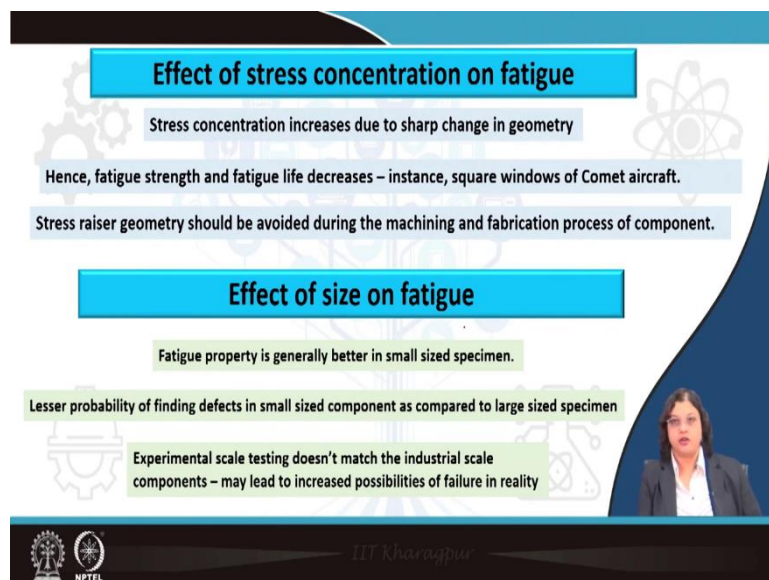
We can see that it is propagating along certain direction, and then we have the final fracture regime. If we are looking at a very higher magnification, then we will be able to look into the striations, okay. So, that signifies that fatigue has occurred in the material.

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Now, once again there are different ways by which fatigue life can be affected and these are the stress concentration or there could be size effect or surface finish is a very important parameter, we need to have a very high-quality surface with reduced surface roughness to ensure it has a increased fatigue performance. Corrosion also leads to fatigue failure and then there could be some effective temperature as well as metallurgical variables as well.

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Coming to the effect of stress concentration and on fatigue, we understand that stress concentration increases due to sharp change in geometry. And hence, the fatigue strength and fatigue life decreases because of the presence of stress concentration one of the famous

instance of this is the presence of square windows in the comet aircraft that lead to the generation or the initiation of fatigue crack.

Stress raiser geometry should be avoided during the Machining and Fabrication process of the component to ensure higher fatigue performance. And if we are talking about the effect of size on fatigue actually fatigue property is generally better in small size specimen because lesser probability of presence of defects in small size component ensures higher fatigue life.

Experimental scale testing however does not match in most of the cases with the industrial scale components, and often for the actual service condition we need to perform the test for the entire component in some cases, particularly if we are talking about a very critical application, for example an aircraft application where we need the entire wings or the entire fuselage to be fatigue tested.

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Effect of surface finish on fatigue

- Fatigue properties are very sensitive to surface finish.
- Fatigue cracks initiate at the surface because of machining defects.
- Polishing of the specimen/component is recommended for better fatigue properties.
- Shot peening generates compressive residual stress and thereby enhances fatigue performance
- Carburizing, Nitriding, Flame hardening and induction hardening are other techniques to change the surface properties which further enhance the fatigue properties.

The slide features a blue header with the title, a list of five points in white boxes, and a video inset of a woman in the bottom right corner. The background has a dark blue gradient with faint gear and atom icons. The footer includes the IIT Kharagpur and NPTEL logos.

Effective surface finish also plays a big role on the fatigue performance, fatigue properties are actually very much sensitive to the surface finish and fatigue crack initiates particularly at the surface. And this is particularly important, if we are talking about high cycle, low cycle fatigue, where we use an unnotched specimen.

So, polishing of the specimen or component is actually recommended for better fatigue properties. Shot peening sometimes are also used for engineering the surface of a component or a material, such that it generates some amount of residual stresses, compressive residual stresses that is beneficial for enhancing the fatigue behaviour of a material. Sometimes, we also do hardening of the surface by incorporating some carbon or nitrogen in the atmosphere,

by doing the carburizing and nitriding or flame hardening or induction hardening such kind of procedures can ensure higher fatigue performance of a material.

