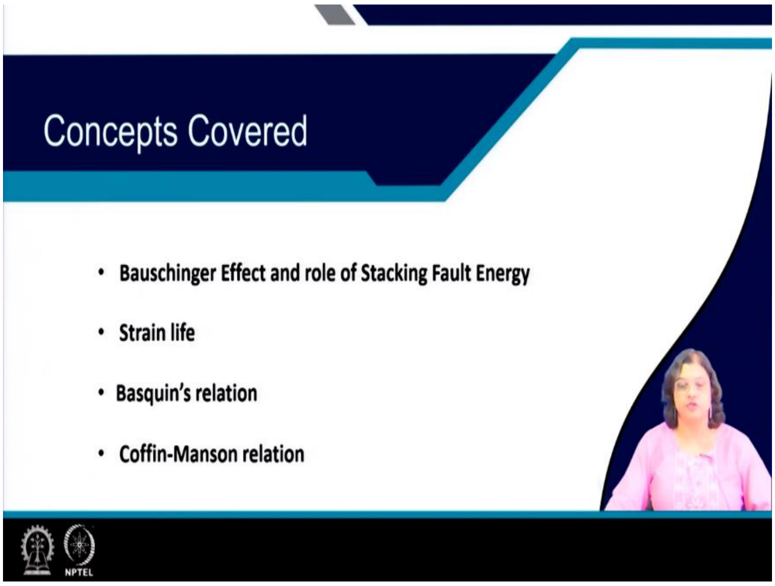
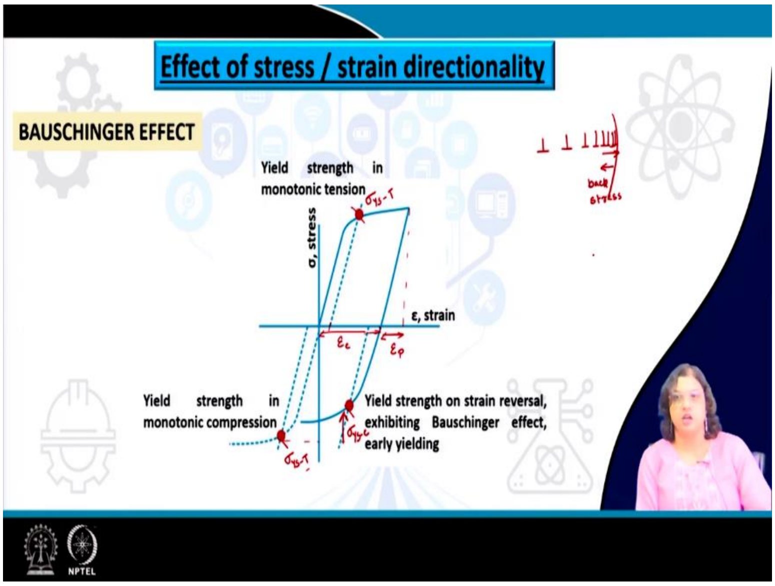
Fracture, Fatigue and Failure of Materials
Professor Indrani Sen
Department of Metallurgical and Materials Engineering
Indian Institute of Technology, Kharagpur
Lecture 37
Strain Controlled Fatigue (Contd.)

(Refer Slide Time: 0:26)



Hi everyone, we are at the 37th lecture of this course Fracture Fatigue and Failure of Materials and in this lecture also we will be talking about the Strain Controlled Fatigue. Particularly what we will be mostly discussing in this lecture is about the Bauschinger Effect as well as the role of Stacking Fault Energy in controlling the fatigue deformation behavior of a material and from there we will look on to the life estimation from strain control fatigue how we can determine the predicted number of life of a component and we will do that through the Basquin's relation as well as the Coffin-Manson relation

(Refer Slide Time: 1:18)



So let us begin with the Bauschinger Effect and the role of Stacking Fault Energy as the very first instance. So, what we have seen in the previous lecture is that when we are applying cyclic loading in the strain control mode there could be hardening or softening of the material just because of the cyclic loading or the repeated loading and this can be reflected in terms of the Bauschinger Effect.

So, what the Bauschinger Effect typically means is that when we are loading a component or a specimen in tension. So, it is giving a yield stress value at the 0.2 % offset value somewhere here. So, that signifies the let us say the yield strength of the material under tensile mode of loading. So, T stands for tension here and we are employing some amount of permanent or plastic deformation to it.

Similarly, what we do for the case of strain control fatigue as we have seen and then we are unloading it till the zero load and because the specimen has undergone some amount of permanent deformation there is some part which is the plastic part of strain as well as there is some part which has been recovered. So, that signifies the elastic strain.

