Fracture, Fatigue and Failure of Material
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Lecture 05
Griffith Criteria - Modification

Hello, everyone and welcome to the fifth lecture of this course Fracture Fatigue and Failure of Materials.

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So, in this class we will be discussing some more about the Griffith criterion which we have introduced already in the last lecture. And from there we will also look into the drawbacks and the ways by which Griffith criteria has been modified to cater to larger kinds of materials. As Orowan's modification and finally Irwin's consideration.

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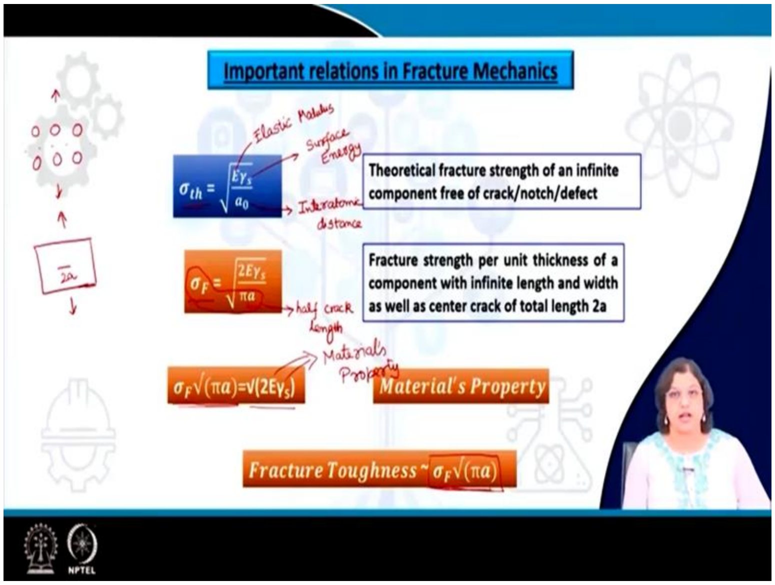

So, in the last class we have seen that how fracture strength of materials can be determined. At the first hand, we have seen that how the theoretical strength of a materials are determined in case of a perfect lattice structure. Let's say we have atomic structure like this which are perfect without any defect. And if we are applying tensile stresses on this at some point fracture will initiate and this fracture strength is known as a theoretical cohesive strength or theoretical fracture strength of a material.

And that is related to the elastic modulus, the surface energy as well as the inter atomic distance. So, let me write this term E is the symbol for elastic modulus and gamma s is known as the surface energy whereas a_0 is the inter atomic distance. So typically, the theoretical fracture strength of a materials is related to

$$\sigma_{th} = \sqrt{\frac{E\gamma_S}{a_0}}$$

And in case there is a defect in a material as we know that all the materials or components are supposed to have inherent defects in it.

In that case Griffith has tried to find out the fracture strength of a material in case of an infinitely long and wide component having a center crack of total length 2a and the fracture strength in such case is related to

$$\sigma_{\mathrm{F}} = \sqrt{\frac{2E\gamma_S}{\pi a_0}}$$

where a is the new term here different from what we have seen for the case of theoretical fracture strength and this a represents half crack length.

So, in case of fracture mechanics we often will use this term a and it is different from a_0 where a_0 is the inter atomic distance so this needs to be carefully considered. Now if we look into this Griffith Criterion more carefully we will see that if we are putting the relation in such that

$$\sigma_F\sqrt{\pi a} = \sqrt{2E\gamma_S}$$

we are simply rearranging the equation.

Now for any material elastic modulus as well as the surface energy are actually the inherent materials property. So, in that sense we can see that this term on the left-hand side that $\sigma_F\sqrt{\pi a}$ is supposed to be a materials property. Although we have also seen that fracture strength of a material is not a material's property; it is related to the crack length and if we are varying the crack length that means the fracture strength of a particular material or a component is supposed to change.

