Texture in Materials
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Module - 09
Theory of deformation texture evolution
Lecture - 45
Basic Mechanics of Polycrystal Plasticity (Contd.)

Good afternoon. We will be continuing with module number 9 which is theory of deformation texture evolution and this is lecture number 45. We will continue with basic mechanics of polycrystal plasticity. So, this is part 2.

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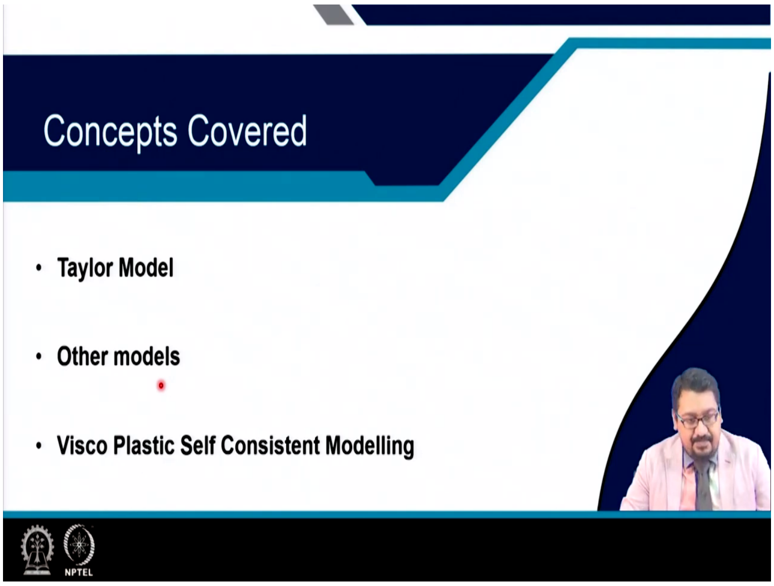

So, the concepts that will be covered in today's lecture will be the Taylor model. The other models which comprises in between the Taylor model and the Sachs models and we will give a little bit of information about visco plastic self-consistent modelling.

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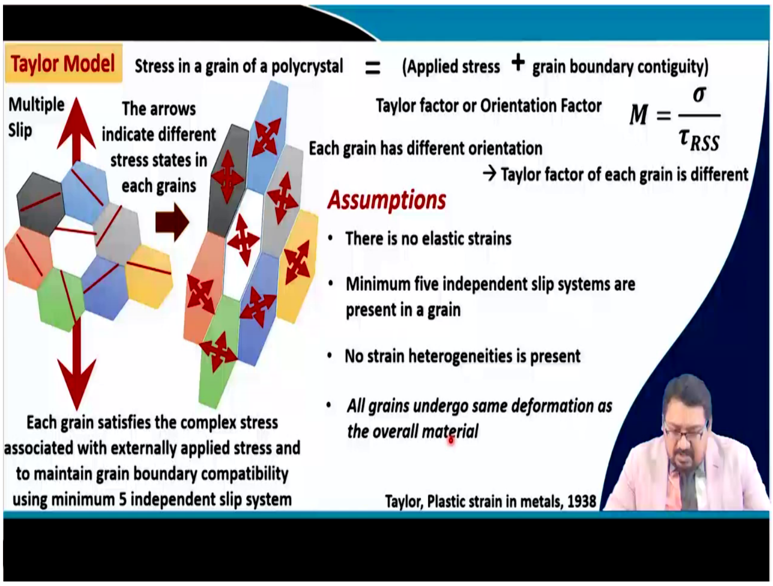

Now, comprising of the Sachs model, where we considered a polycrystal material with grains having different orientation. In this case also, we will take the same condition where there are multiple grains in a polycrystalline material and each grain comprising of a different orientation.

But in this case in Taylor model, what happens that the stress state in each grain of the polycrystalline material must be equal to the applied stress due to the applied loading in this case as we are talking about the tensile loading case. So, let it be the tensile loading case, because the, because it is the most simplest one.

And then apart from the applied stress, there will be a complex stress state which is associated with grain boundary contiguity. Now, in order to sustain this scenario, multiple slip systems has to be activated in each of the grain and the von mises criteria has to be sustained right.