Now, interestingly when we are loading this under compression again based on this 0.2 % offset method we can determine the yielding the stress required and let us name this as σ_{ys} under compression. So, c stands here for compression and what we have seen here is that this is typically the yield strength under tension we have simply put this in the opposite direction to just make a comparison between this compressive and the tensile yield strength and we can very well see that there is a difference between the two and this difference is actually negative which means that the compressive yield strength is actually lesser than the tensile counterpart.

And this is not only related to the direction of loading, I mean if we are loading in compression first and tensile in the second one then we will see the exactly same behavior as if the yield strength determined in second step of loading is always lesser than the yield strength determined under first step of loading.

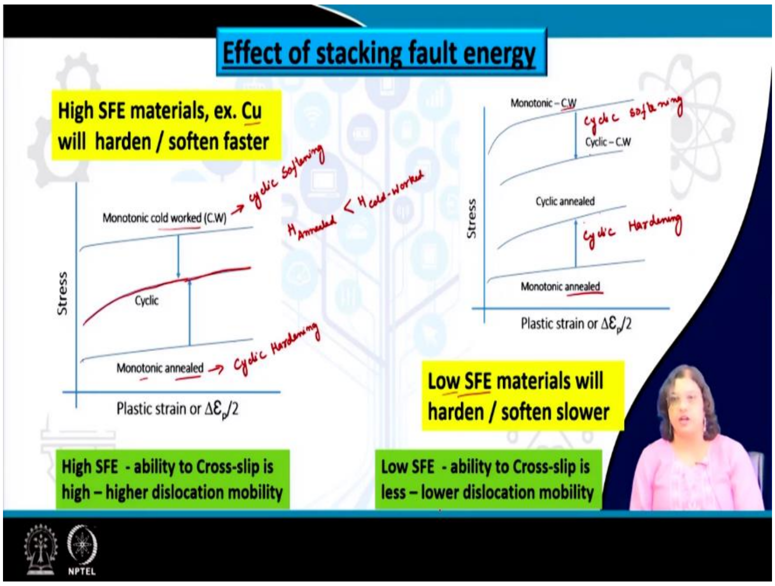
So, this is very typical for the case of cyclic loading and this is just for the initial the first two cycles. So, this is somehow related to the hardening or the softening behavior of the material. So, what happens is this this anomaly between the yield strength due to tension and compression is particularly related again to the dislocation motion. We have already discussed about the role of dislocation and how the dislocation the entangled dislocation can get rearranged if we are repeatedly loading it and that leads to some kind of softening, softening means that it can yield at a lower value of stress.

So, similarly for the case of Bauschinger Effect also that kind of rearrangement of dislocation due to the reverse mode of loading is one of the reason, the other reason very dominant reason is the presence of the back stress. So, what happens is that when we are loading it under tensile mode. So, in the forward loading the dislocations that are generated which are getting hindered by any kind of boundaries or any kind of barriers such as the grain boundaries it forms a pile up and when we are loading it in the reverse direction those pile ups which were initially at a larger distance and let us see how it behaves like.

So, let us say we are having the dislocations like this and the spacing between the dislocations actually decreases as we are nearing towards a boundary and that leads to generation of a stress. So, of course because so many dislocations are present here getting agglomerated here. So, that leads to a quite significant amount of stress. Now, when we are loading it in the reverse direction this acts as a back stress and this back stress along with the applied stress leads to an enhancement in the total available stress level and that leads to yielding at a quite lower value of the applied stress.

So, this applied stress is being acted or being summed up with this back stress while compressive loading or loading in the reverse direction. So, this could be once again it does not depend on tension or compression rather it depends on if we are completely changing the direction of loading as well as the first cycle versus the second cycle the second cycle always is supposed to show lesser value of yield strength compared to the first one and the reason is this rearrangement of dislocation as well as the presence of the back stress.

(Refer Slide Time: 6:27)



So, this is an interesting observation that we make for most of the cases and the other thing which also dictates the plastic or the strain controlled cyclic deformation behavior of the material is the stacking fault energy. Now, we have seen that initially hard materials tend to undergo cyclic softening whereas the initially soft materials tend to undergo cyclic hardening. So, if we think actually that means that most of the cases the materials tend to attain some kind of stabilized condition.

So, that is what is seen here also. Now, for the case of high stacking fault energy an example is this copper what we have seen is there are two different conditions in which we have taken the specimen. So, first one is the cold worked condition and the other one is the annealed one. So, cold worked one means that there are several dislocations already existing there and the hardness of the cold work material is always more compared to the annealed one, annealed one means any kind of material mostly annealing leads to reduction in the residual stress level and that leads to lower hardness of the material.