On the other hand, if we are simply rearranging this relation and we are getting $\sigma_F\sqrt{\pi a}$ that is considered as a material's property because this is related to none other than only the elastic modulus and the surface energy of a material. We will see later on that this factor here $\sigma_F\sqrt{\pi a}$ is nothing but the fracture toughness of a material. We will go through further details of that but for now you should be carefully considering this term as the fracture toughness of a material not fracture strength rather the toughness of a material which is combined of the fracture strength as well as the crack length of a material.

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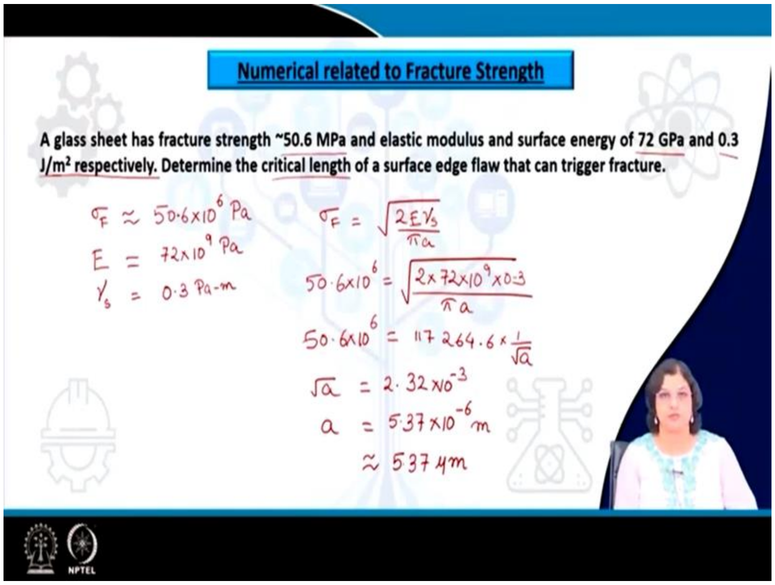

So, let us now solve a problem related to fracture strength determined through Griffith criterion accordingly. A glass sheet has fracture strength of 50.6 MPa and the elastic modulus and surface energy 72 GPa and 0.3 J/m^2 respectively. So, that makes our σ_F as 50.6 x 10^6 Pa and elastic modulus of 72 x 10^9 Pa whereas the surface energy is 0.3 Pa-m.

We have seen in the last problem that we have solved that J/m^2 can be converted to Pa-m. Now what we need to determine from here is the critical length of a surface or edge flaw that can trigger fracture. So, at the point of fracture, it has to maintain this Griffith criteria which is

So, if we simply plug all the values here,

$$50.6 \times 10^6 = \sqrt{\frac{2 \times 72 \times 10^9 \times 0.3}{\pi a}}$$

And we can determine the critical crack length in that case.

So, let us simply solve this problem 50.6 x 10^6 this will be equivalent to the right-hand side here so that comes as 117264.6 x 1/√ a. So that makes √a, equivalent to 2.32 x 10^{-3} or a will be just the square of that, which comes as 5.37 x 10^{-6}. And the unit of a in this case, since we have considered the unit of length as meter; so, this should be a equals to 5.37 x 10^{-6} meter which is equivalent to 5.37 μm. So, a crack as small as 5.37 μm is sufficient to trigger fracture and that leads to a fracture strength of 50.6 mega Pascal.

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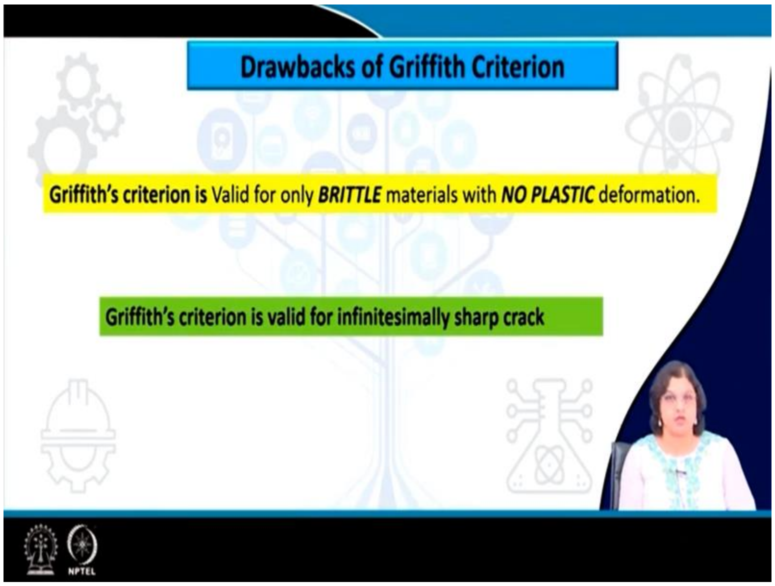

