Fracture, Fatigue and Failure of Materials
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Lecture 14
Plane Stress Fracture Toughness-J Integral

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Hello everyone and welcome to the fourteenth lecture of this course, Fracture Fatigue and Failure of Materials. And in this lecture also we will go ahead and discuss some more on the concepts of plane stress fracture toughness through J integral.

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So, the following concepts will be covered in this lecture, we will be discussing a little more on the J integral and how it can be determined. And particularly most importantly the physical significance of J integral method and the critical value of J that is required for fracture, so what does that actually mean. And we will also look into the ways by which we can determine this critical value of J that leads to plane stress fracture toughness values.

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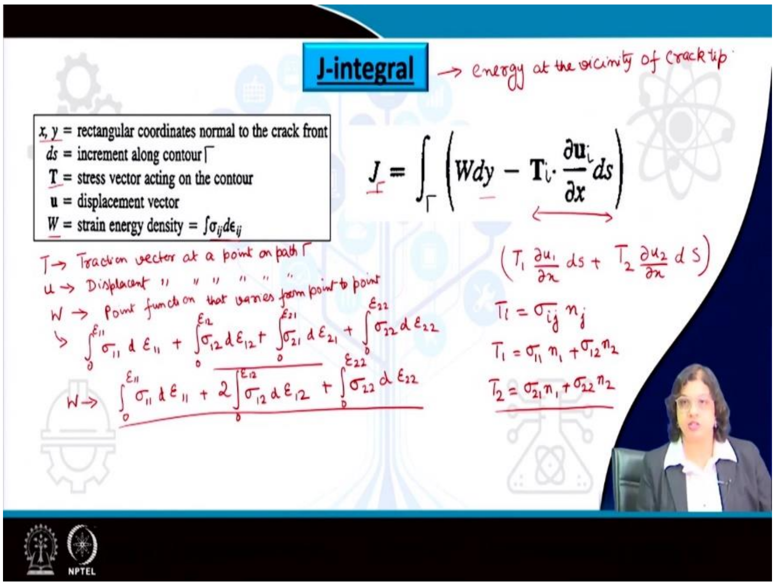

So, let us start from where we left in the last class, we have discussed about the way by which J integral can be used to determine the fracture toughness. It actually signifies the energy at the vicinity of the crack. So, the J integral method is used to determine this energy scenario at the vicinity of crack tip and in the very simple way, for any contour of any shape, we can find out a relation which exists at the different points of the contour this is what it is going to maintain that $J = \int_\Gamma \left(w\, dy - T_i \frac{\partial u_i}{\partial x} ds \right)$, and this is integrated for the entire contour, this equation is valid for any particular point and this is integrated for the entire contour so that we can get the energy of this contour as J.

Now, each of this term should be explained in more details and we have already seen that x, y are nothing but the coordinates which are normal to the crack front in along the direction of y, x load should be added and x is the direction for the crack growth. And ds is the increment along the contour part. Now, T is very important factor here T is the stress or the traction vector at a point on the path. And so, let me also write it down here as basically signifies the traction vector at a point on path in this case sit down.

U on the other hand is the displacement vector again this is valid at any particular point. So, u is this displacement vector at any point on the path and this is supposed to change from point to point. On the other hand, if we are talking about the W part here, which represents the strain energy density or it is actually in more simple words it is the strain energy per unit volume, but it again varies from point to point. So, this is also a point function that varies from point to point so that we can determine this relation at any particular point and if we are integrating this for the entire contour, we are supposed to get the energy for the entire path.

So, let us expand these individual terms for the first one, let's say we do this for W and W is representing the change in the stress with the change in the strain. So, W can be expanded in the following way for one to coordinate path, let's say we can do this as $\int_0^{\varepsilon_{11}} \sigma_{11}\, d\varepsilon_{11} + \int_0^{\varepsilon_{12}} \sigma_{12}\, d\varepsilon_{12} + \int_0^{\varepsilon_{21}} \sigma_{21}\, d\varepsilon_{21} + \int_0^{\varepsilon_{22}} \sigma_{22}\, d\varepsilon_{22}$

So, these two are actually equivalent and that makes us the overall W by simply expanding this as, $\int_0^{\varepsilon_{11}} \sigma_{11}\, d\varepsilon_{11} + 2\int_0^{\varepsilon_{12}} \sigma_{12}\, d\varepsilon_{12} + \int_0^{\varepsilon_{22}} \sigma_{22}\, d\varepsilon_{22}$, that is the overall strain energy density at any particular point.

