Fracture, Fatigue and Failure of Materials
Professor Indrani Sen
Department of Metallurgical and Materials Engineering
Indian Institute of Technology Kharagpur
Lecture 33
Stress Controlled Fatigue (Contd.)

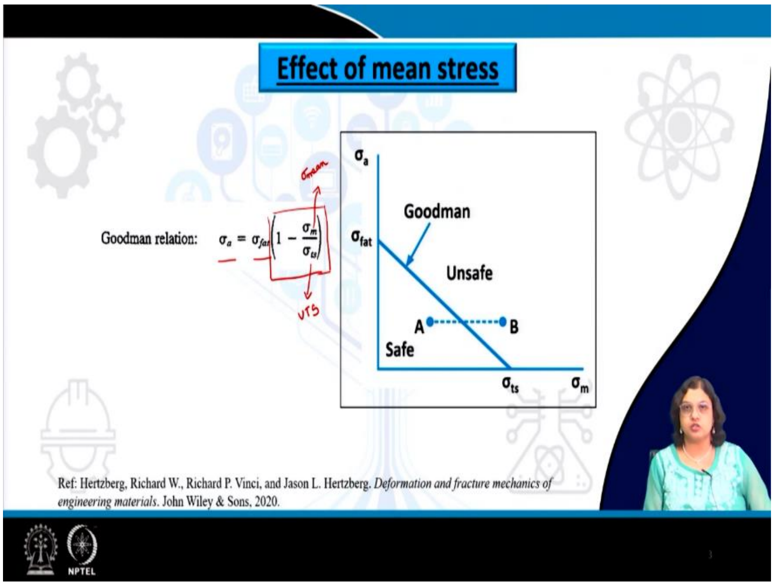
(Refer Slide Time: 00:29)

Hello. Here we are at the thirty-third lecture of this course Fracture, Fatigue and Failure of materials. And in this lecture also we will be continuing on the topic stress-controlled fatigue.

(Refer Slide Time: 00:40)

The particular topics which will be discussed in this lecture are the following. We will be starting with some numericals on the effect of mean stress on the fatigue life and then we will be talking about another very important concept which is very much valuable for the stress control fatigue which is in case we have the variable amplitude loading, how the behaviour will change and we will see that how the cumulative damage leads to the fatigue failure.

(Refer Slide Time: 01:12)

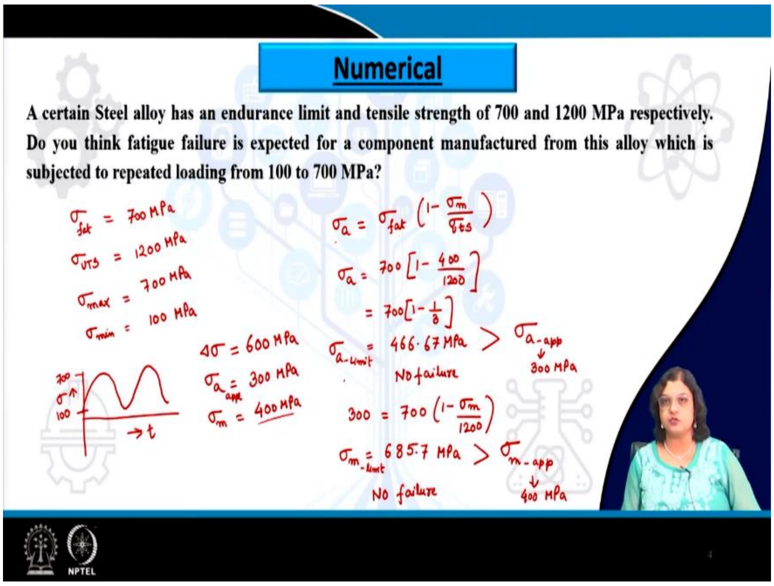


So, let us start with where we have left in the last lecture. We were talking about the Goodman diagram and the effect of mean stress as well as the relation between mean stress as well as stress amplitude with respect to the fatigue strength and tensile strength of the material. And we have seen that for most cases the stress amplitude is related to the fatigue strength multiplied by this parameter here, 1 minus σ_m by σ_{ts},

$$\sigma_a = \sigma_{fat} (1 - \sigma_m / \sigma_{ts})$$

where σ_m is the mean stress and σ_{ts} is the ultimate tensile stress.

