Fracture Fatigue and Failure of Materials
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Lecture: 03
Stress Concentration

Hello everyone and welcome back to the course on Fracture Fatigue and Failure of Materials. So, we are here for the third lecture of this course, and today we will be discussing about Stress Concentration along with some recap of the previous things that we have discussed in the last class. So, let us see briefly what we have discussed in the last class and how we can progress from there.

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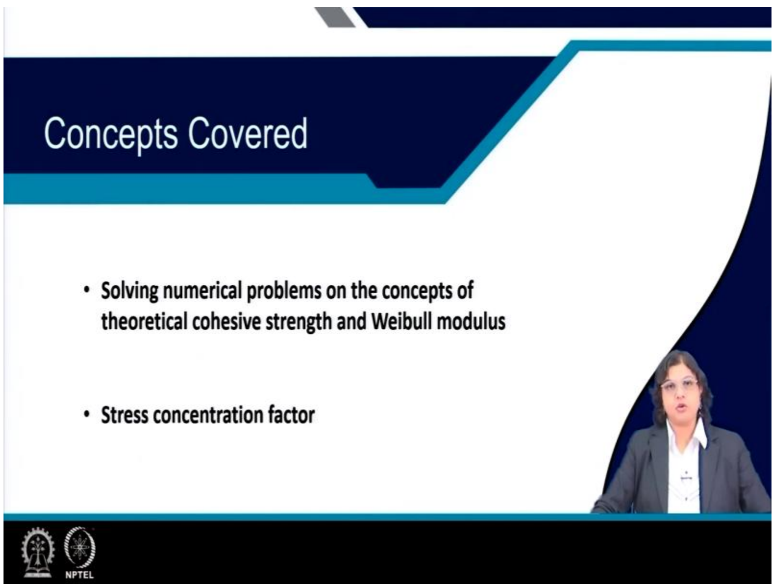

This will be the concepts that will be covered in this class. First of all, we will be solving some numerical problems, which are related to the theoretical strengths as well as the variability of strengths as per the Weibull Modulus concepts. And next we will be talking about the stress concentration factor.

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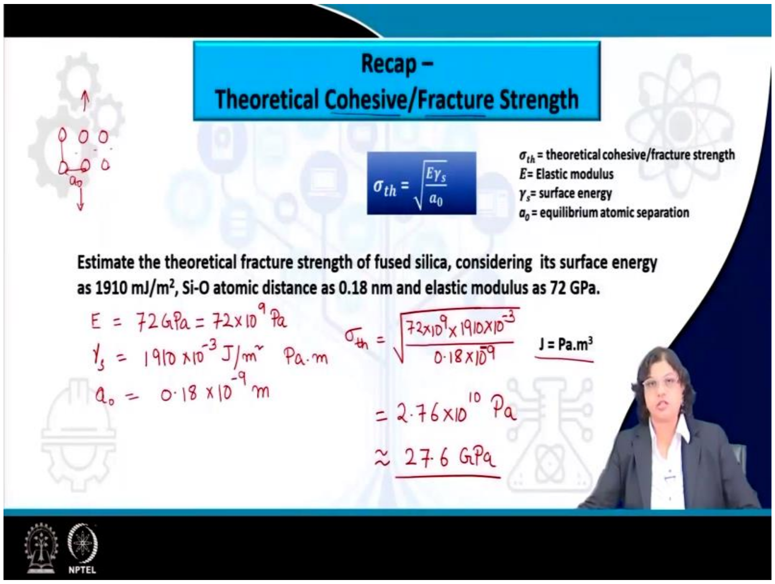

So, coming to the numerical part, in the last class, we have discussed about the theoretical strengths of material theoretical shear strength as well as theoretical cohesive or fracture strength of a material. So, what it essentially means is that, if we have atomic lattice like this applying stress over it at one point this is going to break and form new surfaces.

And that amount of stress that is required to break a perfect lattice system is known as the theoretical cohesive or theoretical fracture strength and this is given by a relation, something like this

$$\sigma_{th} = \sqrt{\frac{E\gamma_s}{a_0}}$$

where E is the elastic modulus, γ_s is the surface energy and a_0 is the inter atomic distance. So, this distance here is a_0 or even this distances is a_0.

So, if such is the case, let us see how in practice we can use this kind of formulation to solve a real problem. Here is one, estimate the theoretical fracture strength of fused silica considering its surface energy as 1910 mJ/m^2, silicon oxygen inter atomic distance is 0.18 nm and the elastic modulus is 72 GPa.

