Fracture, Fatigue and Failure of Material
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Lecture 04
Griffith Criterion

Hello everyone, welcome back to the course on Fracture, Fatigue and Failure of materials. And today we are here on the fourth lecture of this course.

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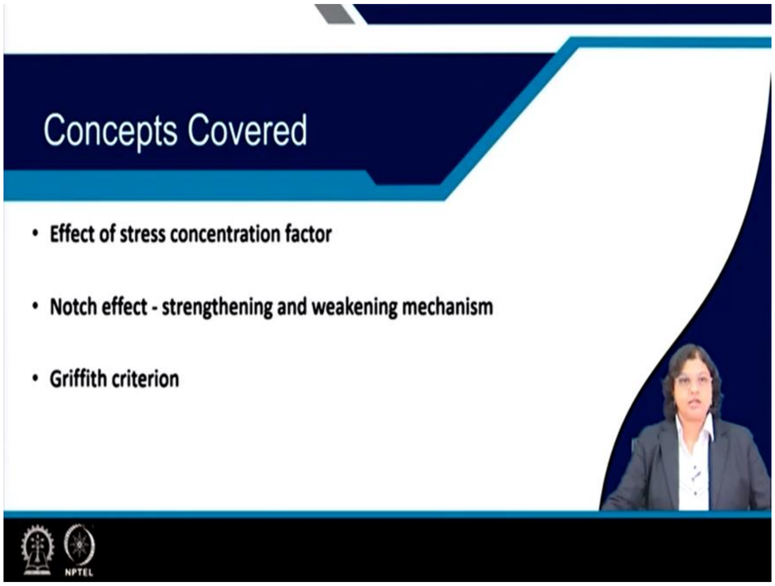

In which we will be discussing the following concepts firstly whatever has been previously discussed on the stress concentration factor I would like to elaborate a little bit more on the effects of that in actual practice. And then, we will be discussing about the Notch effect in presence of a Notch what will happen to a materials behavior that will be discussed. And finally, we will be talking about or introducing the concepts of Griffith criterion.

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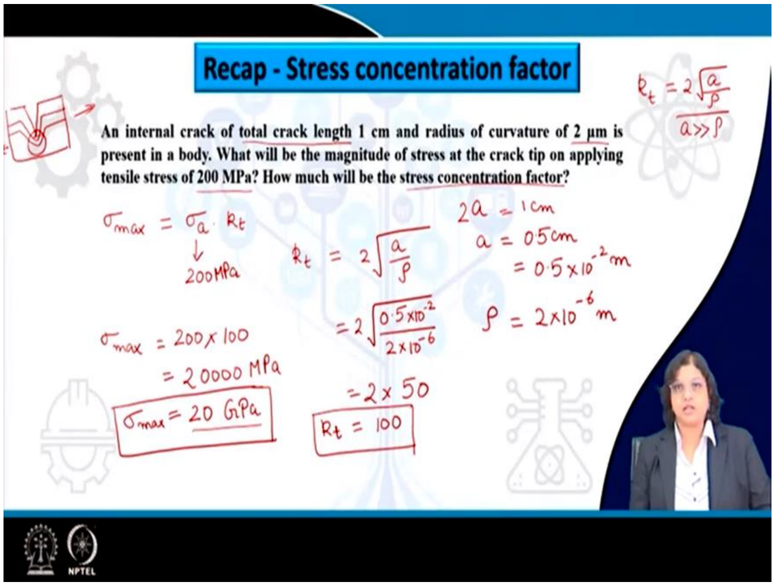

So in the last class we have seen that because of the presence of a Notch or a crack in a material that can lead to some amount of stress concentration factor. So if we have a component like this in which stress are being applied, we have seen that the lines of force are getting agglomerated at this part itself and that leads to stress concentration factor.

And this stress concentration factor can be very well described by a relation k_t which is given by $2\sqrt{\frac{a}{\rho}}$ particularly for the conditions when a or the crack length is much much greater than the crack root radius of curvature which means that for a very long and sharp crack. So here is a problem to understand this concept a little bit more what it says is an internal crack of total length 1 centimeter and radius of curvature of 2 micrometer is present in a body.

What will be the magnitude of stress at the crack tip on applying tensile stress of 200 MPa? How much will be the stress concentration factor? So we need to figure out the σ_{max} from this relation as well as k_t. So σ_{max} is nothing but (σ_a x k_t). And for this case the σ_a is given as 200 MPa. So let us see how much will be the k_t.

