Texture in Materials
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Module - 02
Basic Crystallography
Lecture – 05
Utilization of Stereographic Projections

Good day to everyone. So, this is module 2 Basic Crystallography. This lecture is basically on the Utilization of Stereographic Projections and also other aspects of crystallography.

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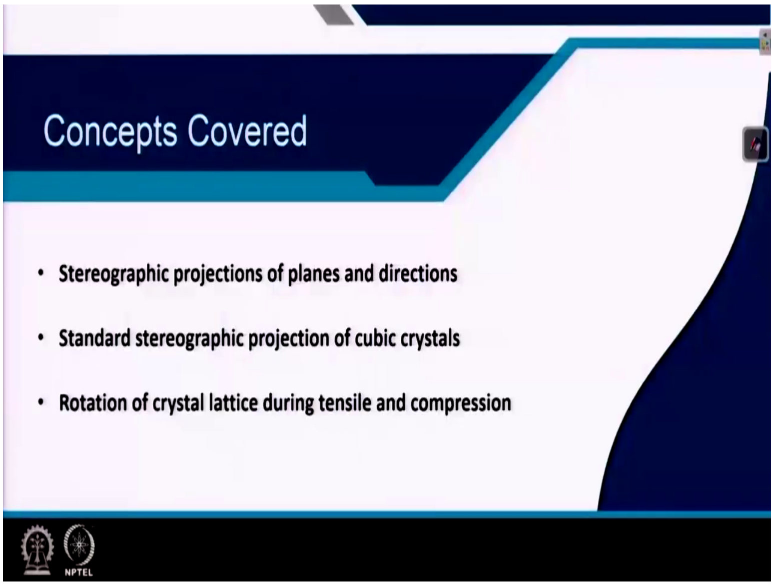

So, the concepts that will be covered in this lecture are the stereographic projection of planes and directions. Let me get the laser pointer, standard stereographic projection of cubic crystals, rotation of the crystal lattice during tensile and compression.

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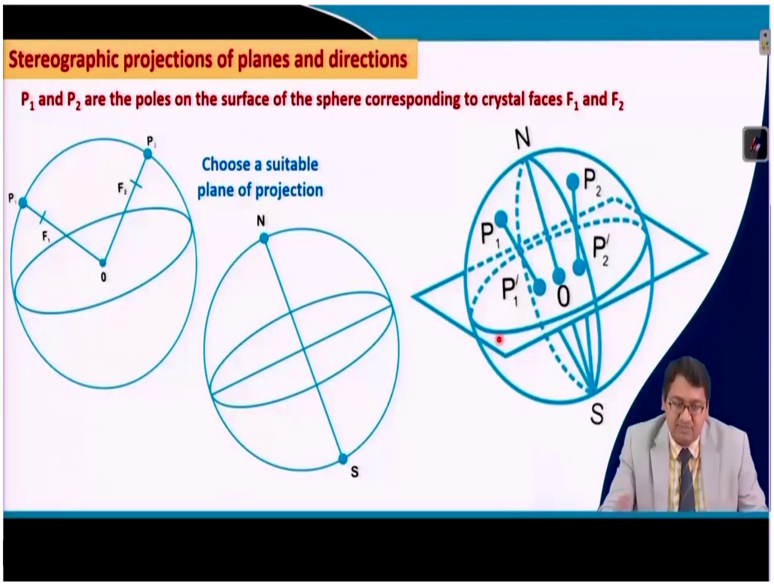

So, when we talk about stereographic projections, we basically talk about projecting the poles of the planes or the great circles small circles of the planes on the circumference of the 3 D stereogram and then projecting those spots or great circles or lines on a 2 D stereographic projection on a projection plane which is kept tangential to a certain axis and we have shown that axis as a and b earlier. So, now that if there are two planes crystallographic planes which are present at the centre of the sphere and say that one can say that if their faces are said F 1 and F 2. Then perpendicular to those planes could be projected on this three-dimensional stereogram as P 1 and P 2 right. So, P 1 and P 2 are the poles on the surface of the sphere corresponding to the crystal faces F 1 and F 2. Now in order to draw the two-dimensional stereographic projection basically, we need to choose a suitable plane of projection and say that in this case if we choose N and S as the axis along which we will mean try to project the faces of the poles of this crystal faces.

Then let us say that N could be the plane of projection or the plane which is tangential to the N could be the plane of projection if S is the point of projection. Now I would like to tell here the plane of projection is tangential to N and that means, the plane of projection is basically normal to the axis N S or the direction N S right. So, it is perpendicular to N S. Now it is important to let that the plane of projection can be exactly at the point N at the circumference of this 3 D stereogram on the other hand it could be even below this surface up to the centre of the sphere.