So, all the informations are provided in this and we are going to solve the problem accordingly. So, the elastic modulus is mentioned as 72 GPa which will be equivalent to 72 * 10^9 Pa and γ_s, that is a surface energy is given by 1910 * 10^{-3} J/m^2. So, we need to be very careful about the units and for that, we need to know the relation between the different units.

And in this case, this kind of relation would be important that joule is equal to Pa * m^3, so, that makes this unit as Pa m, and the other thing that we have is a_0 which is 0.18 * 10^{-9} m. So, the usual protocol is that we keep the stress unit as Pa or sometimes MPa and any kind of distance is typically expressed in terms of meters. So, that is how we have expressed everything.

And now we simply need to implement this formula. So, what it says is

$$\sigma_{th} = \sqrt{\frac{72 * 10^{9} * 1910 * 10^{-3}}{0.18 * 10^{-9}}}$$

So, that if we simply solve it. So, this is coming around 2.76 * 10^{10} Pa or we can simplify it as 27.6 GPa.

So, that is the theoretical strength of fused silica that we have calculated from this relation 27.6 GPa. So, that is pretty high enough and we will understand this in the next part of this lecture, when we will calculate the actual fracture strength of a material.

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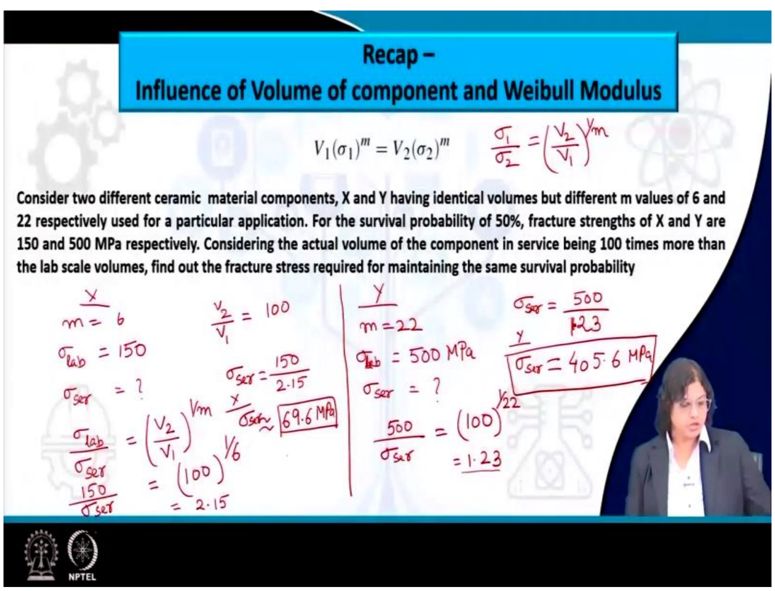

The next thing that we have discussed in the last lectures is influence of volume of the component as well as the stress straight and Weibull Modulus on the variability in the stress or the strength of a material and we have seen a very important relation which is related to the stress condition, the volume as well as the Weibull Modulus all taken together which is given by a relation like this

$$V_1 \sigma_1^m = V_2 \sigma_2^m$$

where 1 and 2 are the two different conditions and m is the Weibull Modulus we are all aware of this. So, again this relation can be simplified as

$$\frac{\sigma_1}{\sigma_2} = \left(\frac{V_2}{V_1}\right)^{1/m}$$

So, write the relation here as for our convenience which we will be using in solving the problem. Ok, so, the problem states here, consider two different ceramic material components X and Y which are having identical volumes, but different m values, the first one for X is 6 and for the second one the m value is 22.

And let me also write the conditions here. So, for X we have m equals to 6 for Y we have m equals to 22 and for the survival probability of 50 % fracture strength of X and Y are 150 and 500 MPa respectively. So, that means, that the sigma value here for the condition of X is 150 for the condition of Y is 500. So, it is important to note also here that as the Weibull Modulus is higher for the condition Y for the sample Y we see a higher fracture strength as well.

So, fracture strength is also related to Weibull modulus and by solving the problem, we will also get to see that how Weibull Modulus higher or lower value how does that influence the variability in the fracture strength of the material. Ok, so, this value is 150 or 500 MPa actually has been obtained from the lab scale testing.