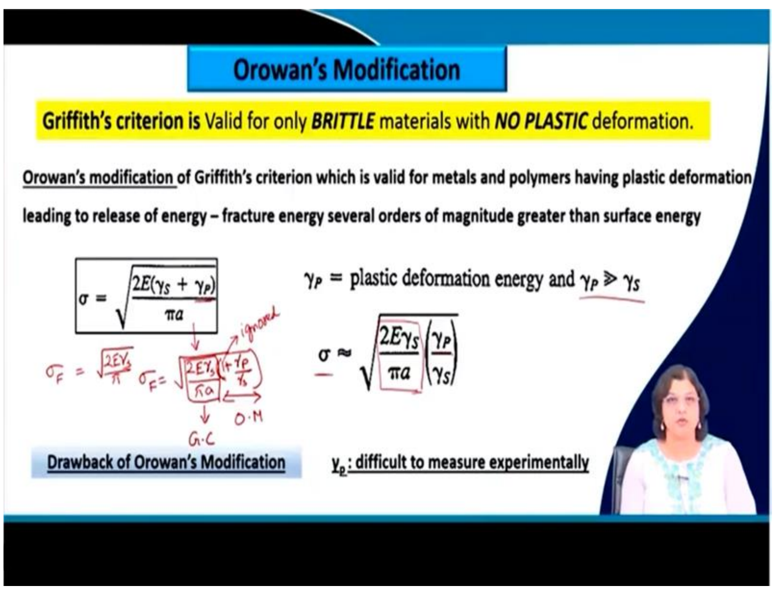

So far, we have seen that how through Griffith criterion, the fracture strength of a material can be determined and now is the time to look into if there are any drawbacks this is suitable for only brittle materials with no plastic deformation. And there is another drawback of a Griffith criterion which says that this is valid for only very very sharp crack or almost infinitesimally sharp crack.

So, let's just see the first point and let's see how this has been modified, Griffith criterion has been modified to suit the need for large different categories of materials. So Orowan has particularly modified Griffith criterion taking care of materials such as polymers or metals which have significant amount of plastic deformation prior to failure.

So, for such cases Griffith criterion is being modified and the fracture energy in this case is several orders of magnitude greater than the surface energy, that is what has been realized and Griffith criterion has been simply modified by adding the term gamma p here, where gamma p is nothing but the plastic deformation energy.

So, we have seen that according to Griffith criterion sigma F is given by simply

$$\sigma_{\mathrm{F}} = \sqrt{\frac{2E\gamma_S}{\pi a_0}}$$

and to that this γ_p term has been added to take care of the plastic deformation energy. So, if we are rearranging this equation this can be done as

$$\sigma_{\mathrm{F}} = \sqrt{\frac{2E\gamma_s(1+\frac{\gamma_p}{\gamma_s})}{\pi a}}$$

We can simply write it in this way just to make sure that the first part here is nothing but the fracture strength that is required as per the Griffith criterion.

So, in that case we have another factor $1 + \gamma_p/\gamma_s$ which is what the modification done by Orowan. But we also know that γ_p is typically much much larger than that the magnitude of the gamma s. So, in that case the term 1 which has been added here does not make much of a sense because γ_p/γ_s will be a large number and in that case 1 can typically be ignored.

Leaving us that sigma according to Orowan's modification is $\sqrt{(2E\ \gamma_s/\pi a)}$ so this is nothing but what originally Griffith has estimated and with that we need to multiply it with gamma p by gamma is this ratio of the plastic deformation energy to the surface energy of any material.