(Refer Slide Time: 02:00)



Now on this basis, let us solve a few numericals to get our concepts more clear and here it is. So, what it says is a certain steel alloy has an endurance limit and tensile strength of 700 and 1200 MPa respectively. Do you think fatigue failure is expected for a component manufactured from this alloy which is subjected to repeated loading from 100 to 700 MPa?

So, what has been given to us already is the endurance limit which is nothing but the fatigue strength of the material. So, let us write this down. σ_{fat} is 700 MPa and σ_{UTS} or σ_{TS} is 1200 MPa. What we can also see from here is σ_{fat} is sufficiently less significantly lesser than that of the tensile strength of the material but the ratio is still quite high with respect to whatever table that we have seen in the last lecture.

Now this alloy, this specimen is being subjected to repeated loading with the maximum stress of 700 MPa and minimum stress of 100 MPa. So that means that it is a tension-tension fatigue and what we can see here is that it starts from some tensile stress and it keeps on like this. So, this extent here is 700 MPa and here we have 100 MPa.

The y axis is σ and the x axis in this case is time or we can also say this as the number of cycles. So, let us find out all the parameters at the very first hand. So, first of all, for this case we have $\Delta\sigma$ or the range is 700 - 100 because both are on the tension side. So, we have the stress range as 600 MPa and that makes our stress amplitude half of the stress range which is equivalent to 300 MPa.

Mean stress, however, is (700 + 100) / 2. So, that makes it 400 MPa. So, if we are using the Goodman relation here which says that $\sigma_a = \sigma_{fat} (1 - \sigma_m / \sigma_{UTS})$, Now, you may think that since all the parameters are already known to us, we know σ_a, we know σ_{fat}, we know σ_m as well as

σ_{UTS}. And what do we need to find out? Actually, we can find out any of these parameters keeping the other parameters as seen from here.

For example, let us say we are talking about the limiting stress amplitude, So, we put all the values from here except σ_a. And we need to find out that as per Goodman diagram, what would be the maximum value of σ_a that we can put and whether that value is matching or smaller or higher than the one that is subjected to it. So, let us talk about σ_a at the very first instance. Let us see how much this turns to. So, we know that the fatigue strength is 700 MPa and this is 1 minus σ_m is 400 and σ_{UTS} is 1200 MPa. So, that makes it as

$$\sigma_a = 700\ (1 - 400 / 1200)$$

$$= 700\ (1 - 1/3)$$

$$= 466.67$$

You do not need to be very specific or you do not need to add more than two digits after the decimal. Please make sure that you are not including eight digits after the decimal which you can get from your calculator because this does not make sense. It is also if we can simply write 466.6 MPa. Fine.

Now, what we are seeing is that this is the limiting σ_a, or let me write this as $\sigma_{a\text{-limit}}$ and that is 466.67. However, the applied σ_a so if we name this as the applied because ultimately, we are applying the loading sequence from 100 to 700 and that means that the applied σ_a is 300 MPa. So, what we are seeing is that the applied σ_a is actually much lesser than the limiting value. So, that means that we are in the safe zone. So, this is greater than the σ_a that has been applied and that means that we are in the safe zone and there should not be any failure in it.

Now, that is one approach. Some of you may also try on the other way when we are inserting the σ_a value and we want to find out the limiting σ_m value. So, let us try that also. So, σ_a in that case will be 300 σ_{fat} is 700 and 1 minus σ_m. So, σ_m is the unknown here, and σ_{UTS} once again is 1200.