But in actual practice considering the actual volume of the component in service being 100 times more than the lab scale volumes, and what we need to find out is the fracture stress that is required for maintaining the same survival probability at service. So, if we maintain the survival probability of 50 % even for service. And we know that in actual practice we are using 100 times bigger volume for the component than that used for the lab scale, we need to find out that what will be the actual strength of the material in service.

So, let us name this as sigma lab from the lab scale testing as 150 and 500 for component X and Y and what we need to find out is the service strength. So, in this case what we need to use the relation as

$$\frac{\sigma_{lab}}{\sigma_{ser}} = \left(\frac{V_2}{V_1}\right)^{1/m}$$

and we know that for both the cases V_2 / V_1 is nothing but 100.

So, that makes this $\frac{150}{\sigma_{ser}} = (100)^{1/6}$

Ok, so, if we solve this once again, this turns out for sigma service for condition X will be something like this. So, let me solve this 100 to the power of 1 by 6. So, this is coming around 2.15 and sigma service will be then 150 by 2.15 so, this is coming around 69.6 MPa. Ok, so, this is for condition X.

Ok, now let us solve this for condition Y and what we see here is $\frac{500}{\sigma_{ser}} = (100)^{1/22}$

so, $(100)^{1/22}$ is equivalent to 1.23. So, that gives us σ_{ser} = 500/1.23 and that is equivalent to 405.6 MPa. So, this is σ_{ser} for condition Y. Now, there are two important things to see here first of all for both the conditions what we see is and the service condition the fracture strength has been significantly reduced.

So, in case of X we see that coming from 150 which is the value that we have received from the lab scale testing. So, if we pretend that this is the value that we are going to get in actual practice we will be totally wrong, and we need to find out what the actual fracture strength would be at service depending on the volume of the component that will be used in service.

And we see that this has been reduced by more than twice the value actually by a factor of 2.15 and it comes down to 69.6 from 150. On the other hand, for the case of Y what we see here that the fracture strength at service is also reduced, but in this case the factor by which it has been reduced is only 1.23 and the service value for fracture strength is 405.6 MPa.

Ok, so, once again based on the Weibull Modulus, so since we have higher Weibull Modulus values for Y, we see that the value, of course, for the service condition has been reduced, but the extent by which it has been reduced is comparatively much less with respect to that for condition X. So, lower is the Weibull Modulus higher will be the scatter and lower will be the fracture strength value.

And in any case we see that fracture strength values of Y is much higher than that of X again because of the Weibull Modulus. For the case of metallic materials, typically, we have higher Weibull Modulus and that leads to less scatter or less variability in the strength value. So, whatever we determine in the lab scale more or less is valid even for the service condition provided the service condition is not very very high with respect to the lab condition.

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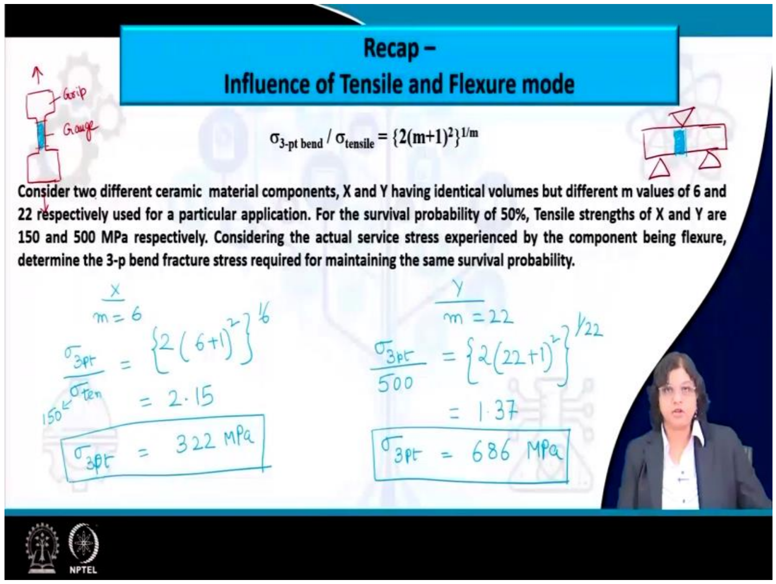

Ok, so, with this will move on to another problem that we have discussed in the last lecture, which is the difference in the fracture strength values if we are changing the mode of loading, right. So, for tensile or flexure mode of loading, we have seen that the flexure mode always gives us higher fracture strength in comparison to that for tensile loading.

