Fracture, Fatigue and Failure of Materials
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Lecture 34
Stress Controlled Fatigue (Contd.)

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Hello everyone. We are here in the 34th lecture of this course Fracture, Fatigue and Failure of Materials. And in this lecture also we will be discussing some more about the stress-controlled fatigue.

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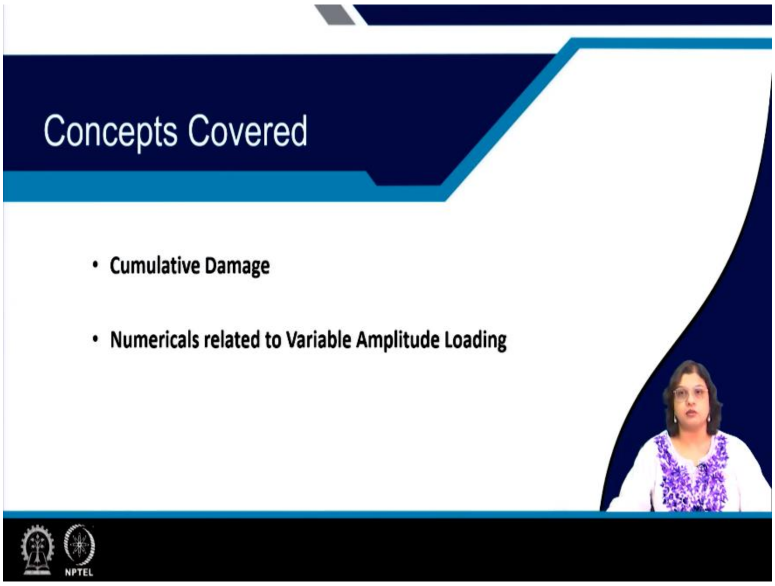

So, the concepts that will be covered in this lecture are the following. We will be talking about cumulative damage. So that means that if we are applying the loading in different stress amplitude levels, we know that fatigue is something in which the damage is being accumulated at every cycle. So, we will now like to see that how if we are varying the amplitude, how will that affect the total number of cycles for failure? And that will be typically determined based on some particular relations and we will be also solving some numericals based on that relation to have our concept even more clear.

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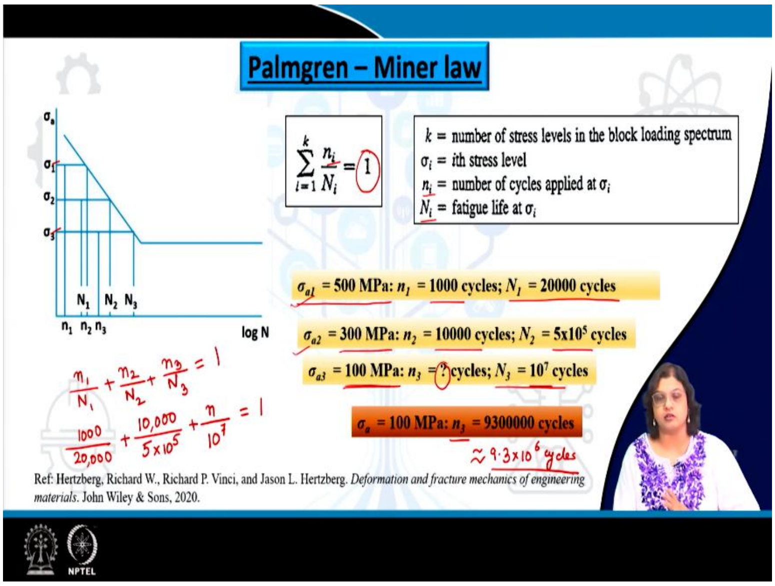

So, let us see what we have discussed in the last lecture is about the Palmgren-Miner's law which says that if we are applying varying amplitude loading then we need to consider the number of cycles at each of this load amplitude which is represented by the small n and the total number of cycles for failure at that particular stress amplitude which is represented by capital N and the ratio of this as well as the summation of the entire number of blocks. So, all this together should come to 1 as per the Palmgren-Miner's law. And based on this we can figure out that how many number of cycles are remaining for a particular stress amplitude level if we know the previous history of loading.

For example, let us say we are applying a stress amplitude of 500 MPa somewhere here and for this we already know that the number of cycles for failure at this particular stress amplitude of 500 MPa is 20,000 cycles. However, we are applying only 1000 number of cycles. That means that we are applying the number of cycles equivalent to 1-20th of the total expected number of cycles at this particular stress amplitude.

So, after that we are reducing the stress amplitude to 300 MPa at which a fresh sample is supposed to survive for 5×10^5 number of cycles. But in this stress amplitude level also we are not doing or we are not cycling it till the end. Rather we are imposing only 10,000 number of cycles and the third level or the third block is at 100 MPa here at which we already know that the expected number of failure for a fresh sample is 10^7. And we need to figure out that how many number of cycles a specimen can survive, provided it has already gone through step 1 and step 2.

So, to solve that, we simply need to use the Miner's law there and we should do it in this way that $\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3}$ the total summation should be 1. So that means what we have here is $\frac{1000}{20000} + \frac{10000}{5\times10^5}$ and this is what we need to figure out. The value of $\frac{n}{10^7}$ and the total summation should be 1. So, if we solve this, you will see that the value of n for the step 3 should be something like 93 lakhs cycles or we can simply represent this as 9.3×10^6 cycles.

So, we can see that although the total number of cycles that a specimen at this stress amplitude level of 100 MPa can survive up to 10^7 number of cycles but because of this previous loading history, where we have applied some highest stress amplitude already to it, so based on the fraction of the life that is remaining in this specimen, it is supposed to survive for only 9.3×10^6number of cycles. So, this is of course very, very important before we put this into service that we should not expect a life of 10^7, rather it has been reduced by some numbers because of the previous history.

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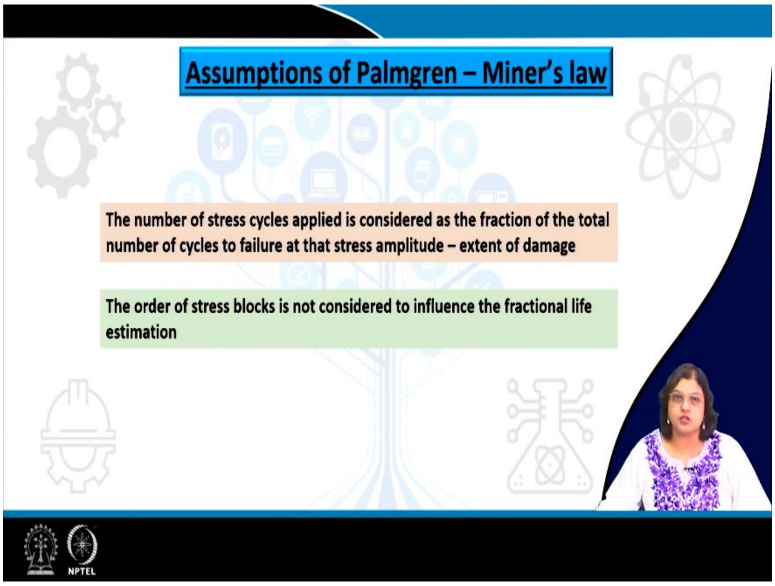

So, there are some assumptions which are already considered in the Palmgren-Miner's law or commonly known as the Miner's law. First of all, we consider that the number of stress cycles applied is the fraction of the total number of cycles to failure. So, we understand that there is some damage accumulation and we consider that for each particular blocks whatever number of fractions of cycles we are applying based on the total number of cycles for failure that fraction of damage is also being accumulated.