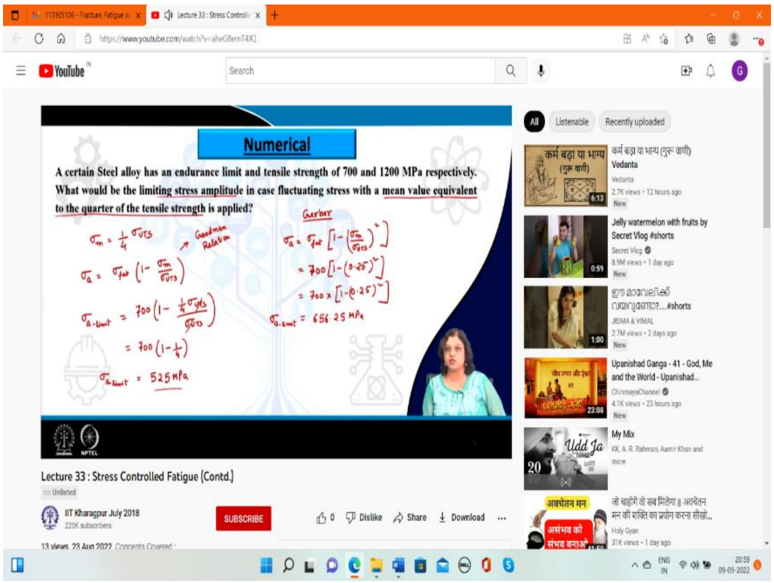
$$300 = 700\ (1 - \sigma_m / 1200)$$

And if we solve this, we should get the σ_m value something like 685.7 MPa.

So once again, the limiting value. So, this one is $\sigma_{m\text{-limit}}$ that we are estimating from the relation of Goodman relation, which is the maximum possible value of σ_m at which fracture will occur. So, we see that this is much higher than the applied σ_m which is only 400 MPa.

So, that also means that we are on the safe zone and that means that no failure will occur. Just to be clear, you should also write that $\sigma_{a\text{-}applied}$ is nothing but 300 MPa. So, this also signifies that there will not be any failure or the component will survive the test. So, that is how we can determine about whether a specimen will survive or not and we can design the experiment or in the actual practice we can use this concept fine.

(Refer Slide Time: 09:30)



So, let us see another extent of this. It says about the same material and having the same values of fatigue strength as well as tensile strength as the previous one. And in this case, we are applying however different kind of maximum and the minimum stress such that the mean stress is equivalent to the quarter of the tensile strength.

So, we can see that the mean stress is equivalent to the quarter of the tensile strength, which means that σ_m now is one-fourth of the σ_{UTS}. So once again, as per the Goodman relation, we know that $\sigma_a = \sigma_{fat}$ $(1 - \sigma_m / \sigma_{UTS})$. And our target now is to find out the limiting stress amplitude. So, we need to find out σ_a limit.

So, $\sigma_{a\text{-limit}}$ is very straightforward. We can use these values already. So, σ_{fat} is 700, σ_m is one-fourth σ_{UTS} divided by σ_{UTS}. So certainly, this will cancel each other out. So, we are getting

$\sigma_{a\text{-limit}} = 700\ (1 – 1 / 4)$

$= 525$

Now this has been solved based on the Goodman relation. One may also wonder that what would be the case, what would be the limiting value if we are using the Gerber relation, for example because Gerber relation also uses the same parameters. It uses σ_m and σ_{TS} as well as σ_a and σ_{fat}. So, let us see if there is any difference at all. So, what we can see here is σ_a equals to σ_{fat}. As per the Gerber relation, there should be