So k_t if we use this relation from here is given by $2\sqrt{\frac{a}{\rho}}$ and in this case a or the total crack length please carefully consider this that the total crack length is given as 1 centimeter. So that means a will be something like 0.5 centimeter. Again carefully considering the unit, since any crack length should be expressed in terms of meter so that should be 0.5×10^{-2}m. Rho is given by 2×10^{-6} meter.

And that leads to k_t equals to $k_t = 2\sqrt{\frac{(0.5*10^{-2})}{(2*10^{-6})}}$ which is given by a relation. So this comes to 50 and that makes k_t =100. So that means stress concentration factor for this condition would be 100. So that is a answer to the second question and coming back to the first one.

So σ_{max} will then be σ_a which is 200 MPa x 100, so that makes 20000 MPa or we can simplify as 20 GPa. So this is also very important to understand that although we are applying a stress value of only 200 MPa what we are getting in practice at the tip of the crack somewhere here the stress will be as high as 20 GPa.

So that is a pretty high value so that can lead to the possibility of failure even if we are applying a stress of only 200 MPa. And this is also important to note that in case this value the σ_{max} reaches the value of the theoretical strength of the material locally at this point that can initiate the fracture. We will look into more details of this in due course but this gives us an idea that how the longer and sharper cracks can lead to early failure.

And I would also recommend the students to do similar kind of numerical problems to see the effect of crack length as well as Notch root radius with by varying these values on your own.

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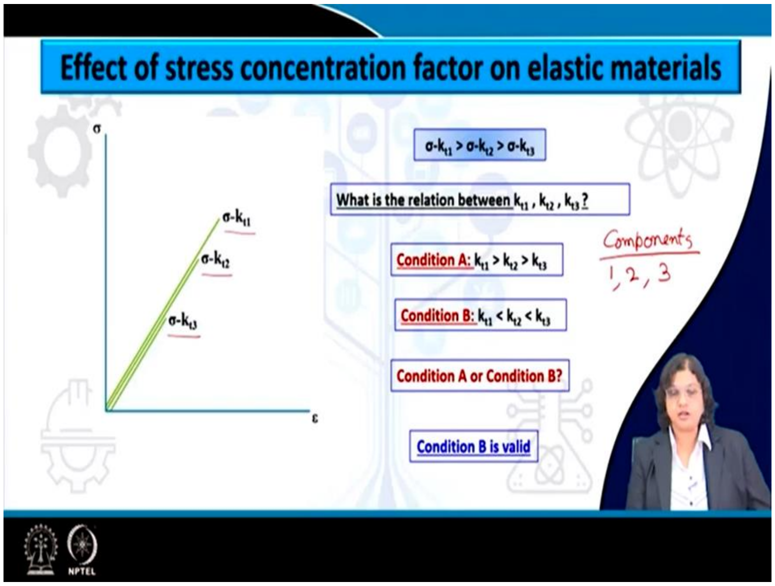

So, this is an example of showing the stress strain behavior of three different components or three different brittle components which fail at certain values of stress. Sigma k_{t1} for the first condition and sigma k_{t2} and sigma k_{t3}. As we can see here that sigma k_{t1} is obviously higher than that of k_{t2} as well as k_{t3}. So this relation is very much clear. Now what would be the relation between k_{t1} that is the stress concentration factor for t1, t2 and t3.

So, there could be three possibilities. The first one would be let us say if condition A is valid then we can see that $k_{t1} > k_{t2} > k_{t3}$. So if there are three components one, two and three, if the stress concentration factor for one is the highest and three is the lowest, are we going to get higher values of σ for condition one or component one in compared to that of three.

So, if this condition is valid for condition A or we can have another condition which says that the stress concentration factor for component 1 is the least and the stress concentration factor for component 3 is the most. So for clarity let me just write this that there are three components 1, 2 and 3 and we need to figure out that which of these conditions are valid so that we get a higher value of σ for component 1.

So although it appears like k_{t1} is higher from this condition, in reality what we should expect is that condition B is the valid one which says that stress concentration factor for component 1 is the least. That is why it can fracture at the highest amount of stress or the strength for the component 1 which is having the least value of stress concentration factor is the highest one.