Although we consider similar values of gauge length or span length for both the tensile and the fracture mode. And the reason for this we have specified is the condition, for example, if we have a tensile loading, this is how the tensile specimen looks like. It is known as a dog bone kind of specimen, this is the grip part. So, this is the grip part and this is the gauge part, gauge part is of actual interest because here is actually the stress is acting which leads to the deformation behavior and whatever deformation, whatever stress that has been applied is applicable on this entire gauge length. So, this entire part, just to make it a little bit more specific, let me color this with blue and this is the part over which the maximum stress is being applied.

On the other hand, if we are talking about the flexure mode, this is how it is. So, we have specimen of similar magnitude as that of the tensile one, of course, the drawing is not on scale, but in this case what we see is, in this case only a small part of the entire span length is where the higher or the maximum stress is applicable. So, this is the only a smaller amount of material or a smaller volume of component is under the influence of maximum stress and as a result. We also have seen that higher is the volume more are the chances that may be one of the defect or a few of the defects, which will be longer and sharper and oriented perpendicular to the loading direction that can lead to fracture. So, eventually what we end up is having higher fracture strength under flexural loading compared to that under tensile loading and this is again particularly valid for brittle materials, let us see, how we can use this for practical purpose.

So, here is a problem which says that, consider two different ceramic materials again the same component X and Y having similar m values of 6 and 22 respectively, which are used for any particular application, and in this case, for the survival probability of 50 % tensile strength of X and Y are 150 and 500. So, like the previous problem, I have kept it exactly the same, but in this case, apart from the service volume, if we are changing the mode of loading in case of service, the service one being flexure loading, then we want you to determine the 3 point bend fracture stress required for maintaining the same survival probability.

So, let me solve also this one more time. So, this is the condition for X and we have also seen that m in this case is 6. So, $\frac{\sigma_{3pt}}{\sigma_{ten}} = \{2(6+1)^2\}^{1/6}$

So, sigma tensile here is nothing but 150, so let me solve this as 7 square into 2 to the power of 1 by 6. So, again that factor comes as 2.15 and that leads sigma 3 point for condition X is this factor into 150, so which is 322 MPa. So, this is for condition X on the other hand for condition Y we have m equals to 22, so that makes sigma 3 point divided by 500 sigma tensile in this case is 500 and that is 2. So, relation like this and this factor would be so, this factor comes to 1.37 and that makes the sigma 3 point as 500 into this factor which is equivalent to around 686.

Ok, So, as discussed we have seen that fracture strength under 3 point condition is obviously higher than that of the tensile condition that we have seen. But, again for both the cases we have seen, higher is the Weibull Modulus less is a scatter this has been increased by only 1.37 times whereas, for the case of X component X because of the lower Weibull Modulus of 6 only we have seen that 3 point bend fracture strength has been increased to quite higher value more than twice the actual value.

So, this is what we solved and I would request the students also to do it simultaneously so that you get familiar with what actually the service condition, how does it differ from the lab condition and this again, this being a part of the course on not only fracture mechanics, but also the failure, we need to be very-very careful and understanding about the concepts and its actual impact.

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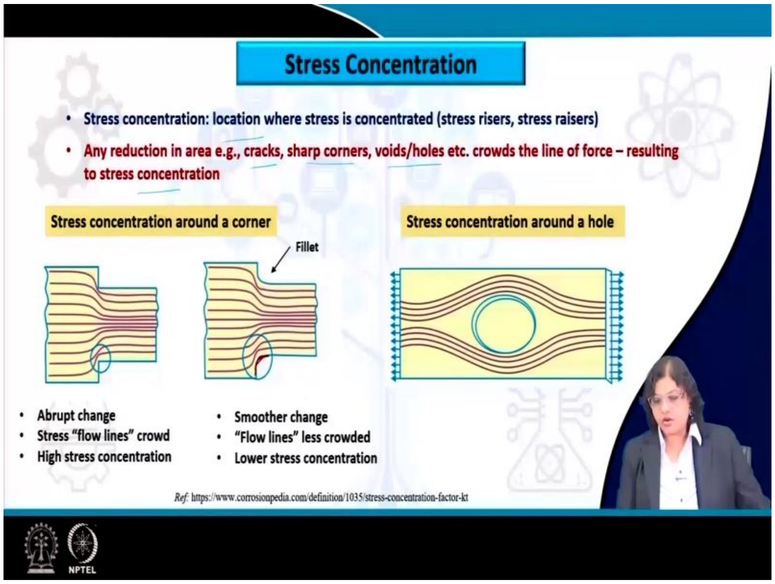