Now, like the Schmid factors criteria, the Taylor factor or an orientation factor could be determined. And as the Schmid factor is determined for a single slip system, the Taylor factor for multiple slip system and that is comprising of the von mises criteria having the minimum 5 independent slip system can be given by M which is equal to sigma by tau RSS.

So, if we look and compare the capital M which is the orientation or the Taylor factor with respect to the small m which is the Schmid factor, we can see that the small m Schmid factor is equal to tau RSS by sigma for a single slip system and capital M is equal to sigma by tau RSS for multiple slip systems.

So, one can say from this comparison that if the results shear stress becomes; means the one slip system which will starts to slip that will that it will reach the higher result shear stress at a lower you know loading stress will be the one which is having the highest Schmid factor.

In case of Taylor factor, the ones with the lower Taylor factor will start to deform faster. So, at a glance it will look like the Taylor factor is inversely proportional for the with respect to the Schmid factor.

But it is inversely related, but are not directly inversely proportional, but they are inversely proportional to the functions of each other. So, if the Taylor factor is lower, then the deformation will sustain faster and if the Schmid factor is higher, then the deformation will occur on that slip system. So, they are basically inversely proportional to their functions and thereby, one can say that deformation of certain slip systems will occur if the Taylor factor is low and that is what I wanted to say.

Now, under the situation of this Taylor factor model where minimum 5 independent slip system has to be sustained, then for each of this grains present, the stress state in order to maintain the grain boundary contiguity will be different. So, we can see that I have put arrows which are differently oriented for each of this grain and showing or indicating different stress state present in each of these grains.

So, each grain basically satisfies the complex stress state associated with the externally applied load or the externally applied stress and the stress that is associated to maintain the grain boundary contiguity or the grain boundary compatibility by using at least 5 independent slip systems.

So, as I said that the each grain will have a different orientation. So, the Taylor factor that can be calculated for each of these grains will be different and thereby, the 5 slip systems will which will be activated to have the lowest value of Taylor factor will also be different for each grains.

So, you see the assumptions that are taken here are; first one is that there is no elastic strain associated with the deformation; that means, that the elastic strain is comparatively extremely small as compared to the plastic deformation. So, no need to take account the elastic strains in the material.

Second is as I said minimum 5 independent slip systems are required to sustain this Taylor model condition. The strain state is homogeneous and there is no heterogeneity present in the strain in the material from one pore in inside the grain in the grain in the grain boundary in the center of the grain the strain is equal.

Finally, all grains undergo the same deformation as the overall material. So, strain in each grain will be equal to the strain in the overall material. So, a detailed account of Taylor model is given by Taylor in 1938 in the paper plastic strains in metal.

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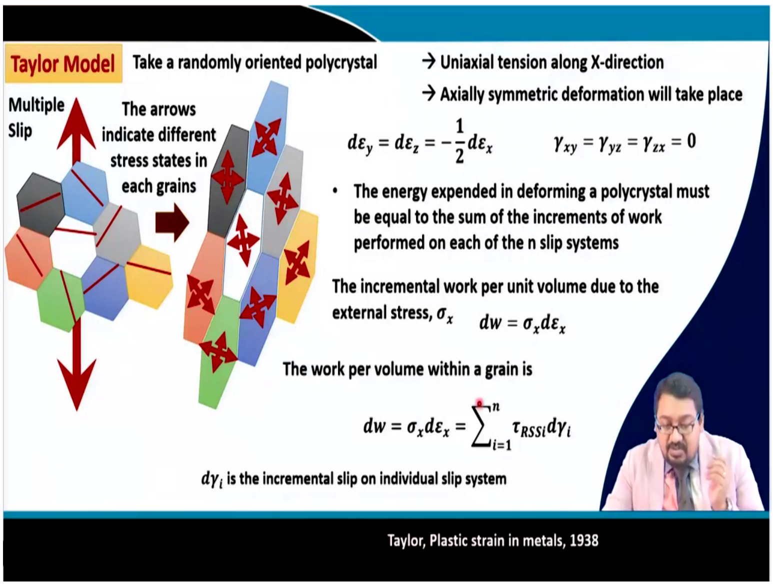

So, the if we take a randomly oriented polycrystal and give a tensile deformation to it as we are given, then let us say that uniaxial tensile is giving in the X-direction. And so, as the texture is random as the polycrystalline material is randomly oriented polycrystal.

