Fracture, Fatigue and Failure of Materials
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Lecture 32
Stress Controlled Fatigue (Continued)

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Hello, everyone. We are in the 32nd lecture of this course Fracture Fatigue and Failure of Materials and covering the module two, Fatigue. And in this lecture we will be talking about the stress controlled fatigue.

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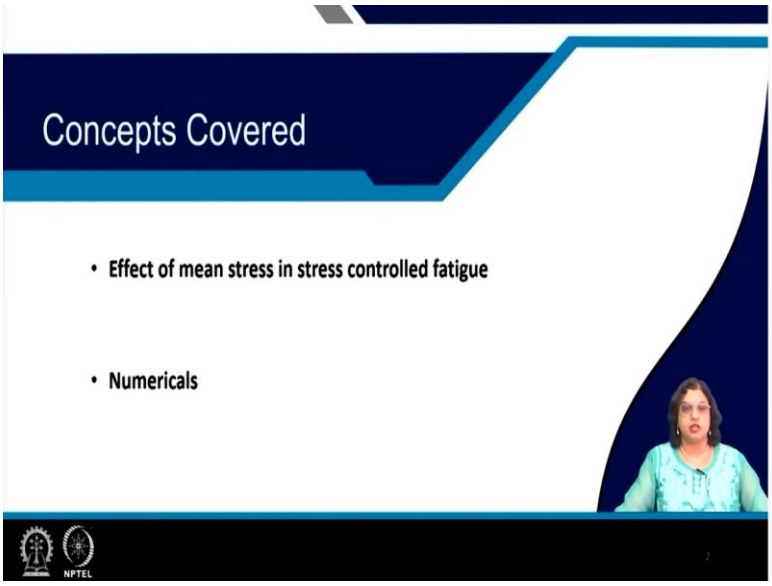

So, the following concepts that will be covered in this lecture is particularly the effect of mean stress in stress controlled fatigue, how the total life is getting changed or getting affected if we are changing the mean stress of fatigue and we will also do some numericals related to that to have a better concept of this.

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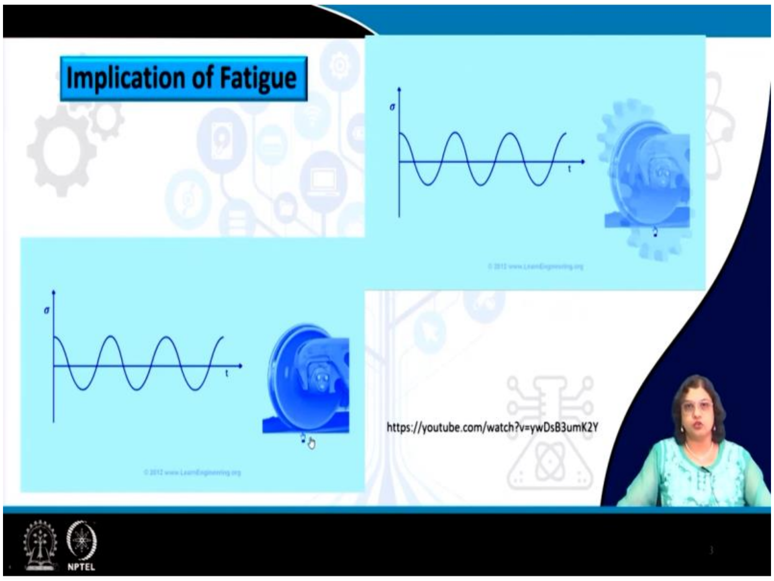