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Effect of metallurgical variables on fatigue

Fatigue life is generally improved by

- Increasing the tensile strength : Grain boundary strengthening, cold working, hot working etc.
- Controlling/refining/bimodal microstructure: Thermomechanical processing, heat treatments, avoid inclusions

Effect of corrosion on fatigue

- Corrosive attack produces pitting on metal surface
- Pits act as notches and stress raiser and finally accelerates the fatigue crack propagation
- Protecting metallic or non-metallic coating is recommended to improve fatigue properties.

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Now, there are several effect of material metallurgical variables on fatigue as well, fatigue life is generally improved by if we are increasing the tensile strength of a material, or controlling or refining the structure, particularly if we are having bimodal microstructure by performing the thermo mechanical processing or heat treatments that can lead to enhance the fatigue performance of a material.

When we say that corrosion plays a big role in fatigue, actually the or the corrosive products that are forming that may actually lead to affect the fatigue in different ways. For example, if there is an corrosion reaction that is occurring that produces pitting on the metal surface, and this pit acts as the notches or this acts as the location of stress concentration where fatigue crack initiate, and will lead to easy propagation. So, protecting a metallic or non-metallic coating as a corrosion resistant coating is very much recommended to improve the fatigue properties.

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Effect of temperature on fatigue

With increase in temperature, the tensile properties such as yield strength and ultimate tensile strength decreases.

Creep, oxidation and diffusion mechanism get activated at higher temperature.

Fatigue strength decreases with increase in temperature

The slide features a blue header with the title, three white text boxes with black borders, and a video of a presenter in the bottom right corner. The background has a light blue tree-like graphic and various icons like gears and a hard hat.

Temperature also plays a big role on the fatigue performance, actually with increase in temperature the tensile properties such as the yield strength of the material decreases, right. So, creep oxidation and diffusion mechanism, all these are getting active, if we are performing the test or if some component is being applied used at a high temperature. Fatigue strength usually decreases with increase in temperature.

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Failure Analysis

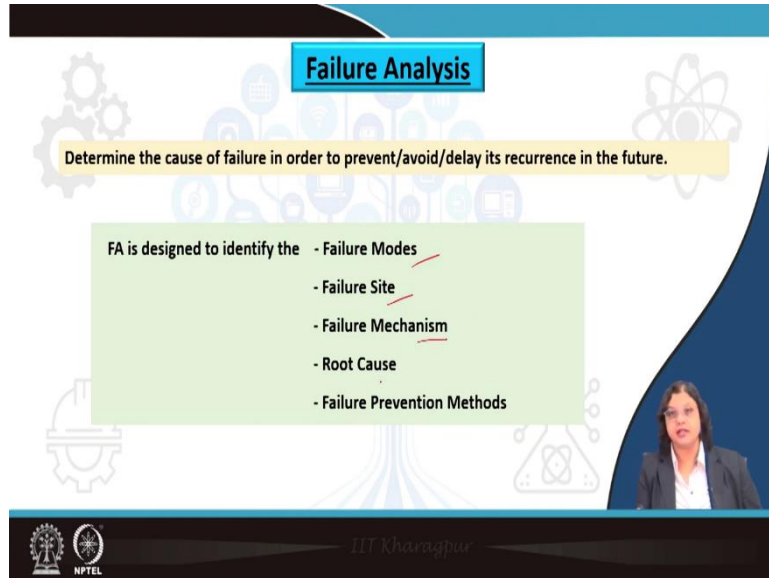
SYMPTOMS

VISIBLE CAUSE

ROOT CAUSE

<https://www.leanteamsusa.com/lean-training/chronic-problem-removal-training/root-cause-tree/>

The slide features a blue header with the title, a tree diagram with three levels (SYMPTOMS at the top, VISIBLE CAUSE in the middle, and ROOT CAUSE at the bottom), and a video of a presenter in the bottom right corner. The background has a light blue tree-like graphic and various icons like gears and a hard hat.