Secondly, there is another major assumption which is considered here is that the order of the stress blocks, right, whether we are applying a higher block first, higher stress amplitude first followed by a lower one or a lower stress amplitude followed by a higher one that sequence is not being considered in the Miner's law.

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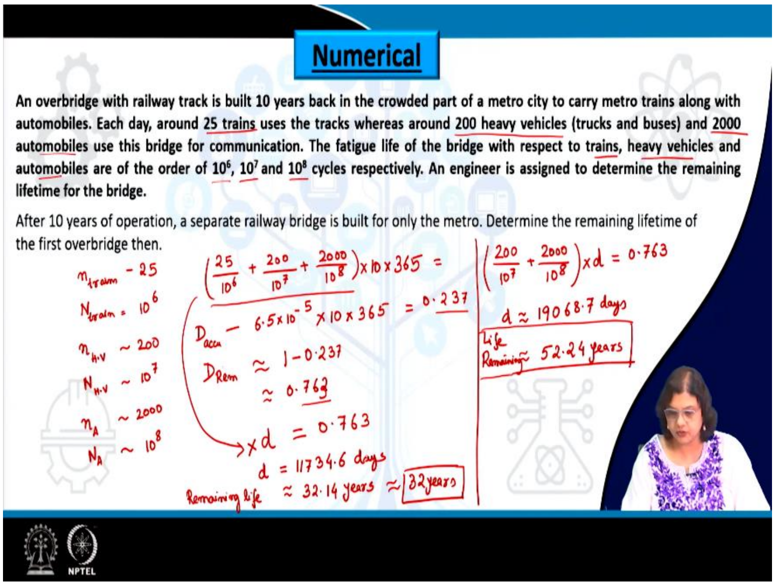

So that may lead to some variation in the results that we will see later. But first of all, let us solve a numerical based on the typical Miner's law and we will see here through this numerical that how the Miner's law would be very relevant and important while designing any engineering structure.

The problem here is based on an overbridge which also has a railway track as well as the roads that is built 10 years back in the crowded part of a metro city and that is supposed to carry the metro trains along with the automobiles. So typically, each day around 25 trains are using this bridge and there are around 200 heavy vehicles such as trucks and buses and all such things and around 2000 automobiles or light vehicles, cars and other vehicles, they use the bridge for communication.

Now, fatigue life of the bridge with respect to each of these different kind of vehicles such as the trains, heavy vehicles and automobiles are of the order of 10^6. So, 10^6 is the total number of times a train can travel or 10^6 number of trains can travel through the bridge before it fails, so that is typically for the trains.

For the heavy vehicles, this number is 10^7 and for light vehicles or automobiles, this number is 10^8. So, an engineer is assigned to determine the remaining lifetime for the bridge. The bridge is already in service for 10 years and we need to figure out that how many more years it can survive. So, let us solve this for the different cases.

First of all, for the trains, we know that for each day the number of trains that uses the bridge is around 25 and the total number of times that a train can cross through this bridge is 10^6. On the other hand, same for the case of heavy vehicles. Let us name this as small n and HV stands for heavy vehicles. So that is around 200 and capital N_{HV} is 10 to the power of 7. Similarly, n for the auto is 2000 and capital N is 10^8. So, we can do this using the Miner's law.

So, this should be $\frac{25}{10^6} + \frac{200}{10^7} + \frac{2000}{10^8}$. Now, this is for one particular day and we know that the bridge has already been in service for 10 years. So that means for 10×365 days. So, we need to figure out what this number is so that we can see that how many number of cycles or what is the fraction of life that has already been used up. So, if we calculate this, it comes around 6.5×10^{-5}, this fraction itself. So, that multiplied by this factor is 0.237.

If that is the case, that means this fraction has already been used up and what we have in remaining is 1 minus this factor. So, let us name this as the total accumulated damage. So, damage accumulated is given by this factor. So, remaining D remaining will be 1 minus this factor. So, that will be something like 0.762 or round about with 0.763. So, if such is the case, we can then figure out by doing this factor again that how many number of cycles it can survive. So, we can simply insert this relation and along with this factor, we can also include the number of days that will lead to this number of 0.763. And if we can do that, you will see that the total number of days that it can survive is something like 11734.6 and if we divide that with 365, you will get a value of 32.14 years.

Now, make sure that you are not overestimating the number of days or number of years. You might be tempted to use more digits after the decimal. But as I mentioned earlier also we have to be a little bit using the conservative approach. So, instead of 32.14 or something years, we can simply mention this as 32.1 years or to be on the safe side, that 32 more years it can survive. But it will not be appropriate for me to consider for example, let us say this number of days, we should not make this as 11735 because there is a digit more than 5 after the decimal. So, we should not round this about to 35 at the end because even if we do that, I mean, one day can lead to a catastrophic failure, and we should be very much careful about this. And we are talking about the real engineering applications of a material based on the fatigue life.

So, from applying this Miner's law, we could see that the remaining life of the bridge is something like 32.1 years or 32 years to be more conservative. Then the second part of this problem says that after 10 years of operation, a separate railway bridge is built for only the metro. So, that means that carries only the metro trains and excluding the heavy and the automobiles. So, determine the remaining lifetime of the first overbridge.

Now, the first over bridge is meant for carrying only the heavy vehicles as well as the automobile. So, that means that this fraction of life which is still remaining with the first overbridge then will be used up for carrying only the heavy vehicles as well as the automobile. So, we can do this part once again by employing this relation, but it will be much more simpler this time.

What we have now is $\frac{200}{10^7}+\frac{2000}{10^8}$. And this multiplied with the number of d or days that should be equivalent to 0.763. So, if we do that, we will see that d comes around 19068.7 days and if we divide that with 365 that means that this will be equivalent to 52.24 years.

So, you can see that if we have a second bridge for carrying only the metro trains, then we can increase the life of the first bridge by almost 20 years. So, that is quite some enhancement in the life and we often need to channelize the different kind of vehicles used for a particular bridge in case we know that we are running out of time or the total life is already about to be over. So, this is the remaining life for the next part of the problem.