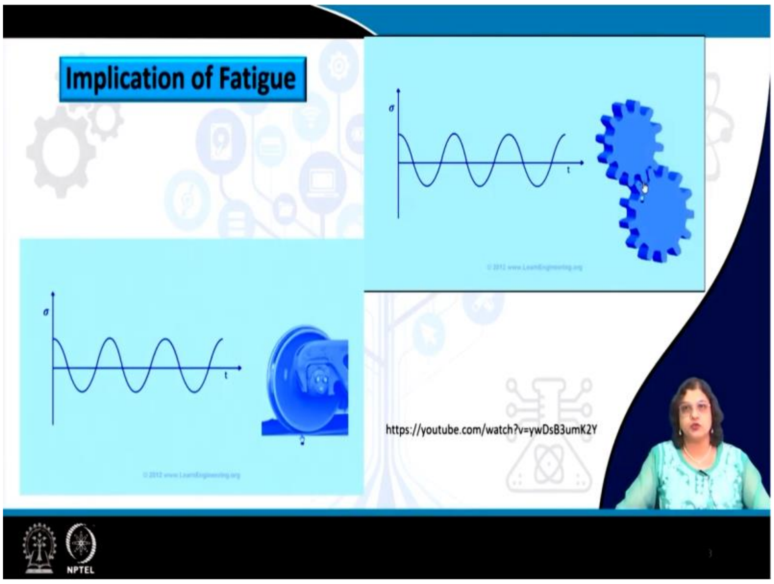

So, fatigue, as I have mentioned in the last lectures is seen in our day to day lives as well as various engineering structures as well. So, two of the very well-known examples are what I am showing here. One is the wheel and axle, the track of train and you can see that wherever it is in contact, it actually is under some kind of stress here and by the time the wheels comes to the other end, the stress is being released, but at the same time all the other points which are covering, which are passing through this point of contact is getting under the stress and then getting relieved again and this continues till the vehicle runs.

So, this leads to a very important kind of fatigue known as a Rolling contact fatigue which is a type of damage that comes due to the cyclic loading. On the other hand, this video here shows the two gear wheels and wherever they are in contact, the teeth which are in contact, there is also a fatigue loading that is happening. So, you can see that this motion and once this wheel is out of this contact, this load is being released, but it comes back again once it is in contact with that, once again as it moves on, as it keeps on cycling.

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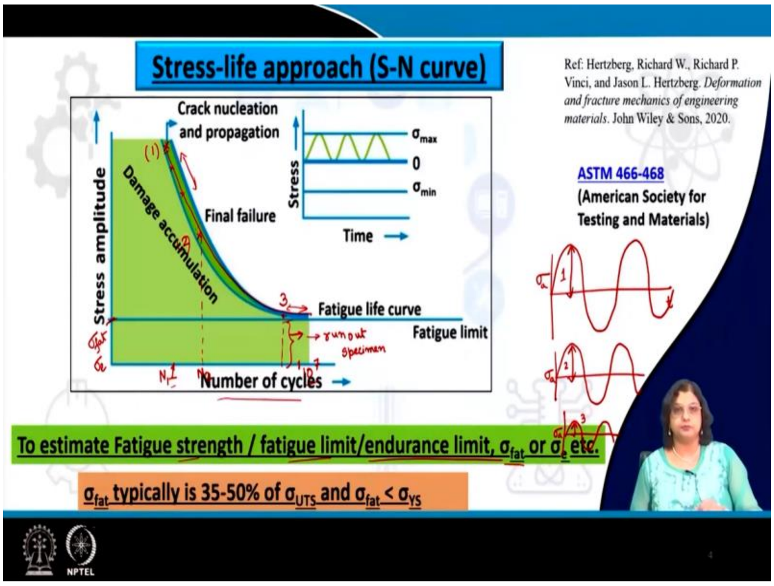

So, if such is the case, we have now also seen that how we can understand the fatigue performance of this through the SN curve, through the Wohler curve and which looks something like this. The y-axis shows the stress amplitude and the x-axis shows the number of cycles and we are getting a typical form of curve which looks something like this.

So, there are various standards for this and typically we use the ASTM standard 466 to 468 and what it says is for the metallic materials particularly, we start the testing at some amount of stress amplitude which is equivalent to 60% to 70% of the ultimate tensile strength of the material. So, we do some testing at such a high stress level. And obviously when the stress level is so high then the number of cycles that we are achieving is very, very small.

So, let us number this as the step 1, and depending on the kind of number of cycles that we are getting, the next attempt will be to apply the stress amplitude which is slightly lesser than the previous one so, the step 1. Typically, we use reduction by around 20 to 50 MPa and that depends on the material, that depends on the number of cycles we are achieving at the previous stress amplitude level. Because our target at any point is to achieve the number of cycle as 10^6 or 10^7.

So, if this value here N_1 is very, very less, we can decrease the stress amplitude to quite some extent, provided all the three or four samples that has been tested at this step 1 should have such low number of cycles. So, for the case of step 1, let us say this is the stress amplitude versus time curve, and it shows something like this. The second step, there could be many other steps. But let us talk about something which is in between. So let us name this as step 2.

And what we do here is we reduce the stress amplitude at this point and it may look like something like this, which is quite some lower stress amplitude level compared to the previous one. So, we can see here, the stress amplitude for 1 is quite higher than that for 2. And obviously, the corresponding number of cycles that we are getting for the step 2 is much higher than that we have obtained for step 1.