So, now, let us move on to, now that we know that there are defects in a component in a system defects are unavoidable. So, now, we need to find out that what would be the stress condition in case there is a defect, how does the stress gets influenced by the presence of that and so, what happens is that because of any kind of non-uniformity, not only defect. But any kind of non uniformity in the structure, which means that there is an irregularity in the component that leads to some stress concentration, if we are applying stress, the stress is actually not evenly distributed rather the stress would be concentrated at those locations. And this can be considered as stress risers or stress raises which increases the amount of stress.

So, any reduction in area this could be due to the presence of cracks, but not limited to this could be also because of the presence of sharp corners or voids or holes as per the design of the component any such kind of non uniformity in the structure leads to crowds in the line of force and that leads to stress concentration. So, you can see here these are the two examples of tensile specimen that I have drawn in the last slide.

So, this is one half of that you can see that this is the grip part and then comes the gauge part, but wherever it goes there is a corner that comes, ok, and at this corner, there is a stress concentration, why, because there is an abrupt change in the shape and that leads to a change in the flow lines. On the other hand, if we have those kinds of corners, but the corners is kind of blunt, in this case that we are seeing here, this however, leads to lower stress concentration.

So, we can see here that this one is much more blunt compared to the very sharp corners here and as a result, this leads to lower stress concentration, but there definitely will be some amount of stress concentration this is also valid in case we have a circular or elliptical or spherical void inside that also will lead to stress concentration, generation of a stress concentration.

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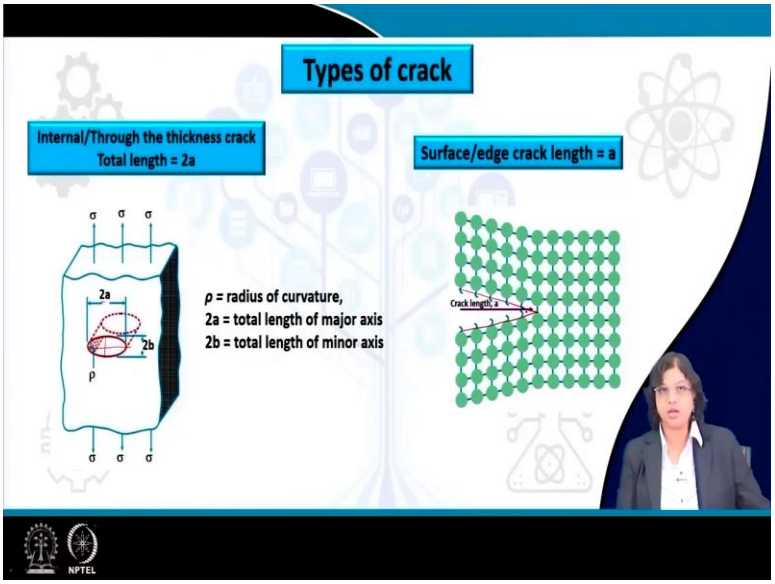

So, let us see that why this happens, but before that, we need to know also since we will be more and more discussing about the presence of cracks and how that influences the fracture behavior. Let us at the very first hand discuss about what are the different types of cracks that we will be considering and the most typical ones are the one which are considered as a central crack.

These are known as the internal crack are sometimes also known as through the thickness crack which means that the crack passes throughout the thickness and this one is an elliptical crack, which has a major axis of 2a and minor axis this one here is 2b and the radius of curvature of Rho. So, these are the typical parameters that we need to describe a crack.

On the other hand, there could be also age crack also known as surface crack in this case the crack is actually half of the total elliptical crack. So, let us say the other half is this part and we had truncated to the surface. So, that makes the crack as half the crack and in this case the crack length is considered as a not 2a but a.