So, that is what we can see from the monotonic curves for both these cases we can see that the let us say the hardness of the annealed one is always lesser than the hardness of the cold worked condition or for that matter any kind of worked material or rolled material etcetera. So, not only hardness we can see that the yield strength, ultimate tensile strength all these stress values are actually lesser for the annealed condition compared to the cold work condition.

So, that means if we are now cyclically loading it the annealed one is expected to undergo cyclic hardening. So, anneal one is expected to undergo cyclic hardening because this is a softer thing, softer material and the cold worked one is expected to undergo cyclic softening and this is the cyclic stress strain curve that we can see let us make this a red one to have a better understanding and what we can see is that both of this the annealed one and the cold worked one is coming to the same level of stress strain values under cyclic loading.

So, that is the stabilized condition which both the material under annealed condition or the cold work condition tries to attain under cyclic loading and in comparison to that let us now talk about a low stacking fault energy material which will also have similar kind of behavior. So, the annealed one is having lower hardness compared to the cold worked one. So, the annealed one is expected to undergo cyclic hardening and the cold worked one although it has higher monotonic strength it is expected to undergo cyclic softening.

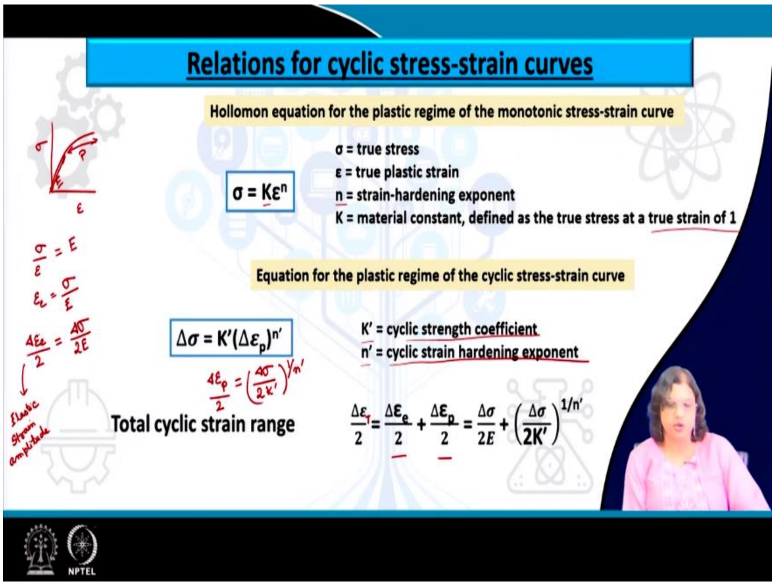
So, that is what we are seeing here the annealed one is undergoing cyclic hardening and the cold worked one is undergoing cyclic softening but what we can see here in comparison to the previous one is that both of this are not attaining the same level. So, there is some differences between the cyclic annealed one and the cyclic cold work one and not only that actually the high stacking fault energy tends to attain this stabilized condition much faster.

So, whatever the condition is whether this is cyclically hardening or cyclically softening this happens much more faster for the case of high stacking fault energy whereas for the case of low stacking fault energy this kind of changes in the behavior is quite slow the reason behind this kind of behavior is the fact that the high stacking fault energy means actually it has more ability to cross slip the distance is lesser.

So, it will be easier to overcome that and that makes the cross slip activities quite easier and that makes the high dislocation mobility. So, obviously whatever all this kind of changes in the behavior cyclic hardening or softening is because of the dislocation activity because now we are applying this strain control fatigue is mostly when we are applying the strain levels exceeding the yield point.

So, that means that all these dislocation activities are getting faster the motion of the dislocation is higher and that makes the high stacking fault energy to get cyclically hardened or softened faster. Compared to the low stacking fault energy in which case the ability to cross slip is less and that makes the dislocation mobility also quite low and that leads to slower attaining the condition of cyclic hardening or softening and in some cases, it cannot even attain the stabilized condition when whether it is cold worked or annealed it has come to one particular value.

(Refer Slide Time: 11:56)



So, then let us look into the relation between the stress strain curve. So, what are the different kind of relation. First of all, if we are talking about the stress strain curve, it has two segments like the elastic part and the plastic part. So, let us say we have a stress strain curve like this whether it is monotonic or cyclic whenever we have a stress strain curve we have some part which is the elastic part and then the other part which is the plastic part.