Once again depending on the number of cycles that we are achieving, as our targeted value is to achieve 10^6 or 10^7 cycles as the ideal one, we keep on reducing the stress amplitude. And of course, at any particular stress amplitude we test multiple samples such that the specimen should have some scatter and we take account of the scatter.

So, let us come to a point which is close to the endurance limit now, assuming that we have decreased the stress amplitude to quite some extent and let us name this as 3. And the kind of stress amplitude that we are applying here is certainly much, much less than that we have applied for 1 or 2. So, in this case, of course the stress amplitude is much lesser than that for 2.

And the number of cycles that we are achieving is close to 10^7 it may exceed 10^7 also, or in some cases when there is no failure and we have already reached the number of cycles attained 10^7 number of cycles and there is still not failure, we can simply stop the test, and we signify that data point with an arrow, which signifies that this is a run out specimen, run out specimen means the one which has not fractured even if we have achieved the targeted number of cycles.

So, depending on the number of cycles that is required for the application purpose, as I mentioned, this could be 10^6 or 10^7 or 10^8 but at any point, our target to find out the fatigue strength or endurance limit, that is this point here is when all the three or four specimens that we have tested at this particular stress amplitude should survive. None of the specimen should fail or should have a lower life, then 10^7 number of cycles.

If it does, that means that we still have to repeat the test. So, as I mentioned that there is a significant amount of scatter particularly because of the finishing in the surface of the sample as such. So, we need to make sure that the value of stress amplitude that we are quoting as the fatigue strength or the endurance limit at that point, there should not be any failure.

So, this is a conservative approach and we assume that there will be no failure, if we are talking about any stress amplitude beneath this fatigue strength level, to make sure that we have to have all the specimens either run out or they have achieved 10^7 number of cycles.

So, the main purpose of this kind of SN curve or Wohler's curve is to estimate the fatigue strength. Sometimes this line depending on whether we are talking about the ferrous alloy or non-ferrous alloy particularly for ferrous alloy we have seen that it maintains a very sharp distinction between this slope as well as this slope here.

So, we name that as endurance limit for other kind of materials which shows continuous reduction in the stress amplitude with respect to the number of cycles, there we consider the limiting number of cycles as 10^6 or 10^7 and the corresponding stress amplitude is termed as fatigue strength or sometimes fatigue limit also, and denoted by sigma and fatigue as a subscript or σ_e.

So, these are the common nomenclature that we use for fatigue testing. Now, typically this sigma fatigue value is 35% to 50% of the ultimate tensile strength of the material. Now, this is also a very rough approximation that we often do that we consider the fatigue strength is half of the tensile strength of the material. And fatigue strength actually is even lesser than the yield strength of the material.

So, this is very interesting to note that yield strength of the material, which is the minimum amount of stress that is required just to achieve plastic deformation. So, forget about failure just to achieve a permanent deformation we require the yield strength whereas, fatigue strength is even lesser than the yield strength value, which means that if we are applying the amount of stress which is much, much lower than the ultimate tensile strength, so, there is no scope for failure even there should not be any scope for permanent deformation.

Even though if we are applying this number of cycles for multiple times, we can achieve the failure. So, that makes fatigue of higher concern especially for critical applications such as in aerospace or biomedical where we cannot take any chances and we should understand and we should appreciate the fatigue properties and fatigue behavior, fatigue strength very, very carefully.

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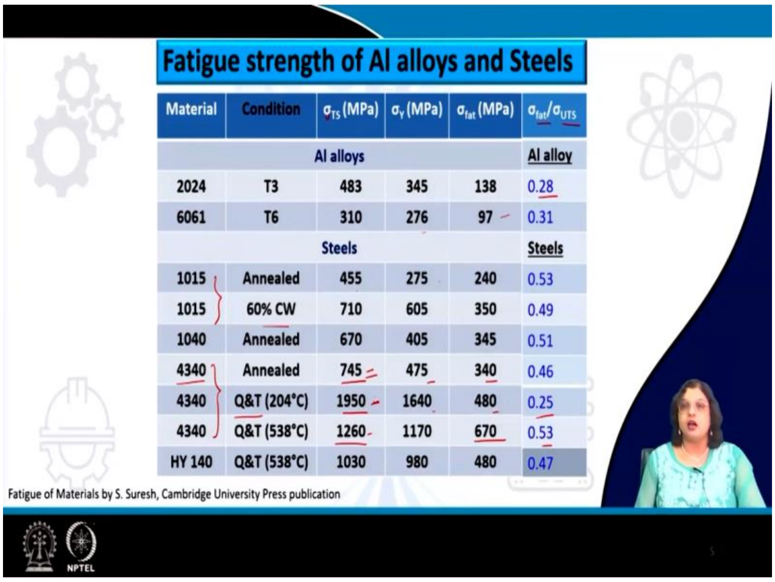

