Fracture, Fatigue and Failure of Materials

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Lecture 06

Stress Intensity Factor

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Hello everyone and welcome back to the sixth lecture of this course fracture, fatigue and failure of materials. So, in this class, we will be continuing from the last lecture, where we have introduced the concept of strain energy release rate, and then we will progressively move towards the stress intensity factor.

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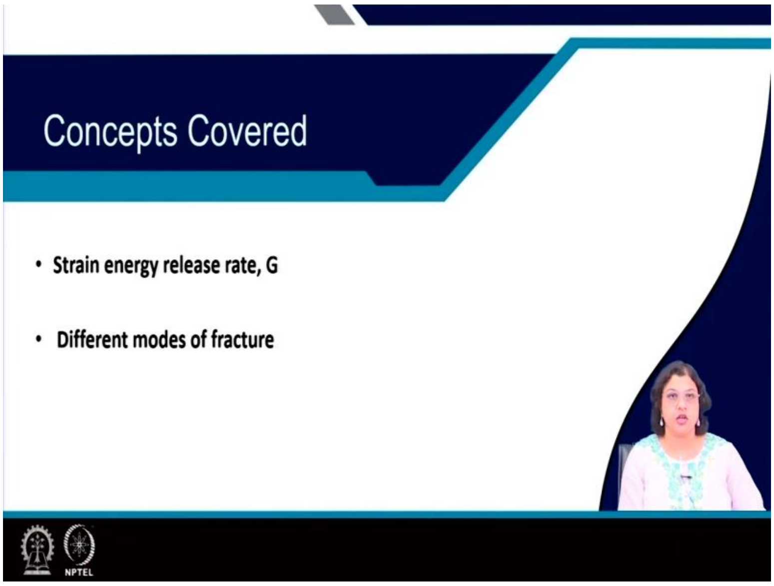

So, let us see the concepts that we have in store for this lecture are as follows, the strain energy release rate in details and how we can use this to experimentally determine the fracture strength of a material. And from there we will look on the different kinds of fracture, different modes of fracture and finally, we will see how the stress intensity factor is determined.

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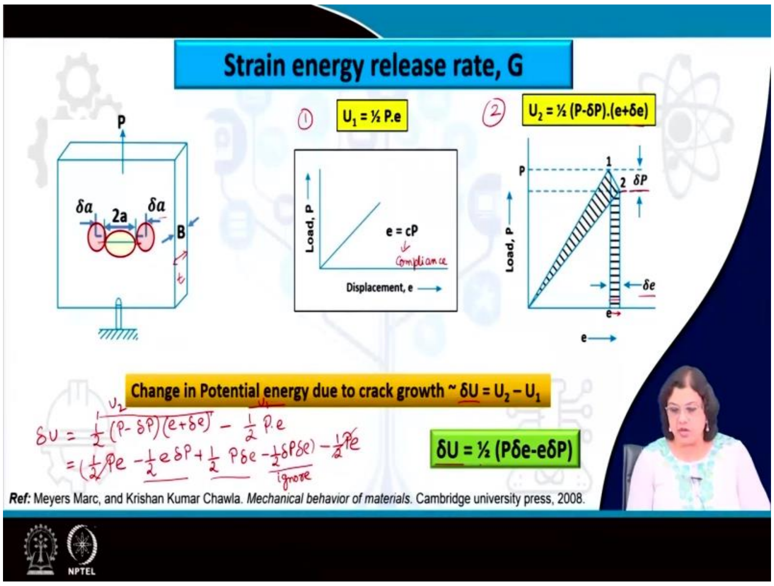

So, coming to the strain energy release rate, we have seen in the last lecture that strain energy release rate is nothing, but the potential energy that is stored in our system or in a component as the crack grows, whatever energy is being released, that is considered as the strain energy release rate and that is relevant for determining the fracture strength of a material as per the

Orowan's modification considering any kind of energy change that is happening including not only the surface energy, but also the plastic work energy.

