Texture in Materials
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Module - 04
Texture representation
Lecture - 21
Size of Euler Space in Relation to Crystal and Sample Symmetry

Good day everyone, we are continuing with the module 4 which is Texture Representation and this is lecture number 21, in this lecture we will try to understand that how the Size of the Euler Space varies in Relation to the Crystal Symmetry and the Sample Symmetry.

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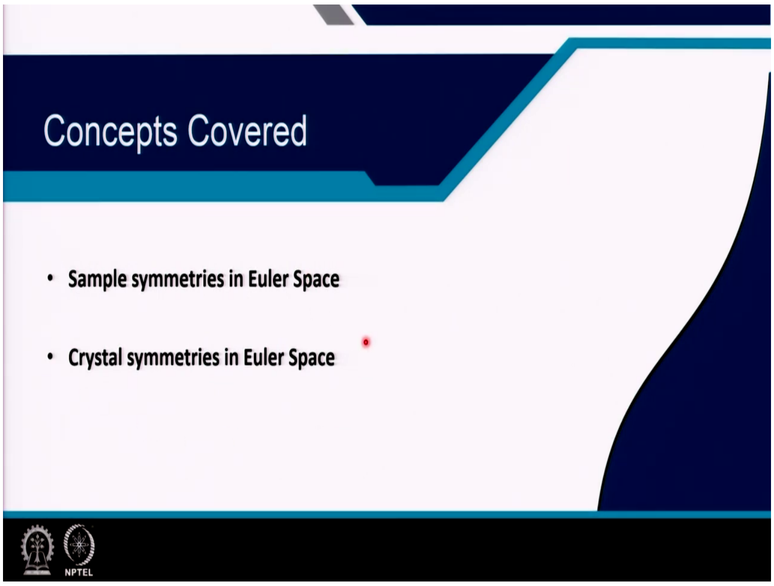

So, the concepts that will be covered here are sample symmetries in Euler space and crystal symmetries in Euler space.

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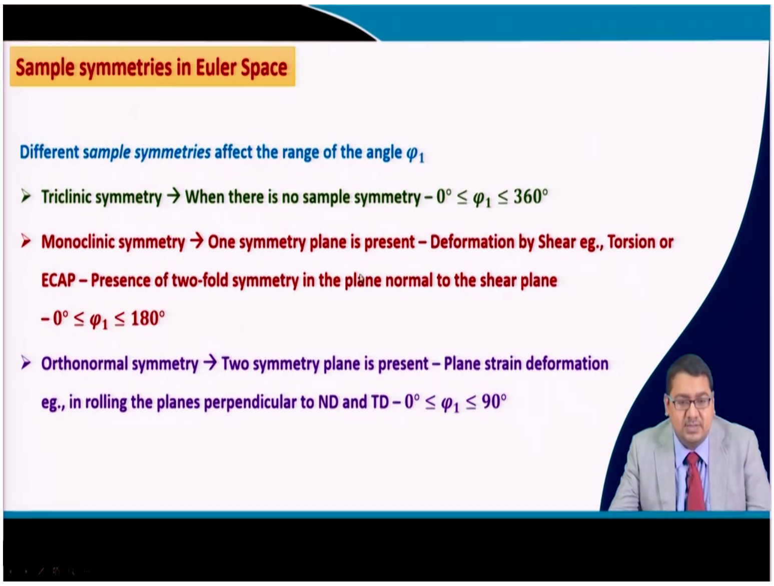

Let us try and understand the affect of sample symmetry on the Euler space, now that sample symmetry means that any sample any polycrystalline material is either processed by some means, means either it is casted it can be high pressure die casting or any other casting process continuous die casting process or it has been deformed plastically deformed. So, it can be extrusion or rolling or it could be any severe plastic deformation process like equal channel angular pressing or forging process. So, depending upon the type of process depending upon the type of process the Euler angle the angle phi 1 of the Euler angle phi 1 phi phi 2 is reduced depending upon the symmetry of the processing of any sample. Therefore, this is known as sample symmetries and that why phi 1 is affected by sample symmetry.