A presentation slide titled "Failure Analysis" in a blue box. Below the title, a yellow box contains the text: "Determine the cause of failure in order to prevent/avoid/delay its recurrence in the future." A green box lists what Failure Analysis is designed to identify: Failure Modes, Failure Site, Failure Mechanism, Root Cause, and Failure Prevention Methods. A small video inset of a woman is in the bottom right. The footer shows IIT Kharagpur and NPTEL logos.

Failure Analysis

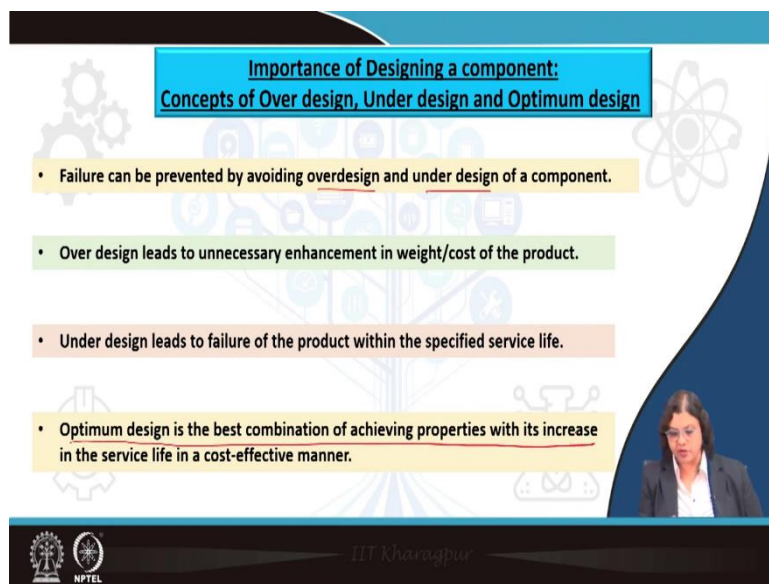
Determine the cause of failure in order to prevent/avoid/delay its recurrence in the future.

FA is designed to identify the

- Failure Modes
- Failure Site
- Failure Mechanism
- Root Cause
- Failure Prevention Methods

Now, coming to the concept of failure analysis, actually all our understanding so far will be very much applicable, if we want to pursue a detailed failure analysis. So, the major reason for performing failure analysis is to determine the cause of failure, and the motto is to prevent failure or at least if that is not possible, we can try to delay it to the maximum possible extent, so that we can ensure a higher performance or higher life of a material. So, failure analysis is designed to identify the failure mode in general, the location of failure, mechanism of failure, what is the actual reason root cause of failure or the different ways by which the failure can be prevented.

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A presentation slide titled "Importance of Designing a component: Concepts of Over design, Under design and Optimum design" in a blue box. Below the title, four bullet points are listed in colored boxes: Failure can be prevented by avoiding overdesign and under design; Over design leads to unnecessary enhancement in weight/cost; Under design leads to failure within specified service life; Optimum design is the best combination of achieving properties with its increase in service life in a cost-effective manner. A small video inset of a woman is in the bottom right. The footer shows IIT Kharagpur and NPTEL logos.

Importance of Designing a component: Concepts of Over design, Under design and Optimum design

- Failure can be prevented by avoiding overdesign and under design of a component.
- Over design leads to unnecessary enhancement in weight/cost of the product.
- Under design leads to failure of the product within the specified service life.
- Optimum design is the best combination of achieving properties with its increase in the service life in a cost-effective manner.

Importance of Designing a component plays a very big role, there are different ways by which we can design a component. For example, it can have an over designing, or an under

designing. So, over designing actually leads to unnecessarily enhancement in the weight or the cost of the product. On the other hand, if we are under designing it, that leads to failure of the product within the specified service life also, so that makes the probability of failure even higher. So, what we need is to have an optimum design which is the best combination of achieving properties within its increase in the service life in a cost-effective manner.