So, for the elastic part we know that typically the Hook's law is valid. So, which says stress by strain is actually dictated by a constant the elastic modulus of a material or the elastic strain can then be considered as $\frac{\sigma}{E}$ or in other case if we are talking about the strain range or for that matter the strain amplitude then that is $\frac{\Delta\varepsilon_e}{2}$. So, $\Delta\varepsilon_e$ is the elastic strain range and divided by 2 is the strain amplitude.

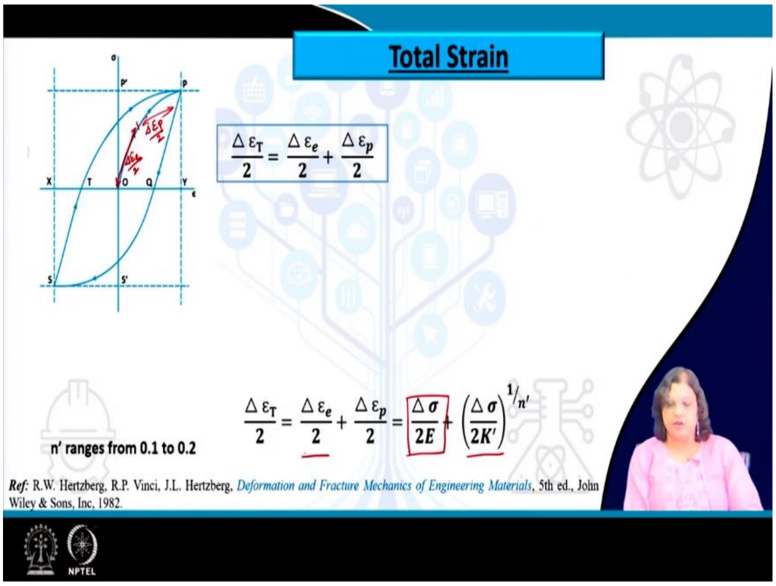
So, let us name this as elastic strain amplitude and that is equivalent to $\frac{\Delta\sigma}{2E}$ again $\Delta\sigma$ is the stress range for cyclic loading and $\frac{\Delta\sigma}{2}$ signifies the stress amplitude and divided by e that gives the total the strain elastic strain amplitude of the material.

Now, for the plastic part actually we have the Hollomon equation. Similarly, for the monotonic cases which says that the stress and the strain are related to this constant K which is defined as the true stress value at a two strain of 1 and the ε^n. So, n is nothing but the strain hardening exponent we have seen.

Now, for the case of the cyclic loading similar kind of relation is followed but instead of K or n we actually use the symbol K' and n'. So, K' signifies the cyclic strength coefficient and n' signifies the cyclic strain hardening exponent. So, here also we can see that the plastic strain amplitude can be determined as something like this.

So, that will be $\frac{\Delta\sigma}{2}$ and then this K' should come in the denominator and this overall with $\frac{1}{n'}$. So, overall, the total strain amplitude which is the summation of the elastic part and the plastic part will be given by this relation $\frac{\Delta\sigma}{2E} + (\frac{\Delta\sigma}{2K'})^{\frac{1}{n'}}$. So, that signifies the total strain amplitude of cyclic loading condition.

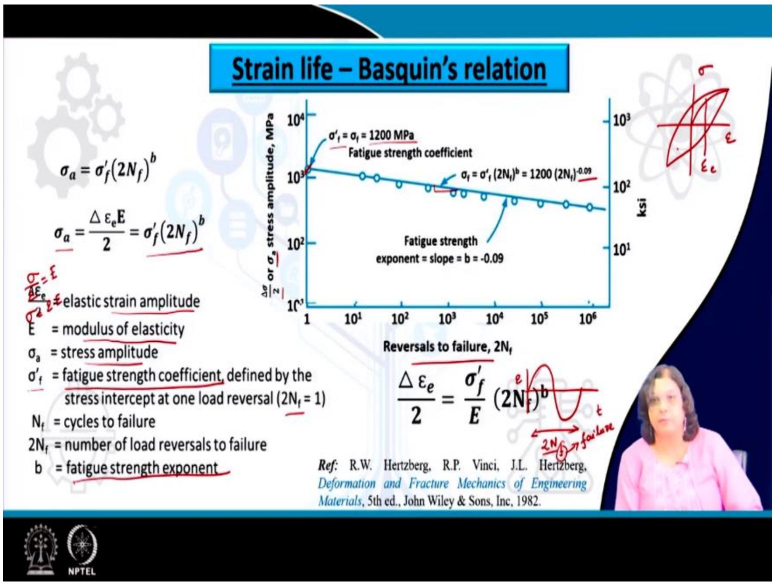
(Refer Slide Time: 15:29)



So, if we are talking about the hysteresis loop once again the elastic part. So, up to this part will be given by this relation here which is nothing but $\frac{\Delta\sigma}{2E}$. And the plastic part so, above this part here that will be dictated. So, let me also write down the elastic and the plastic part distinctly.