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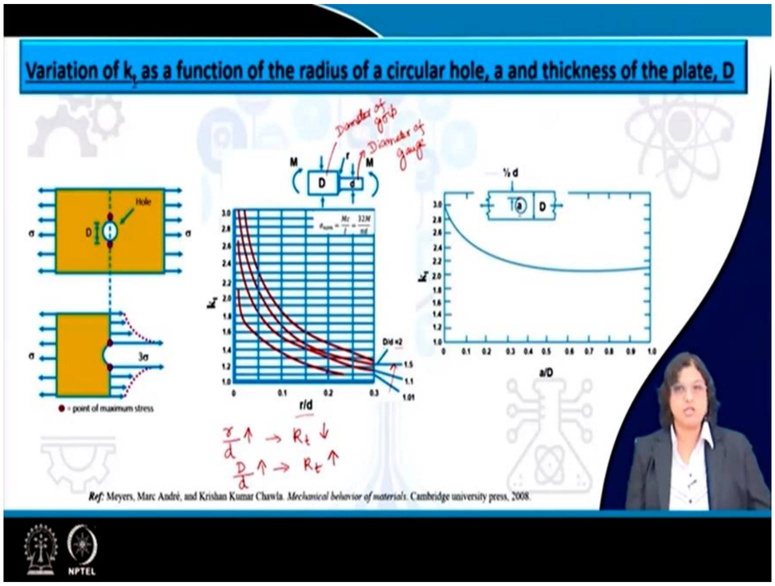

So here is the example of some components which presence of a hole or a semicircular defect there how the stress concentration factor would vary or maybe in case of a tensile specimen as we have seen that there is a sharp corner that leads to stress concentration. And considering all these parameters like the diameter of the grip section versus the diameter of the gauge section as well as the radius of curvature in this case diameter of the hole.

So all this are the leading factors determining what would be the stress concentration factor. So this are some of the experimental results that has been shown here and you can see that how this stress concentration factor varies with the ratio of r/d where r is this radius of curvature and small d is the diameter for the gauge section.

So we can very well see here that this lines are coming down as we are increasing r/d. So that means that as the r/d ratio increases what we are seeing is that k_t or the stress concentration factor decreases. So this is what we are seeing from any of this lines for any particular values we can see that as r/d is increasing, k_t is decreasing.

So this trend is valid now between this four line, four curves here the difference is that the D by d ratio capital D by small d ratio, capital D is the diameter of the grip section. Let me just write it here diameter of grip and small d is the diameter of gauge section. And this could be any other kind of irregularity instead of grip and gauge and you can use similar kind of relation. So in this case what we are seeing is as D by d this D/d ratio is increasing towards this direction what we are seeing is that for let us say for any particular ah value of r/d. So let us say point two what we are seeing is that for capital D/d ratio of 1.01 we get a value as low as 1.1 for k_t as well as if we are changing this D/d ratio to 2 we are getting a k_t value as high as 1.5. So that means that as D/d increases that leads to enhancement in k_t as well.

So, this is how this kind of ratios or this relations between the different parameters can be controlled if we are looking for a particular value of stress concentration factor depending on our service requirement. Here is another example where we have a circular hole and the radius of that hole is represented by a as well as this distance is given by capital D and this distance as small d.

So that means where the hole is starting, so based on that we can also figure out the relation. So there has been several experimental work that has been done and this also varies from materials to material but at least we know that this can be controlled or there can be some relations between this different factors.

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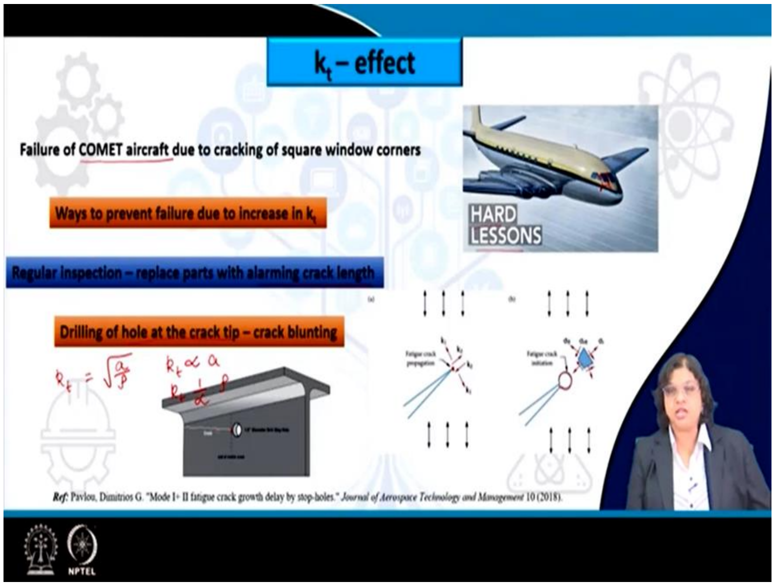

