

Non - Metallic Materials
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Module - 11
Corrosion and degradation of non - metallic materials
Lecture - 58
Design of Ceramics

Welcome to my course Non Metallic Materials and today we are in module number 11; Corrosion and degradation of nonmetallic materials and this is lecture number 58, where I will be describing Designing of Ceramics.

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So, first we will talk about the requirement of reliability of ceramic parts; ceramic material and why they are significant. And then what are the different approaches to design the ceramic article and why they are important. And we will be describing the probabilistic design and two parameter Weibull distribution that will be introduced to estimate the so called design stress; so, that will be covered in this lecture.

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Reliability requirement

Reliability requirement depends on applications

- Heat shield tiles on the space shuttle vs household floor or wall tile
- Glass windows on a deep sea submarine and glass window in house. Following factors must be considered

1. The acceptable failure rate for the application
2. Type of warranty for the system and its subcomponents
3. Expectations of the potential customer
4. Safety requirements defined by industry or government regulations

Design of ceramics

Left: Material strength does not interfere with the design stress required by the applications

Right: One would question the reliability of this material

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So, the term reliability that depends on a particular application; so that is more important. So, if you consider a heat shield type which is used in space shuttle and compare it with a normal household floor or wall tile; then the reliability requirement for the space shuttle tile is much more as compared to the household tile.

Similarly, if you are considering a glass window of a submarine and glass window of your home or if your car, the reliability requirement for the former is much more as compared to the automobile window or window pane in your house. So, here you can see the left graph; you can see the material strength of any material, it may be metallic, it may be nonmetallic material.

So, this material strength here you can see it peaks at a frequency level. So, number of occurrence maybe it is 50; 50 of this sample can withstand this particular strength. And there are less number of a sample they can withstand the lower strength than the peak value. And higher strength also they are exhibited by again limited number of materials, but this kind of distribution this is still ok, if you consider that your application require a design stress in this region.

So, as you can see that this stress level is much lower as compared to any on; any of this stress level. So, peak stress level or slightly lower; slightly higher, it is far apart from the design stress requirement. So, we call that we have a margin of safety here and this is perfect for the use, this material can be used for this particular application.

But, if you consider that this curve; here you can see that there is some kind of overlapping; so the material strength here; although this is larger as compared to the design stress, but the fact that some of the material fails in this region which is in your design stress window.

So, this reliability is not acceptable at least; it is not as good as this type of materials. So, this is just an illustration that what exactly is meant by the term reliability. So, the acceptable failure rate for a particular application that is important; that how much how many samples I can accept that it will if even if it fails it is ok for me.

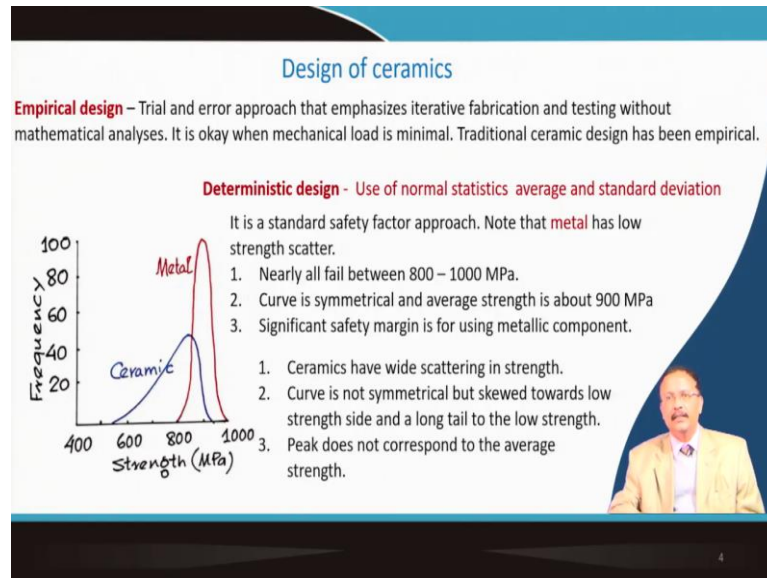
Sometimes, it is very tight not a single sample you would like to fail sometimes; it is not that tight for example, pavers' tiles or your floor tiles or wall tile. So, once in a while if it breaks; it is ok with you.

Type of warranty of the system and its sub component; so, if it is a very high valued product and you have warranty on it; then the reliability is a major issue that you would like that all your material will sustain; I mean they will not fail so that the warranty is valid. Expectation of the potential customer; there are some brand value that is there.