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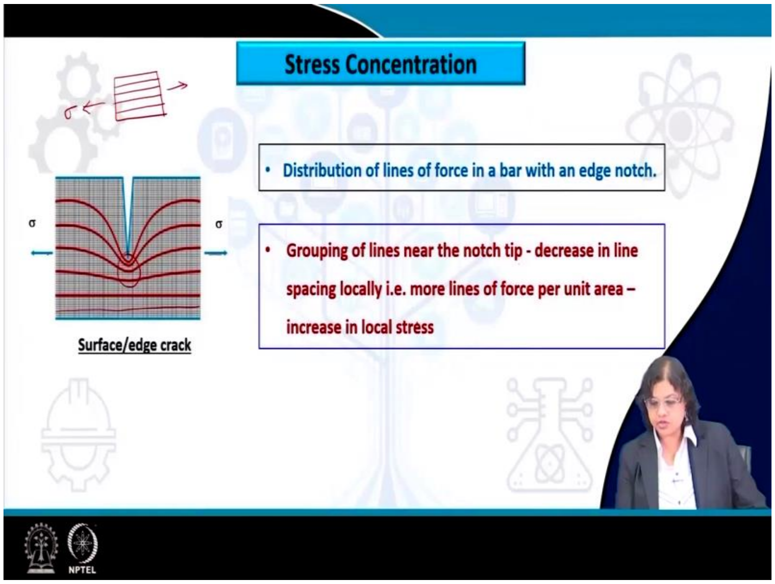

Ok, so, what happens, when there is a crack present in a component, this is what exactly happens, we are applying stress on both direction and these are the lines of force, we know it is very similar to the magnetic lines of force, the lines of force in case of a perfect component would be something like this.

So, we are applying stress and the stress is being distributed uniformly all throughout the part and that makes the lines of force perfectly parallel in case of a ideal perfect component with no non-uniformity or irregularity. Now, due to the presence of a crack, what happens is that these lines of force are getting concentrated near to the tip of the crack.

If you are moving far beyond from the crack tip, we will see that the lines of force, again comes back to its parallel rhythm. But at this point because the lines of force are getting squeezed, so if we want to find out the stress, obviously stress is nothing but force per unit area. So, at this portion the stress is being magnified, right, and that is nothing but stress concentration and we need to figure out that how much is the stress concentration values.

So, as I have explained that there is a distribution of lines of force in a bar with an edge notch and that kind of groups near the notch tip, notch or crack tip and that decreases the spacing in this lines of force and that increases the local stress.

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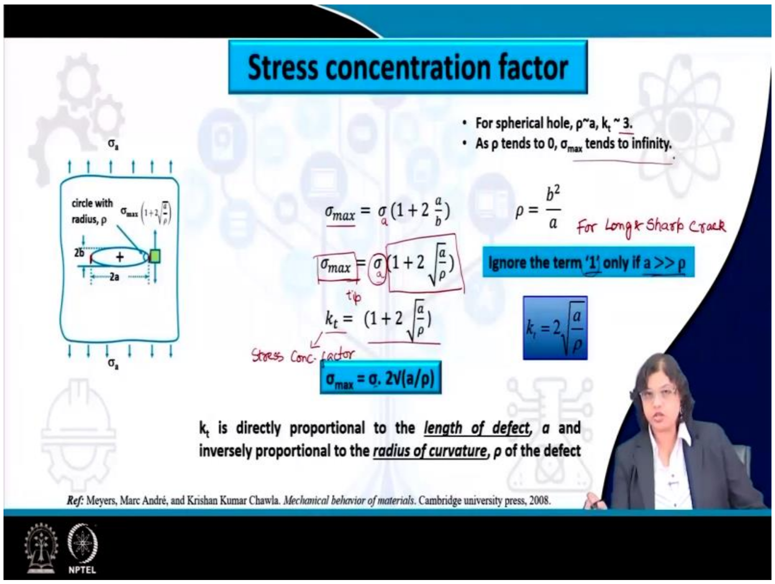

So, if we want to quantify that, this is how we do the σ_{max} at this point at the corner of the crack, at the tip of the crack or corner of any change in the component structure, we are going to get σ_{max} instead of σ_a . So, σ_a is what we are applying the stress. So, this could be σ_a, and what we are getting at the tip of the crack is nothing but σ_{max} and this is held by a relation as

$$\sigma_{max} = \sigma_a (1 + 2\, a/b)$$

this is as per the geometry of the crack.