So, the practical importance of stress concentration factor has been seen for ages and we have now known to design our component for actual practice such that the stress concentration factor can be minimized to the maximum extent possible. This has been also seen for different failure events, one for example, is the failure of the COMET aircraft which was the first amongst the jet liners and there was a series of fatal air crash that happened with this COMET airliners and later on their service has been stopped.

So, there are different reasons for this failure which will we will be emphasizing during the failure analysis part of this course. But for now we can at least pay attention to the fact that you can see in this picture here that there are square windows and if we now carefully pay attention to the next time you are making an air travel or if you look into the picture of an aircraft you can see that the windows that we presently have are oval.

So, what happened is that this square windows here are the source for the stress concentration. Stress concentration arises from these corners of the square windows and that leads to fracture of eventually the fracture of the aircraft. So that was one of the reason and we have learned from a very hard lessons that how stress concentration factor can be dangerous.

Of course, we always wanted to prevent that and we want to control this k_t so that it should not lead to such kind of unwanted failure. And there are several ways by which we can control this k_t. First of all we need to inspect regularly all the components so there should be regular inspection and replace parts with alarming crack length. Any kind of structure for which failure is used to be very, very critical we always go for regular inspection.

If we notice some amount of crack there depending on the crack length and the service amount of time that is required we do a detailed estimation of how much more life is left. And if there is an alarming length of crack which means that the crack length is sufficiently large or long we tend to retire that from practice.

Another very smart way of getting rid of stress concentration factor is drilling a hole at the crack tip. So we have seen that basically k_t is related to a and ρ right crack length as well as the radius of curvature which means that $k_t \propto a$ and $k_t \propto 1/\rho$. Which means that either we have to reduce the crack length or we need to make the crack tip blunt.

Now reducing the crack length is not always in our control but what we can do is to make the crack blunt. And there are ways by which we can actually drill a hole you can see that this is a sharp crack where there are certain values of k stress concentration factor. And if we are drilling a hole here just at the tip of the crack so now the crack which was initially very, very sharp here now because of this hole it has acquired a very blunt feature.

And as a result the stress concentration factor can be reduced dramatically. Of course we need to be very, very careful to implement that you can see here the crack is growing and then because of the presence of the hole the growth of the crack is being stopped because the stress concentration at this point is reducing. Which means that the stress at the tip of the crack is reduced extensively and that leads to stoppage of the crack growth.

We have to do this very, very carefully so that we can drill the hole exactly at the crack tip and not a little behind that which otherwise would not have any effect at all.

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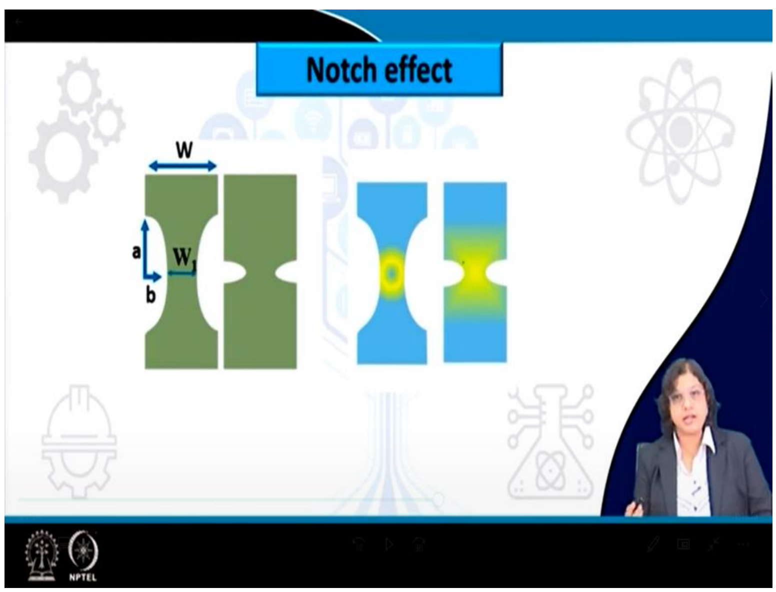

So this leads to another important effect which is known as the Notch effect so what it says is. Now that we know that there are defects and we are slowly trying to figure out that if there are defects how can we correlate this defect to the strength of the material or the fracture strength of the material or the deformation behavior in general. So to that extent Notch if there are present we need to find out the, what would be its effect on the deformation behavior of the material.