Just if we look deeply phi 1 is an angle, which is a rotational angle along the axis RD in case of the rolled sample that I always give an example. So, a rolled sample right having plane strain deformation. Therefore, a rolling is a plane strain deformation more it is a plane strain compression it has a rolling direction RD it has a normal direction perpendicular to the rolling plane ND and it has a transverse direction, which is perpendicular to both RD and ND.

Now if we look into a rolled sample or plane strain compression sample the direction RD contains two planes one plane, which is the RD and the ND plane that is the plane perpendicular to the TD and the RD and the TD plane, which is the plane perpendicular to ND and both of this plane have mirror symmetry or 2 fold symmetry. So, this sample symmetry will affect the angle phi 1 if it has a 2 fold symmetry along RD in two planes then it will reduce the phi 1 into 2 by 2 that is by 4; that means, 360 will be divided by 90.

Now let us take one by one the first one is the triclinic symmetry and what does it mean by triclinic sample symmetry, it means that there is no symmetry at all. So, when there is no sample symmetry present in the sample due to it is history of processing then we have to take phi 1 from 0 degree to 360 degrees. In case of monoclinic symmetry, monoclinic sample symmetry there is one symmetry present along a particular axis and these kind of symmetry is usually observed in case of shear type deformation right like torsion experiments or in case of equal channel angular pressing.

So, that in these cases there is a presence of 2 fold symmetry in the plane normal to the shear plane and thus the phi 1 is reduced from 0 to 360 degrees to 0 to 180 degrees in case of the monoclinic sample symmetry. And the example that I was giving of the rolling is of the orthotropic or orthonormal sample symmetry. So, in this case what happens that two symmetry planes are present and this occurs in plane strain deformation modes in rolling as I said the planes which are perpendicular to ND and TD containing both containing. The RD direction reduces because of the presence of to two fold symmetry sample symmetry reduces the phi 1 from 0 to 360 to 0 to 90 degrees.

Therefore, this is all about how Euler space is affected by the presence of processing symmetry of the sample.

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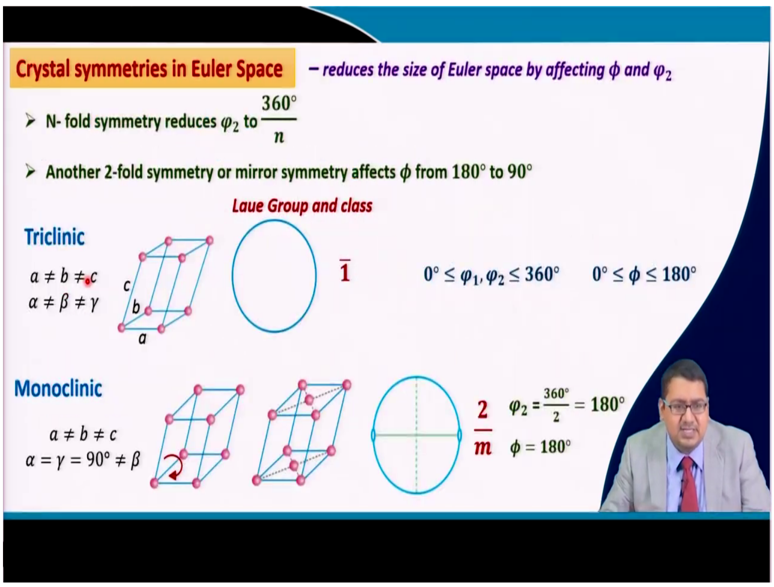

Now let us take how crystal symmetries are will affect Euler space. So, we can understood that the crystal symmetry will reduce the Euler space in phi and phi 2. So, if there is an N fold crystal symmetry N fold rotational symmetry present in the crystal then the phi 2 is reduced from 0 to 360 to 0 to 360 divided by that n. If there is another 2 fold symmetry or mirror symmetry present. Then it affects the phi, which changes from 180 degree we remember that the Euler space the phi is not from 0 to 360. But it is 0 to 180 because of the presence of the identity a glide plane along phi equal to pi and therefore, the therefore, phi is equal to 0 to 180 degree in any Euler space without applying any sample or crystal symmetry.