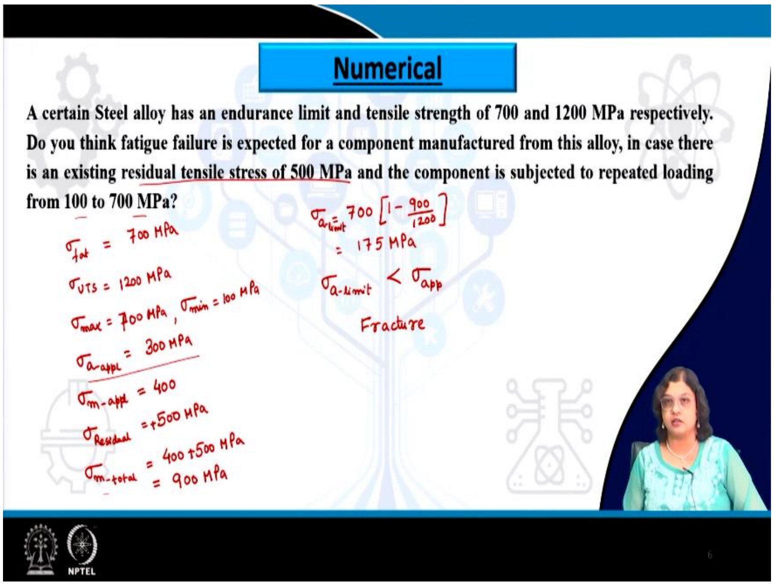
$$\sigma_a = \sigma_{fat}\,[1 – (\sigma_m / \sigma_{UTS})^2]$$

$$= 700\ [1 – (0.25)^2] = 656.25$$

So, what we can see here is that the expected or the limiting value of σ_a as per the Gerber relation is quite high which looks nice if we are designing it. But we have to be careful that if we are using such a high value of σ_a while designing the experiment, there should be absolutely no chances of any failure. So, Gerber relation always kind of overestimates and we have to be careful enough to understand that there will not be any failure at any value of stress amplitude lower than this one.

Goodman relation on the other hand is quite a safe one which does not use such a high value or overestimated value. We can use the same relation as the Gerber relation in the previous numerical problem to see how much is the difference between the limiting values of stress amplitude and mean stress.

(Refer Slide Time: 13:51)



Now that is the case and in the third problem here, which is once again on the same material. So, everything remaining same and the component made of that material is subjected to the same levels of stress. But the only difference now we have here is that it has a residual tensile stress of 500 MPa.

Now let us see that whether this will survive the same test condition as numerical one or not. So, once again, we have the fatigue strength as well as the UTS same values as earlier. So, this is 1200 MPa and this is 700 MPa and the σ_{max} and σ_{min} are also same value as the previous one but we have one difference here which is the residual stress. So, as per this maximum and the minimum value.

So, σ_a applied is supposed to be 300 MPa as well as σ_m which is the most important factor here is 400 MPa. But because of this residual stress, now σ_r which is the residual or sigma residual that is equivalent to 500 MPa and plus 500 MPa plus signifies the tensile nature of it. So, overall σ_m total turns into 400 plus 500 MPa which is quite some high value. So, this is 900 MPa.

So, we have to now of the Goodman relation we have to use the σ_m as 900 MPa. So, that means that σ_a equals to if we are solving the same way we should write the Goodman relation as $\sigma_{a\text{-limit}} = 700\,[1 - 900/1200]$