Now there is a drawback of this Orowan's modification as well. So, this is based on the practical estimation of this γ_p. Now γ_p is very difficult to measure experimentally. We can qualitatively analyze the plastic deformation energy but it is very difficult to quantify this number or this value from the practical experiments and that makes finding the σ value as per or once modification quite critical if you are talking about experimental validation of this. So, we will see how this has been further modified.

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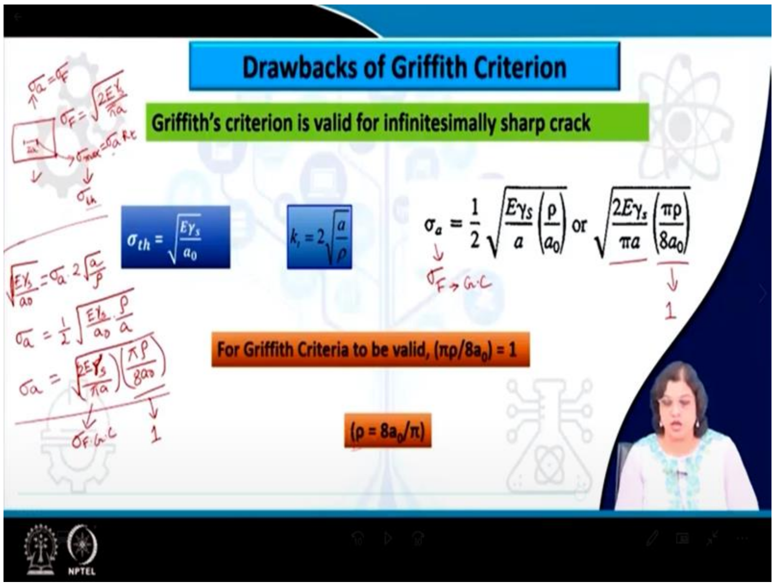

But before that let us focus on the second drawback of Griffith criterion which says that it is valid for only very very sharp crack. So, let us see that by sharp crack again sharp is a qualitative term we need to understand that how sharp the crack needs to be, so that the Griffith criterion gets valid. So, for that we have to understand that the mechanism by which fracture occurs.

So, according to Griffith criterion we have seen that if we have a component which is infinitesimally long and wide with respect to the crack length which makes the crack length as very very small compared to the overall width of the specimen and if we are applying tensile stress to it, fracture will occur as per this relation. Now, we are applying a stress σ_a and at the point of fracture the sigma a will be equivalent to σ_F.

But at the tip of the crack, we are applying the σ_a as the overall stress but at the tip of the crack, both these points here we know that the stress gets maximized. So, σ_{max} actually gets σ_a x k_t where k_t is the stress concentration factor. So, in any case if the σ_{max} achieves the theoretical value for fracture strength σ_{th} at that point only it gets sufficient energy to break the bonds here and to create new surfaces. So, that is the initiation point of fracture.

So, this means that we need to find out the applied stress that is necessary to get the σ_{max} exceed or gets equivalent to the theoretical cohesive strength of a material, so that fracture can initiate. So, the theoretical cohesive strength as we all know is nothing but $\sqrt{(E\gamma_s/a_0)}$ and if we are using this relation here instead of σ_{max} , we can rewrite this equation as, let me write this

$$\frac{1}{2}\sqrt{\frac{E\gamma_s}{a_0}\cdot\frac{\rho}{a}}$$

full equation here $E\gamma_s/a_0$, this is whole root over and that is σ_a x k_t , where k_t is nothing but $2\sqrt{(a/\rho)}$. We can simply rearrange this relation to see what actually is σ_a . So, σ_a will be given by

Again, if we are rearranging this relation so that it gets familiar with this fracture strength related to Griffith criterion. We get something like this $2E\,\gamma_s / \pi a$ and since we are introducing this term π here, we need to multiply that to balance so that gets $\pi\rho / 8\,a_0$.