So, the second term itself, if we are expanding this, this should be like $T_1 \frac{\partial u_1}{\partial x} ds + T_2 \frac{\partial u_2}{\partial x} ds$. So, that represents including the traction at any point as well as the displacement at any point and how that can be used to determine this factor here. If we also need to expand the T vector here itself, so, T is the traction vector can be also written as T_i is related to the sigma value at that point and the coordinate.

So, that makes $T_1 = \sigma_{11} n_1 + \sigma_{12} n_2$ and $T_2 = \sigma_{21} n_1 + \sigma_{22} n_2$. So, we can expand all these factors all these parameters and we can have this relation for any particular point over the entire path and we can integrate this to get the overall energy. Now, this is all the mathematical expression for the J integral term and this is valid for any kind of energy or any other situation that we want it to correlate with. But, when we are talking about the crack situation, we should explain it on the basis of the presence of a crack and the region in the front of the crack.

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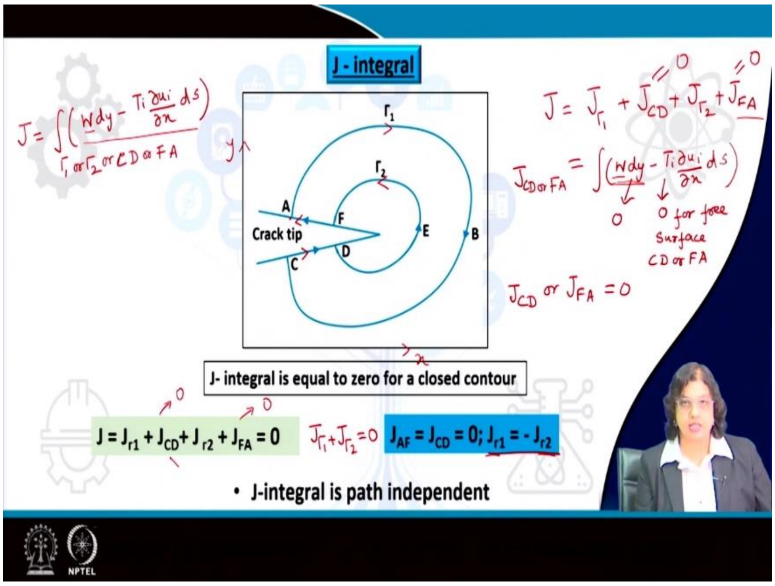

So, let us see how it looks like. So, this is what is a crack tip and this is the contour in front of the crack ahead of the crack. So, this is the y and the x directions. So, for each of the segments, so, there are a few segments in this contour there are actually two contours that we can see. It is starting from the point A here, so, this is represented by contour Γ_1 and then it is coming to the point C which is the free surface of the edge of the crack and it is moving along C D. So, this is one segment and then it is moving along the contour Γ_2 and finally, F and A and coming back to the starting point. So, that makes it four different contours all by itself.

So, the overall J is related to the contour of Γ_1 plus it is in moving from C to D. So, like this and then the contour of Γ_2 and finally, J for the contour F and A. So, we need to fit this relation for all the different segments. So, for each of this we should let me write it here, the relation as J is given for any kind of contour. So, let's say this could be Γ_1 or Γ_2 or a C D or F A for any of this the same relation should be valid which is given by $\int_{\Gamma}\left(w\,dy - T_i \frac{\partial u_i}{\partial x} ds\right)$

Now, for contour tau1 and tau2 we have seen that how individual of this can be expanded and we can find out the overall relation and the overall values. For the case of C D and F A, these are the two segments which are standing on the free surface of the crack weight. So, there it gets very interesting and we can once again expand this as the following. So, J of C D or even J of let me write a J of F A, this should be given by this relation once again. So, $\int_{\Gamma}\left(w\,dy - T_i \frac{\partial u_i}{\partial x} ds\right)$.

