Fracture Fatigue and Failure of Materials
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Lecture 02
Theoretical Strengths and Defects

Hello everyone. So, here is a second lecture on the course Fracture Fatigue and Failure of Materials. And in today's lecture, we are going to discuss some more on the module fracture and we will be discussing about the different kinds of theoretical strengths as well as the role of defects on fracture.

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So, these are the concepts that will be covered in this lecture. Starting from the theoretical shear and cohesive strength as well as the statistical nature of fracture. We will be talking about the Weibull analysis which is particularly relevant for brittle mode of fracture and then the different forms of defects that are controlling the fracture strength will be discussed and we will be analysing the survival probability in case of tensile and the flexural test as the last part of this lecture.

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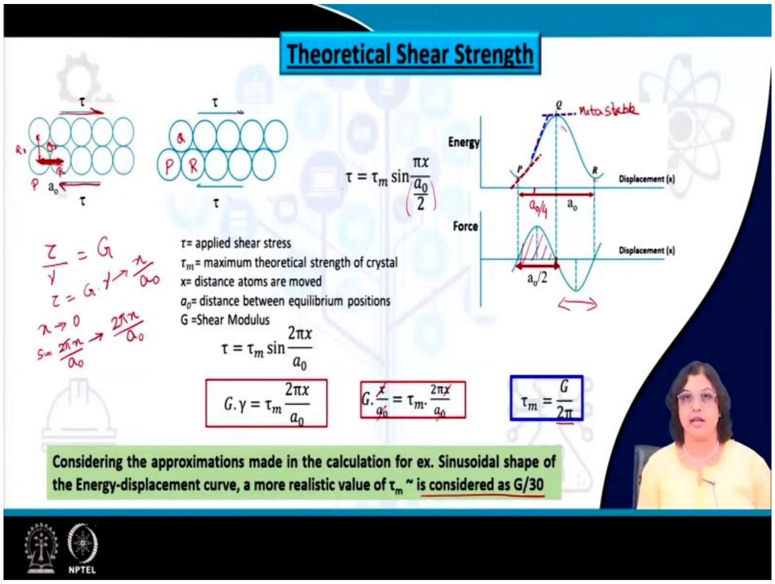

So, coming on to the theoretical shear strength. So, all materials are made of atoms and which are packed in certain orders. So, we can see here that there are a pack of atoms having a distance a_0 which is a inter atomic distance along the horizontal direction along the vertical direction also it has a distance of a_0.

And, when we are applying shear stress to this, with the magnitude of τ acting along both the direction you can see the upper one acting in this direction whereas, the lower one acting in the opposite direction. So, what is happening is that the position of this atoms are shifting and as a result, the behaviour also changing. So, let us see how this is happening.

So, initially let us name this atom as position P and R this atom as R and we have a central position as Q. Now, as we are applying the shear stress, this atoms are shifting to the right in this case as we are applying the stress along the direction of right. So, now, what happens is that, this has come to the position Q that is the central position.

Whereas the P and R are being filled by the neighbouring atoms and as a result, what is happening can be expressed in terms of a energy versus displacement curve. So, this signifies the position P for the first atom here and the position R and then Q has come on top of that at the central position. So, if we look into this more carefully, so this is how the energy will vary as the displacement is happening on atom at position P and R this are the stable position having zero energy requirement.

If we want to shift the atom from position P to Q, we have to continuously add energy. So, this is how this is happening, you can see this is the slope of the curved energy versus displacement curve, which essentially gives the force and if we are looking into this more carefully, what we see is that the slope from this red one to the blue one increases.

So, this is at a distance halfway to that of Q and at the point Q which is exactly the central location. So, this is the metastable location where we see that the energy bar displacement has come to zero. So, the slope here is almost horizontal. So, this is a metastable position. The force requirement here is zero. On the other hand, if we are moving from P to Q, we have to continuously increase the force required up to the point of half of that distance that is $a_0/4$.

So, this distance here is $a_0/2$ up to the point Q and up to the point of $a_0/4$ we are seeing the maximum force requirement, the energy requirement there is highest and then it gradually decreases, comes to zero at the point of Q and while moving from Q to R, we have to apply a negative stress in this case, so that to remain stable at any particular location. So, this is what we are seeing in the negative part of this curve.

So, if we try to quantify this value, we should use a sinusoidal relation which says that

$$\tau = \tau_m \sin\frac{\pi x}{a_0 / 2}$$

So, this is a typical sinusoidal equation here τ_m signifies the maximum theoretical shear strength of a material and τ is the applied shear stress, whatever stress that we are applying m is the maximum quantity of that and this $\frac{\pi x}{a_0/2}$ signifies that over we are calculating this over this span since the force comes to 0 at $a_0/2$ so we are considering this only up to $a_0/2$.

So, if we are simplifying this equation, this is how it comes as

$$\tau = \tau_m \sin\frac{2\pi x}{a_0}$$

Now, we know that this relation as shear stress by shear strain is given by G, the shear modulus. So, we essentially get

$$\tau = G\gamma$$

So, this is how we are simplifying this relation also when x tends to 0. So, for a very small displacement we are getting $\sin\frac{2\pi x}{a_0}$ is being simplified to just $\frac{2\pi x}{a_0}$ for very small angle we do not need the sine to be considered anymore.

On the other hand, the γ which is a shear strain is given by the displacement divided by original displacement, original distance. So, that is nothing but x/a_0. So, if we are implementing all this simplification in this relation what we are getting is nothing but

$$G.x = \tau_m \frac{2\pi x}{a_0}$$

we are simply cancelling out x and a_0 term here and we are getting this relation as

tau m equals to

$$\tau_m = \frac{G}{2\pi}$$

In a more simplified manner, instead of considering this as G/2π we typically consider this as G/30. So, this 30 is not exactly same as 2π, but what we essentially mean here is that, this maintains a similar order of magnitudes. So, if we know the G value, the bulk modulus or the shear modulus value of a material, we can kind of predict the theoretical shear strength of the material that it should be something like G/30 of this.