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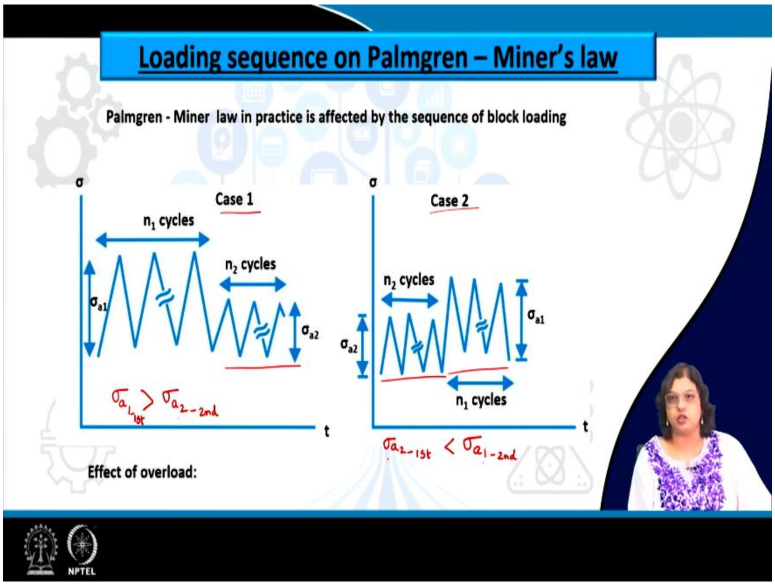

So, now let us look into the Palmgren-Miner's law and how it can be affected if we are considering the sequence of the block loading. So, we know that Miner's law is particularly valid for variable amplitude loading, fine, but if we are loading the highest risk amplitude first followed by the lower one versus the opposite conditions, there should certainly be some effect. So, these are the two cases which are shown here.

Case one shows that we are applying highest stress amplitude first followed by a lower stress amplitude. So, σ_{a1} here is greater than σ_{a2}. On the other hand, case two is also the opposite or the reverse of this. In this case, we are applying sigma a_2 first which is of lower magnitude and this is followed by the higher magnitude of sigma a_1. So, in this case we are applying, let us write this as step first and this is as second.

In this case, however, we are applying, it looks the same like sigma a_2 but now this a_2 is the first one and that is less than sigma a_1. Now the magnitude of this sigma a_2, sigma a_1 are the same as case one. But the difference between case one and case two is the sequence in which a_1 and a_2, sigma a_1 and sigma a_2 are being used for the loading and that may influence the Miner's law and the fraction of life estimation.

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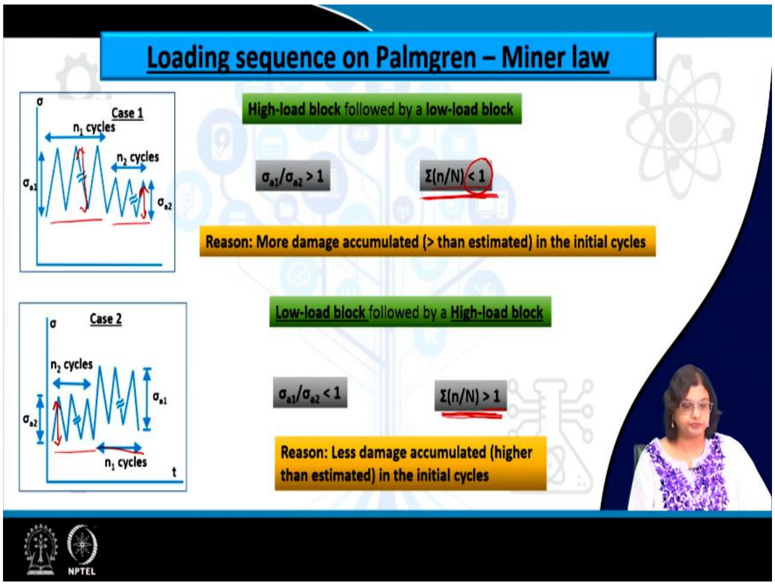

So, let us see how it does that. For the case of one condition one, when we have the higher loading sequence, actually that will lead to the $\sum({}^{n}/_{N}) < 1$. So, this is important to see that instead of 1, it actually should be reduced from unity. And that is particularly because of the fact that when we are applying the highest risk amplitude, more damage is accumulated.

So, that means that whatever damage is expected to occur more than that is being occurring or accumulating because we are applying higher stress amplitude, a component is more prone towards failure and whatever we know that there are a lot of scatter in the fatigue data and any kind of defects which are already present in the component but which we are not aware of. So, such kind of defects can act in a much more detrimental manner if we are applying higher stress amplitude. So, that led to the overall a little bit of higher accumulation of damage compared to the lower stress amplitude one.

And because of this higher extent of the damage accumulation, we end up having the value of this summation less than unity. So, that means that on the other hand, if we are applying the first sequence with lower stress amplitude and the second sequence of this load block with the higher stress amplitude, that will lead to a condition when the $\sum({}^{n}/_{N})$, that turns out to be greater than 1. So, this is based on the fact that if we are applying lower stress amplitude, then the total damage accumulation could be even lesser than whatever is expected.

We consider that as per the Miner's law it has been considered that for each kind of whatever is the loading amplitude same amount of damage will be accumulated for even a higher stress amplitude or a lowest stress amplitude which of course is not very realistic approach and we are taking care of that in this modification where we have seen that if we are applying the higher load sequence first then that will reduce this ratio less than unity and if we are applying the lower load sequence one then we are having this value could be greater than 1.

Now we have to be a little bit careful here also. We just mentioned this as higher and lower as the qualitative term. Now how high this or how low is this based on the actual fatigue strength of the material that is very important. If we are talking of this sequence of very high magnitude then only this condition will be valid. And similarly, if we are talking of for the case 2, if we know that this amplitude at the first loading is very-very less which means that it is not able to accumulate damage at all that can lead to such condition of n/N the summation being more than unity.

Because as per Miner's law we still assumed that it is supposed to have some amount of damage to be accumulated which may not be true in the actual practical situation. So, again fatigue being a conservative approach, we are mostly concerned with such kind of condition when we know that the total summation could be less than 1 and we have to be extra cautious in solving such kind of design problems. If it anyway turns out to be more than 1 which means that the fatigue life should be more than whatever is estimated. In that sense we are still on the safer side. Although in reality apart from using the conservative approach we have to be a little bit careful about the expenses as well and that also in most of the cases that also is determining factor which kind of approach should be used for practical significance.

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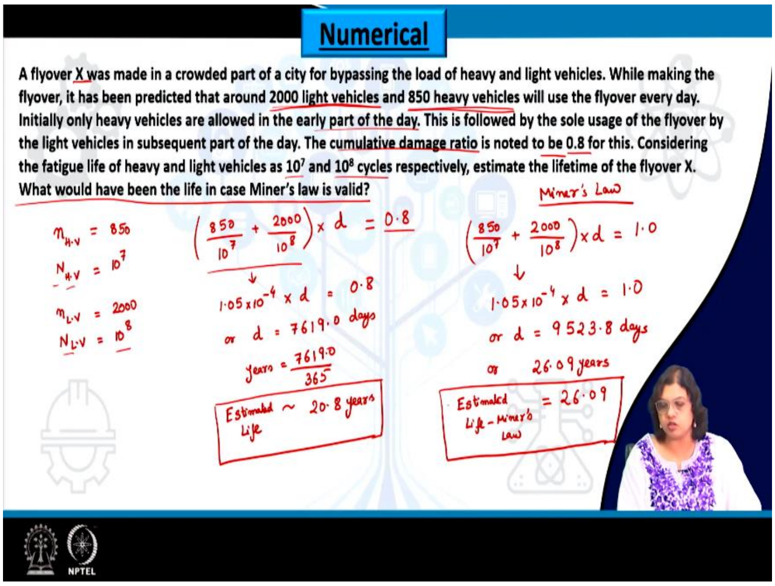

Let us solve another numerical to see how if we are modifying this Miner's law how that can be utilized for the actual practical application. So, in this case also we are talking about a flyover X which is made in the crowded part of the city for bypassing the load of the heavy and the light vehicle. So, we are not talking about the trains anymore. Let us focus only on the heavy and light vehicles that runs on the flyover in a crowded city.