For example, we have taken this plane this plane which is looking like a great circle which is protruding out from the centre of the sphere and perpendicular to the north-south direction as a plane of projection and then how the projection of these poles on the 2D stereographic projection would look like. So, here is the projection plane that we have drawn, S is the point of projection and if from the point of projection the faces F 1 and F 2 are producing two poles P 1 and P 2 on the surface of the sphere and if you trace back this ray of light which is falling on P 1 and P 2 to this point of projection it will show that it is falling on P 1 dash and P 2 dash. Therefore, on a stereographic projection which it looking like this because it is shown in a three dimensional way. The P 1 and P 2 on the 3 D stereogram will be projected as P 1 dash and P 2 dash on the 2 D stereographic projection right.

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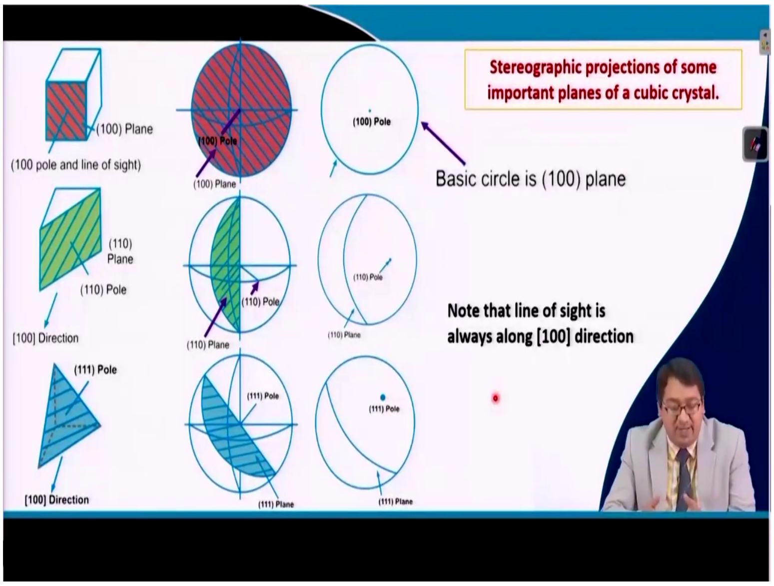

Now, as I said that there are important planes and those important planes are plotted on the stereographic projection and here are few examples by which I would like to show you how the various important planes like the 100, 110 and the 111 planes would be projected on a three-dimensional stereogram and to a two-dimensional stereographic projection. And here is an example of the one g 100 plane of the unit cell. And you can see that perpendicular to this 100 plane is the 100 direction pointing out towards the 100 poles and which is actually in the line of sight. You must note here that in all these cases that we are presenting we are always using 100 directions along the line of sight ok always it is in the line of sight.

So, that if we are drawing a sphere like this. So, I have tried to draw a sphere on the two-dimensional plane and so, that we have this sphere and the here is the top side of the sphere. So, it looks like this. So, that the circumference of the sphere which is like this and a line that is coming out like this that is the 100 poles must be perpendicular and it is basically towards the towards perpendicular to the plane of this screen. Three-dimensionally it is looking something like this and if you look at the plane, this plane basically protrudes outside towards the circumference of the sphere and it forms a plane like this forming a great circle when it is crossing the circumference of the sphere. So, perpendicular to it the 100 pole comes out like that and when we are observing this three dimensional into a two-dimensional stereographic projection, the basic circle could be observed which is the great circle corresponding to the 100 plane and the play and the pole 100 poles will look like a dot or a spot at the centre of the sphere making it a 100 pole. Another example is on the 1 0 110 plane. The 110 plane basically lies at an angle of 45 degrees to the 100 plane and it is shown by this green shaded plane 100 and the pole will be at 90 degrees to this plane and so, it will be protruding out like this.

So, a pole will be 110 pole and if we look into this 3 D stereogram we will find out that if this 3 D stereogram the 110 plane will form a great circle which will be somewhere like this at 45 degrees to this pole which is basically the 010 planes great circle and this great circle is also the great circle corresponding to this pole which is basically 001s. So, now, it will be at 45 degrees to the 100 plane. So, it will be shown somewhere here and therefore, if we could project this to two-dimensional stereographic projection, this will convert to a great circle something like this and the pole will form at exactly 90 degrees of the Wulff net here as 1 0 110 pole. The pole in the 3 D stereogram is shown somewhere here, right? On the other hand, if we look at the 111 plane. The 111 plane will look something like this. The projection of the 111 plane on the 3 D stereogram will look something like this and then its pole will form at 90 degrees to it and will protrude out of the circumference of the sphere at some position like this right. And when we will observe this in a 2 D projection it will form a great circle and spot a pole 111 pole at 90 degrees to each other. So, this is how the planes the poles and the great circles of the plane are being shown.