So, here are the results for some of the commonly used engineering materials particularly metallic systems. So, we have some aluminum alloys and based on the heat treatment condition, we see that there is some changes in the ultimate tensile strength UTS or σ_{TS} and that yield strength of the material. And that also leads to some change in the fatigue strength of the material.

Same goes for the steels, although these are different categories of steel, but even for the same categories of steel if we are doing some kind of heat treatment or thermo-mechanical treatment, we can see that there is some significant changes in the strength of the material and that is reflected in the fatigue strength of the material as well.

So, that means like any other mechanical properties fatigue strength is also being controlled by the thermo-mechanical processing which in turn we know that thermo-mechanical processing controls the microstructure of the material. So, certainly fatigue is also been controlled by the microstructure of the material.

We will look into that slowly through this course. But for now, it is very important to appreciate that for the same category of steel, you can see that for this 4340 kind of steel we can achieve tensile strength as low as 745 MPa to as high as close to 2000 MPa. So, that is a huge jump in the tensile strength just by doing some kind of thermos-mechanical treatment.

And similarly, the fatigue strength has also been changed, but interestingly we can also see that fatigue strength is not following the same trend as the tensile strength of the material. We can see here that the jump in the tensile strength is quite high from 750 for example, to 2000 is more than twice the value and what we are getting in the fatigue strength is only around 140 MPa, so, not so significant.

On the other hand, if we are achieving the tensile strength something like 1260 by some quenching and tempering operation at 538 ℃, we see quite a high value of fatigue strength so, obviously that means that something is changing in the microstructure, which is affecting the strength of the material in some way as well as the fatigue strength could be in another way.

So, we have to look into the microstructure for any kind of detail inside if we want to develop, we have to understand the microstructure property correlation very, very carefully, clearly. Now, what basically we are seeing from this table is that the ratio between this fatigue strength to the ultimate tensile strength of the material could be as low as 0.2 something like this or it could be as high as close to or more than 0.5 sometimes could be even 0.6 and so on.

So, we can achieve such kind of differences based on the trend of the material, how the dislocation movement occurs in the material, what is the properties of the material, how the microstructure is being modified, how the microstructure is controlling the movement of the dislocations and so on. One more thing I would like to clarify here that although I am talking about dislocation just in the earlier slide, I mentioned that fatigue is occurring at a stress lower than the yield strength of the material.

We can see that for all these cases, for any of this alloy, let us talk about this two which or these three elements, we can see that fatigue strength is lower than the yield strength of the material. So, that means that there should not have been any scope for ductile deformation at all or dislocation movement at all, which however, may not be the case because there could be some localized deformation, localized plastic deformation also that may have happened.

And that is once again dictated by the microstructure of the material and if we are altering the microstructure, not necessarily its role in changing the yield strength and changing the fatigue strength will behave exactly in the same manner, it will behave in a manner let us, for example, if we are refining the microstructure strength will increase, we know about that the same trend is expected to be seen for the case of fatigue also, but with what extent that is, needs to be understood for different materials.

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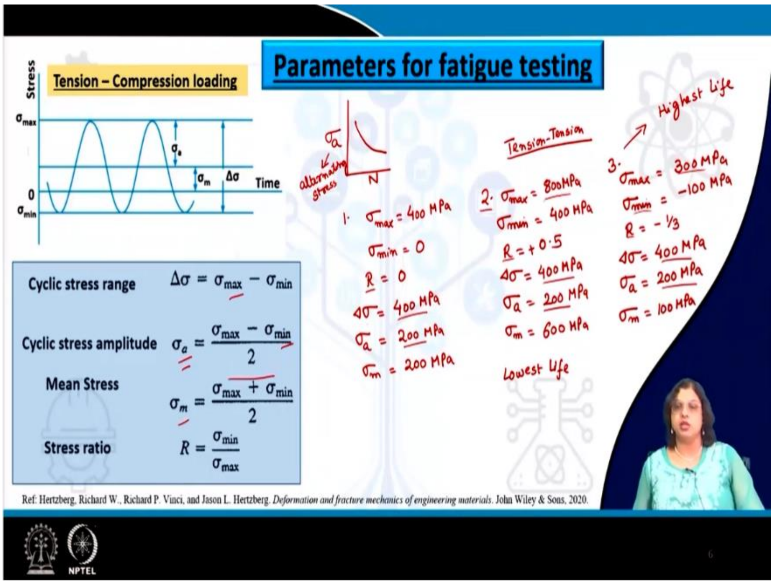