$$= 700\,[1 - 3/4]$$

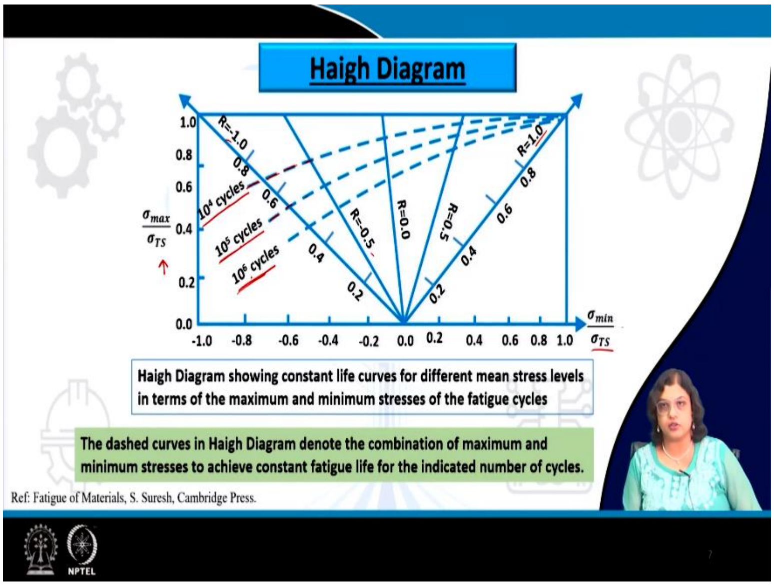
$$= 175$$

Now this is the limiting value now. Because there is already an existing residual tensile stresses present in the component that leads to the limiting value of stress amplitude quite low, much lower than the applied stress amplitude value of 300 MPa. So, now we see that σ_a limit is actually less than σ_a applied. So, that means that we are actually applying much higher stress than it is supposed to be. That is the maximum possible stress that it can withstand. So, obviously that will lead to fracture or failure.

So, we see that the same component, same loading condition, everything else remaining same, if there is a residual stress present there, that can lead to a completely different scenario. And while the previous one without the residual stress did not fail and it was quite in the safe regime, the one with the residual stresses and that too the tensile residual stresses actually lead to fracture.

Now, had it been a compressive residual stress, then the mean stress would have been reduced actually because the compressive one is having a minus sign here. So, compressive residual stress in that sense would be actually beneficial to enhance the life of the material.

(Refer Slide Time: 18:26)



So, here is a very interesting and important way of representation of the fatigue characteristics of a material. This is through the Haigh diagram. So, what it says is the y axis here is the ratio of the σ_{max} / σ_{UTS}. What it means is that the maximum stress has been normalized with respect to the ultimate tensile strength of the material and the x axis on the other hand, represent the minimum stress normalized with the ultimate tensile strength of the material.

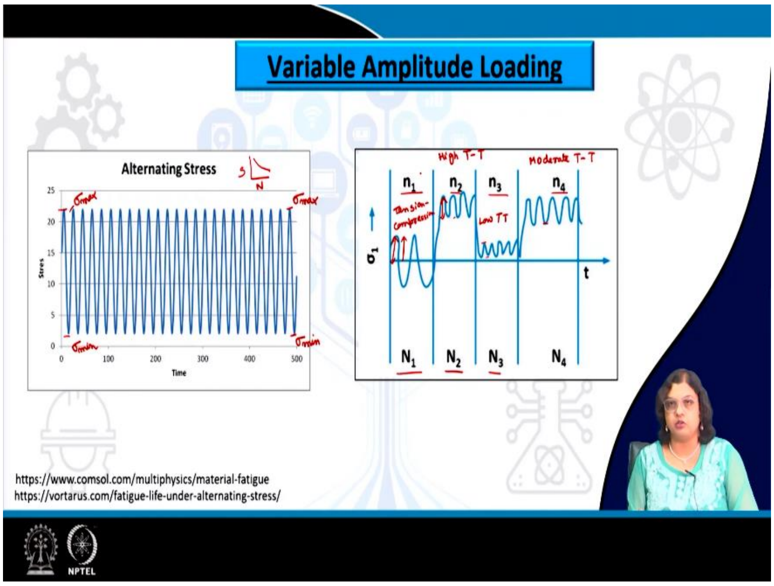
So, if we are representing the fatigue characteristics in this way and then we are doing this for the different values of r equals to minus 1, minus 0.5 and so on up to r equals to 1. What we are seeing is that this dashed line which signifies the life. So, basically the Haigh diagram is showing the constant life curve for different mean stress levels. So, it actually gives us the combination of the maximum and the minimum stresses to achieve the constant life or the same life.