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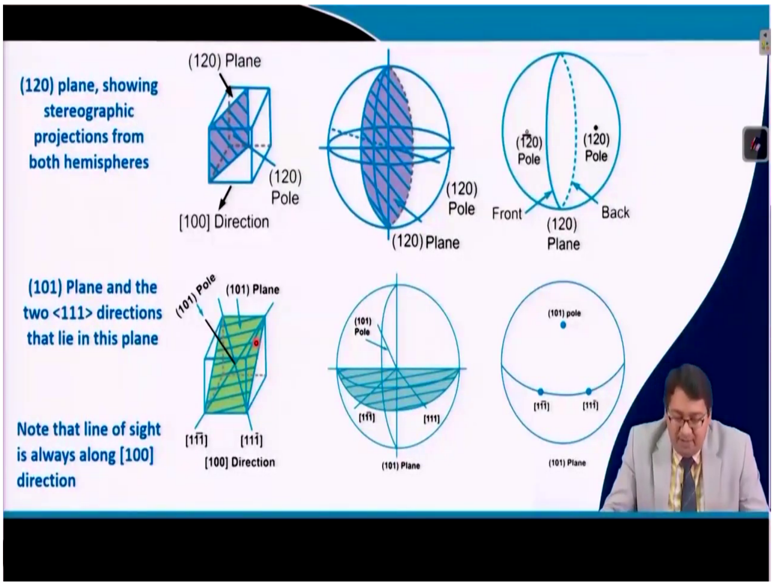

Now, that there are examples of other planes, for example, the 120 planes can be observed in this unit cell and you can see that how the 120 planes basically look. The pole of this 120 perpendicular to it can be shown here in the 3 D stereogram the 120 forms somewhere in between 010 and 0 011 and it forms something here. So that this is the great circle and this is the plane of the 120 shown here and if we look if we can look on the opposite side of this 3 D stereogram, one could observe the other great circle of this 120 here. So, that if we consider the pole of 120 that is the pole forming by normal to 1 to 0 plane, that is the normal to this the pole which is protruding from the circumference of the sphere will be shown here and this is the 120 pole. If we look at the opposite direction one can observe that in the opposite direction also the 120 plane can be projected and now because it is in the opposite direction the plane is actually 1 bar 2 bar 0. And therefore, it is projected on the opposite pole which is basically 1 bar 2 bar 0 pole.

The same thing can be easily represented in a two-dimensional stereographic projection with the help of a great circle and a closed spot or closed pole 120 pole which is because it is in the front side and an open circle close circle and an open circle which is in the opposite side which is representing 1 bar 2 bar 0 pole corresponding to this dotted lines. Therefore, the front and the back of the great circle can be observed at the same time the closed circle and the open circles can be used to show the poles in the front and the back side of the stereographic projection. Another example is the example of the 101 plane. And this example has been shown to show that how if any plane contains one or two different directions how it is represented in the stereographic projection. Now this example the 101 plane contains two important directions that are the one 1 bar 1 bar direction and the one 11 bar directions and it can be shown by the schematic in this unit cell. If we draw this 101 plane on the 3 D stereogram, we can observe that this forms somewhere here as a great circle. The two directions 111 bar and 1 1 bar 1 bar is shown in the forming as a pole protruding from the centre of the sphere to the surface at the same great circle at this position and this position.

On the other hand, the 101 planes pole is protruded at 90 degrees to it and so, it can be shown that it is protruded somewhere here and forming somewhere here at this position. Now, this same thing which is shown in the 3 D stereogram has been projected into the 2 D stereographic projection and thereby the 101 plane could be observed as a great circle like this on which two directions of 111 families is been projected as poles. On the other hand, the 101 plane has been projected by a pole 101 at 90 degrees to this great circle.

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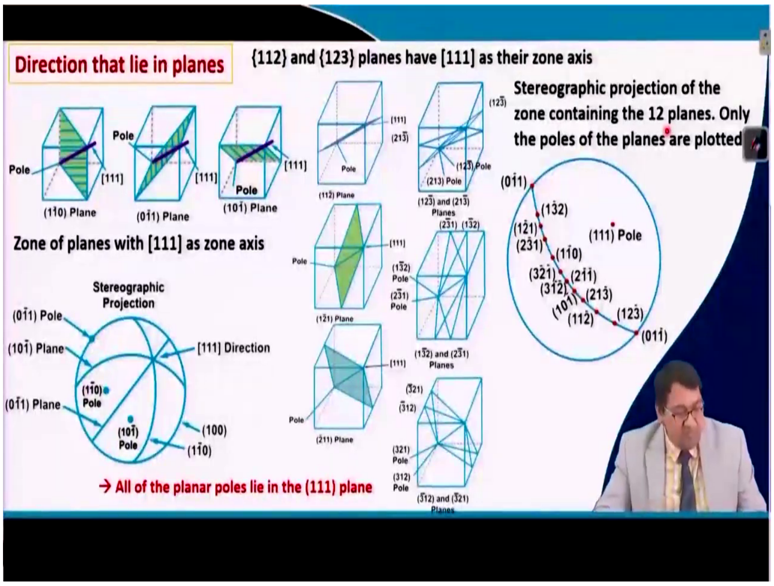