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**Importance of Designing a component:
Concepts of Safe Life, Fail Safe and Factor of safety**

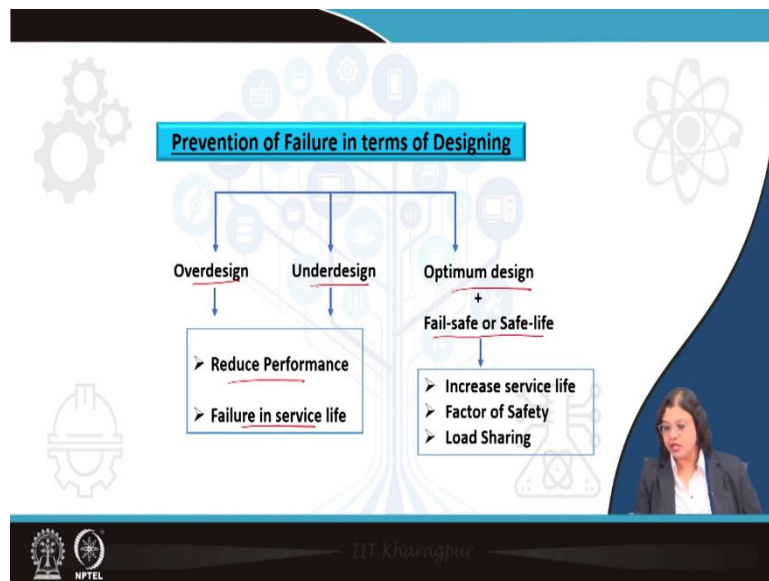
- Safe-life design and Fail-safe design principles are adopted to enhance service life/avoid failure
- Safe life is helpful in reducing unscheduled maintenance and repairs.
- The factor of safety (FOS) is the most important aspect of product design.
- Factor of safety is the structural capacity of a system which determines the load carrying capacity of a system beyond its actual load.
- Fail safe design is based on load sharing principle to avoid catastrophic failure

The slide features a blue header with the title, a list of five bullet points in colored boxes, and a video inset of a woman in the bottom right corner. The background has a faint gear and atom pattern. The footer includes the IIT Kharagpur and NPTEL logos.

So, there are different other kind of design parameters which we often use such as the safe life or the fail safe designs. So, safe life design and the fail safe design principles are adopted to enhance the service life, so where safe life is helpful in reducing unscheduled maintenance and repairs, and it safe life is being ensured by a factor known as a factor of safety. So, factor of safety is the structural capacity of a system which determines the load carrying capacity of a system, beyond its actual load.

We typically keep the factor of safety level quite high, then they actually applied load, such that the specimen and the component should not fail at the highest limit of load that is being applied. On the other hand, fail safe design uses an approach of load sharing, by which even if one or some of the component fails, the other component will act as a backup, and will share the load, and in that way ensures no catastrophic failure of the entire component.

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So, these are the different ways by over designing, we can reduce the performance or the failure can happen within the service life and the same is true for under designing also, but if you are using the optimum designing along with the fail safe or the safe life criteria, then we will be able to increase the surface life, that can be obtained by either enhancing the factor of safety or the load sharing principle.

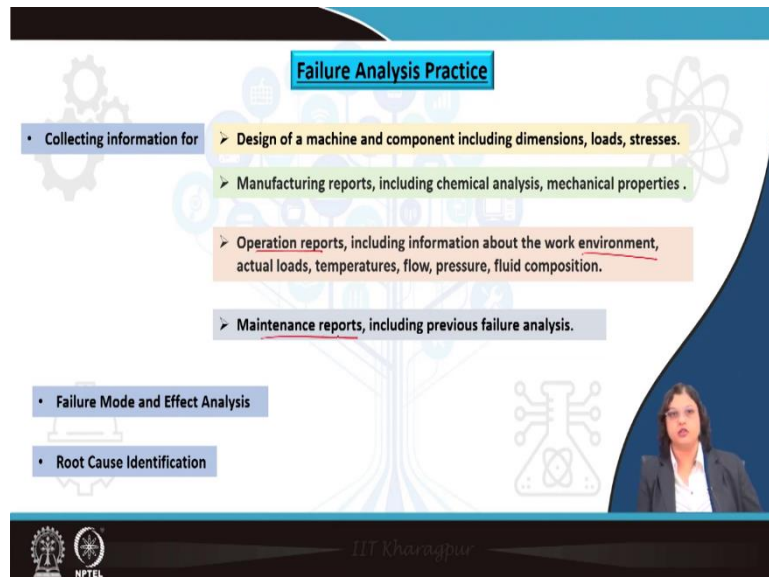
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Equipment	Factor of Safety
Aircraft	1.5-2
Boiler	3.5-6
Pressure vessels	3.5-6
Engine components	6-8
Cast iron wheels	20
Bridge structure	5-7
Building structure	4-6
Static turbine components	6-8
Rotating turbine components	2-3