So, you have if you have 2 fold symmetry or mirror symmetry then phi reduces by half of 180 and becomes from 0 to 180 to 0 to 90 degrees. So, let us take the example of the triclinic system that we gave earlier. As we know the triclinic with is not equal to b is not equal to c and alpha is not equal to beta is not equal to gamma has a Laue group like a circle. Because it has no symmetric at all and therefore, no symmetry is known as triclinic symmetry and a Laue class of 1 bar.

Thus that because there is 1 fold symmetry right no symmetry means 1 fold symmetry or rotation of 360 degrees therefore, the phi 2 not the phi 1 the phi 2. So, just delete this phi 1 this is a typo. So, it changes from it does not change and remains 0 to 360 degrees right and there is no mirror symmetry or 2 fold symmetry either in any other plane. Therefore, phi remains from 0 to 180 degrees.

In case of monoclinic system whereas, though a is not equal to b is not equal to c, but there is alpha equal to gamma equal to 90 degrees right though beta is not equal to 90 degrees. Now that if we look closely to any of this two crystals either primitive or the base centred one you can see that the angle between a and b which is gamma is 90 degrees and angle between b and c which is alpha is again 90 degree. So, if we take the axis b axis then there is a 2 fold symmetry in that axis and the perpendicular to that axis can also have a mirror plane. So, either it has a 2 fold symmetry or a mirror plane leading to formation of the Laue group such that and the Laue class 2 by m. The 2-fold symmetry reduces the phi 2 from 360 and divided by 2 equal to 180.

So, phi 2 becomes from 0 to 180 degree whereas, phi remains the same because the 2 fold symmetry and the mirror symmetry exist either the 2 fold or the mirror in the same plane right.

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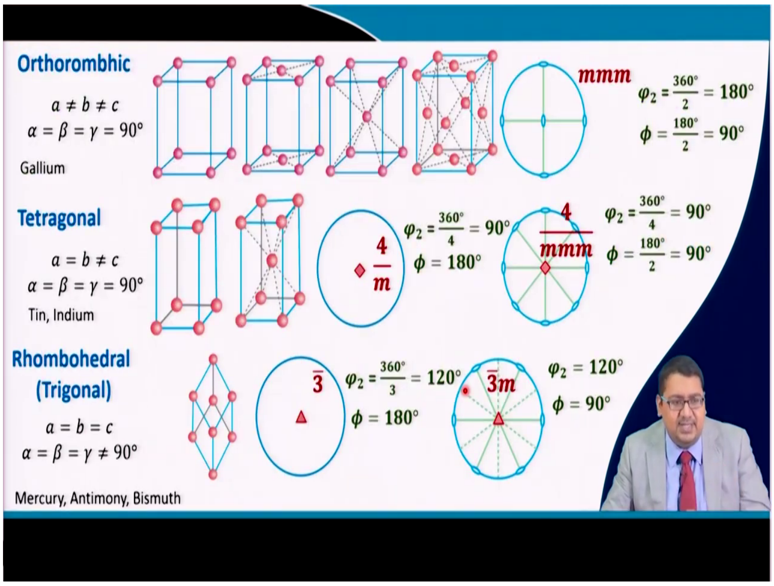

Let us go ahead and look into the orthorhombic system in case of the orthorhombic system you can see that alpha sorry a is not equal to b is not equal to c, but in this case alpha is equal to beta is equal to gamma and this is equal to 90 degree. This makes it 2 fold symmetry in a axis b axis and c axis and either three 2 fold symmetry or one 2 fold symmetry, 2 mirror, two 2 fold symmetry, 1 mirror or 3 mirrors it is given by m m m in Laue class and the Laue group is shown like this. Thus, if you take the first 2 fold symmetry it divides phi 2 by 2 means 360 by 2 and makes it 180 degree. So, phi 2 becomes 0 to 180 degree. If we take the second mirror symmetry then it reduces phi from 180 degree by 2 to 90 degree. Therefore, phi becomes 0 to 90 degree.