So, the axial symmetry in the deformation will be there and thereby, the strain the change in the strain in the y direction and in the z direction will be equal to minus of half of the you know if we are pulling along x along the x and then z and the y, they will be you know compressed.

So, change in epsilon y d epsilon y equal to d epsilon z, because of the axial symmetry, because of the random orientation and that is compressive. So, it is equal to minus of half of d epsilon x whereas, the shear strain gamma x y gamma y z gamma z x is basically 0.

So, in the Taylor model, it is considered that the energy expended in deforming a polycrystal must be equal to the sum of the increments of the work performed on each of the n slip systems. So, the incremental work per unit volume due to the external stress sigma x let us say it is sigma x is equal to delta w. So, if the incremental work per unit volume is delta w for an external stress sigma x, then delta w is equal to sigma x times delta epsilon x.

And if the work per unit volume in the grain is equal to a summation of i equal to 1 to n, where these are the number of slip system present times tau RSS that is the resolved shear stress for the ith slip system times d gamma y that is the incremental slip on the individual slip system then, d w is equal to this value is equal to sigma x d epsilon x.

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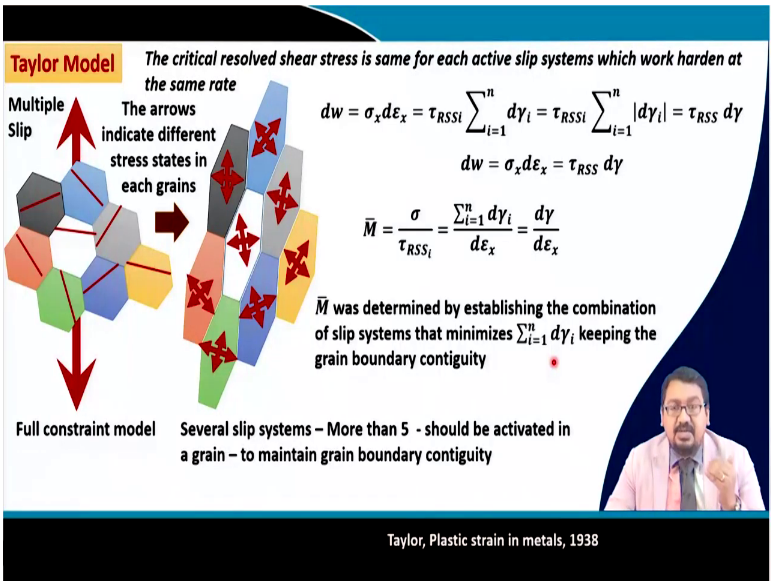

So, the critical resolved shear stress in this case for each of this active slip system. As I said that say for example, FCC. So, the slip system is 1 1 1 1 1 0. So, the critical resolved shear stress is assumed in this case to be same and if these slip systems are hardening work hardening of the slip systems are occurring, it is occurring at the same rate. So, even after work hardening, the critical resolved shear stress may increase, but it will increase same for each of the slip system and still it will remain same for each one of them.

So, the condition of the Taylor factor considers that the shape of the grain and the strain of the grain changes equivalently and it changes, because the stress state in each of the grains are different and it changes in order to maintain the full grain boundary contiguity and therefore, it can be considered that it is a full constraint model.

So, as I said that the work done delta w is equal to sigma x delta epsilon x equal to tau r. So, as the critical resolved shear stress is same for each of the slip system. So, the resolved shear stress for each of the slip system is considered to be same for a certain incremental shear strain.

So, tau RSS can come out of the summation sign and now, the summation sign remains with delta gamma i. And then we can say, that overall if delta gamma i is considered average, then this d w is equal to tau RSS times d gamma right. So, d w is equal to sigma x d epsilon x is equal to tau R S d gamma overall averagely hence in this way the average Taylor factor that is M bar can be calculated which is equal to sigma by tau RSS equal to d gamma by d epsilon x.