So, this is the elastic part and then the remaining is the plastic part and we have seen that how plastic part can be obtained from the Hollomon relation or the modified Hollomon relation for the cyclic loading. So, we can get a total strain amplitude something like this. Now, this is the strain amplitude part and now we are looking for the number of cycles that a material can survive under elastic and the plastic loading condition.

(Refer Slide Time: 16:33)



So, for that we also need to find out the relation between strain amplitude and the number of cycles. So, that is given by Basquin's relation particularly for the elastic part. So, as we have seen that elastic strain so within the elastic limit or below the yield point of the material yield strength of the material it is the stress amplitude that is of most significance and the strain life based on the elastic strain amplitude is determined based on the stress amplitude itself.

So, it says that $\sigma_a = \sigma_f'(2N_f)^b$ and if we are converting this stress amplitude to the strain amplitude one the elastic part is related to Hook's law and so we have seen that how stress and strain are related to E and so stress at any case will be given by $(\varepsilon \times E)$. So, this is what is seen here and since this is the elastic strain. So, we have considered the stress amplitude strain amplitude as $\frac{\Delta\varepsilon_e}{2}$ as I have just explained and multiplied by E. So, that is equivalent to $\sigma_f'(2N_f)^b$.

Now, what are these factors the next question that comes to our mind is what is this σ_f' and N_f and b and all. So, this can be understood from a graph like this. So, if we are plotting the stress amplitude versus number of cycles for failure. So, y axis here is σ_a, σ_a is nothing but $\frac{\Delta\sigma}{2}$ stress range divided by 2 and what we can see here that as per this relation this $\frac{\Delta\varepsilon_e}{2}$ is nothing but elastic strain amplitude and E we all know.

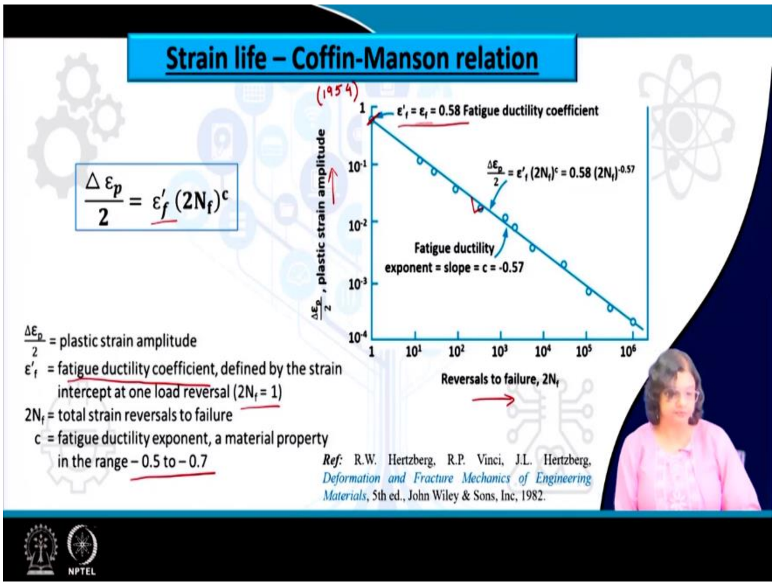
So far, that E is the modulus of elasticity, σ_a is again defined as a stress amplitude what is our concern is the σ_f' and the $\sigma_f'$is actually this point here which signifies the fatigue strength coefficient. So, σ_f' is typically defined as the fatigue strength coefficient which is the stress intercept at one load reversal. So, $2N_f$ is actually one load reversal.

So, if we are applying the strain or stress versus time this is one 2 N f since we are talking about the failure. So, at the point of failure this becomes f but up to the point of failure I mean when it is not failing this is like 2 N because it is moving from one cycle to the other and this is nothing but 2 N at the point of failure this is termed as $2N_f$. So, f here signifies failure.

N_f is typically then the cycles to failure and $2N_f$ is the load reversal since for each cycle of it consists of two load reversal that is why we are plotting here the x axis is $2N_f$ and this is again in a logarithmic scale and b on the other hand is the slope of this curve. So, b is known as the fatigue strength exponent. So, what we have seen here for this particular example the value of σ_f' is 1200 MPa and the value of b is something like - 0.09.