So, this is simply rearranging this relation here. So, all we can see is that this part is nothing but the fracture strength as per Griffith criterion and it will be valid. So, this applied strength will be same as Griffith criterion only when this factor is equivalent to 1. So, this is what we are seeing here rearranging this relation and in case this term here gets equivalent to 1 we know that this entire thing will be as per the Griffith criterion. This is the σ_f as per Griffith criterion as we are seeing here.

So, that makes the condition to be valid to get Griffith criterion active is $\pi\rho / 8a_0$ that is equivalent to 1 which makes rho equals to $8\,a_0 / \pi$. So, this is nothing but the radius of curvature of the crack. So, that means the crack radii or the tip of the crack radii should have $8\,a_0 /\pi$, this or even lesser value. So, we know that a_0 the inter atomic distance is sometimes on the order of Angstrom so you can imagine that row value should be really very very sharp of the order of few atomic distances so that Griffith criterion gets valid.

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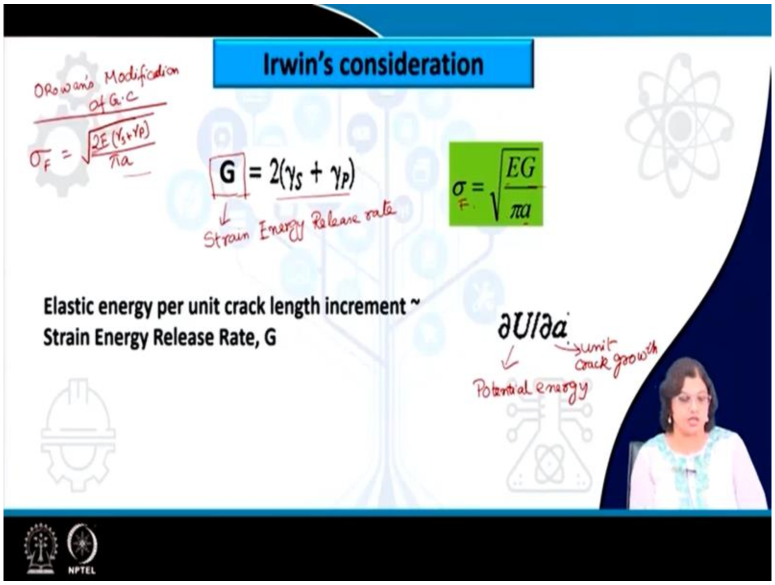

We have also seen that there are some drawbacks of Griffith criterion as well which says that the plastic deformation energy is very difficult to determine experimentally and which calls for some other kind of modification, so that we do not need to determine the plastic work energy but still can quantify the amount of stress that is required for fracture to initiate and here comes the Irwin's consideration.

There is a concept of a new factor which he has introduced, which is known as G or also known as the strain energy release rate. So, this is the stored energy or the potential energy that is stored in a system or a component and when we are applying stress and the crack is growing that leads to fracture, there is this elastic energy is being released over some volume, we have also seen that and that is equivalent to the total energy related to the surface energy as well as the plastic deformation energy. So, this entire, the way Orowon has modified this, so this is how Orowan has modified Griffith criterion, Orowan's modification of Griffith criterion says that fracture strength is not only related to surface energy but it should be related to the plastic work energy as well.

And we have also seen that this plastic energy actually is quite high a number which should not be ignored, in case of ductile materials plastic deformation energy should not be ignored or if we do that that can lead to some discrepancy in the estimated values. So, Irwin has moved a step forward and he considered this entire thing to $\gamma_s + \gamma_p$ as nothing but the G, the strain energy that is being released per unit advancement of the crack.

So, in that case if we are considering that the sigma is then sigma as per the Griffith criterion or as per Orowon's modification is then turning to $\sigma_F = \sqrt{(EG/\pi a)}$. So, this elastic energy or the strain energy release rate is actually equivalent to $\partial U/\partial a$, where U is nothing but the potential energy or the elastic energy that is being released per unit growth of the crack and this says unit crack growth.