Now while making the flyover, it has been predicted that around 2000 light vehicles and around 850 heavy vehicles will use this flyover every day. Initially only heavy vehicles are allowed in the early part of the day and this is followed by the sole usage of the flyover by the light vehicles in the subsequent part.

So, you might have already seen that kind of thing and the late night or the early morning are used by the heavy vehicles such as the loaded trucks and all and then mostly the prime time is being used by the light vehicle. So, similar principles are being followed here. Now, based on this kind of application it has been seen that the cumulative damage ratio is reduced from unity and this has come to a value of 0.8. So, considering the fatigue life for the heavy and the light vehicles as 10 to the power of 7 and 10 to the power of 8 cycles respectively, so same as the previous numerical also. Now we need to find out the lifetime of the flyover X.

So let us do this one more time. So, in this case we have the number of heavy vehicles is 850 and the capital N. So, that means the life cycle for heavy vehicle is 10^7. So, a heavy vehicle can run for 10^7 number of times in a flyover or in the contrary, we can also say that 10^7 number of heavy vehicles can run only for one time in that flyover and the bridge the flyover can collapse.

For the case of light vehicles, this number per day is 2000 and capital N for the light vehicles is 10^8. Also, it is important to note here that since heavy vehicles can survive for lesser number of times, we also should note that capital N is lesser for the heavy vehicles compared to that for the light vehicles. So, if we are imposing the law here, it should be something like this. So, $\frac{850}{10^7}+\frac{2000}{10^8}$ and this needs to be multiplied by the number of days that will lead to not 1. Now we have 0.8.

So, if such is the case, we can solve this and we can see that this factor is coming around $1.05 \times 10^{-4} \times d$. So, that is equivalent to 0.8 or d will be something like 7619.0 days and we can convert this as a number of years. So, years could be if we are simply dividing this with 365 so that will turn around 20.8 years. That is the estimated life of the bridge or the flyover.

So, this time we have not estimated after 10 years of its service but at the beginning we wanted to estimate the life. That is why it is not the remaining life, it is the estimated life which comes around 20.8 years. Again, make sure that you are not rounding it up to 21 years, that will be wrong. So, this is just simply 20.8 years, this bridge can survive. Now, this is based on the modified Miner's law. When we have considered that this factor is not unity but it is lesser than unity. The second part says that what would have been the life in case Miner's law is valid. So, if we are applying the typical Miner's law, we can see that we can use this same relation. Only instead of 0.8, we can write this as 1.

So let us do this one more time and if we are doing this, we can see that this comes around the same, the fraction should be the same. And if we do that, the total number of days now we are having is around 9523.8 days. So, this is equivalent to 26.09 years. So, had we been calculated the life estimated the life based on Miner's law, we see that the life now is quite higher by around 6 years if we have considered the typical standard Miner's law.

Certainly, if we are expecting the life to be 26 years and then it collapse after 20 years, that is something which should not be done. So, we should be very much cautious about this and when to employ what kind of relation should be known to the design engineer so that no such kind of catastrophic event should happen.

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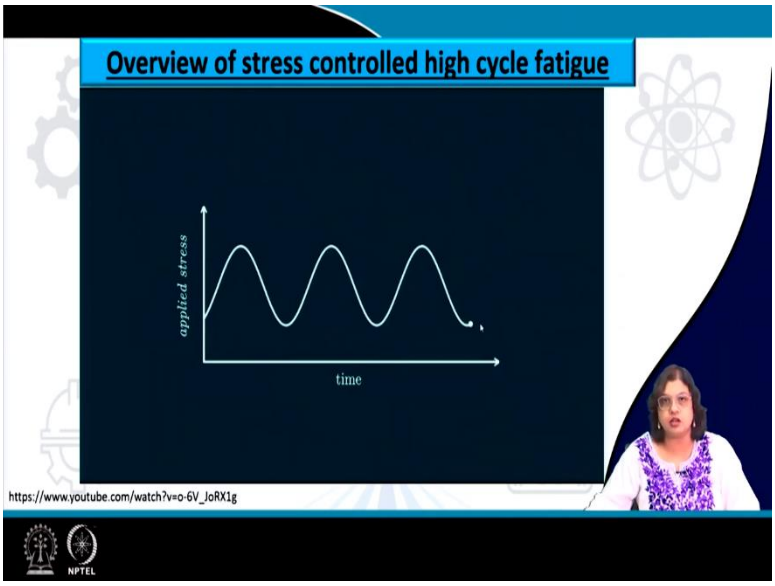


Overview of stress controlled high cycle fatigue
FATIGUE
FAILURE
90%
of mechanical
engineering failures
https://www.youtube.com/watch?v=o-6V_JoRX1g
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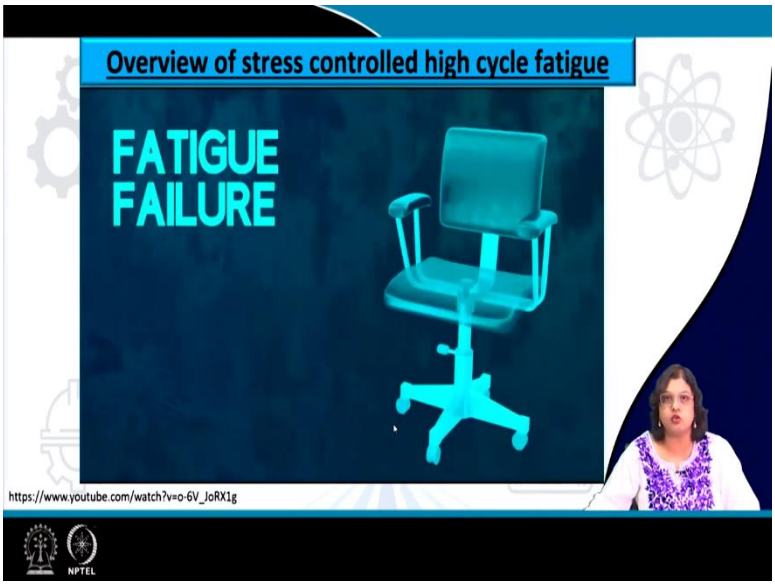
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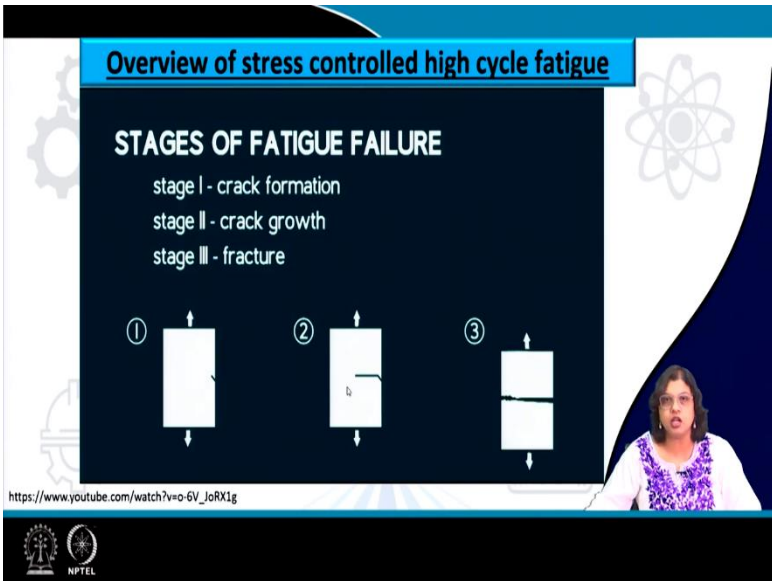

