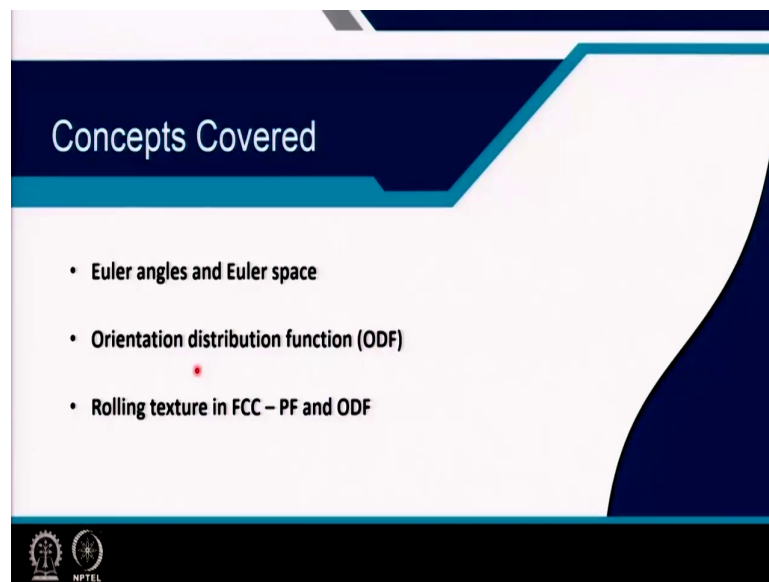


Texture in Materials
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Module - 03
Texture representation
Lecture - 13
Euler Angles and ODFs

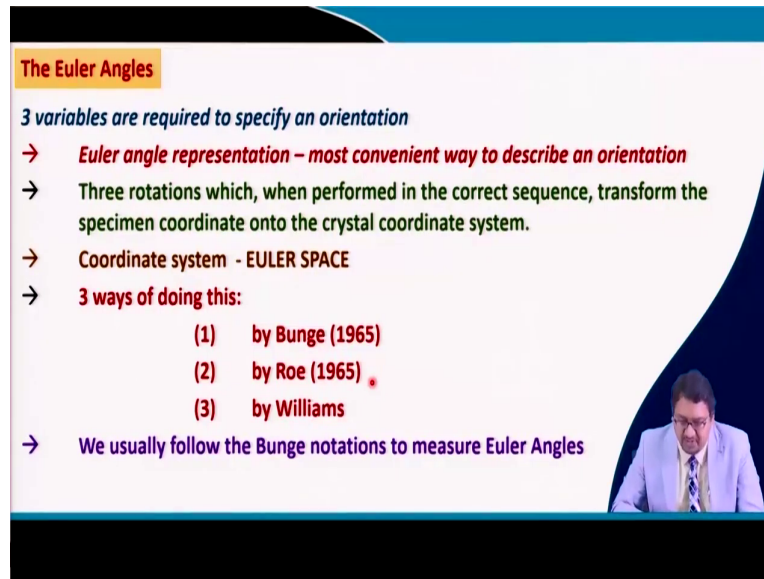
Good day everyone. This is a 13th lecture and in this lecture, we will talk about Euler Angles and orientation distribution functions, which is abbreviated as ODFs.

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The concepts that will be covered in this lecture are Euler angles and Euler space in detail, what is orientation distribution function or ODF and we will give you an example showing the rolling texture of face centred cubic material and relate the pole figures and the ODF and the Miller indices.

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The Euler Angles

3 variables are required to specify an orientation

- **Euler angle representation – most convenient way to describe an orientation**
- Three rotations which, when performed in the correct sequence, transform the specimen coordinate onto the crystal coordinate system.
- Coordinate system - EULER SPACE
- **3 ways of doing this:**
 - (1) by Bunge (1965)
 - (2) by Roe (1965)
 - (3) by Williams
- We usually follow the Bunge notations to measure Euler Angles

what is an Euler angles and we I have been telling this in the past lectures that the orientation location space can be related by 6 variables that is ϕ_1 , ϕ_2 and that ϕ_1 , ϕ_2 is the angular variables and x_1 , x_2 and x_3 are the positional variables. So, Euler angles are actually that ϕ_1 , ϕ_2 that is the 3 variables that are required to specify an orientation.