So, all the calculations or all the estimations as per Griffith criterion has been done based on a component which is infinitely wide and long and then there is a center crack of length 2 a, there is a finite thickness, of course, which is represented by either capital B or t in some cases. So, what happens is that, if we have a component like this and there is a force or stress is being applied.

Now, when we are using this to do some experiment, typically, in any kind of universal testing machine that is used for doing mechanical test, we typically apply load and not the stress directly, we determine the stress based on the change in the area. So, if we are applying certain load, let us say P and what we are getting in turn is the variation of load and displacement.

So, initially we have a crack of length 2 a, as has been encircled in this picture, and what we are getting in turn from the instrument is the variation of load with displacement for a material which does not undergo a plastic deformation, this is how it is we are getting a straight or a linear relation between the load and the displacement and this is related to each other through the parameter the factor compliance. So, c here is compliance.

So, if we want to figure out the energy that is being released in such condition at the initial state, let us name this condition as 1, what we are seeing is that U_1 or the energy that has been released is given by $U_1 = \frac{1}{2} P e$

On the other hand, if we keep on increasing the load further, there is an enhancement in the crack length, very small from both the sides and by an amount let us say δa.

And accordingly, this is being reflected in the load displacement curve also as the following. So, this is for condition 1, this is for condition 2, initially as per condition 1 we have seen that the energy that has been released is given by $U_1 = \frac{1}{2} P e$

on the other hand for condition 2, since there is a growth of the crack, what we are seeing this crack length enhancement is something of the order of this, what we can see is that the load has been reduced because of this crack enhancement and δe, however, has been increased.

So, this part here is the δe, the enhancement in displacement for the component and the load, the reduce load that is required for the condition 2 is δP.

So, the energy that is released for condition 2 is given by again load into displacement, but the load in this case will be P - δP and the displacement is e + δe

$$U_2 = \frac{1}{2}(P-\delta P)\,(e+\delta e)$$

So, if we try to figure out the change in this energy release as the crack is growing by δa terms, we have to simply find out the change in potential energy as δU which is equivalent to U_2 - U_1, so the energy that is released for condition 2 minus the energy that is released for condition 1 or the initial condition $\delta U = U_2 - U_1$

And that will be given by we can simply solve this by rewriting this relation

$$\delta U = \frac{1}{2}(P-\delta P)\,(e+\delta e) - \frac{1}{2}P\,e$$

and if we simply expand this relation what we are getting is

$$\delta U = (\frac{1}{2}P\,e - \frac{1}{2}e\,\delta P + \frac{1}{2}P\,\delta e - \frac{1}{2}\delta P\,\delta e) - \frac{1}{2}P\,e$$

So, we can cancel these two terms here, which leaves us with these three parameters. However, $\delta P\,\delta e$, both of these individual values of δP and δe , both are very-very small. So, the product is of course, again infinitesimally small and we can simply ignore this. So, what we end up having is

$$\delta U = \frac{1}{2}(P\delta e - e\delta P)$$

so these two terms only. So, this is how we can simplify this relation to the energy that will be released as the crack is growing by δa term, this will be given by $\frac{1}{2}(P\delta e - e\delta P)$

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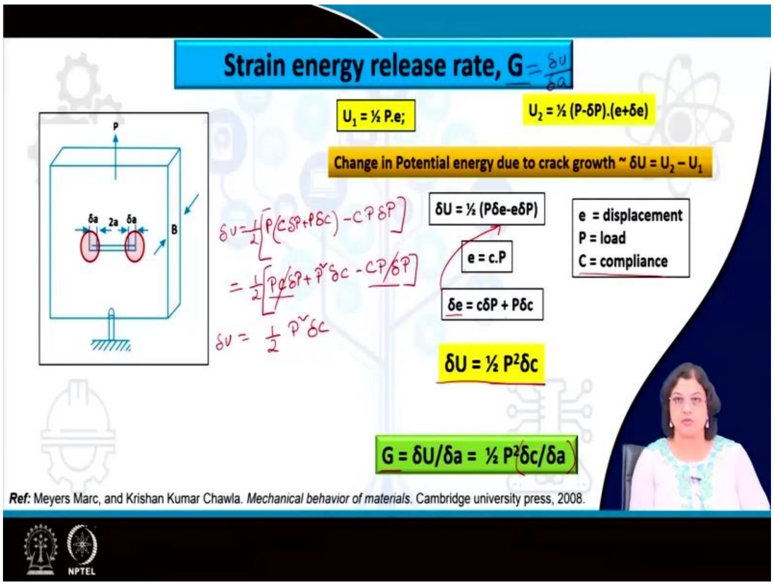