Now, for the case of the C D or F A, for that matter actually there is no growth along the y direction. So, dy essentially is 0, the crack is going along this one, when it does, but dy at any case will be always 0, so that makes the first part. Anyway the product of W and this dy whatever finite value of W is having it is still getting a value of 0. On the other hand, if we are talking about the second term, here also we are seeing this T, this traction vector acting perpendicular to this point.

Now, this is also zero for a free surface. And this C D or F A are the free surfaces of the crack, so that makes it also 0. And again, the product for even displacement it might be having it still gives a product of zero and overall we are getting that J_{CD} or J_{FA} are equivalent to 0. So, these are the individual thing we have made. So, this gets equivalent to zero and this also gets equivalent to zero, but overall the entire energy is still given by this four segment.

And most interestingly, there is another fact that we have to consider that the J integral for any closed contour always comes to zero. So, the summation of this is anyway going to come as zero. So, which makes J as a total the integral for this entire path is $J = J_{\Gamma 1} + J_{CD} + J_{\Gamma 2} + J_{FA}$ as explained here, this total summation comes to zero. And then out of this we have also seen that how J_{CD} gets to zero and J_{FA} gets to zero.

So, that makes actually $J_{\Gamma 1} + J_{\Gamma 2}$ equals to zero or in other words, we can see that J of tau1 equals to $-J_{\Gamma 2}$, which means that even if we are considering it in this direction, or that one that is independent of the path and the magnitude of this Γ_1 and Γ_2 for the J integral part is the same, the magnitude of this free surface are any way coming to zero and the magnitude for the contours whichever path are we following are the same. So, which makes it a path independent term. So, J integral is a path independent it does not matter which path we are considering and at the vicinity of the crack tip near to the crack tip, this is the energy scenario that we can find out based on this relation.

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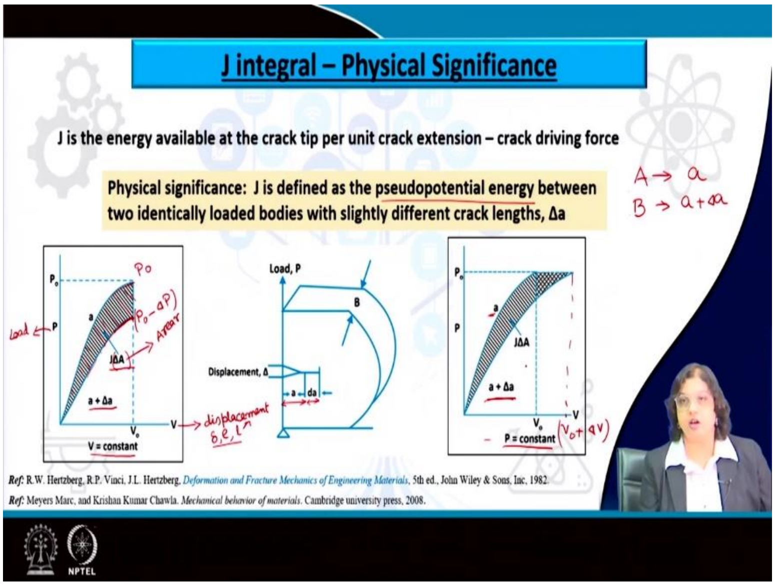

Now, this still comes as a bit of mathematics here and so, that we can use this relation to find out the number, but what does it actually mean physically what is the physical significance of J integral. To understand that, we again have to go back to what we have understood when we talked about the G term the strain energy release rate. So, J integral is also a very similar concept and it represents the of course, the energy which is available at the crack tip per unit crack extension.

So, as the crack is extending per unit time, how much energy is available at the crack tip which acts as the driving force for the crack tip growth. So, it is considered as the pseudopotential energy difference between two identically loaded bodies with slightly different crack length.

So, if one body is having a crack length of a. Let us say we are talking about a component a capital A which is having crack length a and a component B which is having a crack length of a + ∂a, whatever is the difference in the pseudopotential energy between these two components that is given by J or in other words, the same component if the crack grows from a to a + Δ a, how much energy will be released the strain energy that is being released that is considered as J. So, this definition wise also it sounds very familiar to what we have seen for the case of G.