So you can see that these are just examples of different tensile specimens and once again this is the grip and the gauge section that I have highlighted in the previous lectures. And because of the presence of a Notch that can lead to differences in the stress scenario so this is when there was no stress has been applied and when we apply a stress this is how the stress is being distributed.

Either in the entire gauge section or because of the presence of a Notch or cracks that can lead to some kind of stress concentration at the tip and that can lead to a change in the deformation behavior.

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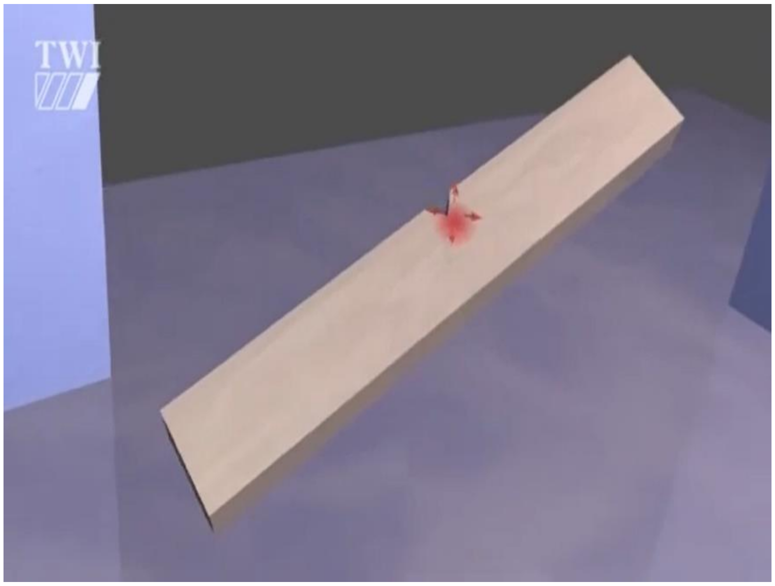

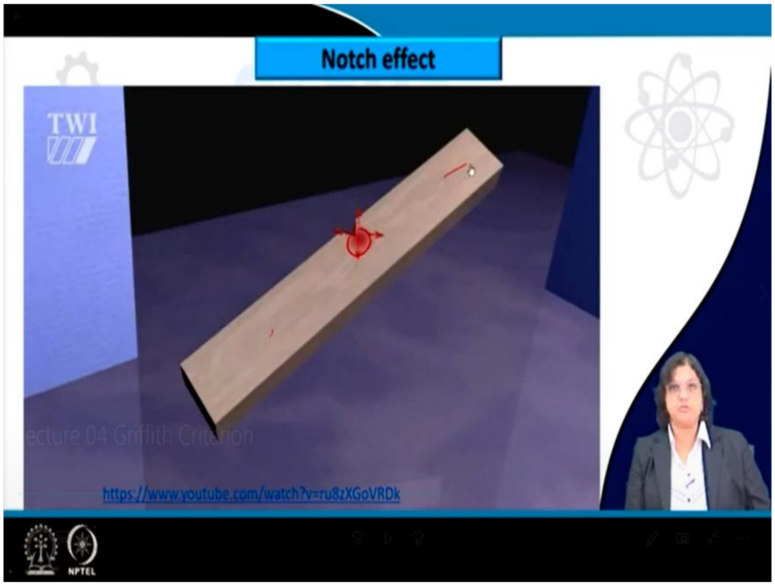

So here is a very small video for you to see. So how the stress is generated at the tip of the Notch. So we can see here that at the tip of the Notch there is a Notch already present here. And when we are applying stress at the tip of the Notch there is some amount of stress concentration right here which is not seen in any other part of the component. And that leads to some change in the deformation behavior.