Of course, in reality, we do not see the theoretical shear strength values to be achieved by a material and that is due to the presence of defects particularly dislocations in the material. So, this was the very simple case when we were applying shear stress in the material, when it comes to fracture, we are more concerned about the application of tensile stress in a material.

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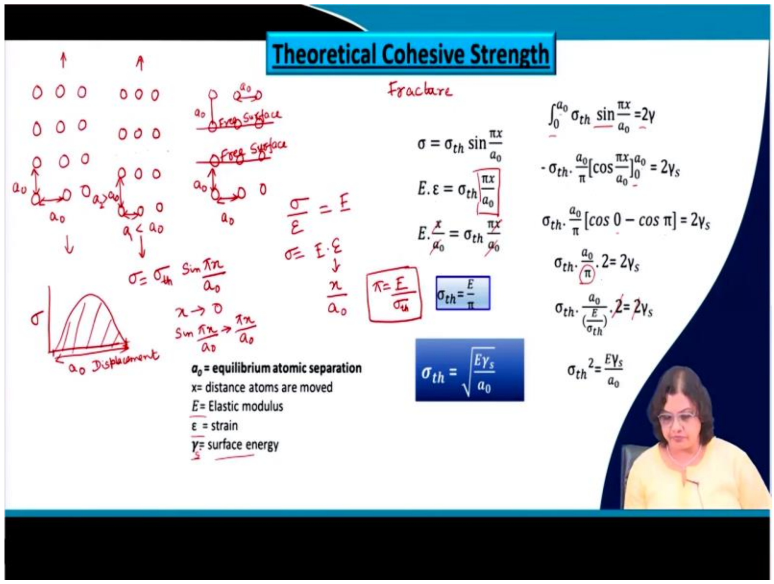

So, let's see what happens if we are applying tensile stress in the material and from there we will be finding out the theoretical cohesive strength of a material. So, for that, let me again draw this lattice. So, this was our initial lattice, for example. So, let say this distance both in the horizontal direction is a_0. Although this has been a free hand drawing, so it may not be on scale, but this is what I mean all the distance between two neighbouring atoms is a_0.

Now, when we are applying tensile stress to it, what will happen, this entire lattice will elongate along the direction of the stress and will squeeze on the perpendicular direction. So, this is how it will become like. So, the entire structure is being elongated along the direction of stress that is in the vertical direction in this case, and it will be squished in the perpendicular direction. So, now we have this horizontal distance instead of a_0, let's say it becomes a_1 and a_1 is less than a_0.

On the other hand, the vertical distance is now a_2. And a_2 is greater than is a_0. Because it is elongated. Now, if we keep on increasing the tensile stresses, what happens is that, based on the presence of some minute defect somewhere it can fracture or it can initiate the crack as we have seen in the last lecture that if you are applying tensile stress cracks initiate from the central part of the specimen from some kind of weak points, weak links or defects.

So, once it fractures from the central part, the lattice will again restored and now, we will be having two fracture surfaces. So, let's say it fractured along this plane and this is what we are going so the lattice distance on both the scale will be restored to a_0 here also. But, what we are getting extra in this case is the presence of two free surfaces. So, we are getting a free surface here as well as another free surface.

So, whenever there is a fracture always we generate 2 free surfaces. So, crack is also nothing but presence of two free surfaces. So, this is what we generate when there is a fracture and everything else remains the same. So, this can be once again explained based on the stress versus displacement curve.

So, if we are drawing this and if we are putting the stress on the y axis and the displacement or distance covered along the x axis similar to what we have seen for the previous case for theoretical shear strength here also we are seeing something like this. The only difference is that in this case instead of $a_0/2$ we are talking about the entire displacement.

So, in this case the curve will cover the entire span of a_0 because either it fractures at this point or not there is no halfway in between. And the relation that will be followed is given by this

$$\sigma = \sigma_{th} \sin \frac{\pi x}{a_0}$$

As I said instead of $a_0/2$ now, we have a_0 only. So, these are all the parameters here a_0 is nothing but the equilibrium atomic separation before we are adding this tensile stress and x is the distance the atoms are moved, E is the elastic modulus and then we have strain and the gamma values which will come later.

So, if we are having this kind of relation once again, we can simplify that for

$$x \rightarrow 0 \qquad \sin \frac{\pi x}{a_0} = \frac{\pi x}{a_0}$$

So, this is how we are getting this. And not only that, we have another simplification to make which is based on the Hooke’s law that we know that stress by strain is nothing but the elastic modulus or the Young's modulus.

So, stress will be given by E into strain and strain once again is the distance travelled by the atom by the original distance. So, it will be given by x/a_0. So, this is how this has been simplified here

$$E\,\varepsilon = \sigma_{th}\frac{\pi x}{a_0}$$

So, $\pi x/a_0$ part is coming because we know that x is a very small value tends to 0 so that makes $\sin(\pi x/a_0)$ as just $\pi x/a_0$.

So, this is how, we are elaborating the equation and we eventually get a very similar kind of relation as

$$\sigma_{th} = E/\pi$$

Often this is expressed as σ_{th} equals to E/10 instead of E/π. We often use it as E by 10 once again it just signifies the order of magnitude that is of importance. So, it is not E by 1000 or E by 100 it is just E by 10 which is a very simplified way of understanding or predicting the theoretical cohesive strength of a material.