Now let us take tetragonal system and tetragonal system is having a is equal to b is not equal to c alpha equal to beta equal to gamma equal to 90 degrees. Now if such a configuration exist that is a equal to b is not equal to c then for both this primitive. And the body centered tetragonal crystal structure you will see in the axis c a 4 fold symmetry exist right and in the same axis of course, if it has a 4 fold symmetry then it will also have a mirror symmetry so, either 4 fold or a mirror symmetry so, 4 by m.

Now in case of complex crystal structure with complex group of atoms present in each position and therefore, it is a motif the phi 2 reduces because of the 4-fold symmetry from 360 degree to 360 degree by 4 which is 90 degrees but phi remains 180 degrees. But in case of simple primitive tetragonal structure or simple body centered tetragonal structure one has

4 by m and then other mirror symmetries or along b axis and a axis leading to phi 2 which is 4 fold symmetry. So, 0 to 90 degrees phi also reduces because of the presence of the mirror symmetry along the other plane that is 180 degree by 2 and becomes 90 degree.

Now in case of the rhombohedral or the trigonal crystal system a is equal to b is equal to c, but in this case alpha is equal to beta equal to gamma. But not 90 degree and then it has a 3 fold symmetry and you can see the Laue group having this and it has there will be a division here right and making this into three symmetry elements and also bar is the Laue class. Because of the 3 fold symmetry present the phi 2 is divided by 3 360 is divided by 3 and phi 2 becomes 0 to 120 degrees. Whereas, phi remains 180 degrees in case the trigonal structure is complex type right and if it is a simple one like for mercury antimony bismuth as I said in the earlier lecture it will reduce the it will reduce the phi by 2. Because of the presence of the mirror symmetry in the other plane, ok.

The Laue class in this case as I said earlier becomes 3 bar time and m 3 bar m and so, this m mirror symmetry reduces the phi from 180 degrees to 90 degrees. Phi becomes 0 to 90 degree for the simple rhombohedral crystal structure of mercury antimony bismuth and other metal.

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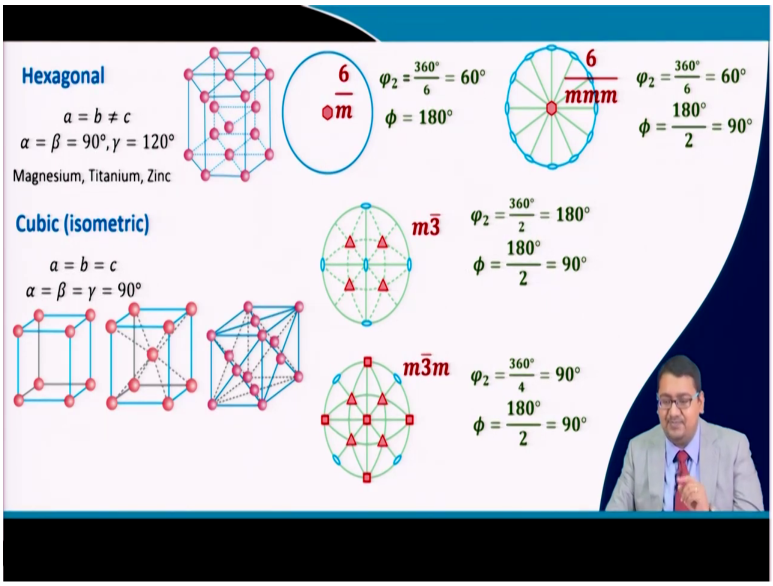

If we go into the hexagonal close packed crystal structure where we have learned in the previous lecture that a is equal to b. But is not equal to c which is the axis the Bessel axis or the axis which is parallel to 000 1 and in this case alpha is equal to beta that is the angle between c and a c and b is equal to 90 degree and gamma which is angle between a and b is

120 degree. So, along the axis c, there is a 6 fold rotational symmetry and along the same axis there if there is a 6 fold then there could be a mirror plane there will be a mirror plane. In case of complex hexagonal crystal structure and not for metals like magnesium, titanium or zinc, but for complex crystal structure then the Laue class is just 6 by m and there will be only 6 symmetry elements.