The dashed line that you can see here, this one or this one, are for particular number of cycles, 10^4 or 10^5 or 10^6 cycles. So, this dashed curve actually denotes the combination of maximum and the minimum stresses to achieve constant fatigue life for the indicated number of cycles.

So, for any particular number of cycles, what should be the combination of maximum and the minimum so that we can achieve this particular stress amplitude and mean stress and to get the same life. So, if we want to have the life constant, so we need to do this for 10^6 cycles, for example. Then through this Haigh diagram we can figure out that what should be the maximum and the minimum stresses that can be applied the limiting values.

Once again, this however needs strong experimental results so that we can construct this Haigh diagram first and then only we can use it for the same material having similar kind of microstructure and everything remaining same. We can use this as a basis based on which the fatigue life or the stress scenario can be assessed.

(Refer Slide Time: 21:15)



So now that we have seen the importance of stress amplitude and mean stress on the fatigue life, next we are coming to a rather practical scenario. Now so far, we have always assumed that whether we are applying tension-tension or tension compression or tension and zero loading cycle, we always assume that every cycle we are applying same values of σ_{max} and σ_{min} and we keep on repeating this until fracture is occurring. That is how we have also determined the SN curve of the material.

So, you can see here this alternating stress versus time that we are applying the same value of σ_{max} and σ_{min} repeatedly until it fractures. So, till the last point we are using the same values of σ_{max} and σ_{min}. Now, in actual practice or in service, this may not be the case. We can have situation like this when we are fluctuating it up to certain extent. So, this is the stress amplitude. Now for n_1 number of cycles, small n_1 number of cycles. The specimen of the material is supposed to survive for capital N_1 number of cycles at this amount of stress amplitude.

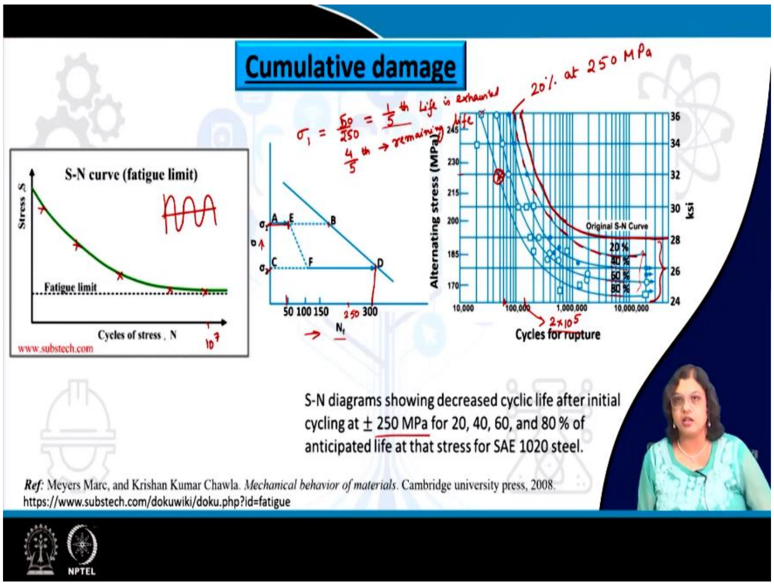
But we are not done yet, we are not doing it till it fractures. Rather we are doing it for only some limited number of cycles and then we are changing this to the amplitude value again. So, initially it was under tension compression and in the second step we are doing this under tension-tension, very high tension and comparatively lower tension. And then in the next step this is even tension-tension but the magnitude of the maximum and the minimum stresses are quite low compared to the previous step.

And at each of this condition, the capital letter here signifies the total number of cycles had the specimen has been cycled only at this particular stress amplitude level. So, the first one that we are seeing here is tension compression. The second one is high tension-tension, T stands for tension. The third one however, is still tension-tension, but this is low tension-tension. And the fourth one although uses high value of maximum stress but the minimum value of stress, although it is tension but it is quite less compared to the second one. So, let us name this as moderate tension.