So, this is for the elastic strain amplitude and obviously the next part since we are talking about fatigue which is having a hysteresis loop something like this we are talking about not only the elastic part but also the plastic part. So, this is valid only for the up to the elastic strain amplitude. So, the total strain, total elastic strain amplitude can be given by this relation $\frac{\sigma_f'}{E}(2N_f)^b$. So, that signifies the total elastic or for that matter it is the elastic strain amplitude.

(Refer Slide Time: 21:11)



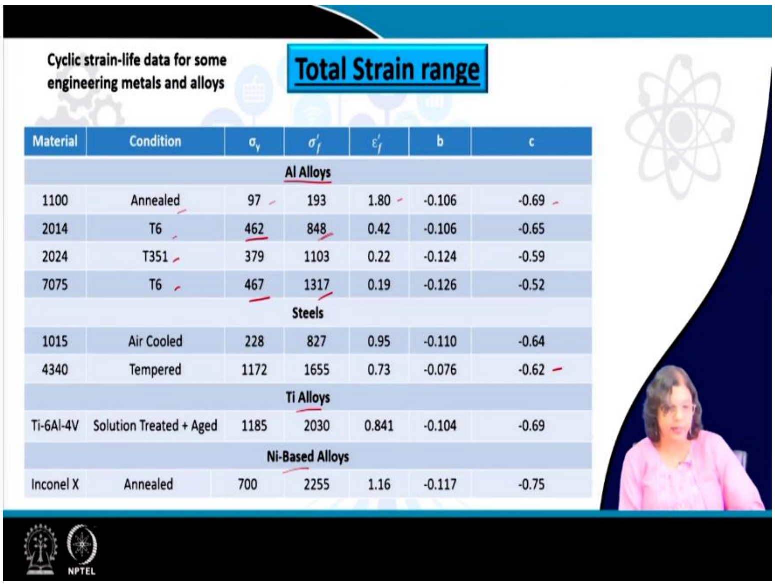
So, next we should move on to the plastic part and let us see that how that is relevant to the number of cycles for failure. So, that has been this kind of relation for the plastic strain amplitude has been found independently by Coffin and Manson both at the same year of 1954 and the relation is again given by the strain plastic strain amplitude equals to $\varepsilon_f'(2N_f)^c$.

So, in this case again if we are plotting the plastic strain amplitude on the y axis and the number of load reversals to failure on the x axis, we will see that we are achieving a straight line this intercept is the fatigue ductility coefficient or that is actually is what is ε_f' and that is defined by the strain intercept at the point of one load reversal.

So, when $2N_f$ equals to 1 that point whatever is the value of strain amplitude that is what is the fatigue ductility coefficient and we have seen that this ductility coefficient is for this particular example is something like around 0.58 on the other hand the slope is given by c and this lies within the value of - 0.5 to - 0.7 of course for both the case of b and c since these are decreasing with the load reversals they are all negative values but what we can see here is that the c has a much steeper slope compared to the elastic part.

So, for the plastic strain amplitude we see the reduction in the values with the increase in the number of load reversals is quite higher or quite steeper compared to the elastic strain amplitude part.

(Refer Slide Time: 23:01)



Material	Condition	σ_y	σ_f'	ε_f'	b	c
			Al Alloys			
1100	Annealed	97	193	1.80	-0.106	-0.69
2014	T6	462	848	0.42	-0.106	-0.65
2024	T351	379	1103	0.22	-0.124	-0.59
7075	T6	467	1317	0.19	-0.126	-0.52
			Steels			
1015	Air Cooled	228	827	0.95	-0.110	-0.64
4340	Tempered	1172	1655	0.73	-0.076	-0.62
			Ti Alloys			
Ti-6Al-4V	Solution Treated + Aged	1185	2030	0.841	-0.104	-0.69
			Ni-Based Alloys			
Inconel X	Annealed	700	2255	1.16	-0.117	-0.75