So, let us see how the different parameters control the fatigue characteristics. For example, we have seen that there are different kinds of parameter, when I say stress, for the case of tensile testing it is quite straightforward. Whatever stress we are applying, the applied stress that if that leads to a permanent deformation, that is the yield strength, if that leads to a fracture that is the ultimate tensile strength.

But in case of fatigue, since we are cycling the load, there are different kind of stress levels, there are maximum stress, there are minimum stress, there are stress amplitude, mean stress etcetera. So, which one actually we are talking about that is very, very important. So, far, whatever SN curve we have seen, the y-axis is in all cases, mostly σa, if nothing is mentioned we assume that the y-axis is σa and x-axis is number of cycles.

Sometimes, this σa which is a stress amplitude is also referred to as alternating stress. So, alternating stress is the magnitude of stress amplitude. So, let us see this with some examples. For example, let us consider condition one. When we have σ_{max} equals to 400 MPa and minimum stress is equals to 0. So, which means that R equals to 0 in this case.

So, if that is the case, what we are getting here Δσ which is the stress range is 400 MPa, and that gives us stress amplitude as 200 MPa, whereas, the mean stress is also of the same order that is also 200 MPa. On the other hand, if we are having the second condition in which we have σ_{max} equals to 800 MPa let us say, and we have σ_{min} equals to 400 MPa. So, that means that R in this case will be 0.5 so, it is plus half.

On the other hand, $\Delta\sigma$ or the stress range once again here is also 400 MPa, which makes stress amplitude also as 200 MPa, amplitude is just half of the stress range that we have seen from the situation here. However, mean stress is different now, and mean stress comes to 600 MPa. So, that is the summation of the maximum and the minimum stress so, 1200 divided by 2 that is 600 MPa.

Let us say, the third condition is such that we have σ_{max} equals to 300 MPa now and minimum stress is in the compressive zone. So, that is 100 MPa. R is negative now, so R is actually -1/3 or -0.33. $\Delta\sigma$ however, still is 400 MPa. So, for all these cases we are seeing that the stress range as well as the stress amplitude are of the same magnitude.

So, all the cases, stress range is 400 MPa and stress amplitude is 200 MPa. But, the mean stress values are different. The mean stress in this case will be just 100 MPa, because now we have this compressive stresses. So, that means 300 - 100 stress that will lead to the mean total stress or 200 MPa divided by 2 so, 100 MPa.

So, one can say that all these three conditions we are applying the same stress range, so, this extent, if we are talking about the total stress range, that is still the same for all the cases. So, why should there be any difference in the number of cycles? And a very good question to ask will be that in which case, even if we are assuming that there should be some differences in the number of, total number of cycles, we need to find out that in which case the number of cycles will be the maximum or the minimum?

So, what we need to understand here once again is that this is very much related to the R value or for that matter the $\sigma_{minimum}$ value or if the $\sigma_{minimum}$ is in compression side or the extent of $\sigma_{maximum}$ in the tensile side. At the very first lecture on fatigue, the very initial slides, I have also mentioned that fatigue occurs only when we have sufficiently high tensile stress.

Now, that tensile stress could be lesser than the magnitude of the yield strength of the material, but it should be a sufficiently higher value. So, what we can see out of all these three condition is that for the condition two, we are seeing the maximum value of the maximum stress, this is the highest value of the maximum stress 800 MPa, so this one is actually going to survive for less number of cycles, because here we are having tension-tension kind of fatigue.

So, this is tension-tension kind of fatigue and tension mode is the crack opening mode. So, once the defect initiates or even if there are any kind of stress concentration that is getting more aggravated, if we are applying the tensile stress and that to of such high extent of 800 MPa will certainly be detrimental. So, this will lead us to the poorest life or the lowest life.

On the other hand, if we have amongst all these three cases, the lowest value of σ_{max} stands for 300 although for all these cases, we have the same values of stress range, but we have the lowest value of σ_{max} and not only that, we do have a compressive stress here also and we also know that for the case of fatigue or for the case of any such kind of even for fracture whenever there is a defect that is involved in the fracture of the material compressive stresses acts as a beneficial one because compressive stresses act to close the crack.