So, we need to simplify it further. So, far, we still do not know how to determine this experimentally and to do that, what we are doing is using this relation $e = cP$ bringing compliance into the picture so, that we can determine this. So if

for δe,

$$e = cP$$

$$\delta e = c\delta P + P\delta c$$

So, again we are plugging this relation in this equation here, which leaves us to

$$\delta U = \frac{1}{2}[P(c\delta P + P\delta c) - cP\,\delta P]$$

So, that leaves us to

$$\delta U = \frac{1}{2}[Pc\delta P + P^2\delta c - cP\,\delta P]$$

So, these two terms are the same one and so, we can cancel this out. So, we end up having

$$\delta U = \frac{1}{2}P^2\delta c$$

where δc is nothing but the change in the compliance. So, this means, so this is the same relation that we have figured out that

$$\delta U = \frac{1}{2}P^2\delta c$$

And we are not done yet in the sense that we have defined G as the change in the energy that is being released per unit crack growth. So, that means, that G is equivalent to $G = \delta U/\delta a$

for δa growth of crack this is what we are getting. So, eventually this means that δU/δa will be given by

$$G = \frac{1}{2} P^2 \delta c/\delta a$$

So, this is nothing but the change in the compliance per unit growth of the crack and that will give us the value of G and if we plug this value of G in this relation for fracture strength, we will be able to determine the fracture strength of the material experimentally.

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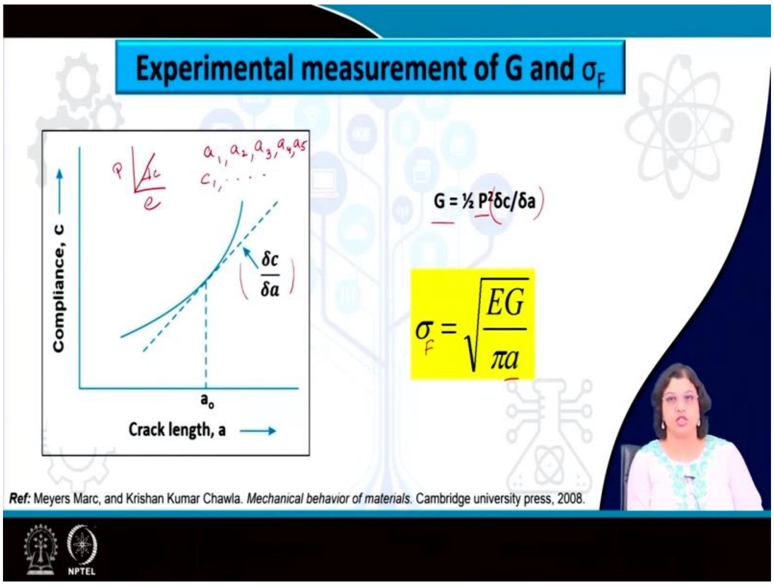

So, let us see how this is being done in reality in practice. So, we typically do tensile tests of different component of same dimensions, but having different crack length and for each of these conditions based on what we have seen earlier the load versus displacement curve, we can find out the c value from the slope here and if we do this for the different crack condition, either we can do this on multiple specimens or we can use the same specimen, but as the crack length grows, we can find out the change in the P versus e curve, the load versus displacement curve and from there, we can find out the c for each of this condition.