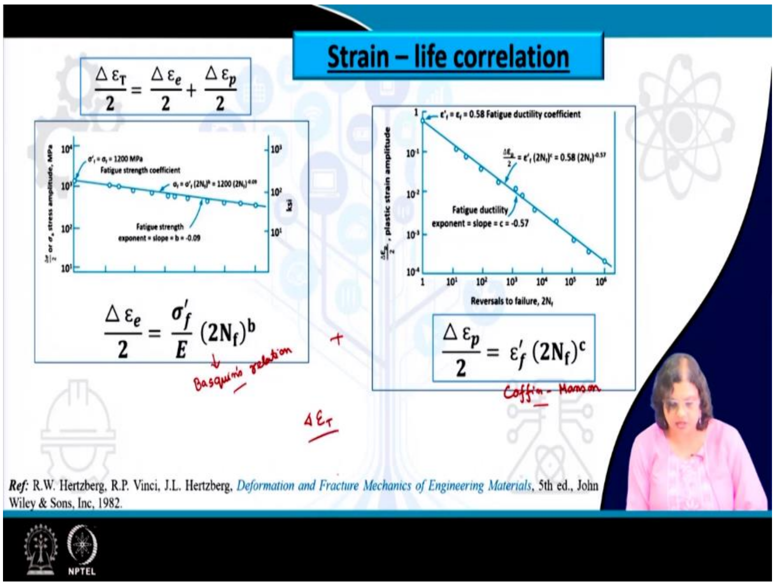
These are some of the examples for some commonly used materials for example the aluminum alloys under different conditions annealed or differently heat treated conditions we can see that it is having different values of yield strength and obviously different values of σ_f' and ε_f' as well as b and c. For any case the values of c as I mentioned is much higher compared to b and we can also see that higher the yield strength of the material.

So, higher is the hardness we are also seeing higher σ_f' but this is not always very religiously followed as for some cases we can see that for the different categories of materials undergoing the same heat treatment for example we see almost similar values of the yield strength but this is not reflected in the σ_f' or the fatigue strength coefficient value is 48 for one case as well as it is much more than that 50 % more than that for the other case, although the yield strength values are more or less same.

So, we cannot simply determine this kind of values just from the monotonic behavior itself we can determine whether a material is prone to undergo cyclic hardening or softening from their monotonic behavior but the exact quantification of the intricate trend of fatigue can be only understood if we are pursuing the test under stress control or strain control behavior, in this case it is a strain control test that we are talking about.

And similarly for the case of steels also we have seen the values of c being quite high and some other examples of titanium and nickel based alloys are also shown for reference just for you to understand or have an idea of the different values of σ_f' or ε_f' that are seen in the commonly used Engineering materials.

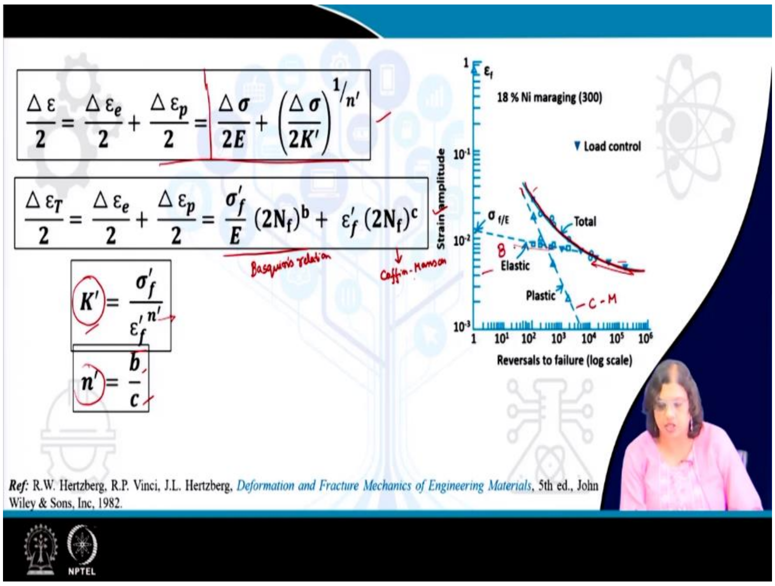
(Refer Slide Time: 25:10)



Now if we want to correlate since we have seen that there are two parts the elastic strain amplitude and the plastic strain amplitude, the elastic strain amplitude is typically given by the Basquin's relation and the plastic strain amplitude versus life that is given by the Coffin-Manson relation.

So, if we try to correlate this we simply have to make a summation of this two strain amplitude levels under elastic and the plastic condition. So, for the Basquin's relation this is what we have seen and we have seen the plastic strain amplitude versus life from the Coffin-Manson relation and we can make a summation of this two to get the total strain amplitude in this case.

(Refer Slide Time: 26:09)



On the other hand we have already seen that if we are talking about the elastic strain amplitude and the plastic strain amplitude we can also figure this out the relation between the two and the total summation as a total strain amplitude based on the Hook's law as well as the Hollomon relation and this is what we have already derived that this is the total value of the strain amplitude that we are getting $\frac{\Delta\sigma}{2E} + (\frac{\Delta\sigma}{2K'})^{\frac{1}{n'}}$ and the K' and the n' signifies the cyclic loading in this case as well as if we are talking about the Basquin's relation and the Coffin-Manson relation then we can also get a summation of this to get us the total strain amplitude.