So the overall stress-controlled fatigue is being explained in a summary in this video here which shows that if we are applying such kind of cyclic loading when the load is varying with time, then it is supposed any kind of material is supposed to have fatigue and finally it can fail. So, we have seen that for most of the cases around 90 percent of the overall failure is because of fatigue loading. And you can see that the chairs that we use of the bicycles, the crank shafts that we use, all can fail due to fatigue failure. So, the stages of fatigue failure is when there is any kind of defects at the surface which can trigger the crack initiation and once the crack is initiated, it then grows up to a certain extent if there is a repeated loading, finally it reaches some particular length and then it leads to the catastrophic fracture.

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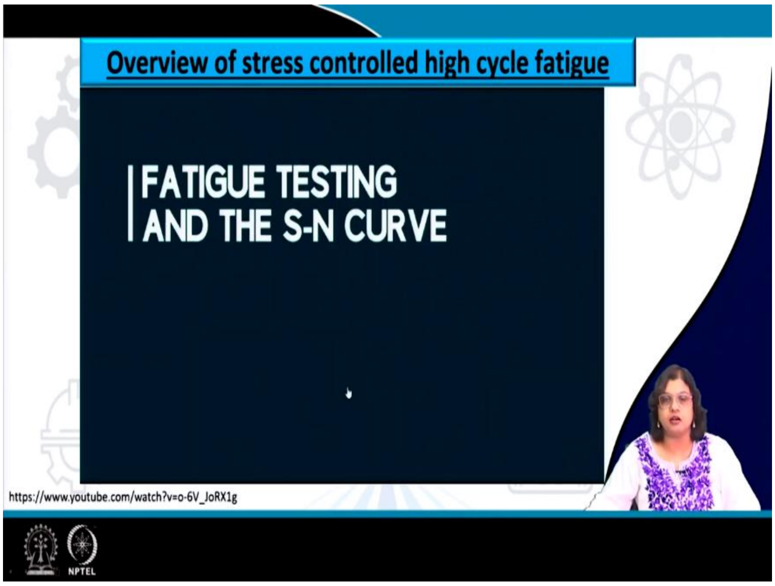

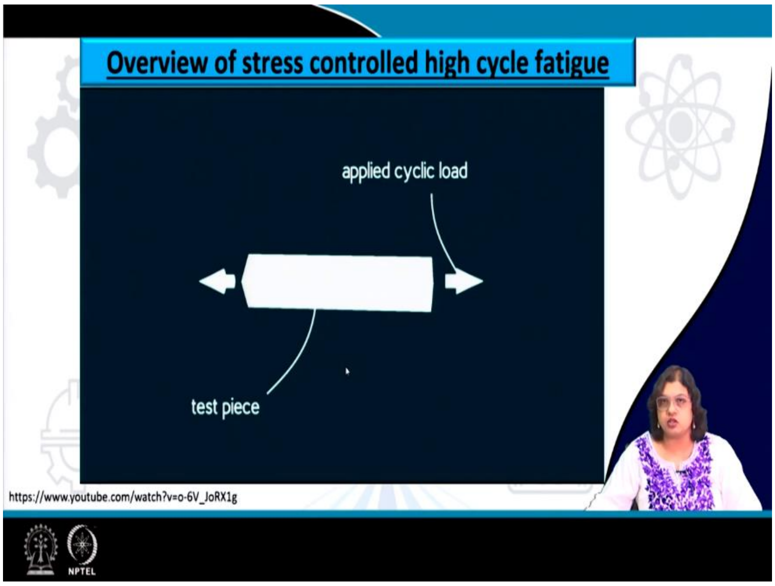
Overview of stress controlled high cycle fatigue
applied cyclic load
test piece
https://www.youtube.com/watch?v=o-6V_JoRX1g
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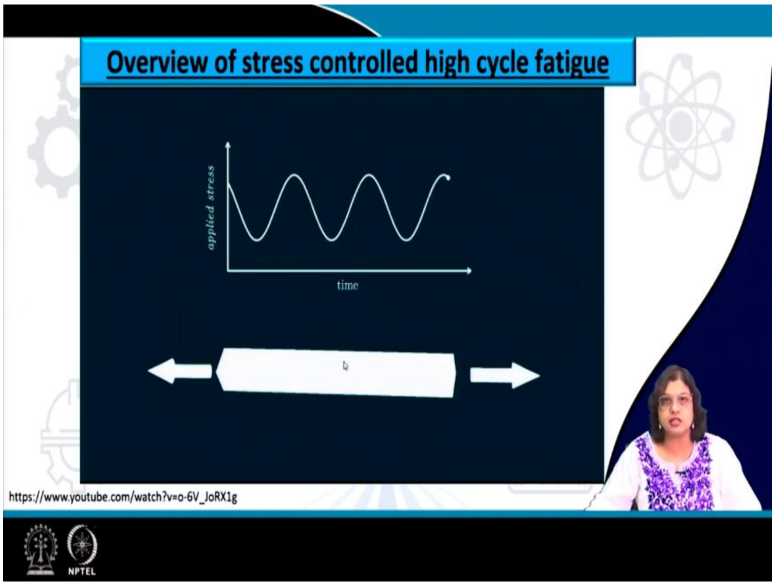
Overview of stress controlled high cycle fatigue
applied stress
time
https://www.youtube.com/watch?v=o-6V_JoRX1g
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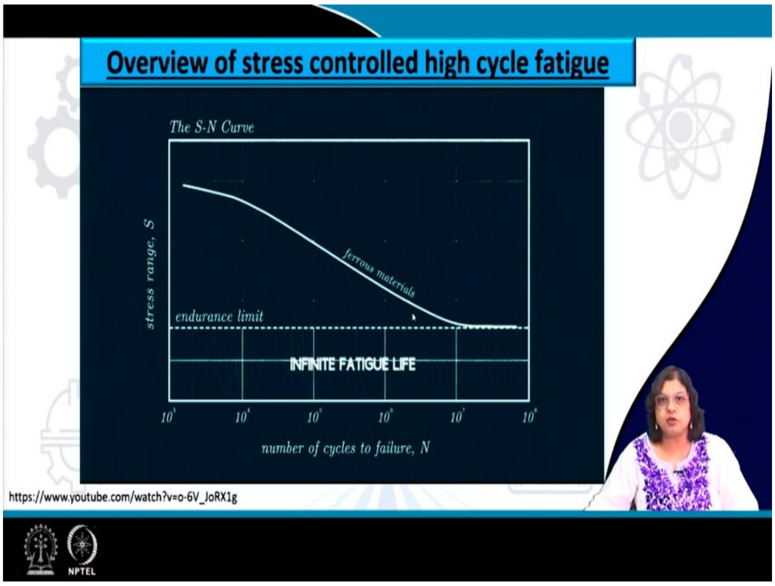
Overview of stress controlled high cycle fatigue
The S-N Curve
stress range, S
ferrous materials
endurance limit
INFINITE FATIGUE LIFE
10^3
10^4
10^5
10^6
10^7
10^8
number of cycles to failure, N
https://www.youtube.com/watch?v=o-6V_JoRX1g
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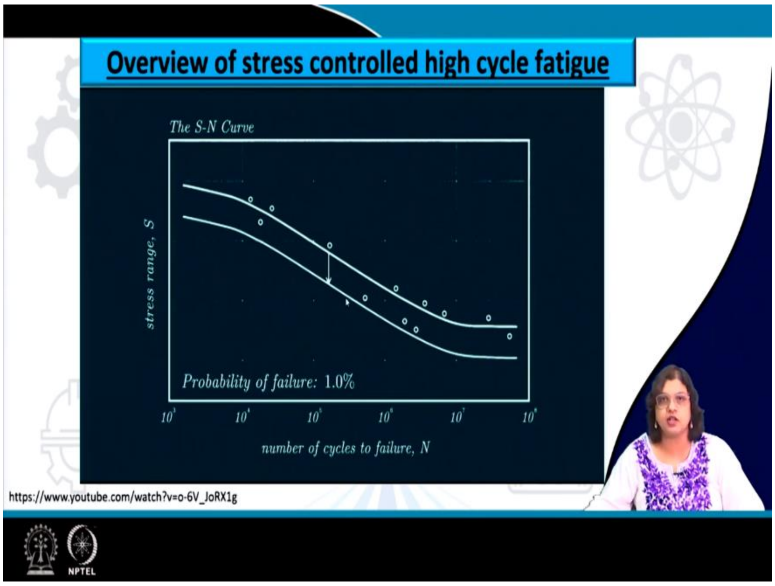