Each of this condition, the specimens has been cycled for only some specific number of cycles while at each of this condition, had this been the only one, the specimen should have survived for the capital N number of cycles. So, if such is the case which we often see in practice or we need to determine sometimes based on the kind of applications that it is supposed to behave.

Obviously, we can understand even without doing the test that this scenario is completely different from this one. And obviously, whatever we have used for determining the SN curve based on this particular amplitude thing will not work in case we have such kind of variable amplitude loading.

(Refer Slide Time: 24:57)



So, we have to figure out another way to do that and for that we have to appreciate the idea of cumulative damage. Now, what happens in a particular amplitude loading? We do this till it fractures and for each of the stress amplitude, we get the corresponding n values and then we draw a curve like this. We are very well aware of this. However, what happens in fatigue, although it might have failed after 10^7 number of cycles but from the very beginning, whenever we are loading it in cyclic condition, there is a damage accumulation. At every step there is a progress in the extent of that damage accumulation. That was a very definition of fatigue that we have discussed in the very first lecture on this module.

So, obviously, the one which has suffered through some number of cycles should not behave exactly in the same way if there has been no previous history of cyclic loading. So, that means that with every number of cycles at any particular stress amplitude, some fraction of the life will be used up and that means that the remaining life which is there that can be used up by the next step or the next condition. So, what we are seeing here is the example is the stress amplitude on the y axis and x axis as the number of cycles for failure capital N,

So, at this point of σ_1, if we keep on applying to this σ_1 stress amplitude, it is supposed to fail at 300 or let us say that this is 250 MPa, but instead of that, we are applying the number of cycles only up to 50. So, out of if 250 had been the total number of cycles that is required for σ_1 to be applied for total failure, so that means that we are applying only one-fifth of the total number of cycles that it is expected to withstand. So, 50 / 250, that is just one-fifth of the life that has been used up at this condition.

So, for σ_1, let me write this as 50 / 250, that is equivalent to one-fifth life is exhausted. Of course, once the damage accumulates to the 100 percent it will fail. So that means that the remaining life is four-fifth. So, that is the remaining life after we are done with the 50 cycles at σ_1. So, that means that when we are starting with σ_2, we are not starting with a fresh sample, rather the one which has only four fifth of its life left. Now, if it can survive up to 300 number of cycles at σ_2, then we can figure out that how many number of cycles it can survive now.

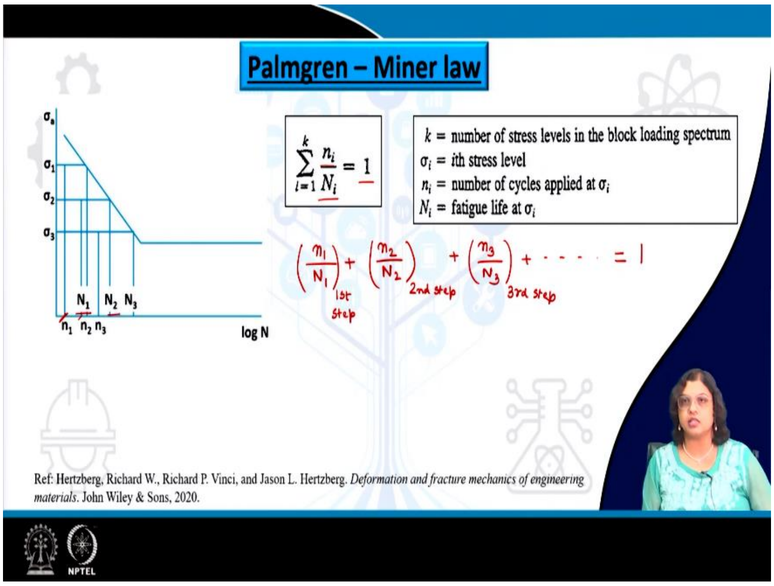
Obviously, a fresh sample should survive up to 300 number of cycles but a used one which has been previously used for one-fifth number of cycles it should not survive up to 300 number of cycles and we can figure out that how many number of cycles it can survive. Now, let us see how the SN curve will look like. So, this is shown here. What it shows is this one here. Let me draw this once again. So, this red one here is the original SN curve of a material.