So, if we know that theoretic the elastic modulus value of a material for example, for the case of titanium elastic modulus value is something like 100 GPa. So, we can assume that then the theoretical cohesive strength also known as theoretical fracture strength. So, theoretical fracture strength the theoretical cohesive strength could be just 10 GPa for the case of titanium.

But we are not done here unlike the shear strength part in this case as I said, as I mentioned that we are generating also two new free surfaces so we also have to take that into account. But before that let me also make another simplification which will be used in this relation which is coming from this relation itself.

So, $\sigma_{th} = E/\pi$. So, in other words we are getting π as nothing but E/σ_{th}. So, apart from this part here, what we need to implement now is the free surface how this free service has been modifying. So, basically whatever energy that is being generated through this process is utilized in making this free surfaces here. The energy that is released by this process has to balance the energy that is necessary to make these two free surfaces.

So, let's see how we can do this we can typically integrate this relation here over the span of 0 to a_0 because we are starting from here at the point 0 and ending up to a_0. So, we need to integrate this relation from 0 to a_0 this entire relation $\sigma_{th} = \text{Sin}(\pi x/a_o)$ and that is nothing but 2γ. Gamma is the surface energy and two appears for we are having two free surfaces that are forming here.

So, if we are simply solving this integration, we can see that there is a minus sign coming due to the sin integration of the sin and cos and we are integrating this over 0 to a_0. And what we essentially get is something like this. So, this 2 part is coming because of cos 0 and cosπ adding up as 2 and now, we are instead of this π term here we are using this this relation here as E/ σ_{th} and essentially what we are getting is something like this $(\sigma_{th})^2$ and this 2 and 2 gets cancelled.

So, which let us $E\gamma_s/a_0$. So, that is a very standard relation let me write this here. So, that gives us a very standard equation for fracture strength of a material theoretical fracture strength of the material or theoretical cohesive strength of a material which is given by

$$\sigma_{th} = \sqrt{E\gamma_s/a_0}$$

where E is the elastic modulus, γ_s is the surface energy often known as γ/γ_s, and a_0 is the inter atomic distances in the equilibrium condition.

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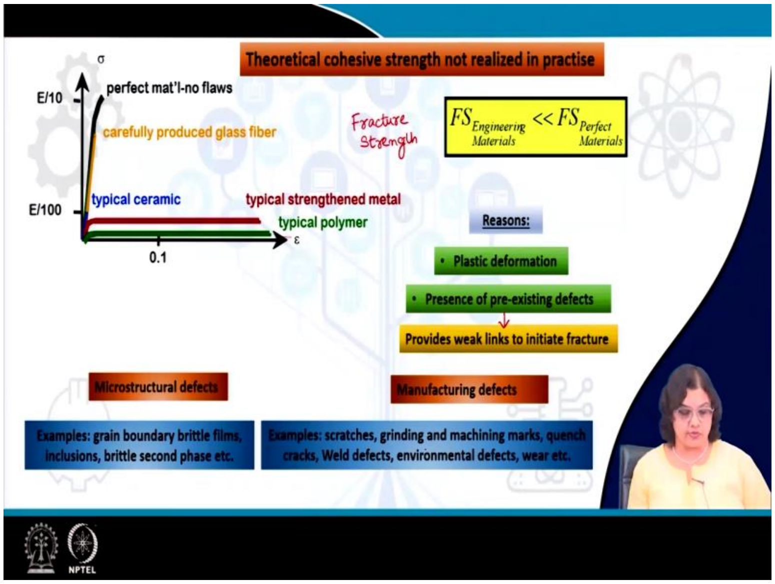

But once again in practice we hardly see the theoretical values to be obtained and what we see is that the fracture strength. So, FS stands for fracture strength here for actual materials for engineering materials is often much less than the perfect materials. As I have given the example of titanium, the elastic modulus of titanium is something like 100 GPA and accordingly the theoretical fracture strength or theoretical cohesive strength should be something like 10 GPa, but in reality, what we get is the ultimate tensile strength of the material is close to 1 GPA.

So, there is so much of reduction in the fracture strength of the material in actual condition and if we are looking into a curve like this, this is how we see for perfect material with no flaws at all, we should get the theoretical value of fracture strain something like E by 10. On the other hand, if we are careful enough to make glass fibre with almost negligible amount of defect, we can reach to almost as close as a theoretical value. Of course, not exactly 10 but maybe some what lower than that values but still pretty high.

If we are talking about strengthened metal for example, this brown curve here, you can see that this is of the order of E by 100 or even lesser, same goes for polymer also ceramic has some something like about E by 100. So, we can definitely understand that there is a huge difference in the values of what we are predicting based on the perfect on the ideal situation and what we are getting in reality.

And the reason for this are several first of all, there could be some amount of plastic deformation which we have not considered so far. And secondly, it could be also due to the presence of pre-existing defects, which is the most viable and this is what we see always that there are the presence of defects and these defects actually does nothing but it provides the weak links to initiate fracture whenever there is defect, that is actually helping the process of fracture to occur and these defects can appear from anywhere.

So, there are different ways by which defects are incorporated in a system although we are determined to make materials or make components as flawless as possible, but there are some defects which are coming into the picture because of the following reason that these defects could be either have microstructural origin or there could be manufacturing defects. So, the examples of microstructural defects are for example, grain boundary, even the brittle films that form at the grain boundary or inclusion or any second phases etc.

So, these are microstructure parts, we sometimes cannot avoid having all this and there could be manufacturing defects. For example, the most typical one are the scratches or even the grinding and the polishing marks etc. which are there. Then there could be quench cracks or weld defects, sometimes there could be casting defects also environmental defects due to wear also generate some amounts of defects. So, all this acts in deteriorating the fracture behaviour of the material.