Let us say we are doing this for five different crack length a_1, a_2, a_3, a_4 and a_5, so just for an example, and if we for each of this condition, if we are getting the c_1 and so, on. So, we should be able to plot this as the variation of compliance with the crack length. As you can see here, if we get the tangent out of it, we will be able to figure out the slope of this δc δa. So, how the compliance is varying as the crack length is changing, we can very well find that out.

And if we can do that experimentally, then our job is almost done in the sense that we can find out the value of G once again if we use this relation

$$\sigma = \sqrt{\frac{EG}{\pi a}}$$

for this crack length, we can figure out what should be the value of σ or the fracture strength as per this Irwin's consideration. So, this is a simplified way by which the fracture strength of a material can be determined.

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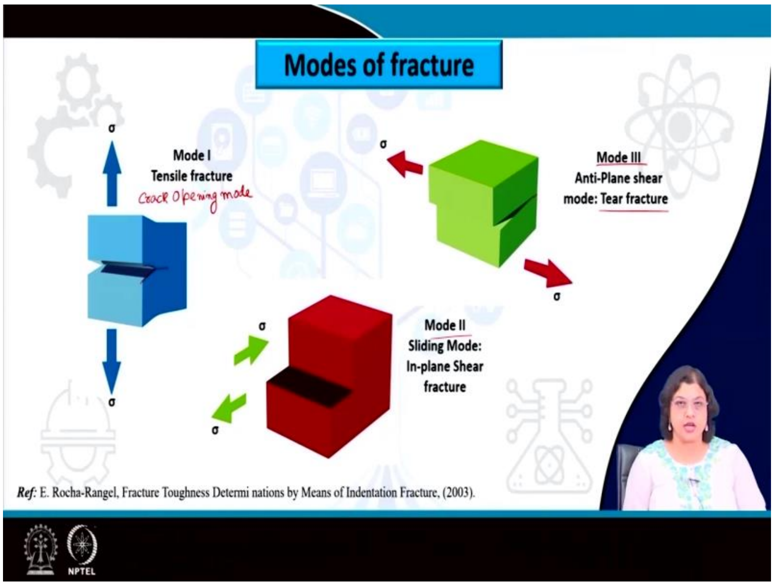

Now, moving on to the modes of fracture. So, so far we have seen in the very first lecture, we have discussed about the modes of fracture as ductile and brittle, the material which is capable of undergoing plastic deformation prior to failure is supposed to give us ductile mode of deformation, while the one which has no permanent or plastic deformation is giving us brittle mode of fracture. But apart from that category, we can also differentiate the fracture mode into three different ways which is dependent on the loading condition.

So, typically the way by which we are loading the component that dictates how the material is going to fracture and how much amount of stress will be generated. So, the most common method that we use typically for experimental analysis is known as the mode 1 or the tensile fracture. So, it means that, we simply have a component where there is a crack and perpendicular to the direction of the crack load is being applied and this is also known as the crack opening mode. So, this is known as the tensile mode or crack opening mode. As you can see, the crack is opening as we are increasing the amount of stress and the crack increases along the direction perpendicular to the direction of the load.

Next, from mode 2 in which we have the sliding mode. So, in this case, one of the plane of the crack or one of the surface of the crack is being shifted in the opposite direction with respect to the other one. So, we are applying stresses in the to reverse direction like is what happens in case of shear loading and this is also known as the sliding mode or the in-plane shear fracture. So, crack faces are sliding past each other.

On the other hand, we have mode 3 also which is known as the anti-plane shear in this case also the loads are applied or the stress are applied along two opposite directions, but, this is a kind of twisting direction that we are doing and this is known as the tear fracture or tear mode or sometimes as anti-plane shear. So, here also the crack faces are moving in opposite directions, but along these two different directions, and the crack is progressing in this way.

So, out of these three modes, the loading situation and the stress scenario at the tip of the crack for each cases becomes different and that also dictates how what should be the fracture strength, but out of these three modes, mode 1 is the most dangerous one or the most critical one which leads to early failure or that means, that the fracture strength will be least under such condition, most of the experiments are being done in mode 1 particularly in mode 1, of course, based on the requirement of the service we often need to analyze based on mode 2 or mode 3 as well.