So, based on this actually we can figure out the total deformation capability of the material one example or experimental data is shown here which shows that up to certain extent actually this is the experimental results that we are getting and the stress strain curve that we are getting the stress versus number of cycles curve.

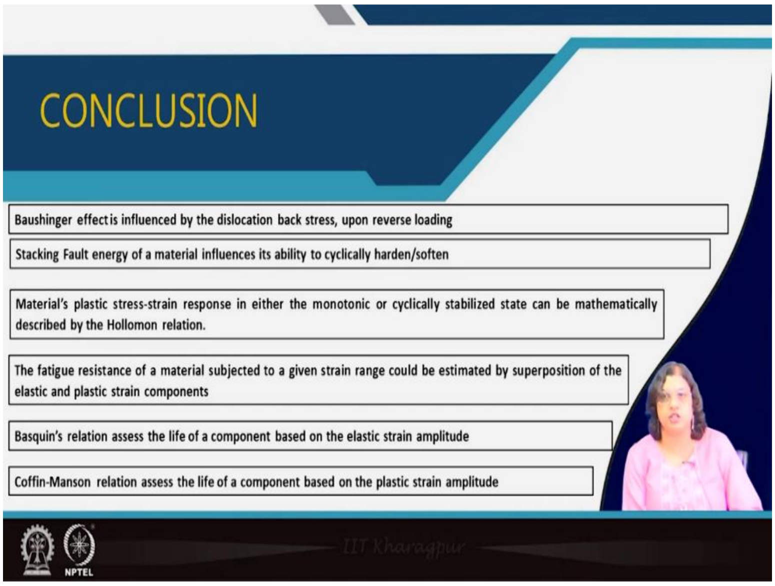
And what we can see here is that this initial part or the one with a higher value of strain amplitude that matches more or less with the plastic relation of the Coffin-Manson relation as well as the elastic part or which is mostly relevant for the low strain amplitude part that is matching or that is more or less resembling the Basquin's relation curve here the elastic curve and if we try to tally these two conditions here based on the Hook's law and the Hollomon relation as well as the Basquin's relation and the Coffin-Manson we also can figure out the relation between this K'.

So, K' is given by the fatigue strength coefficient divided by the fatigue ductility coefficient to the power of fatigue ductility exponent. So, this is the relation between K', σ_f', ε_f' and n' all the different parameters from the fatigue test can be correlated based on summing up the different kind of relations that we are seeing for the elastic and the plastic part and this is very well seen to be tallied with the experimental results as we have seen here.

So, this dashed curve here signifies the Basquin's relation and this one here signifies the Coffin-Manson relation and we have seen that how the total experimental curve can be divided into two parts the initial part with higher strain amplitude is tallying with the Coffin-Manson relation and the lower strain amplitude one is tallying with the Basquin's relation. So, the K' is actually related to the fatigue strength coefficient and the fatigue ductility coefficient to the power of the strain hardening exponent cyclic strain hardening exponent and in turn the cyclic strain hardening exponent is related to fatigue strength exponent and the fatigue ductility exponent.

So, that is how if we are summing up the elastic and the plastic strain amplitude considering not only the Hall-Petch and the Hollomon relation but also the Basquin's and the Coffin-Manson relation we should be able to figure out the interrelation between the different parameters which are used for the cyclic Strain Controlled Fatigue test. So, this is very well tallying with the experimental results also and we have seen that how the highest strain amplitude results are tallying with the Coffin-Manson relation as well as the lower strain amplitude one is tallying with the Basquin's relation.

(Refer Slide Time: 30:20)



So, based on this let us conclude this lecture with the following that Bauschinger Effect is influenced by the dislocation back stress as well as the rearrangement of dislocations upon reverse loading and we have also seen that how stacking fault energy can manipulate the deformation the cyclic deformation behavior particularly if the stacking fault energy is higher it is easier for the dislocations to move and that makes the to attain the cyclic stabilized condition whether it is cyclic hardening or softening that condition can be achieved much faster.

On the other hand, if the stacking fault energy is lower that means that it is difficult for the dislocations to move to cross slip and that makes attaining the stabilized condition quite slower. We have also seen that materials plastic stress-strain response in either the monotonic or the cyclic stabilized state can be expressed with the Hollomon relation on the other hand the elastic part is typically favored by the Hook's law.

Now, fatigue resistance of a material subjected to a given strain range could be estimated by superposition of the elastic and the plastic strain components and the elastic strain amplitude and the life that relation is typically expressed by the Basquin's relation as well as the plastic strain amplitude versus life is expressed by the Coffin-Manson relation.

(Refer Slide Time: 31:56)

So, following are the references which are used for this lecture thank you very much.