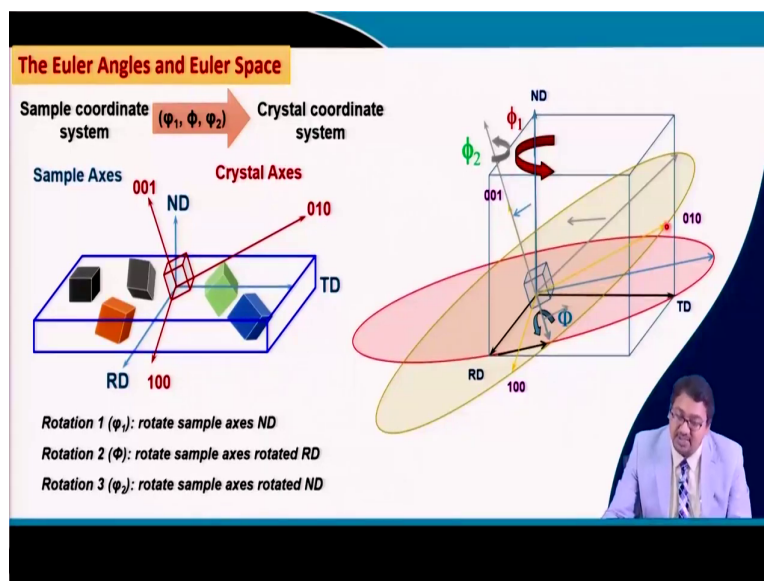
Now, why Euler angle is used? Because Euler angle representation gives you the information of the texture in 3 variables ϕ_1 , ϕ_2 instead of 9 variables of the orientation matrices and therefore, it is very convenient to way to describe the texture. Now, why it became very convenient? Because while a texture is measured, there are multiple points where from where the texture is obtained in a microstructure. Say for example, in the case of electron backscatter diffraction and a software is used to obtain the overall texture information. When texture was developed in the earlier times in 1965 and later during 1979 1975 and 80s. Then the computer programming or the computer hardware were not so equipped enough to handle big data and therefore, the Euler angle representation using 3 angles ϕ_1 , ϕ_2 become became more convenient way to describe the texture because instead of solving 9 different variables, only 3 variables were need to solve per pixel points in obtain the overall information of that texture using the software in a computer.

So, what is Euler angle, what are Euler angles? Euler angles are three rotations which when performed in the correct sequence, transforms the specimen coordinate system into the crystal coordinate system right. So, the coordinate system is basically known as the Euler space. So, the if ϕ_1 , ϕ_2 it is three angles so, this angles can be used to form an angular

coordinate system and can be a space can develop and this space is known as Euler space. Therefore, there are 3 ways of doing this, 1 suggested by Bunge in 1965, another was suggested by Roe and another by Williams. So, one can obtain this ϕ_1 , ϕ , ϕ_2 rotations differently right in order to come make the sample coordinate system equivalent to the crystal coordinate system that is the path by which the sample coordinate system travel angularly to get to the crystal coordinate system and that path is basically the texture and it can be done differently.

One suggested by one way was suggested by Bunge, another way was suggested by Roe and another by William independently and separately, but with time, most of the scientists started following the Bunge's notation of measuring the Euler angle, but it does not mean that the Roe's or the Williams way of measuring the Euler angles were not correct all were correct, but the popularity was gained by the Bunge's way of measuring the Euler angles.

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So, Euler angle or Euler space ϕ_1 , ϕ , ϕ_2 is a relationship between the sample coordinate system and the crystal coordinate system. Say for example, we have a rolled sample with important sample reference directions as I said we will take the rolled sample or the rolled material only as a reference material because most of the students are most more familiar with the rolling and the rolled material.