Now phi 2 becomes equal to 360 divided by 6 right. Therefore, phi 2 is from 0 to 60 degrees whereas, phi remains 180 degree. But in case of normal hexagonal crystal structure of magnesium, titanium and zinc which have a single atom in each point lattice point and then it has a 6 fold symmetry and mirror plane and three other mirror planes two other mirror planes right apart from the one that is having in the c axis. The phi 2 definitely becomes 0 to 60 degrees because 360 divided by 6, but the phi also because of the presence of the other mirror plane becomes equal to 180 degree divided by 290 degrees. So, phi also becomes equal to 0 to 90 degree.

Now let us talk about the most used structure, which is the cubic structure or the isometric structure. Where a is equal to b is equal to c and alpha is equal to beta is equal to gamma equal to 90 degree and we have discussed about this crystal structure in many lectures in this course and we know a lot about it. It has a 3 fold symmetry in the 4 111 axis leading to the formation of this Laue class 3 bar and it also have the mirror plane about that axis and the other axis or you can say the 2 fold symmetry present in the 100 or 110.

In case of complex cubic structure, the Laue class is given by m 3 bar and the Laue group looks like this. Now because of the presence of 2-fold symmetry in the complex crystal cubic crystal structure in 100 phi 2 is divided by 360 divided by 2 so that it becomes 180 degree. Instead of it is not divided by 360 degree divided by 3 and it is not looked upon the 1 1 axis and therefore, phi 2 is kept 180 degree by convention. Because of the presence of 2 fold symmetry along the other planes the phi is divided by 2. That is 180 degree divided by 2 and becomes 0 to 90 degree in this case. But in case of cubic structure for say aluminium or copper or austenitic stainless steel which has a simple face centred cubic crystal or in case of ferritic steel or other metal which has body centred cubic crystal the Laue group looks something like this. It has four 3-fold symmetries along the 111 axis that is by the rotation of 120 degree. 3 bar and it has 4 fold symmetries in 3 100, what you call family of access and then it has 6 110 family of 1 1 sorry 110 family of axis to have another 2 fold symmetries.

It is denoted by m 3 bar m in Laue class, but when we do the symmetry division in case of the Euler space, we use the 4-fold symmetry of the 3 family of 100 axis. And therefore, phi 2 is equal to 360 divided by 4 which becomes 90 degree; that means, phi 2 becomes 0 to 90 degree whereas, phi can also take another mirror symmetry and divided divide itself by 2. Therefore, 180 degree divided by 2 and it becomes equal to 90 degree.

So, in case of the cubic crystal what happens is that we use the Euler space which has phi 1 which depends upon the sample symmetry so forget about it, but phi 2 which can be from 0 to 360 degree. However, because of the 4 fold symmetry of 100 we it reduces to 0 to 90 degree and because of the mirror plane in another axis the 5 also reduces from 0 to 180 to 0 to 90 degree. We have not considered the 3 fold symmetry associated with 120 degree rotation of the 4 111 family of axes right.

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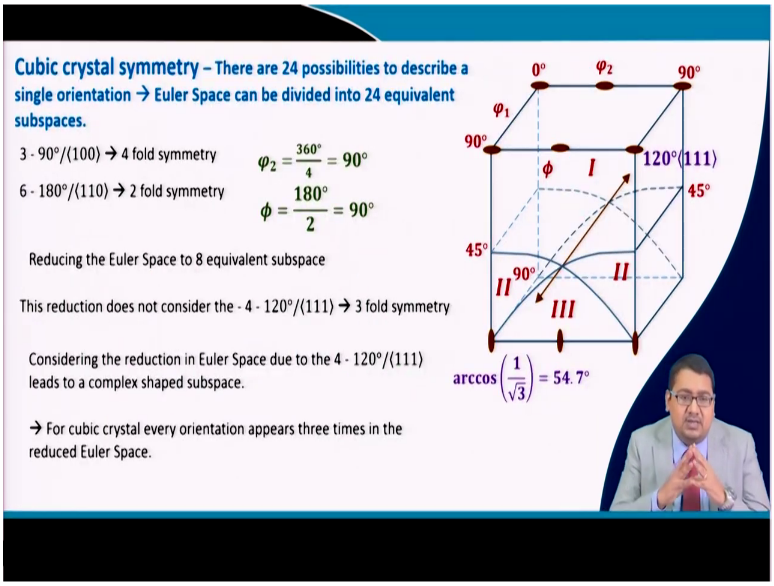