And these are the values of some of the typical values of factor of safety for different kind of applications. And what we see here is that aircraft actually uses very low value of factor of safety compared to any other structure. For example, bridges, we which use quite high value factor of safety or cast-iron wheels which uses even the highest value of factor of safety. And

the reason for this has been explained in the lectures related to factor of safety and failure analysis.

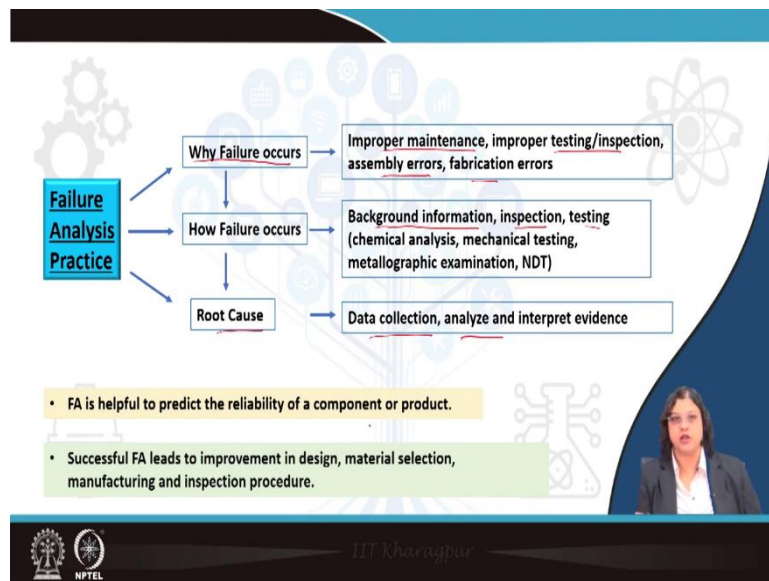
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So, what we do in practice for failure analysis is that we collect information and this can be obtained from the design of a machine in a component, including the dimensions, the loads, the stresses that are being applied, manufacturing reports, chemical analysis, then operation reports including the information about the actual environment maybe the temperature or the load pressure, etc.

And also we need to understand that whether maintenance has been done properly or not, the previous failure analysis report also sometimes come handy, and from there we try to figure out the failure mode and the effects analysis and the root cause identification.

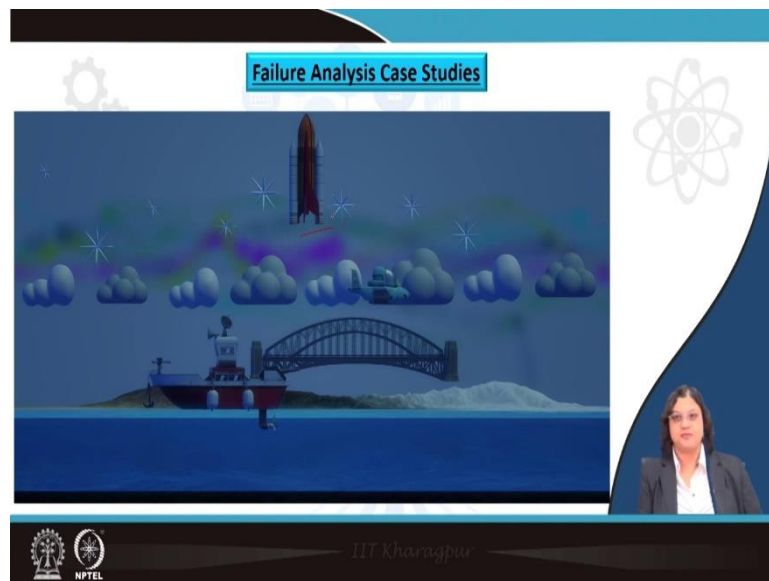
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So, failure analysis practice typically has the following three process, where we try to answer the question why failure has occurred this could be because of the improper maintenance or improper testing or inspection or even assembly errors or fabrication errors, etc. And how failure occurs for that to understand, we need to know they have the background information, the inspection testing processes, metallographic examination, etc.