The important sample reference directions of the rolled material, which are RD, the rolling plane that is perpendicular to ND and the rolling direction, which is RD and TD right. If you

take a sample like this and we have many grains in it or many crystals in it and few crystals have this orientation, few crystals have these, these, these, these and, but even though they have say for example, this orientation, the grains would be spatially separate out right. Now that if this particular orientation is present in a large amount, then the intensity of the texture from this orientation will come highest. So, we will consider that this is the most intensified texture in the material.

If this is RD, this is TD and this is ND and if this crystal has the coordinate system 100, 010, 001 like this, then how to find out the rotation sequence proposed by Bunge that is the rotations sequence ϕ_1 , ϕ_2 , ϕ_3 in order to take this crystal coordinate into the sorry take this sample coordinate system into the crystal coordinate system. So, let us see this schematic which is made of from this, and it is little different, but the RD, TD and ND is similar and the 100, 010 and the 001 of the crystal is also in the similar position at this as this figure.

The Bunge's notation states that there are 3 rotations so, the rotation 1 is ϕ_1 and the sample is rotated along ND that is the normal direction right and then, after some time, a rotation 2 which is ϕ_2 where the sample is rotated along RD right and then, the sample is again rotated along ND and this is known as the 3rd rotation ϕ_3 and let us see what was the Bunge's sequence. Iwe look into the rolling plane of the sample, the rolling plane will contain RD and the TD, and the ND will be perpendicular to this rolling plane. So, here we have drawn the plane which contains the RD and TD in this red and pink color right and let us draw the plane which contains the 100 and the 010-crystal system that is the plane 001 plane where the 001 is basically perpendicular to this plane and this is that plane which contains the 100 and 010.

So, if we rotate ND, and we will use the right-hand thumb rule to rotate the ND, what happens that the RD will start to move in this direction and the TD will also start to move in the this direction. So, how much we will move RD? that the RD will move up to such a point where the two planes basically meet.

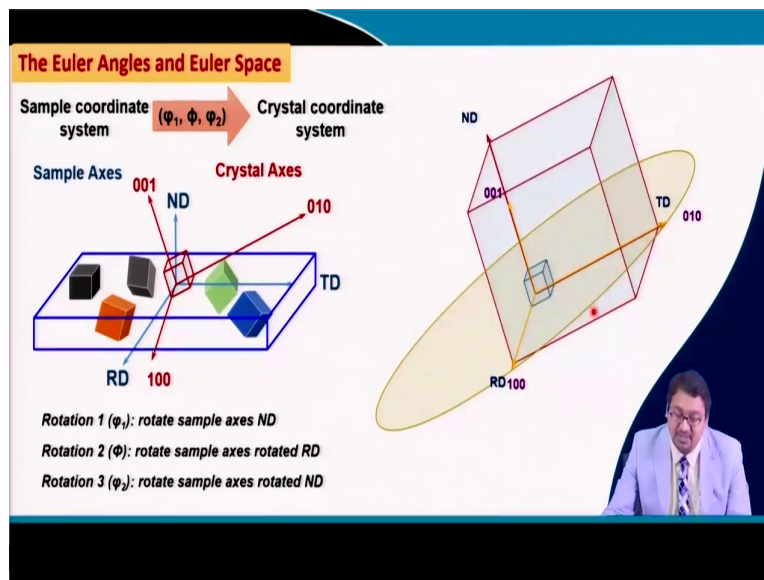
Now, what is that plane? there this is one plane which is the rolling plane and is it contains the RD and the TD on the other hand, this is another plane as I said which contains the 100 and the 010 that is the 001 plane. Now, there is one line; there is one line where both of this plane meets, and this line passes from the origin and it goes somewhere like this right. If this rotation ϕ_1 is given, then the ϕ_1 is given in such a way that the RD rotates by ϕ_1 to go and meet that intersection line between these two planes right and the TD will automatically loop move or rotate maintaining the angular relationship between the RD and the TD.

Therefore, this is the 1st rotation where we need the RD to be in the plane in the; in this plane and also in this plane right.

So, now, that once RD is rotated, the 2nd rotation phi is along this new RD right and when we rotate this new RD by phi, the rotation has to be such that the ND will now move to 001 and how it is possible? Because the RD now lies in this plane that is the 001 plane too so, once the phi is rotated by the right-hand thumb rule, what will happen? The ND will move and go to 001 and that is the phi rotation where the TD will also move up and meet this plane right.

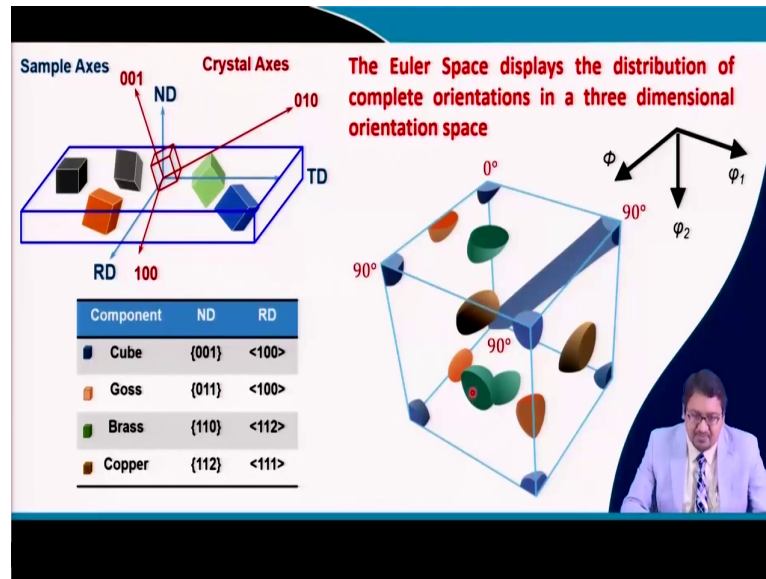
So, here is the rotation and the new TD and the new ND right whereas, the RD remain constant because the rotation took place along the RD. Now, we give the phi 2 rotation which is again using the right-hand thumb rule and is given on the ND. So, as because this is the new ND, the rotation is given and this is the right-hand thumb rule and that what happens that the now, the TD sorry the RD and the TD is now rotated so that it meets the 100 and the 010 and so that the sample coordinate system and the crystal coordinate system becomes the same.

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So this path of travelling, angular path of travelling for the sample coordinate system with respect to the crystal coordinate system that is phi 1, phi, phi 2 basically gives the information of the orientation of the material and this rotation is basically known as the Euler angle and the space that is made by this rotation that is phi 1, phi, phi 2 space is known as the Euler space.

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So, the Euler angles can be used to produce an Euler space that is I have shown a schematic of an Euler space with ϕ_1 , ϕ and ϕ_2 like this. So, it is a space like this with as it is a cubic; cubic crystal and we are working on the cubic crystal for demonstration this time. What we have done is that using the symmetry of the cubic crystal that is 100 has a fourfold symmetry so, after each 90-degree rotation; it should show the same crystal as before the rotation.

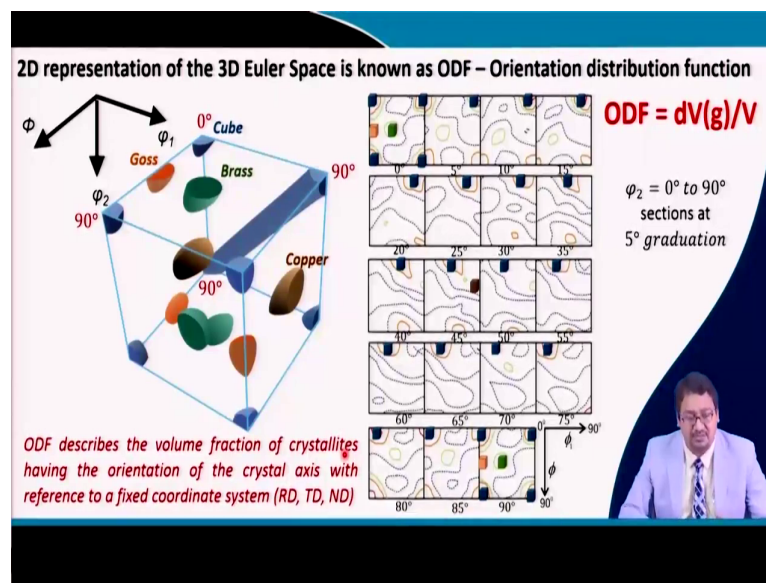
We have taken a Euler space from 0 degree to 90 degree and 0 degree to for the ϕ_1 and 0 degree to 90 degree for ϕ and 0 degree to 90 degree for the ϕ_2 , but that before we will have to discuss this symmetry relationship in terms of crystal symmetry and sample symmetry. And that we will do in the later course of this course, but just to let that the symmetry of the cubic unit cell leads to reduce the size of the Euler space from 0 to 360 degree for ϕ_1 , ϕ and ϕ_2 to 0 to 90 degree at this time. The Euler space, which this means which can be seen like this displays the distribution of the complete orientation in a three-dimensional orientation space right. Say we have shown this with an example for an face centred cubic material which is rolled. that a face centred cubic material which is rolled or in general terms show rolled and recrystallized texture shows a copper texture, a brass texture, a goss texture and cube texture. There are other texture also like the s component is shown, but for this example, I have shown you only this four texture.

So the position of the cube texture is shown by the blue. So, it forms here that is at ϕ_1 , ϕ , ϕ_2 that is 0-degree, 0 degree and 0 degree and it forms here that is at ϕ_1 equal to

0-degree, phi equal to 90 degree and phi 2 equal to 0 degree. Like that it forms here, here, here, here, here and it it forms like a fibre in this position and we look into this in the later lecture that why this kind of fibre is being developed.

Now, if we look at the goss texture, which is given by this orangish colour, and it forms here, it forms here and it forms here. So, that the same texture forms at various position right because of the symmetry of the crystal so, the cube which is 001, 11; 100 forms at the eight positions or eight corners and it also forms at the fibre diagonally at at phi equal to 0 section right phi equal to 0 section. Now, goss which is 011, 100 forms at three different positions. brass which is shown in the bottle green colour also forms at three different positions. On the other hand, copper also forms here and here. But while demonstrating this texture components using a Euler space, visualizing this and the position the angular position of each of this cube, goss, brass and copper component becomes extremely difficult right.

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So, what we do is that we take this Euler space, and we represent this 3D Euler space into a 2D representation and this 2D representation is known as ODF or orientation distribution function. So, what we do is we take this Euler space and most of the time, we slice it like a bread along phi 2 so, we slice it for example, we show this phi 2 equal to 0 section and then, we go below and show the phi 2 equal to 5 degree section and then, 10 degree section and then, 15 like that we go and show the up to the 90 degree section, which gives you a 2D representation of this 3S Euler space in form of an orientation distribution function.

This orientation distribution function basically describes the volume fraction of the crystallites having the orientation of the crystal axis with reference to the fixed coordinate system RD, ND and TD then that is what the definition of the texture is right. If we take the first section at ϕ_2 equal to 0, we obtain this kind of a ODF section which has ϕ_2 equal to 0 and it goes from 0 to 90 degree ϕ_1 and 0 to 90 degree along ϕ right and then; we can take another section which is just below this section and then, we can say that ok this is a 5 degree section.

So, we are taking 5 degree and then, we are taking at 10 degrees and 15 degrees and 20 degrees and like that we take all the sections up to 90 degrees and thereby, we obtain this same information in 2D. Now, that it is easier to perceive this information than to perceive the information that is present in 3D. Therefore, in ODF, we observe the volume fraction of a particular orientation g with respect to the volume fraction of the whole orientation right. So that it gives the intensity in terms of volume fraction of a particular orientation present.

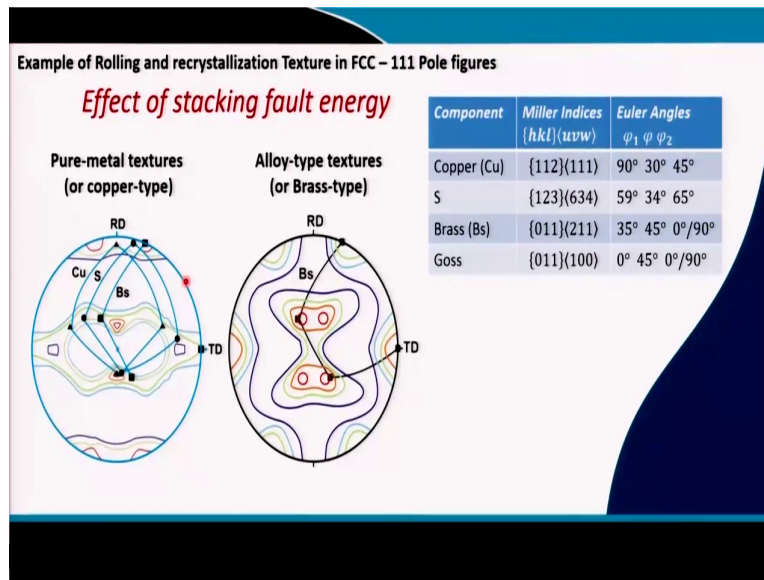
Now, let us see how the cube, goss, brass and the copper texture in this ODF looks like and how we can compare this with this three-dimensional Euler space. the cube texture forms basically here right and so, it is at ϕ_1 equal to 0, ϕ equal to 0 and ϕ_2 equal to 0 right. On the other hand, at ϕ_2 equal to 0 section, the cube texture forms here, here, here and here so, we obtain the cube texture here right.

On the other hand, as because of the symmetry at 90 degrees similarly, the cube texture may develop at the four corners which develops and can be observed from this 3D Euler space, we can see that this cube texture can form and I have shown by the blue coloured square boxes. Now, that like that we can observe that as the ϕ_2 sections are increasing from ϕ_2 equal to 5, 10, 15 to 90 degree, this fibre which forms because of the cube orientation always shows itself as component at with increasing ϕ_2 at various positions. At ϕ_1 equal to 85 degree and 80 degrees and 75 degrees like that, and it goes here, and it becomes 45 degrees here and it goes here and it becomes 85 degree and then becomes 90 degree so that fibre could be observed throughout the whole ODF ϕ_2 equal to ϕ_2 sections right.

Now, on the other hand, goss texture can be observed at ϕ_2 equal to 0 and ϕ_2 equal to 90-degree section and then, brass texture can be observed at ϕ_2 equal to 0 and 90 degree sections and it is also present here which I have not shown right here, this this brass texture right. We can observe the the copper components and which is present here and so, that one can distinguish more easily the various texture components or fibres that are present in the

material using the ODF rather than using the Euler space right. Therefore, because visualizing a 2D picture is much easier for us to perceive than the 3D so, we use the 2D ODF's to observe the texture not the Euler space most of the time.

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Therefore, when we talk about texture, and we mostly are concentrating on cubic materials and we are concentrating on rolled material because there are lots of research on it and many publications on it and we can relate there is a whole bunch of literature of texture relating the rolled material. We give an example of rolling texture of face centred cubic material and here are the 111 pole figures and we see that the texture that forms for the pure material that is the copper type texture is something like that and for the brass type texture, it looks something like that so, there is a subtle difference and then, we see the copper components and the S component and the brass components where distinguished in them.

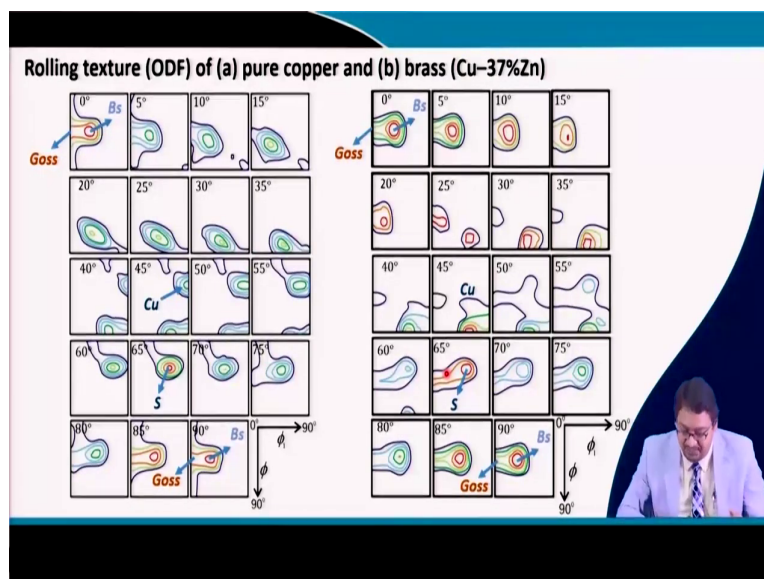
And that this the difference between the orientation, the difference between the texture developed for the copper type and the brass type texture is because of the effect of stacking fault energy and we have discussed a little about this before, but as that high stacking fault energy material during the deformation has the facility of cross slipping of screw dislocation and the climb of edge dislocation leading to easy dynamic recovery and thereby form this kind of texture. Whereas, in case of low stacking fault energy material like single phase alpha brass, they deform and where the distance between the partials 112 partials are so large that it cannot the screw dislocation cannot cross slip to another plane because the partial cannot come together to form a full dislocation and therefore, it cannot cross slip.

And therefore, the climb of the edge dislocation is also restricted and that that leads to; that is leads to restriction in the dynamic recovery processes and thereby the deformation is restricted and then twinning deformation, twinning's also developed and so, the texture basically changes as the stacking fault energy of the material changes and this is known as texture transition.

And so, that the copper type, S type and brass type texture that are shown in the pole figure could be related to the Miller indices and we have seen that each texture component present in the pole figure or the inverse pole figure could be related to the Miller indices and then, these texture components can also be shown in the Euler angles because they are related. The position of the copper texture in terms of Euler angles say it is 90-degree, 30 degree and 45 degrees phi 1, phi, phi 2. For the S, it is 59, 34 and 65 degrees for the phi 1, phi, phi 2. Brass forms at 35, 45 and 0 and 90 degrees at phi 2. Goss forms at 0, 45 and 0 and 90 degrees at phi 1, phi, phi 2 right. So, these are the information of the experimental texture that develops in the face centred cubic material during rolling process.

Now, these texture could be represented using the Euler space and this structure is more easily represented using the orientation distribution function and here, we in the next slide I am showing the texture, the ODF's that is phi 2 equal to 0 to 90 degree section for both the high stacking fault energy material that is copper type texture and low stacking fault energy material that is the brass type texture in terms of phi 2 equal to 0 to 90 degree ODF sections and here it is.

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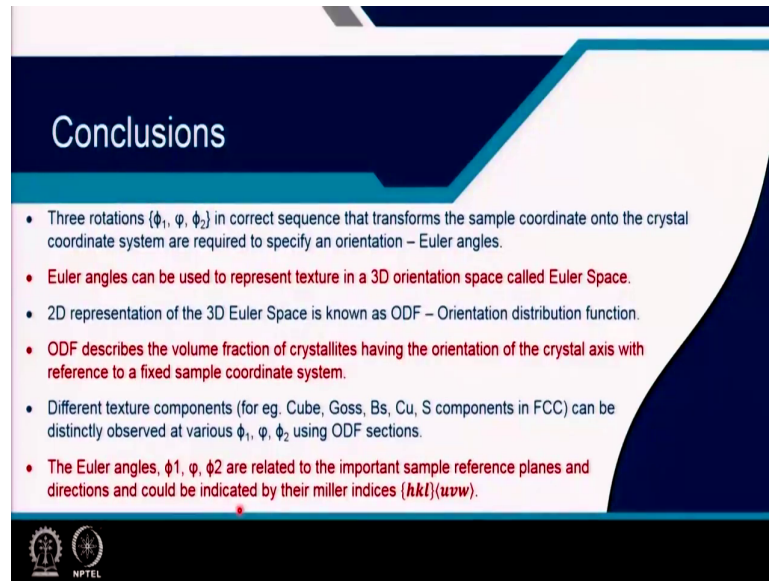