But, it is not that all defects are equally harmful. So, the point is that if we cannot avoid the defects and actually sometimes defects or for example, grain boundary or inclusions or brittle phases are added to improve the strength of the material for some other purposes. So, if we cannot avoid the presence of defects, what we need to understand and what we need to employ is to make the defect less and less harmful. Or we can reduce the number of the defects which are more critical.

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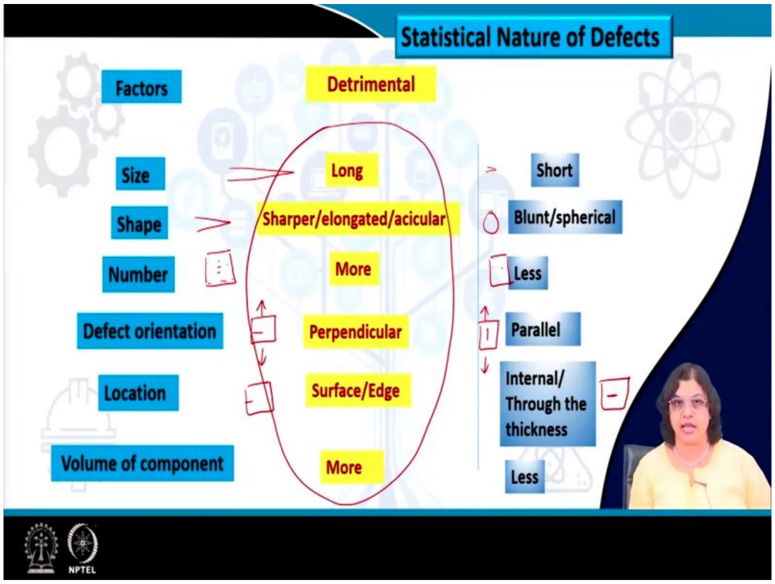

So, for that we need to understand the factors which are controlling the fracture behaviour. For example, the size we can have a small size defect or a large size defect a long one or a short one. Ofcourse, the long one would be more detrimental. So, if we have a defect something like this versus a defect, which are quite short of course, the long one will be the most detrimental. Shape of the defect if we have a sharp defect like this or if we have a spherical void like this a blunt one, the sharp one will be more detrimental.

Number of defects, if we have a component like this and if we have many number of defects there, versus if we have a component with just a few defects. The more the number, the more chances are to get fractured. Defect orientation is also another very important thing. So, if we have a component and we are applying stress along this direction, if we have the defects which are perpendicular like in this way versus if we have the component in which the defects are oriented parallel to the loading direction although being of the same size, the perpendicular one would be the most detrimental one. That will be more prone to propagate and lead to fracture.

Location of the defect is also important, if we are having a defect at the edge versus if we are having the defect at the through the thickness. So, at the centre one versus the edge one also known as a surface defect. So, surface defects are ofcourse, more dangerous. Volume of the component. Now, this is also a tricky one, we generally tend to think that smaller the volume better are the probability of survival and bigger volume is going to fail.

So, this is what is being seen here that more is the volume actually the number of defects that can lead to failure would be more and that can lead to early failure. Compared to that, if we have less number of less volume of the component, then the possibilities of failure also reduces and then it could be even less detrimental.

So, overall, we can say that all these factors are detrimental and we should try to avoid this factor. So, if we know that there are defects, we do not want our defects to be long or sharp or more in number or at the least that it is not should not be perpendicular to the loading direction or located at the edges.

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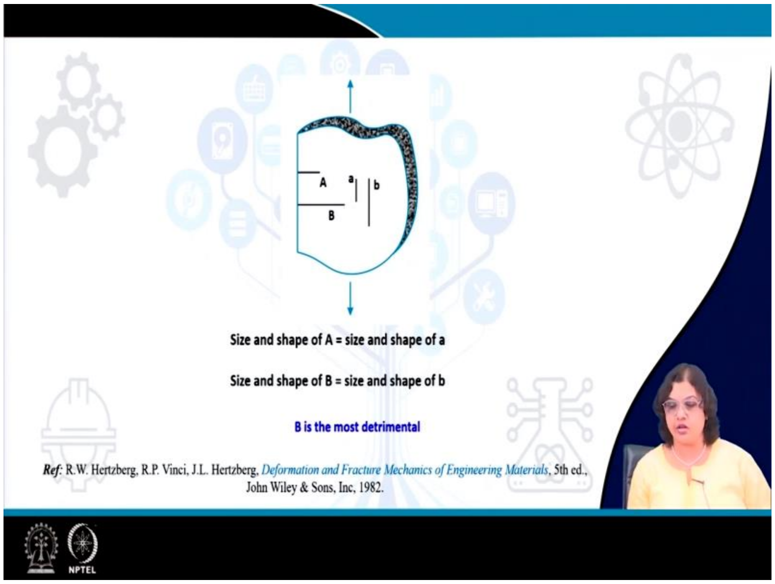

So, here is a simple exercise for you, let’s say we have a component and there are four different defect forms here capital A and small a are of same size and shape and capital B and small b are of same size and shape. So, which one of these defects do you think would be more detrimental? So, what we find here is the capital B one which is perpendicular to the loading direction as well as which is having the, the longer size is the one which would be more detrimental and should be avoided.