But considering that this is the worst-case scenario we often prefer to figure out that what should be the fracture strength that will be obtained even when the conditions are to its worst. And failure analysis and to that matter in fracture and fatigue often we are more focused to find out the characteristics of the material in the worst possible scenario. So, that is why mode 1 is often used in practice.

So, if such is the case, let us see how the stress scenario is there at the tip of the crack or at any distance away from the tip of the crack, we know that at the tip of the crack the stress gets maximized, and as we are moving away from the tip of the crack, of course, the stress maximization of the stress gets reduced to some extent, finally, there will not be any effect of the stress. So, let us look into this in more details.

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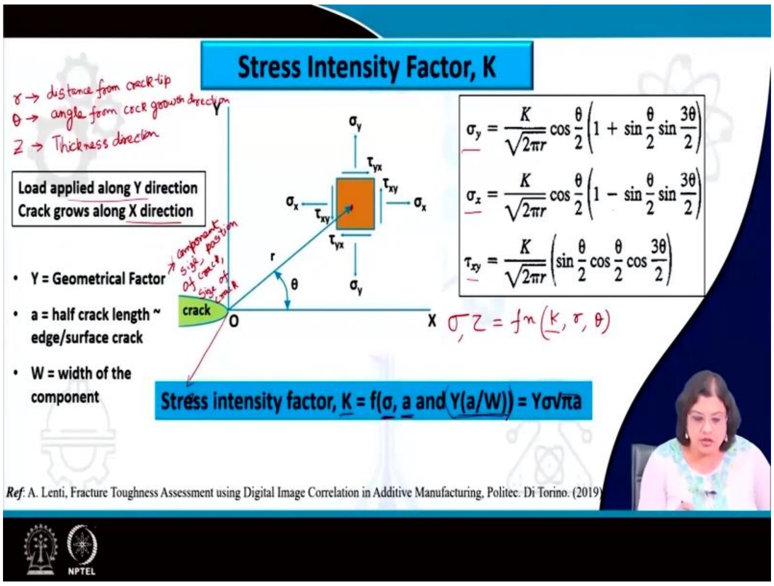

So, this is the situation where we have the x and the y axis and the z axis also along this. So, along the y axis load is being applied and crack grows along the x-direction, whereas z represents the direction of the thickness, so, let us say this as the thickness direction. Now, if there is a crack here and we are applying stresses along the y-direction, we want to figure out that what would be the stress at any point away from the crack tip.

Let us say at a distance r, so, r is the distance from the crack tip so, r is the distance from crack tip and θ is at an angle from the crack tip. So, with this we can figure out any point away from the crack tip, what would be the stress scenario. So, this is the angle from crack growth direction, crack is growing along the direction of x. So, how the stress will be at any distance r, at an angle θ, what will be the stress scenario.

Now, stress at any point will be dissociated in its three counterparts. So, along the x and y and z axis. Now, z is a thickness direction, so there are a lot of importance of the thickness which we will discuss slowly, but, let us talk about the case when there is the thickness is not so important and we are figuring out the stress that is being distributed along the x and the y dissociated along the x and the y and there will be τ component also the shear stress component along this x, y-directions.

So, if such is the case, the stress of all the directions, σ_x or σ_y can be figured out based on this relation. So, σ_y will be given by a relation like this

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \text{Cos}\frac{\theta}{2}\left(1 + \text{Sin}\frac{\theta}{2}\,\text{Sin}\frac{3\theta}{2}\right)$$

σ_x on the other hand has a very similar relation but instead of plus here the minus sign will be used. So, these are mathematical relations determined based on the geometry and this has been experimentally validated also. τ_{xy}, the shear component is also calculated based on that.

The bottom line is we do not need to remember this relation or mug up this relation rather what we need to understand is that the stress, any form of stress, σ or τ these are related or these are a function of three factors here, K, r and θ. So, this is what we are seeing all these are arrangement of K and r and θ, and we should be able to determine the value of stress at any point away from the crack tip.