So that stereographic projections makes it much easier means 2 D stereographic projections makes it much easier to find out the relationship between the planes than the angular relationship between the planes, planes and directions or directions right. Here is another example, now if that there are if there are many planes that lie in a single direction. Now how this can be observed using the stereographic projection? if that if we consider the direction

111 and I have shown the direction 111 and if we consider that this in this direction the 11 r 0 plane, the 0 1 bar 1 plane, the 101 bar plane basically like this right.

So, you can see from the unit cells that this is the 1 1 bar 0 plane, this is the 0 1 bar 1 plane, this is the 101 bar plane and in all of these planes we can observe their poles right. So, this pole represents the 1 1 bar 0 pole, this is the 0 1 bar 1 pole. This is the 101 bar pole and if we draw the stereographic projection of these planes in terms of the great circle if I draw this plane in terms of the great circle it will form a great circle something like this on the front side and the backside cannot be observed right. With respect to this great circle at 90 degrees we can form the 1 1 bar 0 pole here, on the other hand, we can observe this 101 bar plane as a great circle here and perpendicular to it we can observe the 101 bar pole. This plane 11 01 bar 1 plane could be observed again as a great circle and see in this case its looks like a straight line like this and then you can observe the pole of 0 1 bar 1 plane exactly here.

So when we are drawing the great circles of these three planes, it is meeting at a certain junction and this junction is basically known as the zone axis. And we can observe that this zone axis basically represents a pole that represents the 111 directions because the 111 direction is common in between these three planes. Now if we keep this 111 zone axis or the 111 direction common there are other planes also for which this 111 directions could be common. And this could be the 112 bar, 12 bar 1, and the 2 bar 11 planes as shown in these figures. So, there are three more planes which are basically the 112 family planes that contain the 111 direction. Now we can plot these planes in great circles also and they will definitely cross through these poles. But on the other hand, if you would like to show that there are 1, 2, 3 types of planes which also have this common 111 direction and these are the six 1, 2, 3 types of planes which have common 111 direction.

Then one if we would like to show this 6 plus 3 plus 3 1 2 3, 1 1 2, 110 family of planes that is total 12 planes which have the same zone axis 111 it becomes very clumsy to show this 12 planes in terms of 12 different great circle crossing at a curtain pole 1 1 showing the 111 direction rather we can even show it like we can show the 111 poles. And we can draw a great circle corresponding to this 111 direction perpendicular to this 111 pole and then we can show that all the poles of these 12 planes are falling at different positions of these great circles. And therefore, we can get the relationship between the planes and the directions using the 2 D stereographic projection and the complex problems related to these could be solved easily using the stereographic projections. So, this is all that 11 110, 112, 123 planes have same may have same 111 as their zone axis and the stereographic projection of this zone containing the 12 planes can be shown in this case only the poles of the planes can be plotted instead of a great circle and for the 111 zone axis instead of the pole only the only a single great circle could be plotted.

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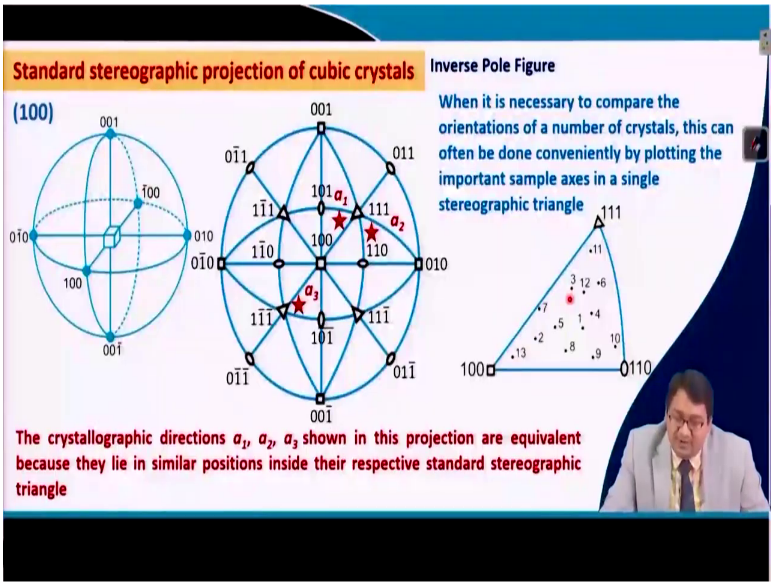

So, now we will be doing the standard stereographic projections for the cubic crystals. Now first we will do the 100 standard stereographic projection. In order to do the standard stereographic projection as we have done in an earlier lecture, we should keep the unit cell in such a way that the origin of the unit cell lies exactly at the centre of the three-dimensional stereogram. And the specification of the unit cell with respect to the three-dimensional sphere should be such that that the x-axis is protruding outside to form the 100 pole here, the y axis is protruding to form the 010 pole, on the other hand, the z-axis is protruding to form the 001 pole. Now as the origin of the unit cell is exactly at the centre, the 100 plane which is coming through the origin will form a great circle that is perpendicular to this 100 pole that is the great circle this one.