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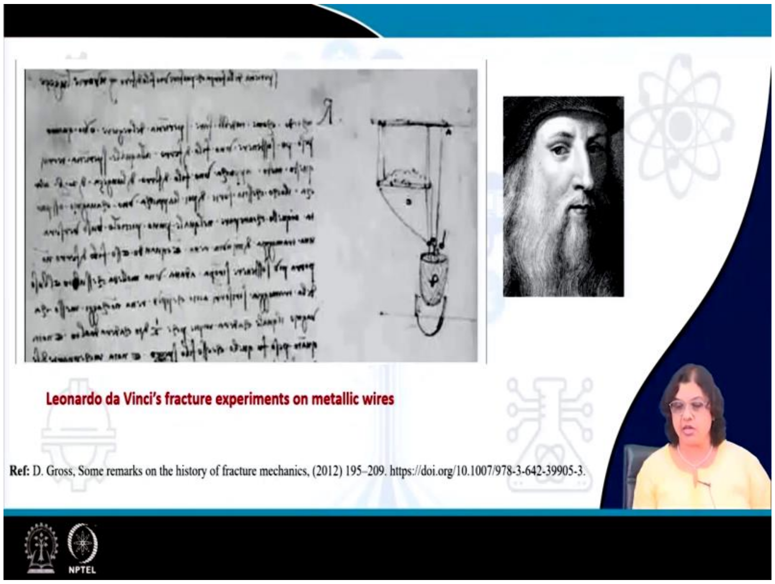

So, here is an experiment that has been demonstrated by the famous Leonardo da Vinci. So, Leonardo da Vinci is actually considered as a father of fracture mechanics. So, he is a pioneer in understanding fracture mechanics and demonstrating that in a very wonderful scientific way, which has led us to understand fracture mechanics in the modern world. So, this is an example from the original diary of Leonardo da Vinci the drawing that he has meant and he has pursued the experiment based on this.

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This paper is for reference for any of you who would be interested to learn more about how this has been described.

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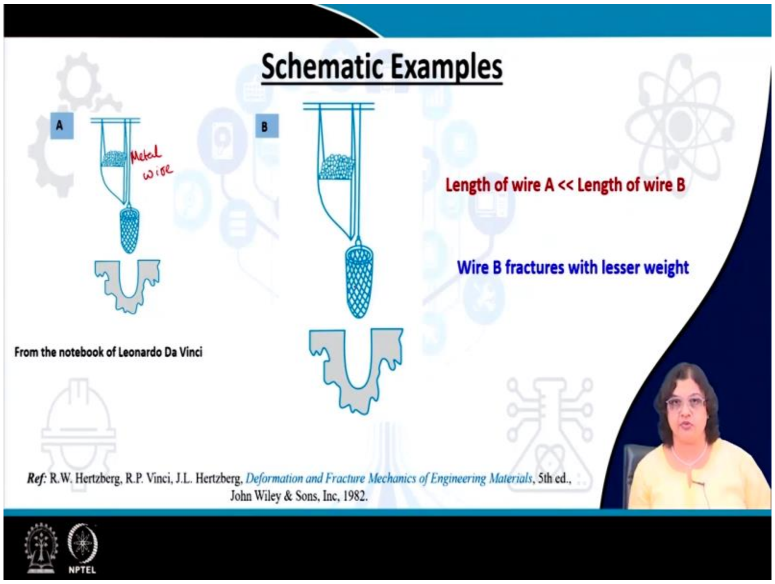

So, this is a schematic example of what Leonardo da Vinci has drawn in his notebook and how he has performed experiment. So, there are two conditions here A and B. So, in the first case, there is some load applied to this and this there are some pebbles actually and this is hang with a metallic wire. So, this is a metal wire, the difference between A and B is the length of this wire.

So, length of the wire A is much smaller compared to the B everything else remaining the same. So, what he the experiment that he performed is in the sense that how much of load does this metal wire can carry with respect to the wire at B everything else remaining the same. And this is just a pit so that if it falls it should be within this distance itself.

So, what he has observed and what has made us understand fracture in a more clear way is that B fractures at a lesser weight, why because the length of the wire B is much higher, of course, these are made of the same material. So, both A and B are of same material, but B has longer length. So, that means that it can have more number of defects, which could be either longer or perpendicularly oriented or maybe sharper, that can lead to failure and that leads it to fail and carrying much lesser weight.

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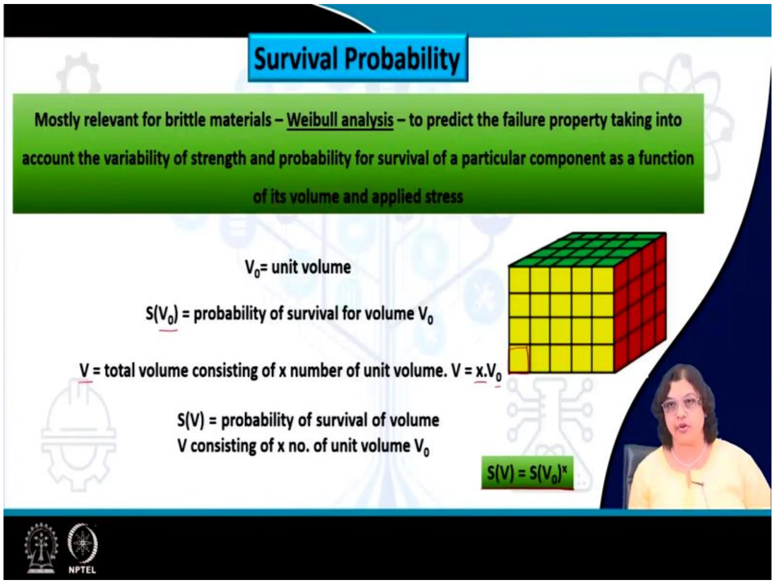

So, this introduces us to another important concepts, which is the survival probability, not everything will survive up to the same level of probability. And this is mostly relevant for brittle materials, where we do understand and find out this relation based on the Weibull analysis to predict failure property taking into account of variability of strength and probability of survival as a function of its volume and applied stress, so all taken together.