So, we have to be very careful at each of these stages and we can determine the fatigue life based on the S-N curve that we have seen. So how this is done, typically we have a test piece on which we apply the cyclic loading. We do tension and compression or we do tension and tension. In this case it is a tension and tension kind of loading. We are using a higher magnitude and a lower magnitude, a sufficient number of cycles so that it fractures.

And we represent this kind of results in the form of an S-N curve where the y axis could be the stress and the stress. In this case it is the stress range but we typically use the stress amplitude or the alternating stress which is just half of the stress range and the x axis is the number of cycles to failure. Typically, we use a logarithmic scale here. This curve is known as the SN curve or the roller curve and we get a curve data points which is compiled using many, many specimens, something like this. So, this is used for predicting the life of a component.

For example, for a particular stress range, as shown here is 100 MPa we get a life of 500,000 cycles and if we consider that it survives for one cycle per minute, that means it can survive up to one year. So based on this, we can figure out the number of remaining cycles. For ferrous materials we have a typical curve like this whereas for non-ferrous materials it can keep on growing and we can determine the fatigue strength based on some particular cycles of 10^{7-8}.

For the ferrous materials, this is quite straightforward. So, we have this endurance limit, below which if you are applying stress level, it is supposed to have infinite life but for regular material it is very important to consider the scattering the data, because we do not want to do any kind of overestimation. And this can be possible if we take the scatter into account and we do the standard deviation.

I have also explained how there are standards ASTM standards to determine the fatigue strength or fatigue life. Typically, instead of the best fit curve, we include the ones which are having the lower life or the lower strength at any particular life based on the standard deviation. And we shift the curve downwards so that we can determine this based on the probability of failure. So, in this case, the probability of failure is only 50 percent. So, half of the specimen is supposed to survive. We can make this close to 1 that means all will survive or there will be no failure at all if we take that into account.