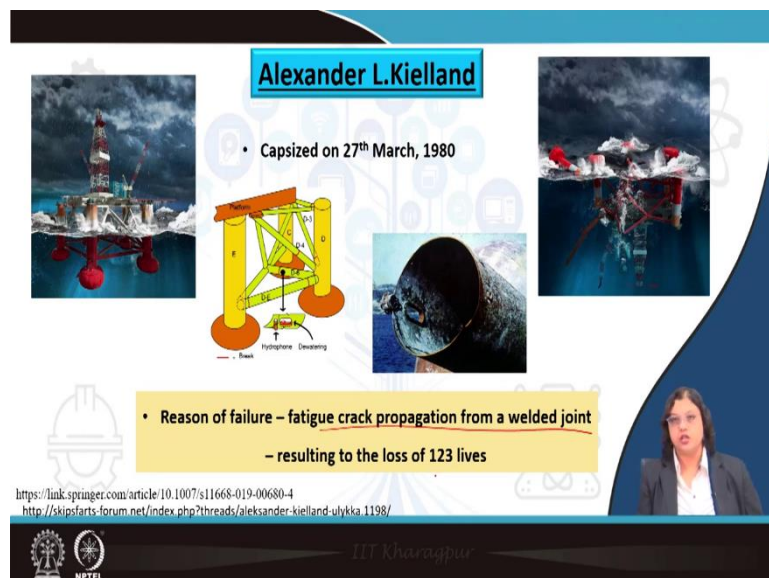
And from overall, the main purpose of failure analysis is to determine the root cause, which can be obtained from all this data collection and as well as to analyse and interpret the evidences. Failure analysis is certainly helpful to predict the reliability of a component or product, and if we can perform failure analysis are successfully, that leads to improvement in the design, the material selection, manufacturing, and inspection procedure.

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So, some of the case studies has been discussed for failure analysis starting from the beneath the sea level with the oil rig, which I could not show here in the schematic, but there then there are the failure of Titanic on the sea level, and then there are bridges, failure of the bridges or the rail failure, and the failure of comet, aircraft, as well as a Columbia Space Shuttle. So, we basically covered all the three dimensions coming from the deeper down of the sea to the space, almost all the span.

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And let us just have a very brief glimpse of all of these. The first one is one beneath the sea level, that is an Alexander L Kielland, which is a semi-submersible oil rig which has been

actually capsized which means, that it has completely toppled down as you can see in this figure, on just one fateful night.

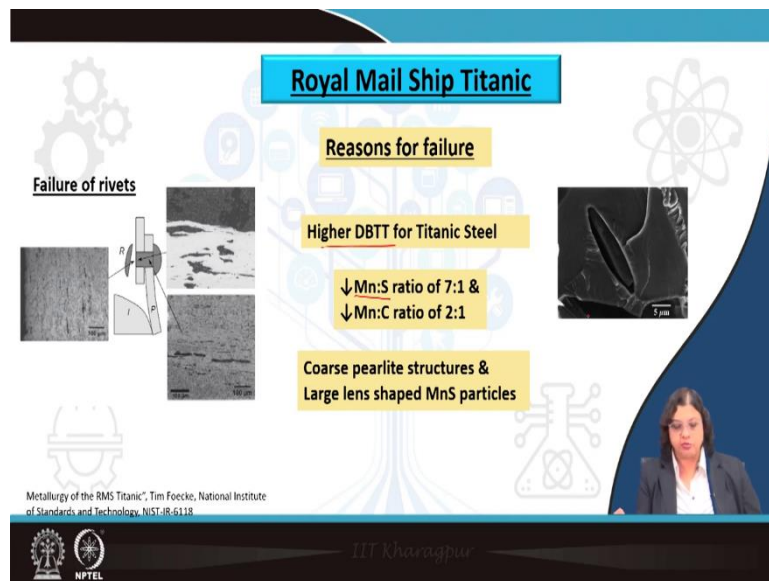
And a detailed failure analysis showed that their reason was actually fatigue cracked propagation from a welded joint, at some location here, which led to the loss of all the 123 lives. So, even failure from a very small location, sometimes can lead to a catastrophic failure if we do not understand this beforehand.