So, if we look into the cubic crystal symmetry there are 24 possibilities to describe a single orientation and that we have observed in the 100 stereographic projection and we can and we can observe that in the 110 standard stereographic projection and even in the 111 standard stereographic projection.

Therefore, the fact is the Euler space can also be divided into 24 equivalent subspaces and any one of those equivalent subspaces should be able to give the correct information of the texture. However, we do not do it because we only take the one again the 3 90 degree rotation of the family of axis 100 which is the 4 fold symmetry. And the 6th 180 degree rotation of the means of the 6 110 family of axes which is 2 fold symmetry.

Thereby we just reduce the mat the Euler space along phi 2 by 4 and phi by 2; that means, we are reducing the Euler space into 8 equivalent spaces. So, any one of this 8 equivalent space should give the information of the texture. So, if you look here if you look here I am drawing here 0 to 90 degree of phi 2 0 to 90 degree of phi 1 and 0 to 90 degree for phi right. And if you look this Euler space actually will be able to give all the information for a material which has means which means a material which has a cubic crystal structure and has an orthonormal sample symmetry right phi 1 0 to 90 degree because of phi 1 0 to 90 degree.

Now that; however, while considering this Euler space we have not considered the 4 111 family of axes having 3 fold symmetry by rotation of 120 degrees along this axes and we have not considered this. Now if we consider the reduction of the Euler space due to this 4 3 fold symmetries of 120 degrees about each of this 111 axis then the Euler space gets divided into complex shape and you can see that how it is. So, there is one space which is divided by this curvy curvical plane and another with this curvical plane. So, which is intersecting at 45 degrees to phi and both these planes are intersecting at 45 degree to phi and it is rotating and going like this.

This because of the presence of this 3 fold symmetry along the 4 111 axes the reduced Euler space the reduced Euler space means the Euler space which is reduced from phi 1 phi phi 2 equal to 0 360, 0 180, 0 360 to 0 90 0 90 0 90. So, the reduced Euler space it divides the reduce Euler space into three part and that one can find out the mathematical relation relating it to the 120 degree and relating it to the angles between the 3 111 plane.

And you can see that there in this small Euler space it has 1 2 and 3 areas equivalent areas though the 2 area the area number 2 is divided, but it is divided because it is half here and half in here or here right because it is a repetition right. Therefore, it is divided into 3 spaces. Now that therefore, for represent while representing a particular orientation in the Euler space for cubic crystal the orientation or the texture component appears three times even in this reduced Euler space a very nice example is of the example of the Gauss texture that I have shown few lectures.

Before where you will see that the relationship between the Euler space and the orientation matrix shows that the Gauss is present at three positions and you will if you look deeply then the one position will be in the volume number 1, another will be at the volume number 2 and another at 3.

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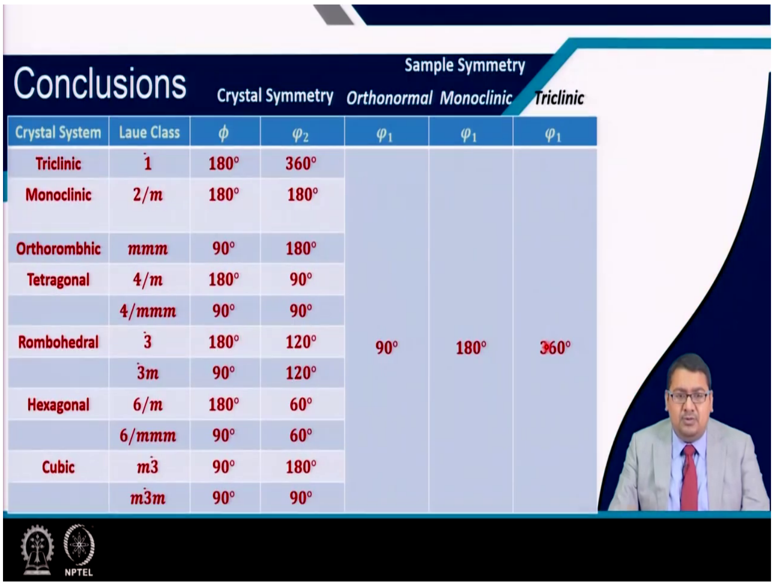