On the other hand, the 010 plane which is perpendicular to the 010 pole in this direction should form a great circle like this here. On the other hand, the 001 plane perpendicular to this direction forming a 001 pole will form a great circle like this. So that is how these important planes of the unit cell could be shown as poles and great circles in the three-dimensional stereogram. And now if we are putting 1 bar 0 0 as the point of projection and the 100 as the plane of projection, then we can observe this three-dimensional stereogram into a two-dimensional stereographic projection which will look something like this. Now, just try to imagine that if you are looking from here and you are looking at the spots which are forming at a plane that is tangential to this circumferential point of the sphere and then let us say that we are making the stereographic projection at the centre of this sphere.

Let us say that instead of out on instead out on the surface of the sphere, we are making the plane of projection exactly at the centre of the sphere keeping that it has to be parallel to the plane tangential to the surface of the sphere at 100. Now if you are doing so, the point projecting from the centre to this 100 pole will be shown exactly at the centre that I am pointing out with my laser pointer and so, this is the 100 pole. On the other hand, the 010 pole will be observed somewhere here right the 001 pole could be observed somewhere here and the rest is obvious the 0 1 bar 0 pole, the 001 bar pole will be here. Now the great circle the 010 great circles will be projected like a line here because this is straight and when you observe it from here, it will observe as a the great circle will be observed as a line.

On the other hand, if we look at the 001 great circles it will be forming somewhere here and then as I said the 100 great circles will form as the great circle circumcising the whole two-dimensional circle stereographic projection right. So that we can obtain the 100 pole, the 010 pole, 001 pole, 01 bar 0 pole and the 001 bar pole. Now how we can obtain the other poles? Now that a plane or a direction not the plane say the direction say a direction containing 011 direction should be somewhere protruding something like that and will come out on the surface somewhere here and so, its plane will be perpendicular to it and could be shown as a 011 plane somewhere like this.

If we protrude the if you if we plot the poles of the 011 plane and it will format you say 45 degrees from the 010 and 45 degrees from the 001 and so, it will form somewhere here and that these are equiangular right 45 degrees from here and 45 degrees from here in the stereographic projection. And another way of drawing this is that, if you add these two miller indices 011 sorry 001 plus 010 it becomes 011 at the centre. Similarly, we can get the poles for the other 110 planes family of planes like if you add 010 with 001 you will get 011 bar. And then if you add these two you will get 0 1 bar 1 bar, if you add these two you will get 0 1 bar 1. Now there are only four 110 planes there are other 110 planes. So, that if you add 100 with respect to 001 bar then you will get 101 bar pole if we add 100 with respect to 010 you will get 110 like that you will get 11 bar 0, 101 and therefore, we can obtain various 110 poles family for with respect to the 110 family.

In a similar way if we add say, for example, 100 with 011 and somewhere we will get the 111 poles with respect to the 111 planes and like that we will get one 1 bar 111 bar 1 bar 111 bar poles and that thereby we can obtain the various poles of different hkl planes as well as directions and we can show them in terms of spots. On the other hand, if you look at these 110 poles and if we go at 90 degrees with the help of the Wulff Net, you will reach somewhere here because the angle between the 110 and 11 bar 0 is basically 90 degrees and therefore, one can obtain the great circle with respect to 110 here.

The great circle of 11 bar 0 here, the great circle of 101 bar here, the great circle of 101 here like that we can get obtain the great circles of the other planes right. On the other hand one can get the great circle of 110 11 bar here and 011 here. So, one can plot the poles and the great circles pertaining to different planes and directions in the 2 D stereogram and here is the way how a 100 standard stereographic projection is being plotted. Now that usually the standard stereographic projection not only shows the poles, but it also shows the symmetry of the poles and these square ellipsoidal and the triangular formations that are shown here are not only shown to show different symbols but shows the point group symmetry of the material. Cubic crystals have three 100 axes and each of these axes has a fourfold symmetry and the square shows that the symmetry of the 100 axes is fourfold. On the other hand, there are six 110 axes and this axis shows two fold symmetry and therefore, the ellipsoidal symbol is been used to represent the twofold symmetry of the 100 poles.