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Failure of Royal Mail ship Titanic is another incident that has made us understand the concept of impact toughness, the concept of ductile to brittle transition, and we all know that failure of this Titanic happened on just in, it is made in voyage itself, when it hit with an iceberg, okay. It was chilled water and that led to the complete breakage of the ship and it sank.

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So, when we did a detailed failure analysis, long after the actual incident, it has been realized that the steel that has been used for making the ship, were actually having all the strength wise it is quite comparable to the modern day steel, but it has very high DBTT, Ductile Brittle Transition Temperature, which means that at the chilled water of the Atlantic, it actually attains the brittle condition, and that is because of the elemental composition of the steel it has a very low value of manganese is to carbon, and manganese is to sulphur ratio that led to such catastrophic event.

Of course, the microstructure plays a big role in any of the mechanical properties, but another very interesting fact that we have discussed in the failure incident of Titanic is that the rivets and how the defects can be arranged by different thermo mechanical processing that can lead to failure in of such a higher magnitude.

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Crash of Jetliner Haviland De Comet



- Series of failure incidents for comet aircrafts – in early 1950s
 - G-ALYP – crashed on 10th Jan 1954
 - G-ALYY – crashed on 8th April 1954

Reason of failure

- Structural failure of the pressure cabin due to fatigue
- Presence of square windows which caused high stresses

<https://www.aerotime.aero/articles/23055-aircraft-brought-future-de-haviland-comet>

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Coming to the failure of the aircraft Comet, again Comet airliners were famous and even is considered as the best design principle that they have introduced long back in the 1950s, the first jetliners, but there were a series of incidents that were happening, and later on people understood the importance of fatigue for aircraft component and how that kind of structural fatigue can lead to the failure of the aircraft without any prior indication.

One of the other interesting features is the square windows, that typically has been used for any structure and also for the aircrafts, that can lead to fatigue crack initiation, and these days what we can see in the aircrafts are all having oval windows to avoid such locations of stress concentration.

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Columbia failure



- On February 1, 2003, the space shuttle Columbia broke up during re-entry at the end of its mission

Reason of failure

- Insulating foam separates from external tank after lift off
- Foam strikes underside of left wing, breaches thermal protection system (TPS) tiles
- Superheated air enters the craft during re-entry
- All 7 crew members died

https://www.researchgate.net/publication/268078377_Space_Shuttle_Case_Studies_Challenger_and_Columbia

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Now, coming to the space, what we have seen is that the Columbia Space Shuttle has collapsed while it is coming back to the Earth, and it broke up during the re-entry. The reasons for the failure is that there is an insulating foam that separates out right during the beginning of the journey, and that gets separated from the external tank after lift-off which strike the underside of the left wing of the aircraft and breaches the thermal protection tiles. Upon returning, when the superheated air enters the craft that lead to the catastrophic failure, and all the seven crew members died.

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Eschede Derailment

- June 3, 1998: Intercity Express collision with a bridge at Eschede, Germany, causing it to collapse.
- Event Chronology: Wheel fracture, Derailment, Finally collides with bridge, Pileup of coaches

https://www.reddit.com/r/CatastrophicFailure/comments/1pqr7train_accident_eschede_june_3_1998_the_train/