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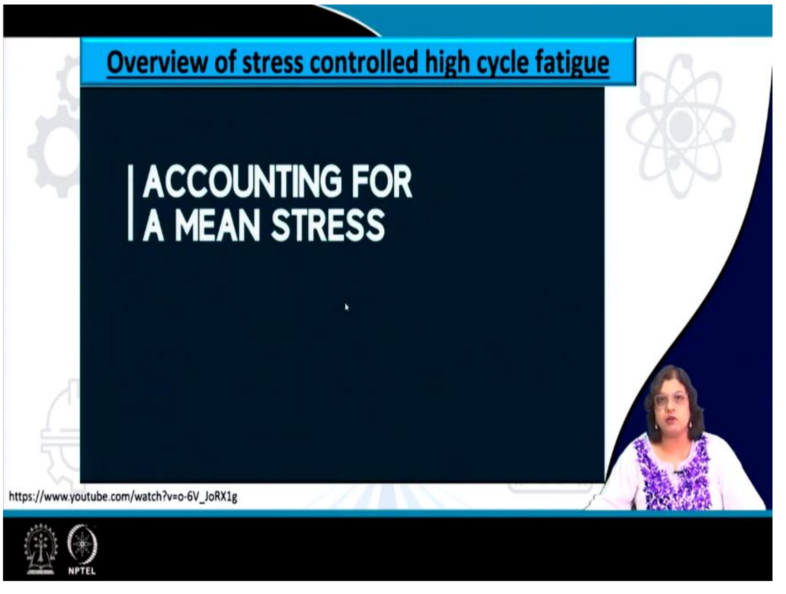
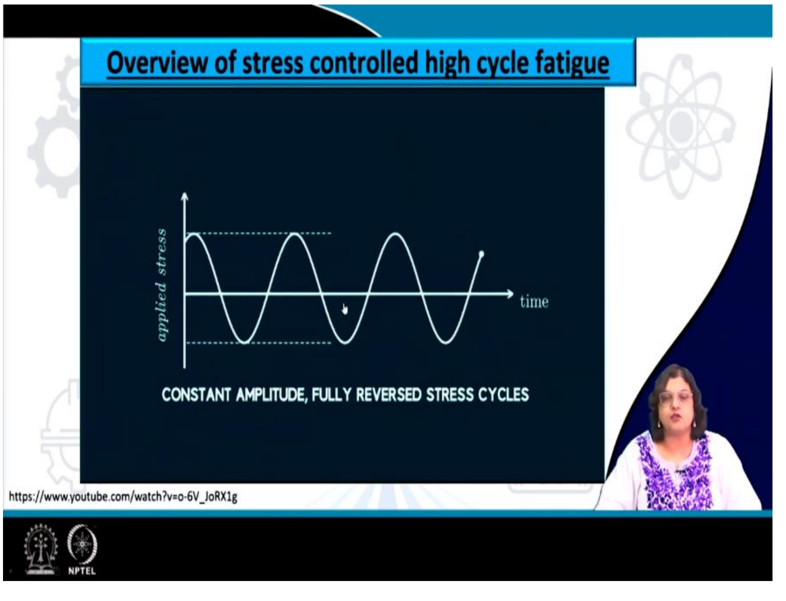
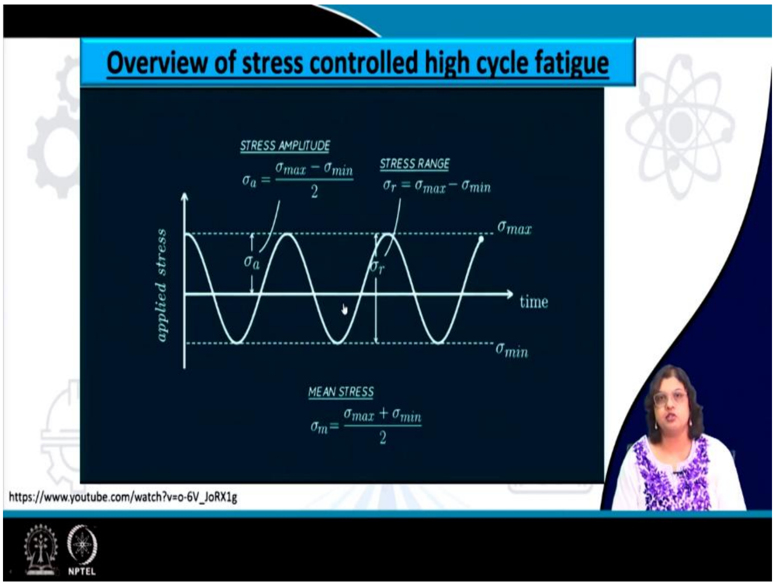
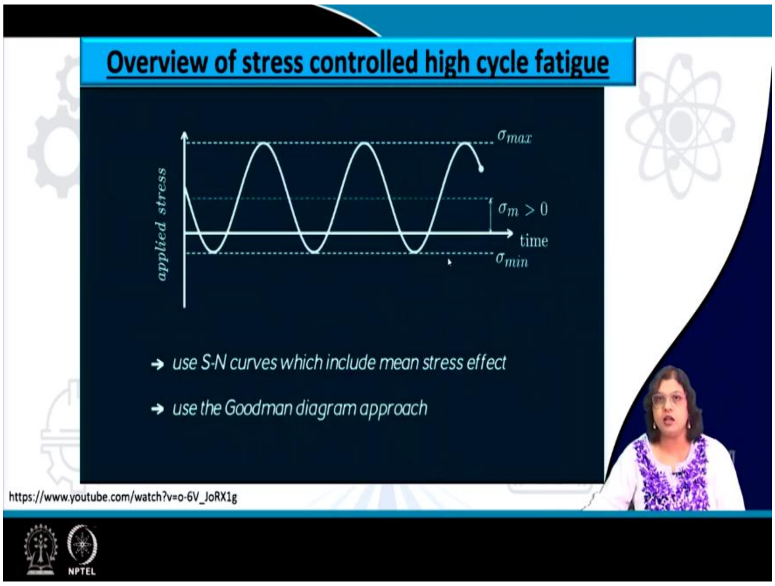
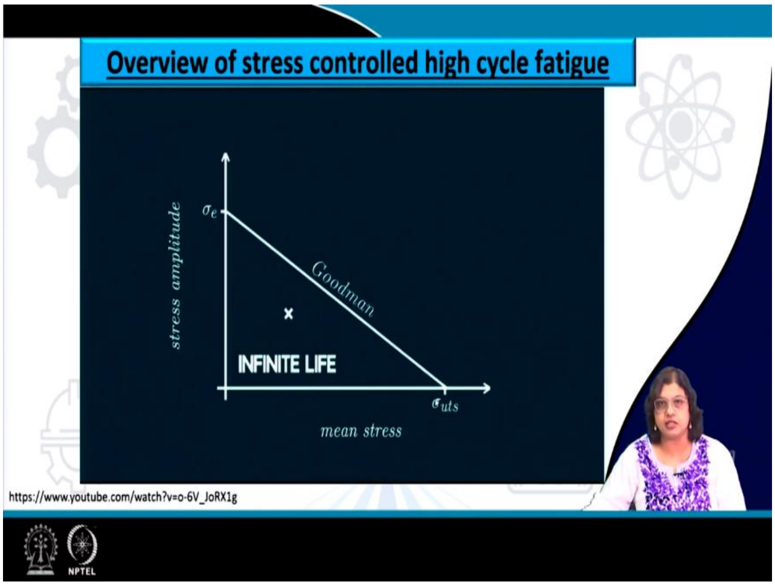
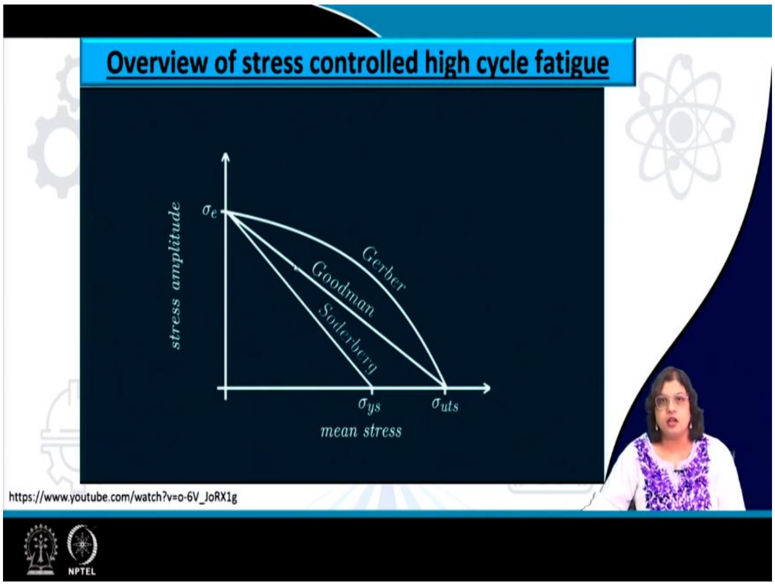





Overview of stress controlled high cycle fatigue
STRESS AMPLITUDE
$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$
STRESS RANGE
$\sigma_r = \sigma_{max} - \sigma_{min}$
applied stress
σ_{max}
σ_a
σ_r
time
σ_{min}
MEAN STRESS
$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$
https://www.youtube.com/watch?v=o-6V_JoRX1g
NPTEL


Overview of stress controlled high cycle fatigue
applied stress
σ_{max}
$\sigma_m > 0$
time
σ_{min}
use S-N curves which include mean stress effect
use the Goodman diagram approach
https://www.youtube.com/watch?v=o-6V_JoRX1g
NPTEL


Overview of stress controlled high cycle fatigue
σ_e
stress amplitude
Goodman
INFINITE LIFE
σ_{uts}
mean stress
https://www.youtube.com/watch?v=o-6V_JoRX1g
NPTEL



Now, mean stress is another important factor which controls the fatigue life. This one is a perfect tension compression and we have seen that it uses the exact amplitude as that is for the maximum stress and the minimum stress. The dimensions are the same but the signs are different. And this as we have already seen that this can be typically defined with the stress amplitude or the mean stress. In this case, the mean stress is zero. So that is the best-case scenario as we know that the compressive part is actually beneficial.

Now, if we are reducing the compressive part of this section, although the stress range remains the same, in this case you can see that the mean stress is having a positive value greater than zero. It can have in the tension-tension mode also and that may lead to further damage.

So, what we have seen is that this mean stress, if we are applying the higher the mean stress, lower will be the number of cycles that it can survive, lower will be the fatigue strength, okay for any particular number of cycles. So, this can be very well understood based on the Goodman diagram where the y axis here signifies the endurance limit or the fatigue strength and the x axis, this point here signifies the ultimate tensile strength.