The 111 and there are four 111 directions in the cubic unit cell and each of these directions pertains to a threefold symmetry and therefore, is shown by the triangle. So, the sorry the stereographic projection is basically made up of a few triangles triangles all are triangles made up of corners which are 100, 110 and 111. There are so many triangles and each one of them is made up of 100, 110 and 111 families. Now, what does it mean? It means that because of the symmetry of the cubic crystals the stereographic projection could be divided into 24 different triangles on the front side and 24 on the backside making it 48 different angles made up of 100, 111 and 110 corners family of poles corners.

Therefore, if we consider a crystallographic direction say, for example, a crystallographic direction a 1 which is at a certain angle in a stereographic triangle with respect to 100, 1 110 1 and 111 and then another direction a 2 another direction a 3 which has the same angular relationship with the 100, 110 and 111 in their respective triangular region. Then these a 1, a 2 a 3 which are shown in this stereographic projection are equivalent points or equivalent poles that lie in a similar position inside of the respective standard stereographic projection because of the symmetry elements of the cubic crystals.

Therefore, instead of showing the whole stereographic projection sometimes a single triangle could be shown and it is sometimes necessary when it is necessary to compare the orientation of a number of crystals. Here 12 and 13 number of crystals have been compared and that the position of some important sample direction with respect to this crystals vary from 1 and 2 and 3 and 4 and 5 and 6 like that throughout the stereographic projection. Sometimes showing a particular crystallographic axis is more convenient when plotting this in terms of sample axis instead of them instead of in a whole stereographic projection but only by taking a single stereographic triangle like this and this system of showing the orientation is known as showing inverse pole figure.

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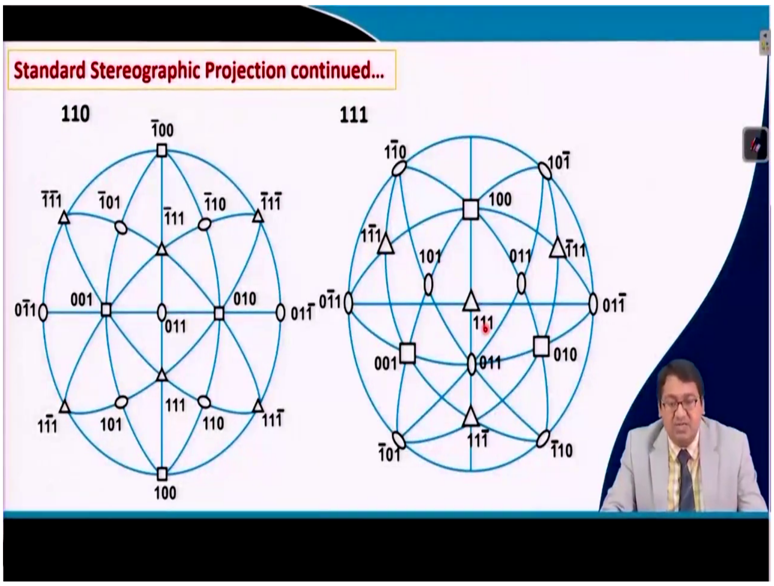

Like the standard stereographic projection of the 100, 110 and 111 standard stereographic projections could be also observed. that when we look into the standard stereographic projection of 100, we found out that not only the square symbol represents that it has the four-fold symmetry, but also the shape of and the positions of the poles and the great circles indicate that fourfold symmetry elements are present in it. In order to make the 100 sorry the

110 stereographic projection, the 100 stereographic projection has to be rotated along with the say, for example, this particular axis ok by 45 degrees. And so, that earlier the positions of the poles that are 001 1 bar means this great circle which was here is now shifted here right. So, when we do this rotation we can obtain the 110 stereographic projection and in this stereographic projection what we can see is that the centre which is 110 has a twofold symmetry and which could be observed from the pole figure itself. This part of the pole figure and this part of the pole figure are symmetric, but this part and also this part of the pole figure and this part of the pole figure is symmetrical. So, it has a mirror symmetry. So, the 110 poles have a twofold symmetry and it does not have a fourfold symmetry.

On the other hand, if one can also calculate and draw the 111 stereographic projection where the 111 stereographic projection has in the centre the threefold symmetry and this threefold symmetry could be represented in the 111 stereographic projection.

We are not going into much detail about how to draw this projection but you might have got a basic idea fundamental idea of how to obtain this stereographic projection with simple practice using a pen and paper, one can easily draw the 100, 110, and 111 type of stereographic projection with these with a little practice you will be able to get it.