So, M was determined in this way by the combination of the slip systems that minimizes the incremental shear strain d gamma y actually the summation of the incremental slip systems d gamma y keeping the grain boundary contiguity and maintaining minimum 5 independence slip system requirement for each of these grains right. So, several slip systems and even more than 5 can be activated in the grain or should be activated in the grain to maintain the grain boundary contiguity using this model.

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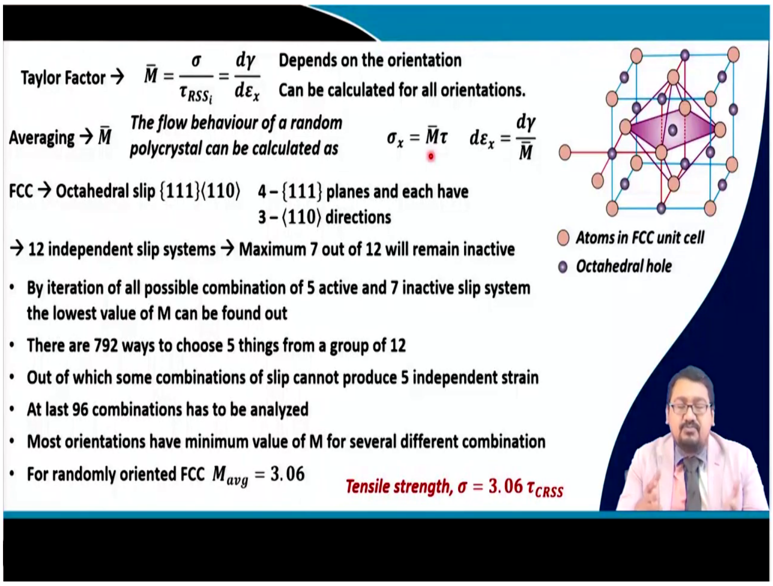

So, Taylor factor as I said M is given by sigma by tau R S or d gamma by d epsilon x depends upon the orientation of each grain right and can be calculated for all the orientations.

So, say for example, we are talking about the FCC material which deforms by 1 1 1 close packed plane and 1 1 0 close packed direction. So, there are twelve possibilities of it right. So, it is a it is an octahedral slip like this right and there are you know 4 1 1 1 planes and 3 1 1 0 direction for each one of them.

So, there are four possible twelve possibilities. So, if we average the value of M that is the Taylor factor M bar the flow behavior of a random poly crystalline FCC material can be calculated and. So, sigma x can be written in terms of average M bar times tau which is tau RSS and or d epsilon x can be written in terms of d gamma which is the incremental shear strain by the average M.

So, one can see that there are 12 independent you know slip systems 4 into 3 4 1 1 1 3 1 1 0 for each of this 1 1 1. So, 12 independent slip system present. So, maximum out means maximum 7 out of this 12 can remain active, if minimum 5 is active then, maximum 7 out of 12 will remain totally inactive.

So, by iteration of all possible combination of 5 active and 7 inactive slip systems, the lowest value of M is basically calculated. So, there are 7. It was it can be found by calculation that there are 792 ways to choose 5 things out of the group of 12. So, by doing permutation and combination, you can find out that there are 792 ways to choose 5 things out of a group of 12, out of which some combination of slip system cannot produce 5 independent strain.

So, at least there are 96 combinations that can be found out by analysis which can produce 5 independent strains which is required for which is minimum requirement for the you know plastic deformation of the polycrystalline aggregate. So, most orientation have minimum value of M for several different combinations too. If we talk about this randomly oriented polycrystal, the M average M bar comes out to be equal to 3.06.

So, if we calculate the strength of the material sigma times M bar time sigma is equal to M bar times tau, then M bar equal to 3.06. So, the tensile strength for a randomly oriented FCC polycrystal could be equal to 3.06 tau CRSS right. So, where CRSS is the one where the you know plastic deformation just starts microscopically.

Now, you can see that we can calculate the tensile strength using the sax model using Schmid factor; average Schmid factor which comes around 2.336 and which is the lower bound approximation of the strength of the material whereas, we can use the Taylor factor model to calculate the tensile strength of the material which is the upper bound approximation.