So, rate in case of fracture mechanics has a significance because in case of when we consider anything as rate instead of the time what we mean here is the growth of the crack. So, per unit growth of the crack how much is the change in the elastic energy that is considered as the strain energy release rate and that will lead us to the fracture strength of a material. In case we know G we also are supposed to know the E and the a, so that we can determine the fracture strength of a material.

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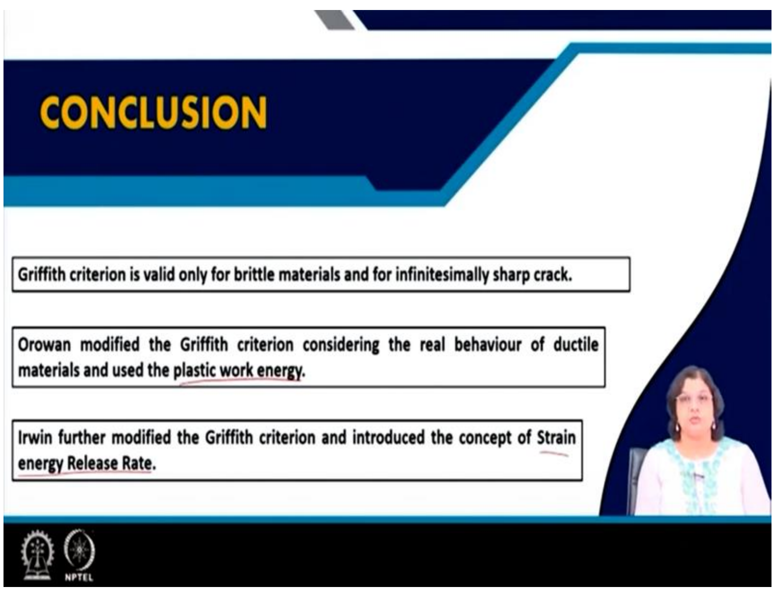

So, this leads to the conclusion for this part of this lecture, which says that Griffith criterion is valid only for brittle materials and for infinitesimally sharp crack. Orowon has modified the Griffith criterion to make it suitable for the behavior of ductile materials, particularly metals and polymers for which there are significant amount of plastic deformation or ductile deformation and that calls for the plastic work energy which typically is much, much higher than the surface energy of a material.

And in case of ductile materials this plastic work energy should not be or cannot be ignored and should be included in the relation. However, we have also seen that from this modification that this plastic work energy is very difficult to determine quantitatively through an experiment and here comes the modification by Irwin who introduced the term 'strain energy release rate' and that is equivalent to the overall surface as well as the plastic work energy which is given by the energy that is being released as the crack is growing.

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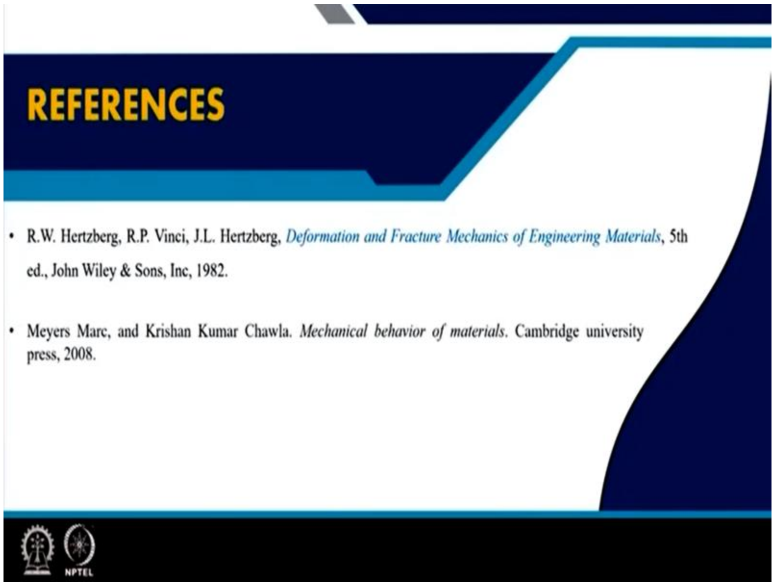

So, these are the following references that has been used for this part of this lecture. And thank you very much for your attention.