This is just an example, let us say a Rubik's cube and for each of this cube, we have a volume of V_0. Now, survival probability for unit sell V_0 is S V_0. And if we are considering the total volume, which is nothing but x into V_0, where x is the total number of cubes in it. So, the total survival probability will be given by a relation like this is V equals to S V_0 to the power of x.

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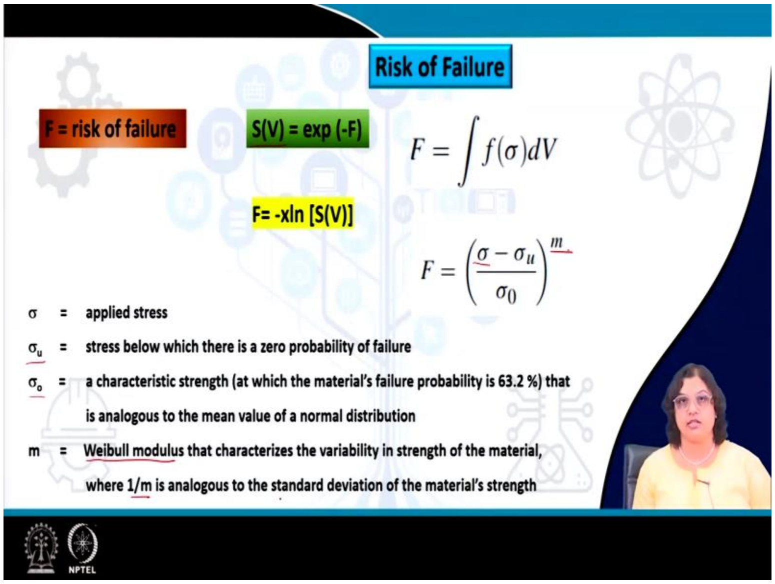

Now, if we try to find out the risk of failure, risk of failure is nothing but the exponential or the x ln survival probability, which can be written in this form and so survival probability can be written in terms of failure and vice versa. Basically, failure is nothing but explained with a relation like this as

$$F = \left(\frac{\sigma - \sigma_0}{\sigma_0}\right)^m$$

So, here are several parameters that we are introducing and I would like to explain that briefly, in the sense that sigma is nothing but the applied stress. So, this is the stress that we are applying σ_u is important term it is a stress below which there is zero probability of failure. So, this σ_u value could be even 0 for the case of brittle material.

So, even if one of the components survives there could be other component of same design and same material, but because of the presence of defects that can fail. So, this could be as low as zero for brittle material and this could be equivalent to fatigue strength in case of ductile material. σ_0 is based on the variation in this fracture strength value and this is analogous to the mean value of the normal distribution. The other term m is another very important parameter.

So, this signifies the Weibull modulus. It characterizes the variability in the strength of the material and 1/m is actually analogous to the standard deviation. So, if we know that there is scatter in the values there are change in the values, we would like to know that how far this scatter is and we should be able to understand the standard deviation. So, that is represented by the term m.

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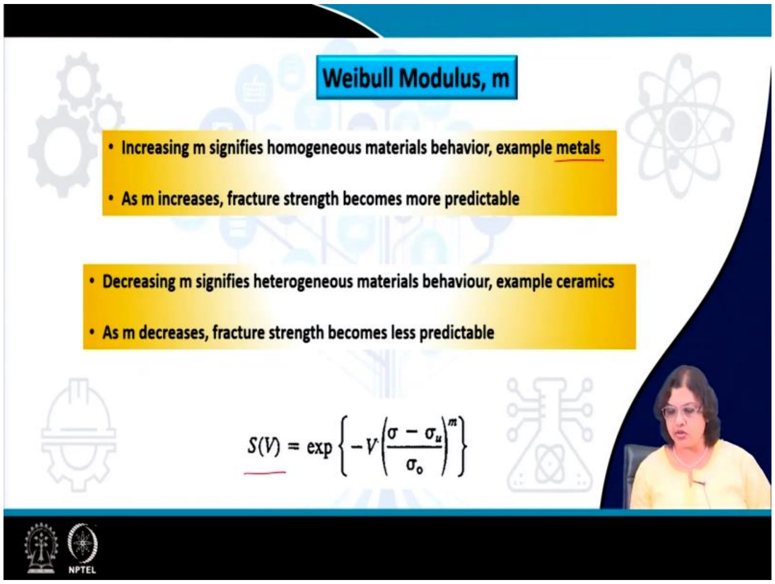

Higher value of m signifies material behaviour is homogeneous that means, scatters are less. So, this is seen for the case of metal. Fracture strength becomes more predictable if there is a higher value of m. On the other hand, decreasing value of m signifies heterogeneous material behaviour for example, ceramics. So, as m decreases, fracture strength becomes less predictable. So, this is the relation between survival probability and considering all these parameters here including m.

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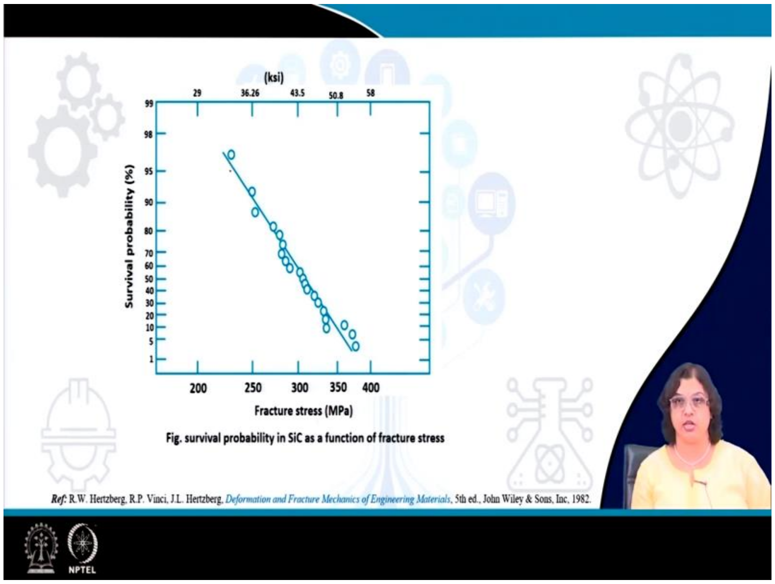

So, here is a experimental graph, which shows how the survival probability decreases if the fracture strength increases or vice versa fracture strength decreases, if we want to talk about high survival probability.