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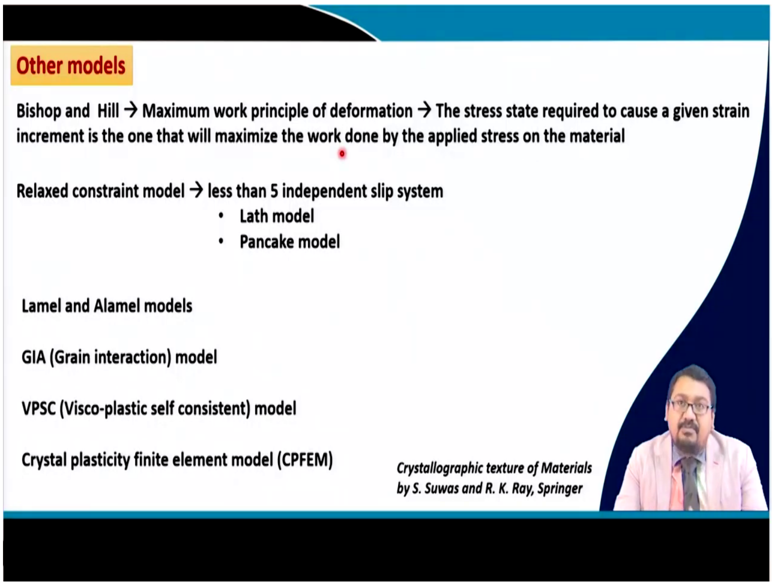

So, there are same other models and there are several developments in this field. And I am just going to show you different names of this model and if you are interested, there is fundamental information given in crystallographic texture of material by ah Suwas and Ray and one can go into detail to their to various publications and literatures.

And also one can go to various books and their manuals are also present in the webs in their websites in various websites of university of the universities which are related the professors which are related in you know developing this model.

So, the first model is the Bishop Hill model. The Bishop Hill model on the other hand comprises that the maximum work principle of deformation. And the stress state required to cause a given strain increment is the one that will maximize the work by the applied stress on the material. So, that is the Bishop Hill model.

There are other models like relaxed constraint model which states that during you know deformation less than 5 independent slip systems can allow a plastic deformation and. So, there are lath model and pancake model. And you can see that when the deformation takes place say for example, if there is a compression and initially, we started with equiaxed grains and the compressed grains looks like pancake right.

On the other hand, if it is a rolling then the grains become elongated or needle type or lath type. So, in both the cases the shape of the grain basically leads to the lower down the requirement of minimum 5 independent slip systems and thereby, deform at less than 5 slip systems independently and thus, the relaxed constraint models.

There are other models like Lamel Alamel model, GIA or the grain interaction model by RWTH Aachen Germany, the visco plastic self-consistent model by Molinari, Toth, Kalidindi and all who have Carlos Tome who have contribution in developing this model.

And then the crystal plasticity finite element model where Agnew and (Refer Time: 21:06) and even Toth and other researchers in this field basically work to see and to develop an understanding on the mathematical by mathematical equations to observe that how the deformation in the material takes place plastically.

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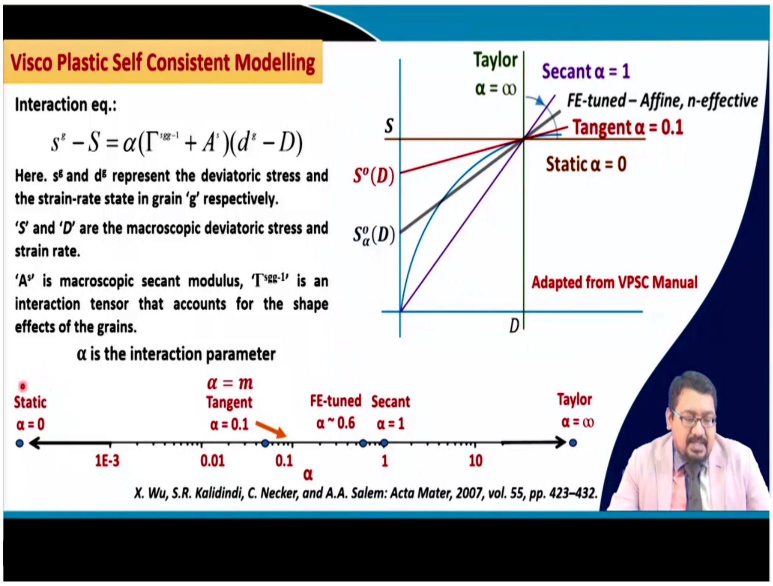