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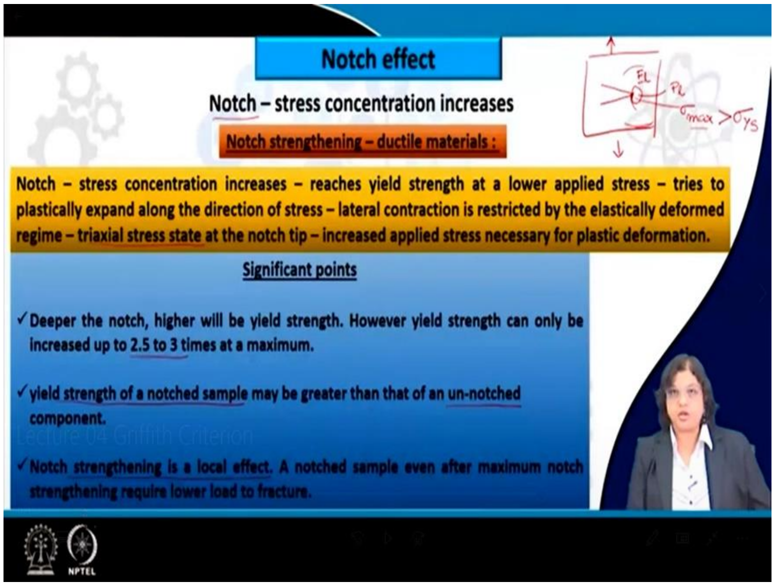

Now when it comes to any material as we have seen that presence of a Notch leads to stress concentration. And in case of ductile material that leads to a very interesting effect which is known as Notch strengthening. So what happens here is because of the presence of a Notch let us say we have a Notch here in a component and we are applying stress over it.

As a result, at the tip of the Notch we know that there will be σ_{max} right. Now in case this σ_{max} its exceeds the yield strength of the material yield strength is the amount of stress that is required for the material to start plastic deformation. So, if the σ_{max} value exceeds the yield strength of the material obviously the material in front of the Notch tip would like to have plastic deformation.

So let us name this as P_l so it will try to expand and as a result if it as per the conservation of volume factor if it tries to expand along the direction of the loading or elongate along the direction of the loading it is supposed to contract on the perpendicular direction to maintain to conserve the volume.

However, that will be restricted by the surrounding material which so far is in elastic condition. So, this elastic part is not aligned to have any permanent deformation at the tip of the crack. As a result although we are supposed to have yielding at this point as we know that σ_{max} has exceeded the yield point that is not enough which means that we have to apply even higher stress to start the yielding criteria.

So, this is being explained here that because of this presence of a Notch there is a stress concentration and once that reaches the yield strength at a lower applied strength it tries to plastically expand. However, the lateral contraction is restricted by the elastically deformed regime. And that leads to generation of a triaxial stress state and overall, we need to apply even higher value of stress to start the plastic deformation.

Which means that now the stress that is required for the commencement of plastic deformation increases or in other words the yield strength increases. So that effect is known as the Notch strengthening, that is seen particularly for ductile materials material which has a tendency to undergo plastic deformation under the influence of stress. But we also have to be careful about the following points for not strengthening.

That deeper the Notch more is the length of the Notch higher will be the yield strength. However enhancement and yield strength can be done only up to 2.5 to 3 times and not more than that. And yield strength of the north sample of course is greater than that of an unknown specimen that is what has been discussed. However, whatever is the case notch strengthening is a very, very local effect because this is what is happening at the tip of the crack.

The surrounding of the material or any other part of the material is not being influenced by this presence of a Notch. On the contrary even there is a Notch and there is Notch strengthening even after maximum Notch strengthening, this specimen will still require lower load to fracture. So that is something very, very important so this is not a regular method of strengthening like we know that there are several ways by which we can enhance the yield strength of the material this is not however is followed for Notch strengthening case as it still requires lower load to fracture.

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So on the other hand there is another way by which we can show this for the case of brittle materials. Here also if there is a Notch there is of course a stress concentration and since there is no plastic deformation. So what happens the stress gets maximized here and that leads to breakage of this so that leads to total failure of this. So because of this stress concentration stress relaxation by plastic deformation anyway is limited for the case of brittle material.

There is no provision for dislocation movement and as soon as the stress concentration or the maximum stress reaches the fracture strength of the material there will be no yielding rather there will be direct fracture. So this is known as Notch weakening and this is as per our common understanding that presence of a Notch will make it fracture at a lower strength.