Fatigue crack origin

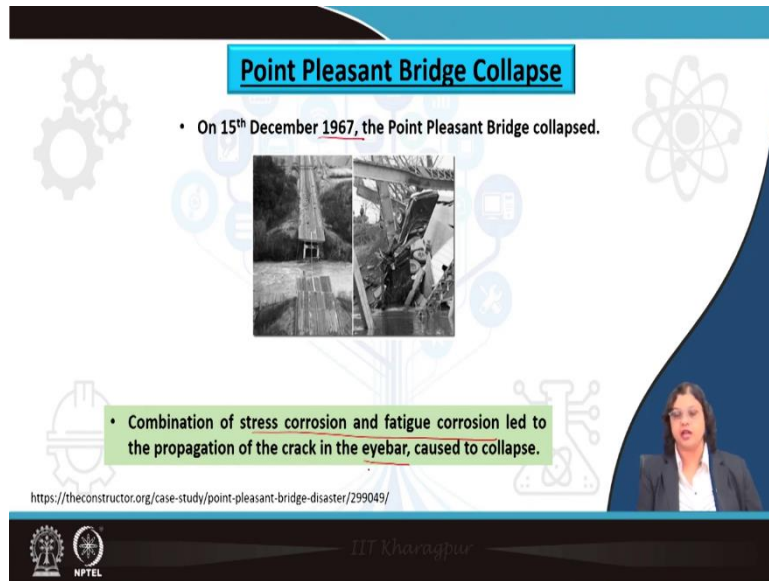
Eschinger, Volker, et al. "The railway accident of Eschede—technical background." *Engineering Failure Analysis* 11.4 (2004): 515-535.

- Fatigue beachmarks, striations were observed.
- Action Taken: Replaced back the monoblock wheels.

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Coming to the railway failure, actually this happened not long back, this happened actually in 1998, and a complete derailment as well as piling up has occurred, and what we can see from the event chronology is that there is actually, a wheel fracture that happened first, and along with that there is a derailment and then it collides with the bridge, and the coaches pile up, so that leads to huge number of life losses, and of course all this failure incidents actually, it reduces the faith on any kind of such a high level advanced structure. And detailed failure analysis showed us the importance of fatigue, railway fatigue from there and as a precautionary measure makes the wheels has been replaced to mono block kind of wheels to avoid such kind of failure.

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The slide features a blue header with the title 'Point Pleasant Bridge Collapse' in a white box. Below the title, a bullet point states: 'On 15th December 1967, the Point Pleasant Bridge collapsed.' To the right of this text is a small inset image showing a car on the bridge. Below the bullet point is another image showing a close-up of the bridge's structure. A green box at the bottom of the slide contains the text: 'Combination of stress corrosion and fatigue corrosion led to the propagation of the crack in the eyebar, caused to collapse.' The slide also includes a URL at the bottom: 'https://theconstructor.org/case-study/point-pleasant-bridge-disaster/299049/'. The background of the slide is white with blue decorative elements, including gears and a molecular structure. A small video feed of a woman is visible in the bottom right corner.

Point Pleasant Bridge Collapse

- On 15th December 1967, the Point Pleasant Bridge collapsed.

Combination of stress corrosion and fatigue corrosion led to the propagation of the crack in the eyebar, caused to collapse.

<https://theconstructor.org/case-study/point-pleasant-bridge-disaster/299049/>

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Point Pleasant Bridge collapse is another unfortunate incident which happened sometime back in 1967, when the bridge collapses. One fine day in the month of December, just before the Christmas and then the bridge collapsed. All of a sudden some of the vehicles which are traveling on the bridge just fell into the chilled river underneath. Once again, a detailed failure analysis has been performed, which showed that a combination of stress corrosion and fatigue corrosion led to the propagation of crack in one of the eyebar.

So, we have again discussed all this in details in the lecture, the very concerning point here is that one of the eyebar failed for the bridge and that lead to the overall catastrophic failure of the entire structure. And we understood the importance of fail safe design such that every time there should be a backup or there should be a load sharing mechanism, such that the catastrophic failure can be avoided to the maximum possible extent.

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CONCLUSION

The concepts of Fracture, Fatigue and Failure are discussed



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Metallurgy of the RMS Titanic", Tim Foecke, National Institute of Standards and Technology, NIST-IR-6118



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So, coming to the conclusion, basically, in this lecture we have seen the concepts of fracture, fatigue and failure, in once again very briefly, and following are the references that are being used for this lecture. I thank you very much, and I hope that you have enjoyed this lecture series, thank you very much.