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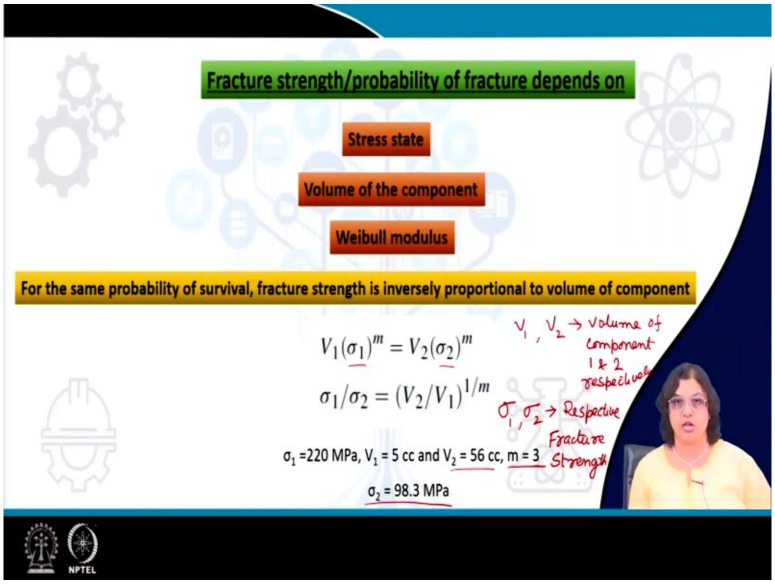

Now, fracture strength or probability of fracture depends on a very essential parameters which are the stress state in which form of stress that we are applying volume of the component as well as Weibull modulus and all this are taken care of by a relation. For the same probability of survival fracture strength is inversely proportional to the volume of the component.

So, this we have explained that as the volume of the component is higher then the possibility is that one of the many defects that is the most detrimental one could be there that can lead to early fracture. So, this leads to a relation something like this

$$V_1 \sigma_1^m = V_2 \sigma_2^m$$

Where V_1 and V_2 are the volume of the component and the corresponding fracture strength are given by $\sigma_1$and σ_2.

So, V_1 and V_2 are volume of component 1 and 2 respectively and sigma 1 and sigma 2 are respective fracture strength. So, we get a relation like this and here is a small example that if we are applying a fracture strength of 220 MPa for a volume which is 5cc. If you are now changing the volume to 56cc in both cases, Weibull modulus is 3 what we are ending up is getting a fracture strength, which is just 98.3.

So, so much reduction if we are increasing the volume of the component. So, this has a very important role if we are talking about finding out the fracture strength at the lab scale for smaller component and then using it for the real life where a big component of similar material is there.

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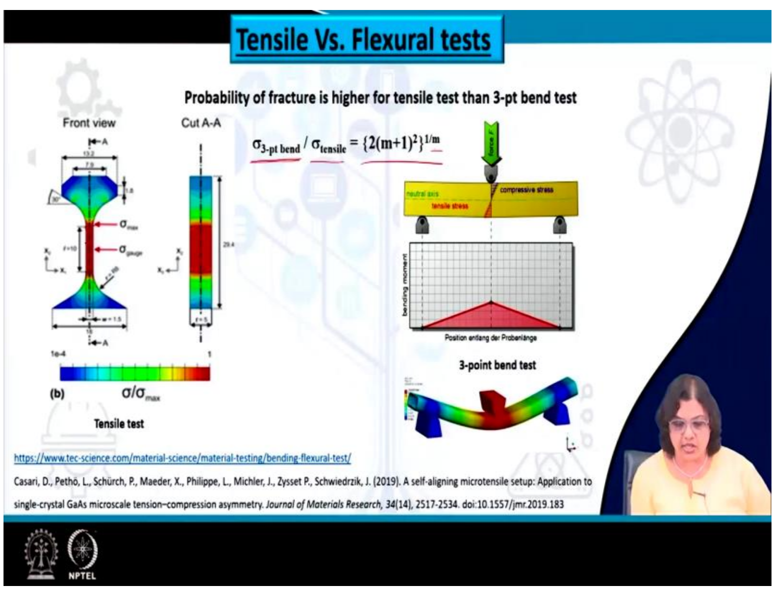

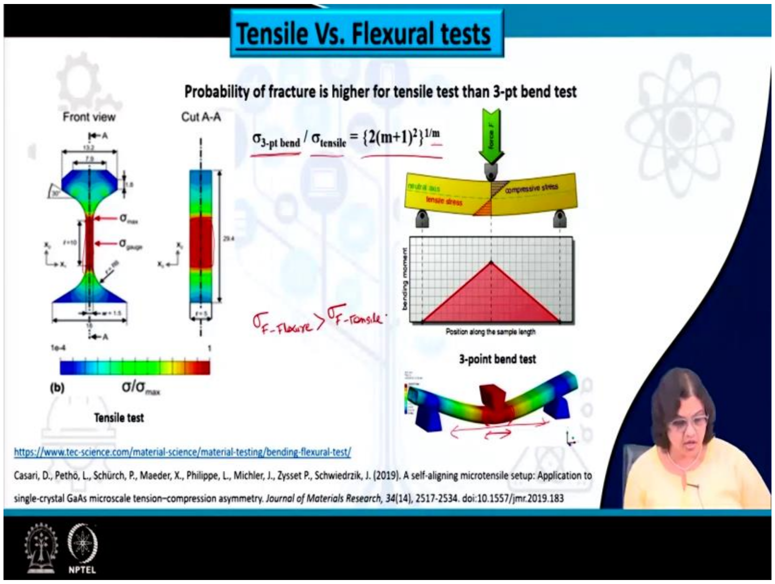

So, here is how it is different for the mode of test. So in this case we are talking about tensile versus flexural test and what we are seeing is that the fracture strength under 3 point bend is that the fractional loading divided by the tensile facture strength is given by a relation which is related to the Weibull modulus of the material