So, compressive stresses will certainly, will act beneficial here, which means that we need to apply this kind of stress fluctuations, again and again for many number of cycles for actually the highest number of cycles to achieve fracture. So, in this case, for condition three, we are getting the highest life amongst all. So, this is in a very straightforward way that we could figure out that other than the stress range and stress amplitude, if you are keeping the stress range and the stress amplitude constant, it is the mean stress that may have affected the fatigue behavior of the material.

So, let us see how the mean stress controls the fatigue strength of the material or the number of cycles for failure, which is the fatigue life of the material.

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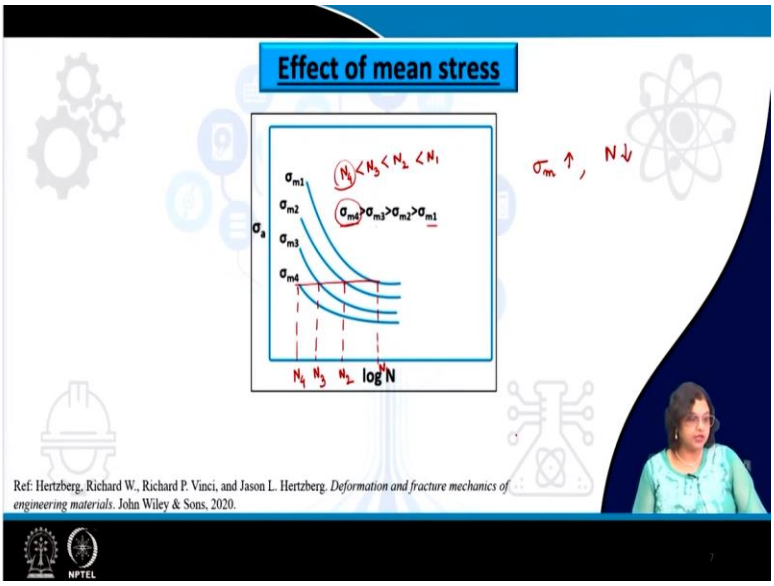

So, what we are seeing here is relation something like this, where the σ_a versus N plot similar to the SN curve and what we are seeing here is for the different values of σ_m which is the mean stress we can see the different SN curves for that the trend is still the same, but the position of the curves are different.

So, that means that for any particular number of cycles, for example, this one here, what we are seeing is for the highest stress, mean stress of σ_{m4} we have seen that σ_{m4} is of the highest value and σ_{m1} is of the lowest value. So, let us see the corresponding number of cycles that we are getting here. Let us name this as N_4, for m3, σ_{m3} we are getting the life of N_3, for s σ_{m2} we are getting the life of N_2 and σ_{m1} we are getting the life of N_1, quite surprisingly we do see some trend for the number of cycles also.

So, if we want to find out this trend, what we are seeing here is that N_4 less than N_3 less than N_2 less than N_1. So, although σ_{m4} or the mean stress is highest for the condition 4, the number of cycles for fracture is lowest for the condition 4, which means that if the mean stress increases N or the number of cycles for failure actually decreases. So, they have an inverse relation amongst themselves.

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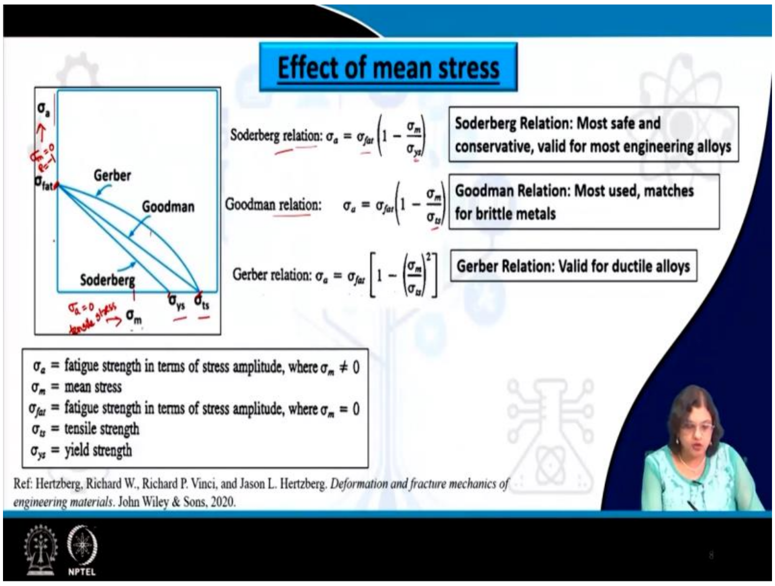