Crystal System	Laue Class	ϕ	φ_2	Orthonormal φ_1	Monoclinic φ_1	Triclinic φ_1
Triclinic	$\bar{1}$	180°	360°			
Monoclinic	$2/m$	180°	180°			
Orthorombhic	mmm	90°	180°			
Tetragonal	$4/m$	180°	90°			
	$4/mmm$	90°	90°			
Rombohedral	$\bar{3}$	180°	120°	90°	180°	360°
	$\bar{3}m$	90°	120°			
Hexagonal	$6/m$	180°	60°			
	$6/mmm$	90°	60°			
Cubic	$m\bar{3}$	90°	180°			
	$m\bar{3}m$	90°	90°			

So, we can finally, give the conclusion this conclusion is an information that we can plot all this information that we obtained now from this class and the last class about the crystal system into we can divide it into Laue class crystal symmetry orthonormal, monoclinic sorry monoclinic and triclinic sample symmetry. If we take the sample symmetry it is triclinic then phi 1 remains 360 degrees if it becomes monoclinic that is having say for example, a shear or torsion or equal channel angular processing kind of thing then the phi 1 is reduced because of the presence of 1 mirror plane to 180 degree. And in case of orthonormal like plane strain compression the phi 1 which has 2 mirror planes the phi 1 is reduced to 180 360 degree divided by 4 so 90 degrees so, 0 to 90 degree.

Then depending upon the crystal system so, triclinic with no symmetry or we can say triclinic symmetry 1 bar the phi 2 is 360 phi remains 180. In case of monoclinic, it is 2-fold symmetry or a mirror plane. The phi 2 becomes 180 degree. In orthorhombic it has 3 axes with mirror planes so, the phi 2 is divided by 2 180. And 90 degree in tetragonal 4 fold symmetry or mirror. Therefore, only the phi 2 is reduced to 90 phi remains the same.

In case of tetragonal with simple crystal, structure not the complex one as before the phi also reduced by 90 degree. In case of rhombohedral system there is a 3 fold symmetry therefore, the phi 2 is reduced to 180 degree and in case for the simple rhombohedral not with the complex structure or complex presence of motif. The Laue class becomes 3 bar m and therefore, the m reduces the phi to 90 degrees.

In case of hexagonal, close packed material if the Laue class is 6 or m 6 by m then phi is equal to 180 degree and 60 degrees, in case of the Laue class is 6 by m m m that is the presence of mirror planes at different mirror planes then the phi is reduced to 90 degrees. Whereas the phi 2 as usual because of the 6 fold symmetry reduced by 360 by 6 which is 60 degrees. In case of cubic crystal, we understood that instead of 3 fold symmetry of the 111 axis which gives 3 bar m and m which is because of the 2 fold symmetry of the other plane gives in a Laue class of m 3 bar.

We use the 2 fold symmetry and the 4 fold symmetry, 2 fold symmetry for the complex crystal structure and the 4 fold symmetry along 100 for the normal simple cubic crystal structures for aluminium or copper or steel. As I gave the example, which reduces phi 2 by 360 divided by 4 because of the 4 fold symmetry of the 100 and the 2 fold symmetries which are present in the other plane.

This is very important to understand that how the sample symmetry and the crystal symmetry affects the representation of texture in the Euler space and that is all for today's class.

Thank you very much.