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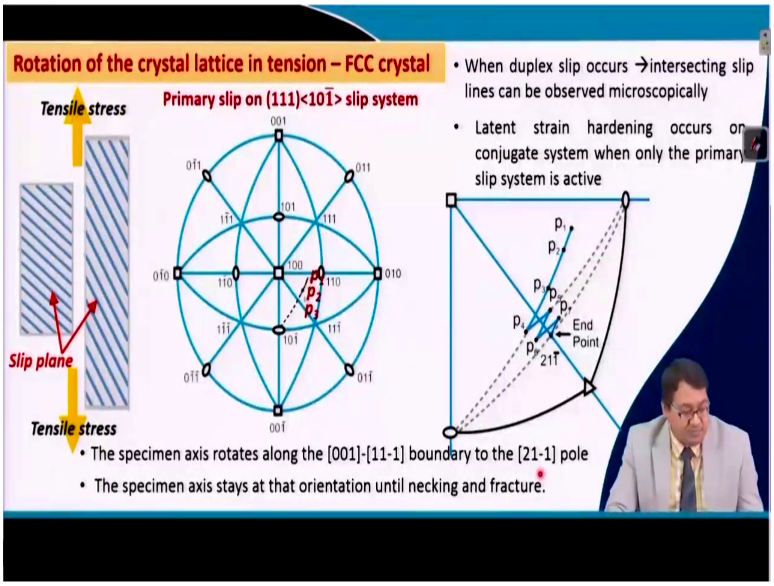

Now, this slide shows the utilization of the 100 stereographic projection in observing the rotation of the crystal lattice when tension or a tensile test is conducted in a face centered cubic crystal. One can show the same thing for tension in a BCC also compression in FCC and BCC. So, here if you look into the tensile sample, that is what happens that if the initial sample is something like this. And the is we have drawn hypothetical slip planes and that they are at a certain angle to the tensile axis and now when the tensile test is being carried out.

The tensile test is carried out, and that these slip planes are rotated by a certain angle, and what is basically happening is that, that during the tensile test the slip directions are moving towards the tensile axis whereas. So, if we take a face-centered cubic crystal, then the slip system by which the face-centered cubic crystal slips means a single crystal slip is 111, 110 family of planes and directions.

If you look into this 2 D stereographic projection and if you say that yes the when the sample is being used like this and the crystallographic orientation of the sample say, for example, is at this position when the deformation started. So, let us choose a stereographic triangle like this and show the position of the hkl what you call the position of the sample axis with respect to a crystallographic direction say, for example, u v w. So, p 1 is the position from where the sample actually started when we start the tensile test.

If at this position when the tensile test is started the primary slip system in which the sample will start to deform in a single crystal should be the one which is having the highest smith factor and we can calculate that for this position for the primary slip system 111 planes in the 101 bar direction the smith factor will be the highest. And while the sample is deforming, the sample axis will move towards the slip direction which is the 101 bar. So, you can see that the sample axis is moving from p 1 to p 2 to p 3 and it is moving towards the 101 bar direction. Now when it is moving and it is going from p 1 to p 2 to p 3 it is coming to a position somewhere like this which lies in the great circle containing the 100, 111 bar poles, and in this case what happens; means the sample axis basically rotates towards this position which contains the 100, 111 bar great circle and then at this position when the sample comes what happens? Is that the smith factor with respect to this primary slip system which is 111 101 bar becomes equal to the smith factor of another slip system which is basically 1 1 bar 1 and 110. So, a conjugate slip system basically develops and both primary and conjugate slip systems at this position will have the same smith factor.

Therefore, the axis which was now moving towards the 101 bar will go towards the 101 bar and because of the inertia it will move from p 1 if you look in this larger figure a larger area it move from p 1 p 2 p 3 and it moves it crosses this boundary and goes to p 4 because of the inertia even though at this position the smith factor for the conjugate slip system which is 1 1 bar 1 bar and 110 is higher.And when it reaches p 4 at a certain point it has to go back because of the higher smith factor of the conjugate system and goes to p 5 and then at this position the primary slip system will get a higher smith factor and therefore, it will go back p 6 and then p 7 and it will end up at a point which is basically 211 bar where the equal and opposite forces exerted because of this shear deformation by the primary and the conjugate slip systems become equal. So, what is happening here is that in a single crystal duplex slip occurs, and in the duplex slip what is occurring? Intersecting slip lines are can be even observed microscopically because of this issue that we just discussed now. So, this situation where the conjugate slip system occurs in the material is known as latent strain hardening right. So, when the conjugate slip system becomes active also with the primary slip system, then latent strain hardening occurs and this leads to the specimen axis to rotate along this 100, 111 bar boundary to go to the end orientation which is basically 211 bar here and the specimen axis stays here until the orientation until this the material necks and fracture.