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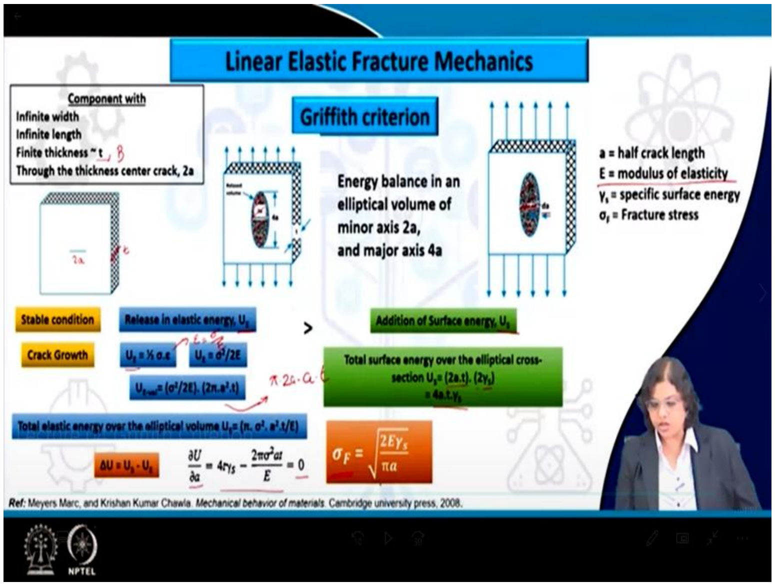

Now from there we are next moving on to the concept of Linear Elastic Fracture Mechanics and I would like to start this with the famous Griffith criterion. Griffith is known as the father of fracture mechanics and he has developed a criterion which is followed for fracture mechanics. And whatever in depth studies that has been done on fracture mechanics is somehow falling or depending on this Griffith criterion itself.

So, this is the most important criterion in fracture mechanics that we are going to start in today's lecture. So, what it essentially says is that we understand that any component any system should have defect and if there are defects then the stress that is required for fracture will not be same as the stress that is required in the theoretical part. So, it will be less much less than the theoretical cohesive or fracture strength this we have understood. So next our target is to find out so what is the magnitude of this stress in a component in presence of defects.

So let us see what it shows we will now start with a component which is of infinite width and infinite length which means very very long and wide component but it has a finite thickness of t and there is a through the thickness center crack 2 a. So, this is what it is it has it is in reality very long and very wide ah component that you can think of and there is a finite thickness.

So this is the thickness t sometimes the thickness is also referred as capital B just for your information and then we have a Notch or a defect of total length 2 a. Now what happens is that if we are applying stress ok there could be two different ways first of all if we are applying stress there will be some elastic potential energy or elastic energy that is stored in the system that will be released over an elliptical volume so over this volume.

So this ellipse has a major axis of 4a and minor axis of 2a so whatever defect was there this crack that has now become the minor axis of the ellipse over which the elastic energy is being released this is not a crack this is the volume of consideration over which all the mechanisms are happening. At the point of stability this release in elastic energy will be same as the addition of any new surface energy.

Now surface energy as we have seen even for the calculation of theoretical cohesive strength that surface energy comes into the picture as the crack grows or as it leads to fracture there are two new surfaces that are forming. So at the point of stability this energy that is being released is same as the energy that is necessary to create a new surface.

And crack growth materializes only when this release in energy is greater than the energy that is required for maintaining that free surface. And this entire thing as I said is happening over this area or over this volume where the crack is increasing by a length infinitesimally small length da. So let us see how it goes this elastic energy or the stored elastic energy is nothing but half into stress into strain so this is all what we already know.

And we can convert this as we know the Hooke's law so strain can be represented as σ/E, E is the elastic modulus here and so that means U_E is given by $\sigma^2/2E$. Now over this volume this is happening this is releasing over this entire volume of the ellipse so that will be given by this relation π so this is for the volume π into half of the major axis 2a and half of the minor axis into thickness. So that is the entire volume of this ellipse which is $2\pi a^2 t$. So this multiplied by this $\sigma^2/2E$ is the total elastic energy for this entire volume.

On the other hand if we are talking about the surface energy so this surface energy is Υ_s and since there are two surface forming so this should be $2\Upsilon_s$ and that is over this entire energy entire area surface area is what is important. So the surface area of importance here is 2 a that is the total length of the crack which is nothing but the minor axis into t the thickness it is happening over the entire thickness.