So, Johnson tile or Kajarias tile; they have a brand value, so you purchase it and it is reliable; all the tiles, it will not get fracture is a unbranded product once in a while they can be cracked or fractured. So, this expectation of the customer is also important and that is also somewhere related to the reliability issue.

And finally, the safety requirement that is either defined by the industry; so, they usually follow a very stringent safety requirement or there are certain government regulations are there that you will have to maintain this kind of you will have to maintain that. So, that is reliability is somewhere related to that as well.

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Now, in case of the ceramic; you know there is a brittle material, so traditionally the ceramic article like a flower vase or a glass or tumbler or teapot. So, traditionally it is a trial and error approach that has been adopted to actually design this kind of part. So, mathematical analysis that was (Refer Time: 06:59) for this type of application because the mechanical load, they are not load bearing structure.

So, once in a while if it is broken; it is ok, but you formulate a particular composition, heating schedule and just blindly follow it and maybe 90 percent yield you will get and those material will survive. So, this total process is based on an empirical design which is some somewhere related to the processing aspect.

Next, there was a deterministic design; so there at least some statistical terms are used. So, for example, average strength or the standard deviation; those kind of statistical term is included here. So, if you compare between a metal part and a ceramic part; in case of a metal part, this kind of variation in the strength they are very very limited.

You see that; between 800 to 1000 with a symmetric kind of plot you will be getting and maximum strength of the samples will be around 900; so, you can safely average it out. So, average strength will be about 900 mega Pascal.

And if your application is something related to 700 mega Pascal, so you have a wide margin of reliability; so, metal part does not have any problem because of this very

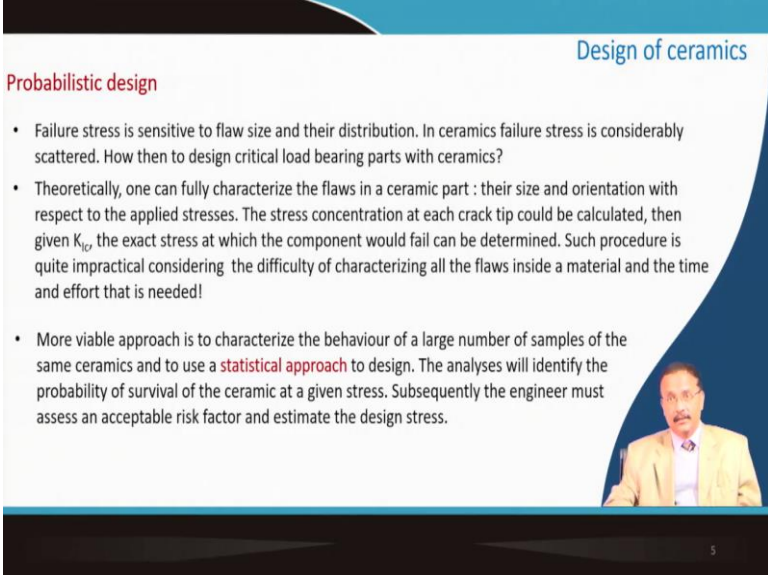
shallow; sorry very narrow kind of a strength distribution across a wide number of some; I mean large number of samples.

But if you consider any ceramic part; it has a wide scattering in strength and that is related to the microstructure. You know that already I talked about when I talked about the mechanical property of the ceramics due to the presence of crack and void and their distribution, the microstructure plays a major role in controlling their strength.

So, usually they have a wide scattering of the strength. So, it can be any way in between 500 mega Pascal up to 900 mega Pascal and see the curve is totally skewed here right. So, here it is very difficult for you to identify the design stress. So, the design stress will be much much less; if you have a skewed kind of pattern and this is also you having a long tale; so it can extend further 2 and 300 something like that.

So, for practical application; this is a real problem, this is a real problem to use any of the centered ceramic material. So, this deterministic approach to design just by going for the average or by the standard deviation which is related to the strength minus average square divided by number of application; so number of occurrence. So, if you do the standard deviation or average; this simply does not work for the ceramic article.

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The slide is titled "Design of ceramics" in the top right corner. Below the title, the section "Probabilistic design" is highlighted in red. It contains three bullet points discussing the challenges of deterministic design for ceramics and the benefits of a statistical approach. A small inset image of a man in a yellow jacket is visible in the bottom right corner of the slide.

Design of ceramics

Probabilistic design

- