

**Texture in Materials**  
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**Module - 04**  
**Texture representation**  
**Lecture - 16**  
**Euler Angles and ODF's (Contd.)**

Hello everyone. So, today we will continue with the understanding that we are creating on Euler angles and ODF's, this is lecture number 16, and we are doing module 4 on Texture Representation.

(Refer Slide Time: 00:48)

**Crystal orientation  $\{hkl\}(uvw)$  in terms of Euler angles  $\varphi_1, \varphi, \varphi_2$**

$\varphi_1, \varphi, \varphi_2$	$(hkl)(uvw)$
(i) $0^\circ, 0^\circ, 0^\circ$	$(001)[100]$
(ii) $0^\circ, 0^\circ, 90^\circ$	$(001)[010]$
(iii) $0^\circ, 90^\circ, 0^\circ$	$(010)[100]$
(iv) $90^\circ, 0^\circ, 0^\circ$	$(001)[0\bar{1}0]$
(v) $0^\circ, 90^\circ, 90^\circ$	$(100)[0\bar{1}0]$
(vi) $90^\circ, 0^\circ, 90^\circ$	$(001)[01\bar{0}]$
(vii) $90^\circ, 90^\circ, 0^\circ$	$(010)[001]$
(viii) $90^\circ, 90^\circ, 90^\circ$	$(100)[00\bar{1}]$

ODFs:  $\varphi_2 = 0^\circ, 5^\circ, 10^\circ, \dots, 90^\circ$   
 $\varphi_1$  and  $\varphi$  are continuous

Today what we will do is that we will try to represent the crystal orientations in terms of the Euler space in terms of the Euler angles phi 1, phi, phi 2 and that is what we are trying to do that we are trying to trace back the Miller indices in the Euler space.

We are trying to construct the Euler space. So, this lecture is related to the construction of the Euler space utilizing Euler angles phi 1, phi, phi 2. So, let me first draw the Euler space right. As in the other slides I am trying to draw the Euler space and this is the Euler space right.

So, this is 0 degree for the phi 1 section up to 90 degrees and we will do this construction for the cubic crystal and therefore, we will always have phi 1, phi, phi 2 which is starting at 0 and

ending at 90 degrees right. Therefore, this is the  $\phi_2$  section. So, any orientation that is 100 010 or and 001.

This is used to represent the crystal coordinate system right and in order to represent texture we try to going make the sample important sample reference system, which in most of the cases we are using RD, TD and ND. Because most of us are familiar with rolling and we try to coincide the important sample reference directions that is RD, TD and ND with parallel to the important specimen reference direction which is the 100 axes right.

So, that in order to construct the Euler space in terms of the crystal orientation hkl uvw what we need to do is we need a sample reference system like we draw here and the crystal reference system as we have drawn here. So, let us say  $\phi_1$ ,  $\phi$ ,  $\phi_2$  here we are keeping it for writing this in form of a table right and here we will keep hkl uvw right. So, we will try to write the hkl uvw corresponding to as much as possible  $\phi_1$ ,  $\phi$ ,  $\phi_2$ .

So, let us take the first condition. The first condition is the condition where  $\phi_1$  is 0,  $\phi$  is 0 and  $\phi_2$  is 0 and the condition of this will be such that RD is parallel to 100, ND is parallel to 001 and TD is parallel to 010. The hkl uvw for this first condition which is the starting condition itself without any rotation is 001 as hkl and 100 as the uvw that is the rolling plane and the rolling direction right. So, now, let us write this position in terms of Miller indices in the Euler space. So, this position is 001 100 right.

We see that when we are finding out the texture components and we gave example of cube component rotated cube component and Goss component in the earlier lectures. And we are rotating  $\phi_1$ ,  $\phi$ ,  $\phi_2$  that is we are rotating the sample coordinate system ND that is  $\phi_1$  rotation right by the right hand thumb rule, then RD that is the  $\phi$  rotation by right hand thumb rule and again  $\phi_2$  rotation which is along ND using the right hand thumb rule. But when we are tracing back the crystal orientation while we rotate the sample coordinate system, we should remember that we have to exact rotate in the reverse sequence right. We need to rotate first by  $\phi_2$  and then by  $\phi$  and then by  $\phi_1$  and this time we have to use the inverse of the right hand thumb rule that is we have to use the left hand thumb rule. Because we have to rotate opposite, we are tracing the crystal orientation in terms of the Euler angles. Therefore, instead of the right hand thumb rule we have to just do it opposite. So, let us see how it is done.

So, let us say that the first condition that we showed is the 0001 condition, which is this one right I will not write it here. So, this was the first condition right this right the second condition. Let us say the second condition this is the first condition this is the second condition is  $\phi = 10^\circ$ ,  $\phi = 0^\circ$  and  $\phi = 2^\circ = 90^\circ$ . So, the  $90^\circ$  rotation is not by the right hand thumb rule, but just opposite to it that is the left hand rule right or the opposite of the right hand thumb rule like this right.

So, let us use the green color to show how this rotation is done right. So, the rotation is now like this right instead of this. So, if we rotate ND by this then what will happen? The TD will come here and RD will go there right. So, let us draw how it will look like how the specimen axes will look like. RD will be now the TD is here and the RD is here. So, for the  $0^\circ$   $90^\circ$  degree the hkl planes for the ND remains the same that is 001 and the RD becomes opposite to 010 that is  $0\bar{1}0$  right.

So, let us show the position of the second position ok. So, here 000 is shown by here this position in the Euler space,  $0^\circ$   $0^\circ$   $90^\circ$  that is  $\phi = 10^\circ$ ,  $\phi = 0^\circ$  and  $\phi = 2^\circ = 90^\circ$  can be shown by this position right and this position is  $0^\circ$   $0^\circ$   $1^\circ$   $0^\circ$   $1^\circ$  bar right. So, we have traced 2 Euler two crystal orientation in the Euler space right let us trace the third one and what is the third one? Let us take  $0^\circ$ ,  $90^\circ$  and  $0^\circ$  right. If we do that; if we do that let us say this is the third one and if we do that we have to take this one and we have to rotate by  $90^\circ$  degrees along RD by the left hand rule. So, we have to rotate just opposite to that of the right hand. So, how the rotation will be. So, initially it is RD and then it is TD and then it is ND here and we are rotating like this like this. So, it is the using opposite of the right hand and so, this will become sorry.

So, this will become RD will remain RD, let me change the color for clarity, RD will remain RD because we are rotating along RD now and the TD will go down and the ND will go here. So, for the third case ND is becoming parallel to 010 and RD remains parallel to 100 right.

I can show the position in Euler's space that is  $\phi = 10^\circ$ ,  $\phi = 90^\circ$  and  $\phi = 2^\circ = 0^\circ$ . So, this position is 010 100 right. Now let us check the fourth condition the fourth condition let us say it is  $90^\circ$ ,  $0^\circ$  and  $90^\circ$ . The condition will be similar to the second condition right where the ND is rotated. Therefore, that it forms TD here and RD here. Therefore, this is equal to the second condition right. Here the I have written it a little wrongly. I am tracing the  $90^\circ$   $0^\circ$  and  $0^\circ$  degree right.

So, it's the same rotation which we gave in the here where we have rotated 00 and 90 now we are rotating 90 0 0 and therefore, the rotation along phi 1 at 90 degree is along ND which is similar to the what we have done in the second one. The Miller indices will remain the same as in the second one. One can see repetition on the Miller indices even in this Euler space which had been shortened from 0 to 360 degrees along phi 1, phi, phi 2 to 0 to 90 degree because of the cubic symmetry right. So, phi 1 equal to 90, phi equal to 0 and phi 2 equal to 0. The position is here which is 001 01 bar 0 right. Therefore, the fifth situation could be 0 degree 90 degree and 90 degree.

So, as I have said that we have to trace the specimen sample axis to the crystal sample axis and in this regard we are tracing the hkl uvw in terms of phi 1, phi, phi 2. Therefore, thereby we are doing it in the opposite direction right. In order to trace the fifth one we are doing we have to do phi 2 section rotation first by the opposite of the right hand thumb rule and then phi section rotation by 90 degrees. The phi 2 section rotation is already given in the by 90 degrees already given in the second condition which is this one right and then we take this and we add phi equal to 90 degree section phi equal to 90 degree rotation.

And then what will happen is that, we are giving along RD opposite of the right hand rotation which is this one right and then when we are plotting this what we are observing that RD remains the same right RD will remain the same and then ND will change and it will form here and TD will form here right. RD remains the same. In this case the ND becomes parallel to 100 whereas, the RD becomes parallel to the opposite of 010 that is 01 bar 0 right.

Now, let us put this in the Euler space. So, it is phi 1 equal to 0, phi equal to 90 and phi phi 2 equal to 90 right. So, the position of this is here and this is 100 01 bar 0 right now the sixth position the sixth position is let us say that it is 90 degree, 0 degree and 90 degree. For the sixth position, let us take the second one and give a rotation at phi 2 equal to 90 degrees right. If we do such then what will happen we are rotating again by ND right. In addition, not like this what we are doing is that we have to rotate it like this right opposite of the right hand thumb rule and thereby what will happen? The TD will come here and the RD will go there right.

So, let us draw it. So, here is the ND and here is the RD and here is the TD right. Therefore, the ND remains 001 and RD became 01 bar 0 right. So, let us write the sixth one and which is 001 01 bar 0 and so, phi 1 90 phi 0, phi 2 90 is this position which is 001 01 bar 0 right.

Therefore, the seventh situation could be 90 degrees along  $\phi_1$ , 90 degrees along  $\phi_2$  and 0 degree along  $\phi_3$ .

So, the seventh situation could be taken as the situation that we have already done given that is a rotation along  $\phi_3$  in the third condition and so, if we take this particular situation that is the iii and plus if we do a  $\phi_1$  equal to 90 degree along ND again. Let us take it take this one. We will write ND and this is RD and this is TD and so, this is after the  $\phi_3$  rotation along RD and thereby if we give this rotation  $\phi_1$  rotation along ND which is by the opposite of right hand thumb rule, then RD becomes goes up and TD goes to RD.

So, this looks like ND will be the same, RD above, and TD here. ND now becomes parallel. I am writing it directly here parallel to the 010 right and the RD is parallel to 001, so let us write this position in the Euler space. So,  $\phi_1$  90 and  $\phi_2$  90 and  $\phi_3$  equal to 0. So, we have got this position this is 010 right 001. Now we need to trace another position which is the eighth position and that is 90, 90 and 90 degrees right. So, in that case it is better to take this 1 because we have already given  $\phi_1$  to 90 and  $\phi_2$  90.

So, let us take the eighth position sorry eighth position. So, that is the rotation that we give in the fifth one that is the rotation of  $\phi_3$  by 90 and then  $\phi_2$  by 90 using opposite of right hand thumb rule plus we have to give  $\phi_1$  equal to 90 along ND right that is an ND rotation using the opposite of the right hand thumb rule. So, which one is the fifth one? Therefore, we will have to use this particular one right.

We have to use this and we have to rotate by 90 along ND using the opposite of right hand thumb rule, which is this, thereby bringing RD above and TD to RD. If we draw this here then ND remains in the same direction RD goes up and TD goes here. ND of this position is 100 whereas; the RD becomes equal to 001 right. So, if that this is the position where  $\phi_1$  is 90,  $\phi_2$  is 90 and  $\phi_3$  is 90. Thereby this position is 100 001.

Now, that we have found out and calculated the all the corners of the ODF's and of course, all the corners of the ODF's are related to the cube texture components, but that one can find out the other texture components also by giving simple additions. Now just to tell you here it is very important to know that when we are giving the rotation along this  $\phi_1$  angle along this  $\phi_1$  angle this  $\phi_1$  angle the rotation is along ND. The ND component will not change the component that will change is the RD and therefore.

This kind of a fibre which goes from here to here is known as the ND fibre and then if we find try to find out the position which is at the centre here, then that in this component will have an hkl which is 001 which is known because the ND is the same. The rolling plane is same, but if we see the rolling direction at here it is 100 and here it became 01 bar 0. The rolling direction if we add 100 and 01 bar 0 it becomes 10 bar 0 right. So that we could trace the Euler angle for which the miller indices is this one and the Euler angle of this position is 45 0 and 0 degree right  $\phi_1$ ,  $\phi$ ,  $\phi_2$ . Now if we trace the Euler angle 0 45 and 0 degree that is this position, then that at the rotation of  $\phi$  is along RD therefore, the rotation axis is RD. So,  $\phi$  represent the RD fibre right. So, the RD will remain same.

100 and 100 for both this component remain the same whereas, the ND component that rolling plane becomes 010 for this and 001 for this Euler angle right. So, like that one can find out the components in between this in this line which is parallel to  $\phi_1$  right. Here also it is the ND component because the 100, which is the ND texture remains the same. 100 remains the same whereas, the rolling direction changes from 0 1 bar 0 to 001. So, this is 0 1 bar 1 right addition of these two like that various other texture components can be observed and can be depicted using this Euler space in terms of Miller indices say for example, the component which is at the centre here is 010 right 101 right. Here also the rolling plane or the is constant.

So, this is parallel to ND and the component which is along RD is 1 0 1 sorry 101 right like that one can trace each and every component of the Euler space; that is each and every position of the Euler space, which is in  $\phi_1$ ,  $\phi$ ,  $\phi_2$  in terms of hkl uvw. Miller indices by using a rotation which is opposite of the regular trend that is the specimen axis in is rotated by  $\phi_1$ ,  $\phi$ ,  $\phi_2$  that is a rotation along ND RD and ND to go to the crystal axis by the Euler criteria following bungees wave of rotation. In order to trace back the hkl uvw plane thereby we have to rotate the specimen axis by first  $\phi_2$  opposite to the right hand thumb rule and then by  $\phi$  and then by  $\phi_1$ . Following the opposite rule that now we have to rotate using the opposite of the right hand thumb that is the left hand thumb rule.

So, as to trace back the hkl uvw of the crystal. This is all today's for today's lecture in this lecture what we have found out is that we can trace back the hkl uvw in terms of the Euler space. So, the Euler space can be represented in terms of the hkl uvw and one can trace enormously and thereby one can find out the relationship between the Euler angles and the hkl uvw I will request the participants to practice this geometrically this geometrical

rotations. So that one can understand how this Euler space has found and then one can even go and trace make orientation distribution functions ok from this Euler space in terms of  $\phi_2$  sections right at  $\phi_2$  equal to 0 degree, 5 degrees, 10 degrees. and we can make it up to 90 degrees and see the positions of the hkl uvw various positions of hk uvw in it. We give a small example I will give you a small example say for example, texture right and we have seen the position of the Goss texture in this Euler space. Now if we are adding 0 01 and 100 and 010 100. We can see that the 100 is common and so, we are adding 010 and 001 right. So, this position which is at 0 45 and 0.

This is a position 0 45 and 0 represent the position of the Goss texture in the similar way. If we are adding this and this where the RD 0 1 bar 0 is constant and we are showing the texture which is the position 0-degree at  $\phi_1$  and 45 degree at  $\phi_1$  and at 90 degrees  $2\phi_2$ .

So, if we are adding then we are adding the rolling plane that is 101 and then the rolling direction is 01 bar 0 and so, you are seeing here this position we can observe the the Goss texture. Again we can add this position and the position of the Goss texture here is  $\phi_1$  90,  $\phi_1$  90 and  $\phi_2$  45 that is this one that is 90 90 and 45. So, this position and therefore, if that again the RD component is same 001 therefore, it's an RD fibre.

Now, you can see that this texture if you add the rolling plane direction that is ND it becomes 110 hkl and 001 uvw. So, you can see that by using one can observe various kinds of texture and in this case we have shown the example of the Goss texture component that can be observed from the trace of the hkl plane with respect to the Euler angles.

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**Conclusions**

- Texture components can be represented using Euler space and orientation distribution functions (ODFs).
- Crystal orientations  $\{hkl\}\{uvw\}$  are related to Euler angles  $\phi_1, \phi, \phi_2$  can be used to construct the Euler Space and thereby ODFs

So, from this lecture course, this sessions on Euler space and Euler angles, we can find out that texture components can be represented in terms of Euler space and orientation distribution function. The crystal orientations important orientation that is hkl uvw if it is in terms of rolling plane and rolling directions can be related to these Euler angles phi 1, phi, phi 2. And can be used to construct the Euler space and thereby the orientation distribution functions, but we should remember that the construction of the Euler space using this hkl and uvw, the specimen axis has to be rotated by phi 2, phi, phi 1 opposite to the right hand thumb rule.

Thank you very much.