$$\sigma_{3\text{-pt bend}} / \sigma_{tensile} = \{2(m+1)^2\}^{1/m}$$

So that essentially gives the probability of fracture is higher for tensile test than the 3 point bend test.

Or in other word, we can say that fracture strength under flexural loading conditions would be higher and in comparison to that for the tensile test. So, why is that for understanding that we need to understand how the volume of the component or the specimen that is under tensile and the flexural loading is appearing.

So, in this case of tensile loading we see that there is the central gauge section, which is under loading. And this is a uniform the entire section is getting same amount of stresses. So, this is the entire gauge section which is getting this high value of stress. On the other hand when we are talking about the flexure test so you see that this is a 3 point bend test loading is applied on the midsection while these to others are the signifies the span length.

So, the span length and the gauge length remaining same in this case of the flexure test it is only the central location very much restricted location where we are getting the highest value of stress. So, obviously the volume which is under consideration for the highest value of stress is much smaller compared to the tensile condition and hence fracture strength for sigma fracture for flexural test is higher than sigma fracture for tensile test.

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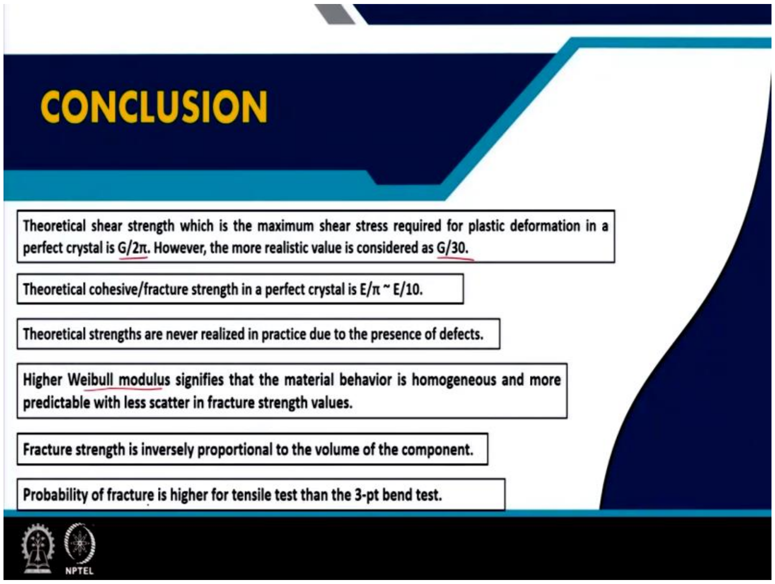

So, this is another interesting understanding that we had. So, overall in this lecture we have discussed about the theoretical shear strength, which is given by a relation G/2π, and in reality, but we consider is around G/30. On the other hand, theoretical cohesive strength or theoretical fracture strength is given by E/π or E/10.

Theoretical strengths are however never realised in practice due to the presence of defects. And we have also seen how the different kinds of defects whether this is sharp or loaded in different directions can lead to deference in the fracture behaviour. We have also understood about the importance of Weibull modulus and higher Weibull modulus signifies that the material behaviour is homogeneous and more predictable with less scatter in the fracture strength values.

And fracture strength is inversely proportional to the volume of the component. We have also seen that the probability of fracture is higher for the tensile test in comparison to 3 point bend test having similar gauge length of the span length condition.

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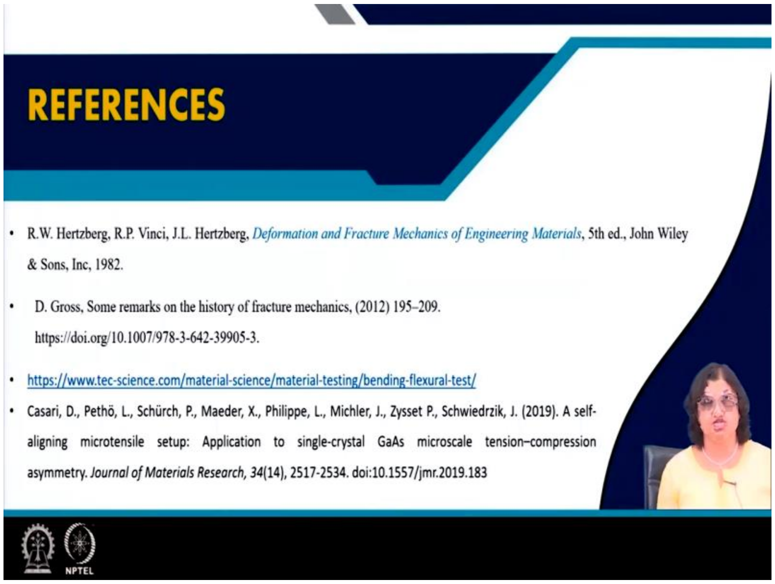

So, these are the references that has been used in this lecture and I hope that you have understood the lecture. Thank you very much.