So, if we look into one of this model, the visco plastic self-consistent model, which are being used in a large scale by various researchers in this area to understand the texture evolution and the strain hardening behavior it the model consists of mainly two equations; one is the interaction equation.

The interaction equation relates the stress and the strain of the individual grains with the overall stress and strain of the material. So, if small s and small d are the deviatoric stress and the strain state for each and every grain or the grain g respectively, then the capital S and the capital D are the macroscopic deviatoric stress and strain for the whole material.

Now, it relates these two with the help of a parameter which is alpha. It is an interaction parameter and this along with this, there are A S which is the macroscopic secant modulus and tau which is the interaction tensor that accounts for the shape effects of the grain that is the shape of the grain.

If we look in this right side, we are showing this in terms of a stress strain diagram for the plastic deformation. Remember, that the elastic part has been omitted. Now, if the material is deforming somewhere like this then, the if the stress remains constant; that means, this line this is the Sachs model right where the stress remains constant and so, alpha is equal to 0.

So, if we look here, the Sachs model state it is a static model and it has it is a no constant model. So, each grain deforms individually and there is no condition of grain boundary contiguity. So, alpha there is alpha is equal to 0.

Now, on the other hand, this vertical line if you see, the strain remains constant. So, this comprises of the Taylor model which is a full constant model. And if you look into this line, you can see that the Taylor model alpha becomes equal to infinity and this is a full constant model.

So, there are models which are you know relaxed constraint and they are self-consistent which are used to model the realistic plastic deformation behavior of various material. And in order to understand texture evolution and strain hardening behavior. In order to understand, the various you know critical resolved shear stress and their ratios for various you know slip systems.

For example, in case of hexagonal close packed material, basal slip system, prismatic slip system, pyramidal slip system and various twinning's may present at the same time during the deformation.

So, there are models the tangent model is the one where alpha basically is equal to m. Most of the time m is taken equal to point 1 and you can see that the model relates the stress and curve with a tangent at the point of the maximum stress.

There is model which is known as the Secant model which relates the alpha means relates the overall stress and the strain state to the stress and the strain state of the grains by alpha equal to 1. In between there could be various models, in order to go more closer to the realistic scenario in the material and these are FE tuned model which includes affine model n effective models.

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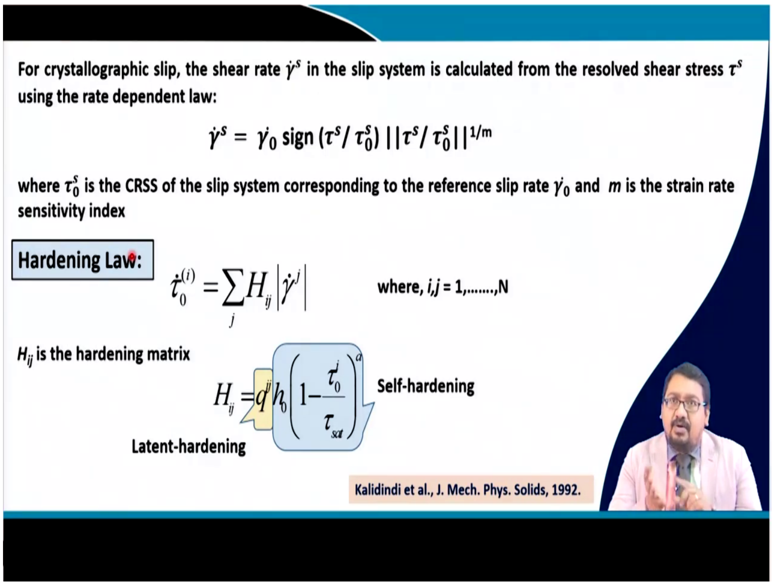

For crystallographic slip, the shear rate that is gamma dot s in the slip system can be calculated from the resolved shear stress tau S right. So, by using a rate dependent law. So, gamma dot S is equal to gamma dot 0 which is the reference slip rate and that is the reference shear strain rate times sin of tau S by tau S 0 where tau S is the resolved shear stress and tau S 0 is the critical resolved shear stress for the slip system and it is to the power 1 by m, where m is the strain rate sensitivity index.