So, there is nothing more to that, but it is simply because of this geometry factor this is coming as σ_a, multiplied by this term and we also know that this b and a are related to the radius of curvature. So, radius of curvature ρ equals to

$$\rho = \frac{b^2}{a}$$

So, if we consider this we can simply, simplify this relation as

$$\sigma_{max} = \sigma_a \left(1 + 2\sqrt{\frac{a}{\rho}}\right)$$

Ok, so, this is as simple as this, which means that if we are applying stress of σ_a, that is being magnified by this term here and at the tip of the crack we are getting a stress value of σ_{max} . So, this is nothing but the stress concentration factor. So, this is the stress concentration factor typically represented by k_t small k_t, and this is the value

$$k_t = 1 + 2\sqrt{\frac{a}{\rho}}$$

Now, if we have a very-very sharp crack, if the length of the crack is too high compared to the radius of curvature in that case, so, if we have a much-much higher than that of ρ in that case, we need to ignore this one term because it does not make sense even if we have this one term, it will unnecessarily complicate this equation and the formulation, but otherwise it will not make much of a difference. So, for very long and short crack, we ignore this one term.

So, this is valid for long and sharp crack or notch. So, for that case k_t is given by

$$k_t = 2\sqrt{\frac{a}{\rho}}$$

and hence, the relation will be simplified as

$$\sigma_{max} = \sigma_a \left(2\sqrt{\frac{a}{\rho}}\right)$$

So, this also gives us to think and to control k_t. So, we always want to find out that what are the ways by which we can reduce this k_t, term because k_t is not a very good factor for the presence of a crack because there will be stress concentration and that can lead to early failure, even if we are applying lower amount of stress because of this stress concentration factor. Here we are ending up getting a very high value of σ_{max} at the tip of the crack and that leads to a failure of a component at a much lower σ_a, value much lower than the actual fracture strength of the material. So, we definitely would like to reduce the stress concentration factor and there are based on this relation, we can see that k_t is directly proportional to the length of the defect and inversely proportional to the radius of curvature.

So, we would like to have a crack which is not very long or maybe not very sharp to achieve a lower value of k_t. For the case of spherical hole, you can determine this yourself that for the case of spherical hole, k_t turns to 3 in this case, however, we do not ignore the one term because spherical hole or a sphere or void is a no way considered as a sharp crack or where a, is much greater than ρ, a, is actually equals to ρ for the case of sphere.

So, that makes k_t equals to 3, and if ρ tends to 0, which means for very very sharp crack we see that σ_{max} tends to infinity.

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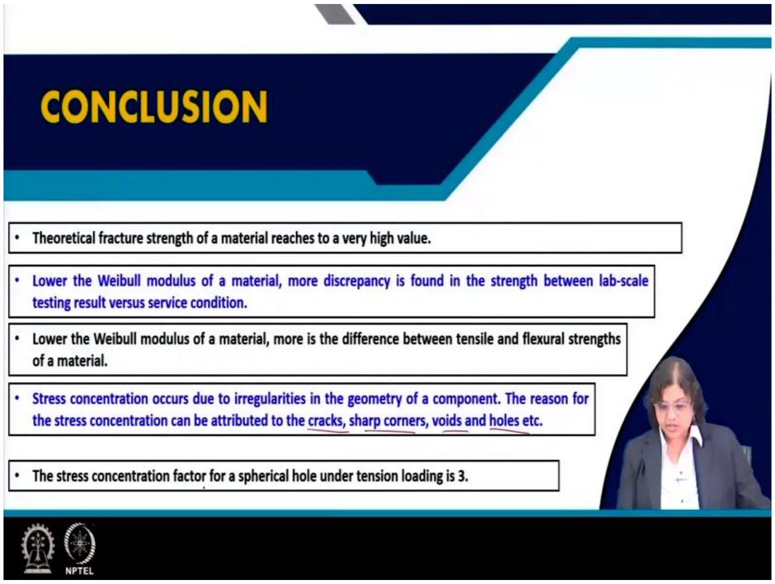

So, this is what we have and coming to the conclusion, theoretical fracture strength of a material reaches to a very high value that we have seen, which is hardly obtained in practice in the numerical problem that we have solved we have seen the theoretical value as high as 2.7 GPa and lower the Weibull Modulus of a material more discrepancy is found in the strength between the lab scale or the actual service condition or between the tensile and the flexural strength of a material. On the other hand stress concentration occurs due to irregularities in the geometry of a component. The reason for the stress concentration can be due to the presence of cracks or sharp corners or voids or holes or any kind of irregularities in the component and the stress concentration factor on the other hand for a spherical hole is nothing but 3.

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So, following are some of the references that has been, used for this lecture. And thank you very much.