So, this is shown here initially there is a crack length of a and then it is growing by a infinitesimally small amount da, of course, it is schematically shown and not on scale. But if such is the case, then how much of the energy that is being released that is termed as J and there can be two ways to obtain that either we can consider that the volume is constant. So, here both of these curves here signifies the load versus displacement curve.

And in one case, so, this is the load and this V signifies the displacement. So, typical load versus displacement curve that we get if we are applying a tensile stresses on any kind of component. The symbol of this displacement sometimes are also used as Δ or e or l, so just to be familiar with that all of these symbols actually represent a displacement. Now, what is important here to notice is that either when the crack is growing, if we are talking about a situation when the displacement is constant, it is being controlled it is a constant value then there is a drop in the load that we are seeing. So, this could be one of the cases and the energy change is represented by this hatched space here.

So, initially, it has a crack length of a and this is what the graph looks like and again this being a elastic-plastic material apart from the elastic part, there is also some amount of plastic deformation here that is noticeable. On the other hand, once the crack grows to $a + \Delta a$, there is a drop in the load and this is the load-displacement curve for the second part when we have $a + \Delta a$ crack length. And once again here also we are seeing the elastic and the plastic part very prominently.

The energy that is being released is given by this hatched area here and this is equivalent to J times the total area. So, this capital A here signifies the change in the area. And on the other hand, there could be the other scenario when we have P equals to constant, so that means that we are applying a constant load in that case, since the load will be constant there will be an enhancement in the V, because of the second case when we have $a + \Delta a$ crack length.

So, the first case is when we have crack length of a, second case is when we have crack length of a plus delta a. In one case we require volume of V_0 and in the second and the load of P_0 and on the other hand, if we are controlling the load the load is supposed to be P_0 here also, but the volume is increasing by a term let's say $V_0 + \Delta V$. On the other hand, in this case, the load is being released to let's say $P_0 - \Delta P$.

And we can determine this area and the difference in the area for this condition of a and a plus del a delta a that can be determined and that is physically that is what is signifies the change in the energy that is being released as the crack is growing. So, that signifies the J integral. Apart from the mathematical term, this is the physically what is happening.

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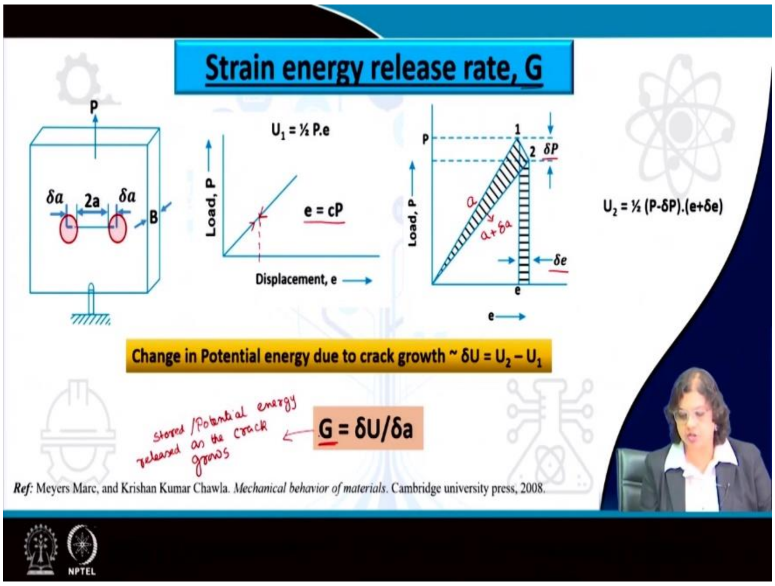

And since, we have talked about this graph, and we have talked about the definition, just for a while, we should look back to where we have first learned about this and where we have first discussed about the concept of G. So, G is again nothing but the strain energy release rate, which signifies the energy or the elastic energy or the potential energy the stored energy that is being released per unit growth of the crack or potential or elastic energy that is being released as the crack grows. So, this is the one that we have determined for the case of elastic condition.

So, in this case, you can see that the loading and the unloading curve follows the same path. So, that is what is a brittle failure or that is what is an elastic condition where the stress and the strain or the load and the displacement they are related by some particular constant relation and at any particular displacement in this case, we can figure out the corresponding stress, it does not change if we are talking about the loading part or the unloading part.