So, a rate dependent law is basically used in order to relate the shear strain which is written in terms of shear strain rate, because this is a visco plastic self-consistent model where the strain is basically calculated in terms of strain rate of a material, because the deformation is visco plastic.

In most of this model, while the evolution of texture by the presence of various slip and twin system is measured, where as I said the shear rate is for a particular slip system. The shear rate can be said for a particular twin system to comprising of the similar kind of rate dependent law for the twinning system.

Now, the hardening law is also incorporated to understand the strain hardening behavior of the material. The most of the hardening laws such as the Holloman equation or the Voce hardening or the Bronkhorst hardening can be utilized.

And here, we are showing the normal you know Voce hardening law which is basically a summation of the hardening matrix.

The hardening matrix comprises of two equations; one is q, which is related to the latent hardening and another equation which looks something like this relating tau 0 to tau saturation is the self-hardening. And this hardening law can be obtained from the publication by Kalidindi et al in journal of mechanics physics of solid 1992.

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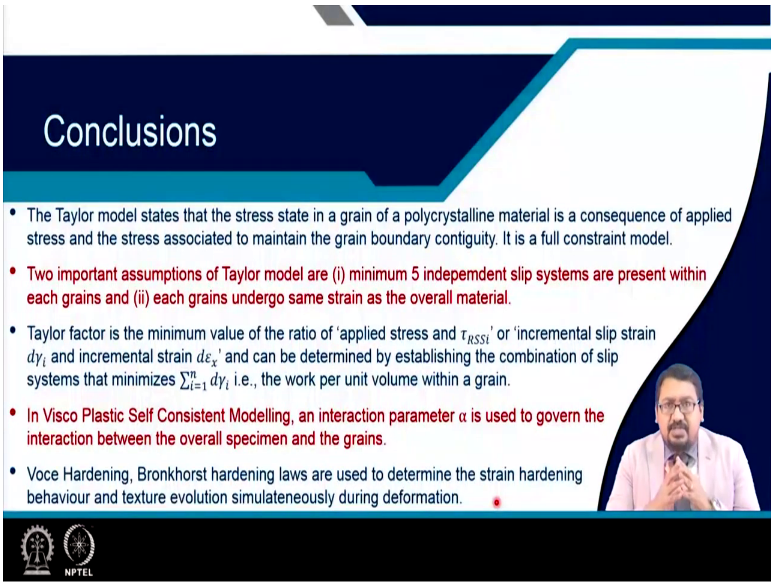

So, from this lecture class, we can conclude that the Taylor model states that the stress state in a grain of a polycrystalline material is a consequence of applied stress and the stress associated to maintain the grain boundary contiguity. And it is a full constraint model.

The second thing is that two important assumptions of Taylor model are that minimum 5 independent slip systems are present within each of these grains and each grain undergo the same strain as the overall material.

So, Taylor factor is the minimum value of the ratio of the applied stress and the resolved shear stress for the slip systems or the incremental slip strain to the incremental strain delta epsilon x and can be determined by establishing the combination of slip system that minimizes this summation of delta gamma i that is the minimum incremental slip strain shear strain that is the work per unit volume within a grain right.

So, in Visco Plastic Self Consistent Model, an interaction parameter alpha is used to govern the interaction between the grains and the overall specimen, in order to obtain hardening or strain hardening behavior.

Voce hardening or Bronkhorst hardening laws are being used to determine the strain hardening behaviour and the texture evolution simultaneously during Visco Plastic Self Consistent Modelling; that is during predicting during simulation of plastic deformation by Visco Plastic Self Consistent Modelling.

Thank you very much.