And so that means that if we are doing the test or the component is being applied to certain combination of mean stress and stress amplitude within this part, then it will have infinite life.

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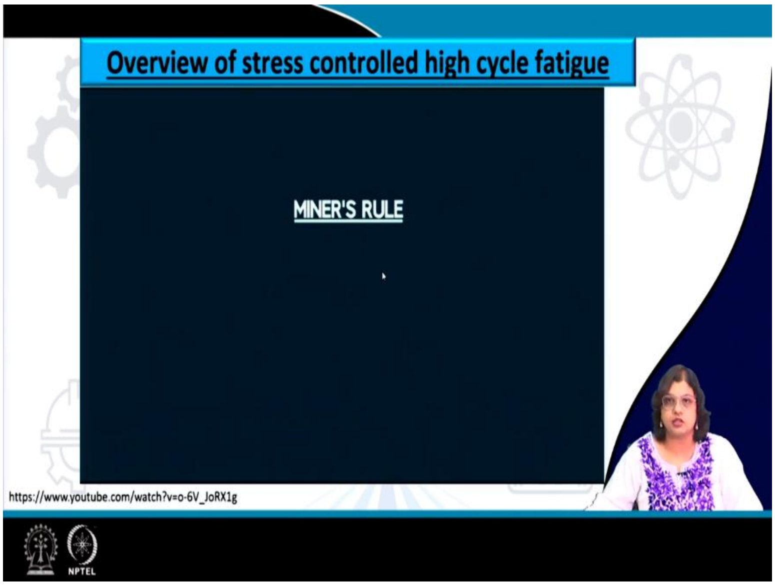

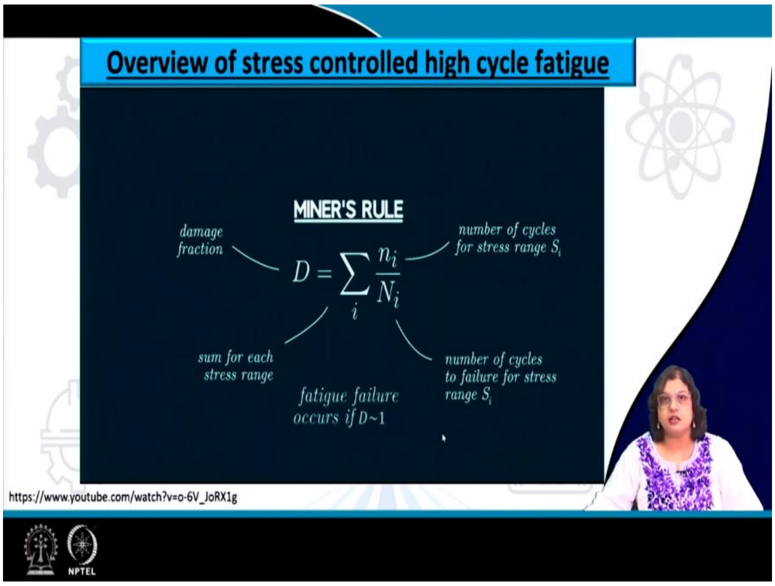

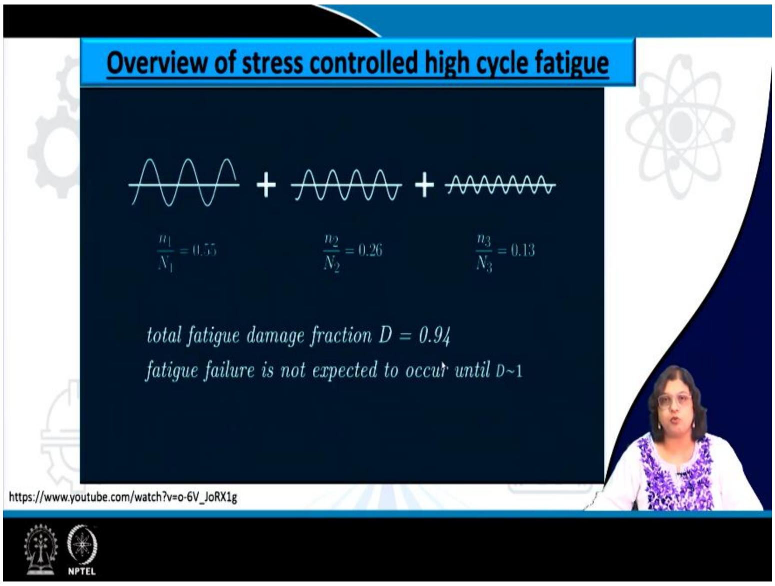

There are other ways to solve this like the Soderberg and the Gerber rules that we have already shown. Now Miner's rule is the one that we have just discussed in this lecture which takes into account of the variable amplitude loading and based on the summation for the ratio of the number of cycles for certain stress amplitude or stress range divided by the total number of cycles to failure at that particular stress range. And this value of d at the total damage fraction should be one at the point of failure.

So, here are some examples which has been shown which says that the total damage is 0.94 and the fracture can occur only when d = 1 and we can determine the number of cycles based on such rules. So, this is just a summary of what has been discussed about the stress control fatigue so far, so that in the next lecture we can move towards the strain control fatigue.

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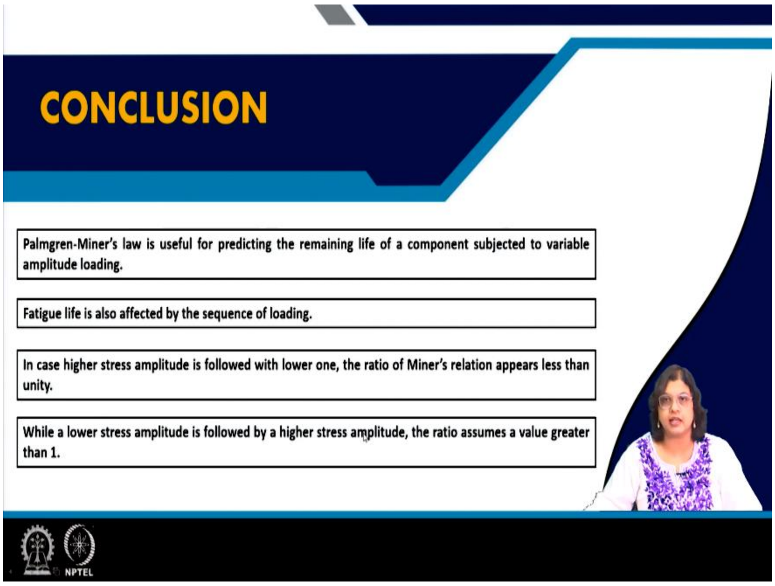

So, let us then conclude this lecture with the following points that Palmgren-Miner's law is useful for predicting the remaining life of a component subjected to variable amplitude loading. Fatigue life is also affected by the sequence of loading, whether we are applying the higher blocks of highest stress amplitude or the lower to that matter. And in case the highest stress amplitude is followed with the lower one, the ratio of Miner's relation appears less than unity or in case we apply the lowest stress amplitude block first followed by a higher stress amplitude, in that case the ratio of smaller n to capital N that can attain a value greater than unity.

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Following are the references that have been used for this lecture. Thank you very much. Bye.