So, this is just to draw the analogy between J and the G. So, this is for the elastic part we are seeing that as we are looking for the situation when we have a crack of length a for condition one and for condition two in this case, we have a + ∂a or a + Δ a, whichever way we can see that there is a drop in the load by Δ P ∂ P and there is an enhancement in the elongation by ∂e. Similar, to what we have seen in the last slide and we can determine how much is the value of this G.

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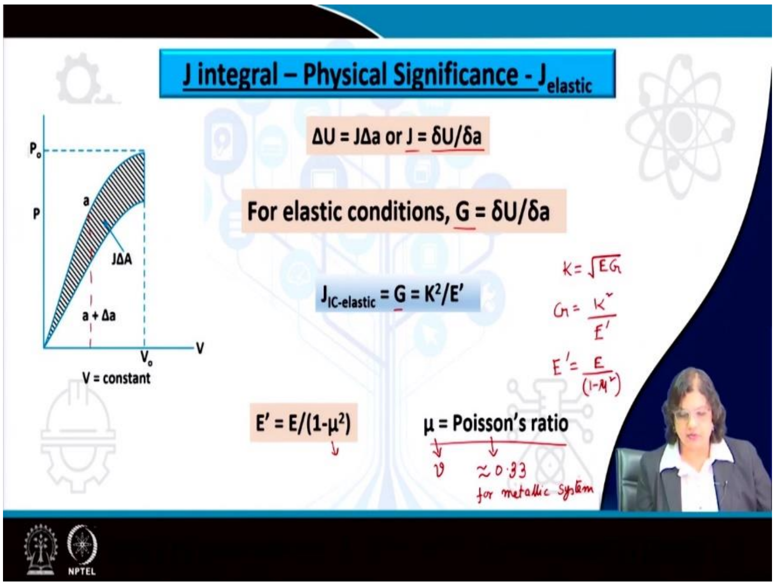

Now, coming to that, we then again it kinds of reminds us to the fact that means that up to for the elastic part at least, this J and G can be correlated. So, this is essentially the same thing that we are talking about, at least up to the elastic part, up to the plastic part something else is happening, and that we can explain. So, for the elastic part, actually, J is nothing but the same thing the change in the potential energy per unit or per crack length. So, that is given by J and for elastic condition, this is exactly the same that we have seen for the case of G.

So, essentially, J for the elastic part is same as that of G and G we have also seen that how G and K are related as per the Erwin's modification. We have seen that K= $\sqrt{EG}$, which gives us G = K^2/E , typically for the plane strain, but since we are talking about the plane stress condition here, we are using this term E prime and that includes actually the 1- μ^2 term, where μ is the poisons ratio. So, E' is given by E by this factors here, 1- μ^2.

Again, for some cases, we also use the symbol of μ for the case of poisons ratio. So, this is just for the sake of understanding that which symbols are being used, but essentially it means that E is being changed by this factor here 1- μ^2. For the case of metallic materials, this μ value is around 0.33 for metallic system or 0.3 and for any other system usually the poisons ratio is, has to be known or we can find that out from the standard references. So, that is what we are seeing for the elastic part.

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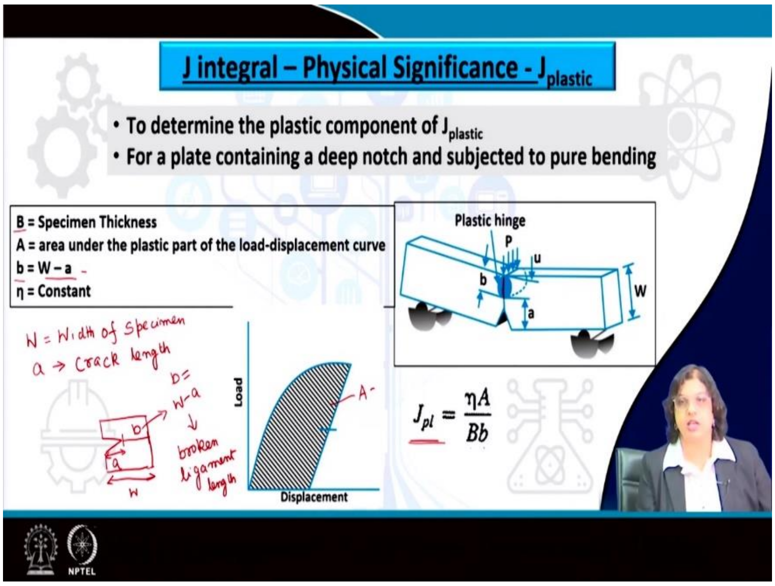