And this can be very well explained if we are plotting σ_m versus σ_a curve. So, in this case of the y-axis is σ_a, the stress amplitude and the x-axis is σ_m. So, that means, if we are talking about only the axis here, what we are seeing here for the x-axis actually σ_a is 0 for this x-axis part. So, σ_a is 0, this can only happen when we are talking about tensile or static stress or monotonic stress. Only then we can have stress amplitude equals to 0, stress range equals to 0 because there is only one stress, it is not fluctuating.

So, at that point if we are talking about a monotonic stress, that means that failure will occur at the point of ultimate tensile strength. There will be yielding at some point, sigma yield strength and failure will occur at the point of the ultimate tensile strength of the material. On the other hand, if we are talking about the y-axis, which is the stress amplitude that means that for this y-axis line, we have σ_m equals to 0, mean stress equals to 0.

Now, mean stress equals to 0 this can only happen when we have R equals to minus 1. So, that means, it is a tension compression cycle and that too with equal magnitude, that is why we have R equals to minus 1 and mean stress equals to 0, at that point. So, this is a completely reversed cycle, if we are applying that failure will occur at the point of fatigue strength of the material.

So, these two are the limiting conditions. On the y axis, we are getting the fatigue strength at the axis per line and on the x axis we are getting the sigma tensile strength or ultimate tensile strength at the axis line. Now, the relation between this fatigue strength and tensile strength as including involving the mean stress as well as the stress amplitude is given by the following.

So, one of, so, all the parameters are being described here, which says that sigma a is the stress amplitude and sigma m is the mean stress, as well as sigma fatigue is the fatigue strength and this is the tensile and the yield strength of the material. Now, the relation between this two involving the sigma m and sigma a could be explained as per Soderberg relation which says that sigma amplitude equals to sigma fatigue 1 minus mean stress by yield strength or this could be also as per the Goodman relation.

So, this is the Soderbergh line, this one here which joins the yield strength of the material and the one the straight line that joins the tensile strength of the material with the fatigue strength that is given by the Goodman relation, which says same kind of relation, but instead of the yield strength, now, we have the tensile strength.

Or there could be another third relation, which is the Gerber relation, which says that sigma a equals to sigma fatigue 1 minus this parameter sigma m by sigma ts that ratio should be squared. So, these are the three relations which define the actual involvement of sigma m and sigma a in achieving the fracture under monotonic condition as well as under cyclic condition.

The Soderberg relation, however, is the most safe and conservative. So, this is valid for most of the engineering alloys as we can see that this is actually the safest one. So, if we can predict it on the basis of the Soderberg relation we know that we will be always on the safe side, if we are following this as a limiting condition.

However, Goodman relation is the most commonly used and this matches for most of the cases particularly for brittle metals. So, that is what we are always very much worried about, about the brittle failure and that means that we always do not need to be so much conservative as a Soderberg we can still follow the Goodman relation and achieve more or less a valid results.

Gerber relation is however valid for most ductile materials but as we can see that it kind of over estimates at some point. So, we have to apply this with care. So, what we actually can see from here is using any of this relation, our target is to find out the sigma m and the corresponding value of sigma a.

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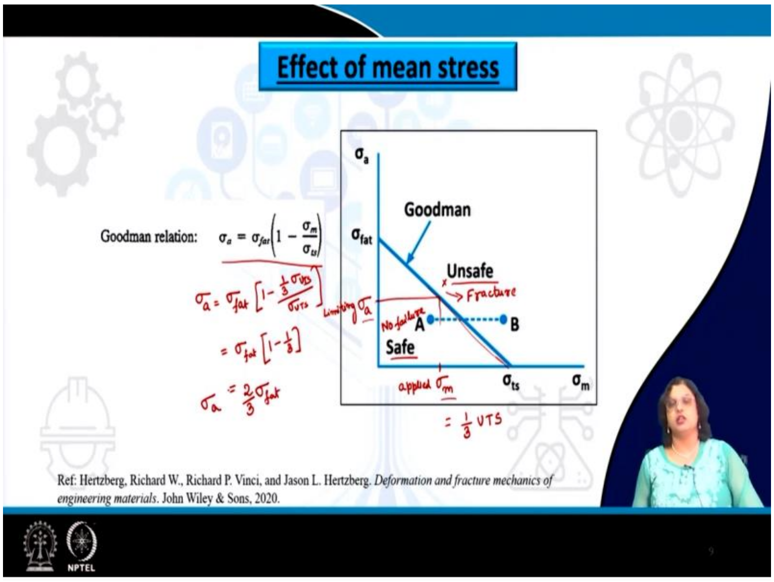