So, r and θ we have already discussed, the distance from the crack tip as well as the angle from the crack tip, what we have not yet focused is on the factor K, capital K, which is nothing but the stress intensity factor of a component. Once again stress intensity factor is also a function of several other parameter like the stress state the a crack length as well as a geometrical parameter Y (a / W).

So, this depends on the component size, the position of the crack, the size of the crack, with respect to the component all these are being considered in the geometrical parameter itself. σ is the applied stress of course, a is the crack length. So, all these are taken together by this term K and this is a simplified relation for K is $K = Y\sigma\sqrt{\pi a}$

So, Y is the geometrical factor, σ is the applied stress and a is the crack length. So, this is related to the component size position of the crack or defect as well as size of crack, so all these are being taken together while considering Y.

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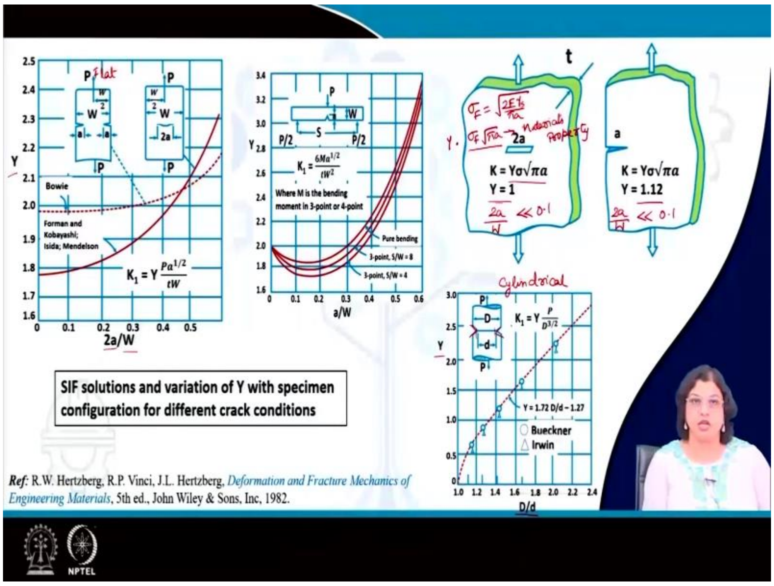

So, this is how Y varies as we are changing this parameter 2a / W, 2a is the total crack length and W is the width of the component. So, it has been seen as well as experimentally validated these are some experimental results that are being shown here that if the position of the crack varies and as we are changing this to a / W, you can see that this parameter Y is increasing.

For example, this solid curve is for the one when we have a center crack of total length 2a and this is the total dimension, the total width of the component and we can see that how if 2a / W is increasing, how Y is also changing and Y is also increasing. In a similar fashion, if we have the edge cracks on the two surfaces, there also Y is increasing, but of course, the pattern the trend by which Y was increasing for the case of the center crack is not following exactly, in case we have two surface cracks.

Again, if we are changing the dimension of the component, this variation of Y can change even further. Also, if we are using a three-point bending compared to tensile loading, this may also affect the variation of Y. So, there are several factors taken together if we change the dimension from a flat one, so, here also we can see that there are two edge cracks on the two surfaces, but this one is a cylindrical specimen and this one is a flat specimen. So, here also we see that the trend by which Y is varying with 2a / W or in this case the ratio of the diameter of this capital D, total diameter versus this unnotched part diameters small d, how Y is varying.

And this also changes for different from material also and we need to consider this based on the different size of the component, the position of the crack, the size of the crack, etc. There are ASTM standards for that as well to consider that. Some of the examples are being shown here, but most routinely what has been, what are being used for fracture toughness estimation or fracture mechanics in general, for lab scale testing are the following two conditions, either we have a center crack in which Y is equals to 1 or we have an edge crack for which Y equals to 1.12.