So this is what is the total surface energy (4a t Υ_s). So at the point of crack growth this is what needs to be maintained delta U equals to surface energy minus the elastic energy term. And for the crack growth of delta a we need to differentiate this with delta a and this should come to 0 to maintain this stable condition up to the point of stability. So, if we do that if we simply differentiate this we can solve this numericals.

And we can see that the fracture strength that is obtained is nothing but this is given by a relation $\sqrt{\frac{2E\gamma_s}{\pi a}}$, where a is nothing but the half crack length here. So, this is what we are seeing this is - what is the most important factor which is considered for fracture mechanics? This is the fracture strength of a material and that is related to $\frac{2E\gamma_s}{\pi a}$.

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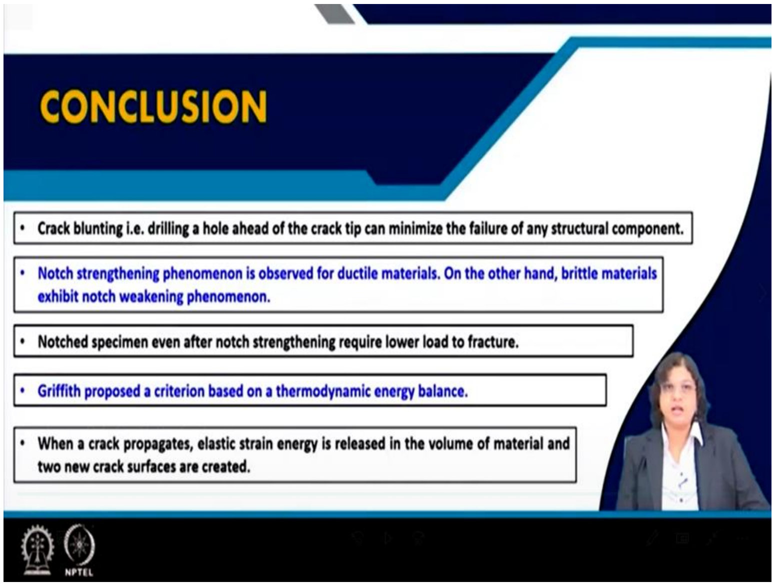

Let us conclude here, so we have seen that crack blunting or drilling a hole ahead of a crack tip can minimize the failure of any structural component. And in case of a Notch strengthening phenomena which is particularly seen for ductile materials, there is some plastic deformation that happens at the Notch tip or the crack tip that leads to a triaxial stress state and that leads to enhancement in the yield strength but that is a local phenomena.

On the other hand, for the case of brittle material we see a Notch weakening. Notch specimen even after Notch strengthening require lower load to fracture and most importantly we have also seen how the fracture strength of a component with an existing crack can be determined as per the Griffith criterion which is proposed based on the thermodynamic energy balance.

So, when the crack propagates the elastic strain energy is released in the volume of the material and that is being utilized by forming two new crack surfaces. So that is being balanced by the surface energy.

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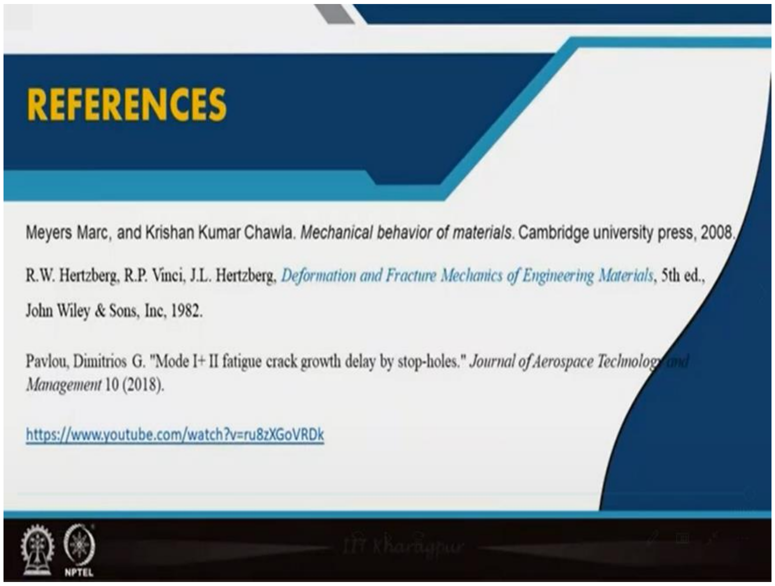

So, these are the following references that has been used for this lecture. Thank you very much.