For example, if we are talking about the Goodman relation, we can do this for any other kind of relation like the Soderbergh or Gerber. But the point is that if we are talking about applying a sigma mean value something like this, then that means that as per the Goodman relation it should follow this line, based on the relation sigma a equals to sigma fatigue 1 minus means stress to sigma ultimate tensile strength ratio and for that matter, the limiting point will be somewhere here.

So, that means that this is the limiting value of stress amplitude. So, if we are applying sigma mean like this applied sigma mean we know that the stress amplitude should not be more than that, if we are at any point outside this domain, so, on the right side of this curve or this straight line, then we are in the unsafe mode and we know that there will be fracture. If our data points like the combination of sigma mean and sigma amplitude is within this triangular space, then we know that we are in the safe condition and there will be no failure. So, this is very, very important.

As we have seen in the previous example, that although we are following the same stress range and same stress amplitude, we may still have different mean stresses. So, it is very important, like when we are even for designing the fatigue experiment, when we want to do the fatigue experiment, we need to define what would be the maximum and the minimum stresses and for doing so, we need to maintain this relation between mean stress and stress amplitude so, that we are in the safe zone and then we do the test and then we determine that how many number of cycles it can survive till failure. So, that is how we try to figure this out.

So, for example, in case we are using this applied mean stress which is equivalent to let us say, one-third of the ultimate tensile strength. So, that means that we can simply use this relation as sigma a equals to sigma fatigue 1 minus and sigma m by sigma ts which should be just one-third. So that makes it 1 minus one-third or simply two-third. So, that is how we can determine the relation if we have the idea of any of these stresses, we can figure out what should be the other extent of the stresses that we can apply to still be on the safe regime.

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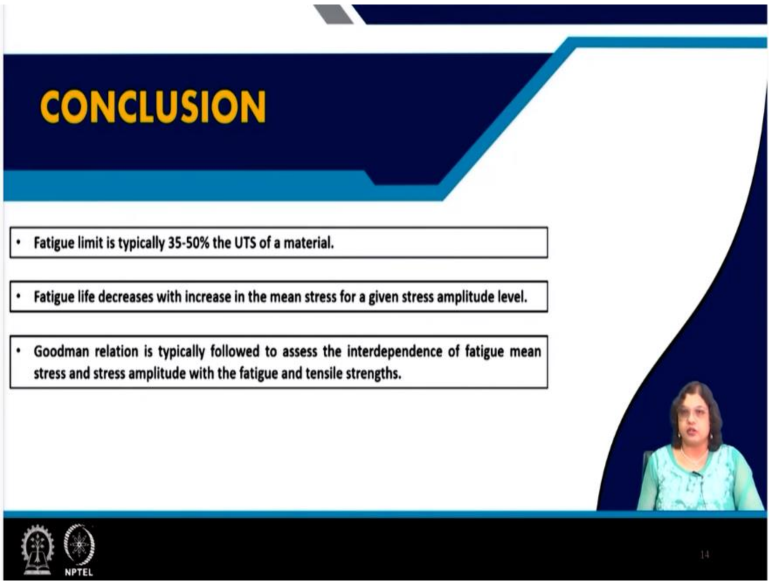

So, what we are seeing here basically is as per the SN curve or the Wohler curve, we have seen that fatigue limit or the fatigue strength is typically around 35 percent to 50 percent of the ultimate tensile strength of the material and in all cases it is actually lesser than the yield strength of the material. So, fatigue still occurs because of the some localized deformation that can lead to crack initiation and that will lead to the final fracture even if at a stress which is lower than the stress limit that is required for the permanent deformation of failure.

We have also seen that how fatigue life decreases with increase in the mean stress for a given stress amplitude and amongst the different relation that could be used to formulate between the mean stress and the stress amplitude involving the fatigue strength and the ultimate tensile strength of the material. We have seen that Goodman relation is the one which is typically followed for most of the cases to assess the relation amongst all these parameters. These are the references used for this lecture. Thank you very much.