Now the same material is being tested again and again and it is a very interesting way by which the test has been done. So, this one, the one here, the second one actually, let us do this dashing here. So, this one signifies the one which has been tasted at the highest stress amplitude of 250 MPa for 20 percent of the total life. So, the second one here actually has been fatigued for 20 percent of the total life at 250 MPa which means that let us say at 250 MPa it is supposed to have a life of 2 lakhs, for example, or maybe 1.5 lakhs. So, let us make this simpler. And this is 2×10^5. So, this is what is the expected life for 250 MPa. So, if we are applying 250 MPa stress amplitude, the specimen of the material can survive for 2×10^5 number of cycles.

Now, in the second specimen we are testing at 250 MPa for up to 20 percent of this 2×10^5 cycles. 20 percent of 2 lakh cycles has been employed at 250 MPa and then the stress has been decreased, the stress amplitude has been decreased. So, this is the dashed curve that we are getting in such cases. The next one, the third one shows up to 40 percent. Next one is up to 60 percent, 80 percent and so on. So, now you can see that if we are cycling it previously at the highest possible stress or any other stress for that matter, then obviously the SN curve that we are generating after that will be completely different from the original sample.

Mind it all the specimens that has been tested here, you now know that SN curve consists of the experimental results from several specimens. And all the specimens has been tested up to 20 percent, 30 percent, 40 percent, whatever is the specified value here and then the stress amplitude has been decreased. So, that means if we are talking about this particular data point here, this means that this specimen has been cycled at 50 MPa up to 80 percent of the total life and then the stress level has been reduced to 225 MPa till it fractures and then it has achieved a life of something like this.

(Refer Slide Time: 31:58)



So, there is a significant difference that we are seeing from a virgin sample versus a one which has been cycled for many number of cycles and that actually can be determined based on the rule known as the Palmgren-Miner law. It says that at each stress amplitude, let us say σ_1, it is supposed to fill up to capital N_1 number of cycles but we are employing a number of cycles of n_1. If such is the case, then the next one at σ_2 which is supposed to survive for N_2 number of cycles and we are implying cycles up to small n_2 number of cycles and so on.

So, if we follow a sequence like this, then according to Palmgren-Miner's law, the summation of this ratio of small n_i / N_i, where i varies from 1 to k, where k is the total number of steps that we are adding. That summation should be equivalent to 1. So, in this case we are having three steps. So, basically what we should see is that n_1, small n_1 / N_1 for the first step plus so let me write this as first step plus small n_2 / N_2 which is the second step and small n_3 / N_3 which is the third step. So, it can continue like that.

In this case there are only three steps that have been shown but the overall summation should have a value of 1 and then we can figure out if we know at least for the rest of the steps except the last one, we can figure out that how many number of cycles it can survive at that particular condition. So we will have further discussion about this in the next lecture.

(Refer Slide Time: 34:01)

For now, let us conclude this session. As we have seen in this lecture that based on the Goodman diagram, the limiting values of the mean stress and stress amplitude can be determined and the safe space can be identified. So, based on that only we can figure out that whether specimen is expected to fail or not. We have also seen that how the Haigh diagram represents the combination of maximum and the minimum stress level that lead to constant fatigue life at any particular number of cycles.

So based on this, we can determine this combination and we can find out what should be the stress amplitude and mean stress and so on. Now with occurrence of variable amplitude loading of course, the same principle as has been discussed in the previous cases with constant amplitude loading will not be valid and we have to take into account that there will be some cumulative damage. There will be some progressive damage at every cycle which we have to consider. And this can be well explained with the Palmgren-Miner's Law which is typically used to assess the life of a component subjected to cumulative fatigue.

(Refer Slide Time: 35:12)

These are the references used for this lecture. Thank you very much.