that the components that is this is for the high stacking fault energy material that is pure copper texture. One can observe the formation of the goss, the formation of the brass texture, the goss is weaker as compared to the brass. Now, the S comp, the copper component has formed, the copper component is the strongest here and this has been shown so, the copper is the most strongest and then, the S component is present. The S component is also present stronger here so, the copper and the S component is stronger, and the brass is weaker and the goss is more weaker right.

On the other hand, that in case of the low stacking fault energy material, what has happened that there is a substantial amount of brass component present in it. So, the brass component is very stronger and the goss is a very weak and then, that the copper texture has not even formed properly, and the S component is there, but it is weaker as compared to the high stacking fault energy materials texture. So, that the 90 degree section, the 90 degree ϕ 2 section and the 0 degree ϕ 2 section shows exactly identical type of texture and because of the symmetry of the cubic crystal. So, you we can observe the sub the subtle difference between the texture developed after rolling of a high stacking fault energy face centred cubic material and a low stacking fault energy face centred cubic material using the ODF sections and this could be related to the pole figure and the Miller indices because they are mathematically interrelated and they are different tools to represent the texture.


And you can see that most of the time, we use ϕ 2 sections and we divide the sections at 5 degree graduations, sometimes at 10 degree graduations or sometimes we show the most important ϕ 2 sections it does not mean that we cannot show the texture using Euler angles in other sections, there are relations which can be observed when we observe the texture in ϕ sections or ϕ 1 sections too.

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Conclusions

- Three rotations (ϕ_1, ϕ_2, ϕ_3) in correct sequence that transforms the sample coordinate onto the crystal coordinate system are required to specify an orientation – Euler angles.
- Euler angles can be used to represent texture in a 3D orientation space called Euler Space.
- 2D representation of the 3D Euler Space is known as ODF – Orientation distribution function.
- ODF describes the volume fraction of crystallites having the orientation of the crystal axis with reference to a fixed sample coordinate system.
- Different texture components (for eg. Cube, Goss, Bs, Cu, S components in FCC) can be distinctly observed at various ϕ_1, ϕ_2, ϕ_3 using ODF sections.
- The Euler angles, ϕ_1, ϕ_2, ϕ_3 are related to the important sample reference planes and directions and could be indicated by their miller indices $(hkl)\langle uvw \rangle$.

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So, that is all for this lecture and we can conclude that three rotations ϕ_1, ϕ_2, ϕ_3 if done in correct sequence can transform the sample coordinate into the crystal coordinate which is required to specify the orientation, and this is known as Euler angles. Bunge, Roe and Williams all did this using different rotations, but all of them are right, but the Euler's angle proposed by Bunge has become the most popular angle.

Euler angles can be used to represent texture in three-dimensional orientation space and is known as Euler space. The 2D representation of the 3D Euler space is known as orientation distribution function that is the ODF. The ODF describes the volume fraction of the crystallites having orientation of the crystal axis with reference to the fixed sample coordinate system. Different texture components for example, cube, goss, brass, copper, S components this is an example for FCC, rolled FCC material can be distinctly observed at various ϕ_1, ϕ_2, ϕ_3 positions of these ODF sections. The Euler angles ϕ_1, ϕ_2, ϕ_3 are related to the important sample reference planes and the direction and could be indicated by the Miller indices hkl, uvw .

Thank you.