So, now, if we think about the Griffith criterion and how Griffith has maintained this, so, actually this both these conditions are valid in case we have 2a / W , much-much less than 0.1. So, for both the cases, this has to be satisfied. So, in case 2a / W is much-much less than 0.1, or in other words the total length of the crack is much-much lesser than 0.1 times the width of the specimen, which essentially means, that the crack length is very-very small compared to the overall width of the specimen for a very minor length of defect, this condition will be valid.

Now, if we are thinking about Griffith criterion, this is exactly what we have seen that for the case of Griffith criterion, if we once again write this relation

$$\sigma_F = \sqrt{\frac{2E\gamma_s}{\pi a}}$$

and if we are seeing that this $\sigma_F \sqrt{\pi a}$

this is what we have seen is a materials property and this is nothing but the fracture toughness of the material.

We will slowly move on to that, but exactly this is what we are seeing here as K, this is nothing but $\sigma_F \sqrt{\pi a}$ with a consideration that Y equals to 1. So, that is what Griffith has also considered and it is being valid for that. So, we will look into the more details of stress intensity factor and how can that lead to fracture toughness and in what conditions fracture toughness can be considered as materials property, we will look into more details of that in the subsequent lectures.

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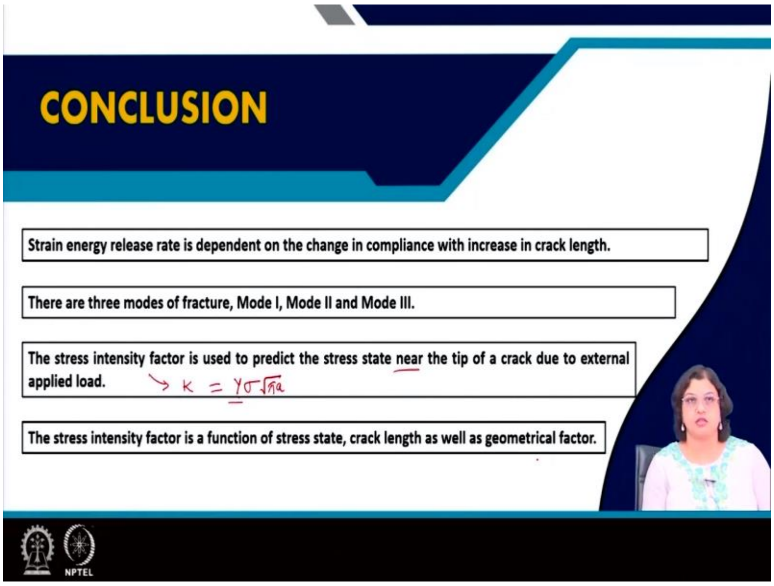

For now, let us conclude here. So, what we have learned from this lecture is that strain energy release rate is particularly dependent on the change in the compliance as the crack length increases. So, that means that essentially, we can determine this experimentally, as the crack length changes how the compliance is varying, if we can figure out this δc δa, this ratio we would be able to find out the value of G and if we can do that, we should be able to find out the value of fracture strength also experimentally.

We have also seen that there are three modes of fracture. Mode one or the tensile mode, mode two or the shear mode or in plane shear mode or mode three or the tear mode or anti plane shear mode, out of this all these three modes different modes, mode one is the most critical one.

Stress intensity factor is used to predict the stress state near the tip of a crack. So, at any point away from the tip of a crack. So, that is why I meant near not exactly at the tip of a crack at the tip of a crack, we know that the stress gets maximized there, but any point away from the crack we can be able to figure out the stress scenario through this stress intensity factor.

And this is given by the stress intensity factor is typically expressed by capital K and by a relation $K = Y\sigma\sqrt{\pi a}$

by the new term that has been introduced in this lecture is Y, which is the geometrical factor that is dependent on particularly 2a / W or in other words, it is dependent on the position of the crack, the size of the crack, along with the size of the component. So, this is what is shown here this the stress intensity factor is dependent on the following three factors.

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These are some of the references that has been used in these lectures. I hope you have enjoyed this lecture. And thank you very much.