On the other hand, when we are talking about the plastic part, we still need to consider the area under the curve and for that case, let's say in case when a specimen is being subjected to pure bending something like this, we can obtain the plastic part also and the area under the plastic curve is determined as A which is the area under the plastic part of the load displacement curve.

And typically, the J value for the plastic part is given by a relation eta A by B and small b. Now, capital B here is the specimen thickness and we can find out the specimen thickness right before the test, we can measure that out and small b here is the broken ligament part, which is the area in front of the crack tip. So, that signifies the length of W which is the width of the specimen and a which is the crack length of the specimen. So, W is the width of specimen and a is the crack length.

So, whatever the specimen is whether it is a compact tension specimen or a bent specimen, whatever the area head of the crack tip or the length ahead of the crack tip is what is important. So, you see, this is the total width of the component and this is the a, crack length. So, in that case, this part here, which is dictated by the total width minus the crack length W - a, so, that is term as the broken ligament length.

So, this is B equals to W - a that is equivalent to the broken ligament length, because this is the section, this is the area, this is the region, where the plastic deformation is happening, this part is not of much significance when we were talking about the plane strain condition, but for the plane stress condition, this is where the plastic deformation is happening, and we want to figure out that whatever mechanism is happening and whatever this length which is of interest, so, that makes it place on this relation for J plastic which is given by eta A by B b. So, we will discuss some more about this J integral and how this can be determined in practice in the next lecture.

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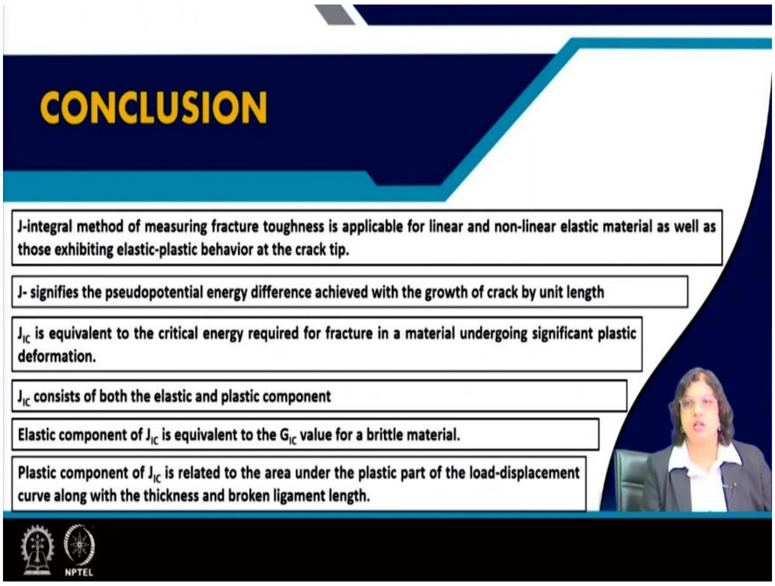

So, concluding this lecture comes the J integral method for measuring fracture toughness is applicable for linear as well as nonlinear elastic material that means, which undergoes elastic-plastic behavior at the crack tip and for that kind of material J of that kind of behavior J integral is the ideal one. It essentially signifies the pseudopotential energy difference achieved with the growth of the crack by unit length. So, as the crack grows whatever is a change in the strain energy or the potential energy that is what is termed as the J.

And J_{IC} is equivalent to the critical energy that is required at the point of fracture that is considered as the critical value of J integral and 1 again stands for the mode 1 here. So, that signifies the J_{IC} for the plane stress fracture toughness. And it has we have seen that it has both the elastic and the plastic component, the elastic component is similar or equivalent to the G_{IC} that we have seen earlier. And the plastic part is related to the area under the load displacement curve along with the thickness inversely proportional to the thickness as well as the broken ligament length.

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So, following are some of the references that has been used for this lecture and thank you very much.