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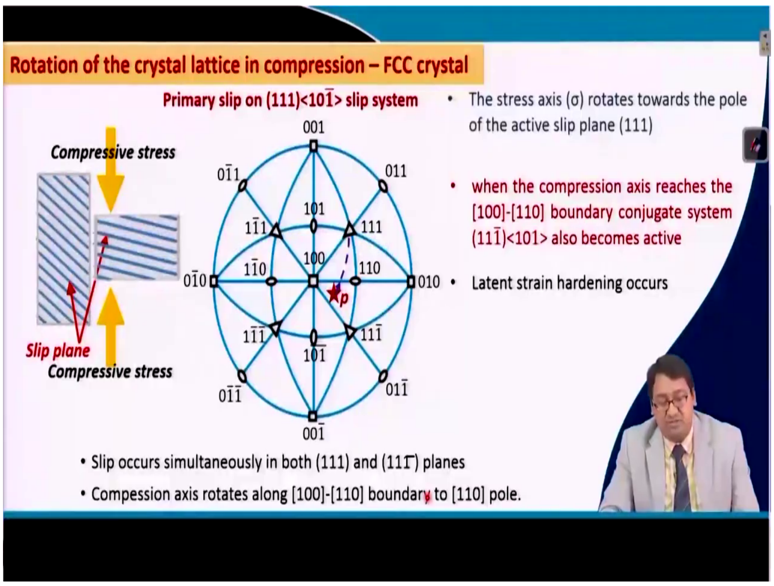

In the case of the compression of the same face-centred cubic crystal what happens? you take the same sample like this and the slip planes are like this and we give the compression and when we do the compression what we found out that the slip planes are rotated so, that their perpendicular becomes near to the compression axis right.

Initially, the planes are like that. So, their perpendicular or the normal directions are away from the compression axis and when you are compressing it and the sample is becoming like this, the slip the normal to the slip plane becomes closer to the compression axis.

And if we look into the stereographic projection in a similar manner and if you say that ok the position of the compression axis is p and then when the slip starts to occur and if the slip starts to occur it will occur in the 111 plane and 101 bar direction which is having the highest smith factor and therefore, the compression axis starts to move towards the pole of the 111 plane right. So, it starts to move here, but when it reaches the 100 110 boundary what happens is that in this case also the conjugate slip system starts to take place. Now, the conjugate slip system is basically in this case would be the 111 bar plane and the 101 direction and therefore, the equal and opposite forces because of this smith factor will lead to the movement of the dislocation slip in a zigzag manner as it occurred in case of the tensile.

Thereby the end orientation will occur like this and the end orientation where the equal and opposite forces by both the slip systems become the same, 110 in this case. So, the stress axis sigma rotates towards the pole of the active slip plane 111 and when the compression axis reaches the 100 110 boundary conjugate system that is 11 bar 111 bar 101 this one starts to become active and then latent strain hardening occurs. So, slip occurs simultaneously in both the 111 and 111 bar planes, compression axis rotates along 100 110 boundary to the 110 pole unless equal. And opposite forces by the two slip system becomes exactly same and then it becomes the end orientation in this case.

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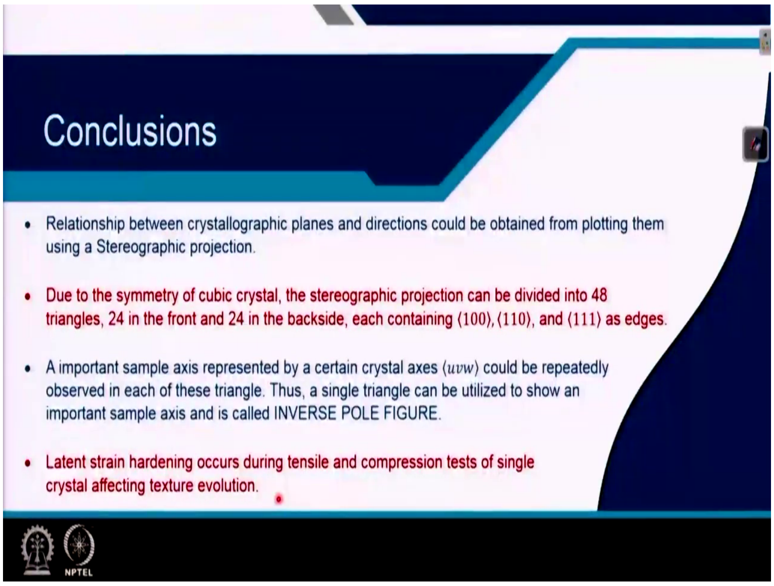

From this lecture, we can conclude that the relationship between crystallographic planes and directions could be obtained from plotting them using the stereographic projection. Due to the symmetry of the cubic crystal, the stereographic projection can be divided into 48 triangles; 24 in the front and 24 in the backside containing each triangle containing 100, 110, 111 as edges.

An important sample axis represented by a certain crystal axis u v w could be repeatedly observed in each of these triangles, thus a single triangle can be utilized to show an important sample axis and this is called the inverse pole figure. Latent strain hardening occurs during tensile and compressive stresses for the single crystals affecting the texture evolution.

Thank you.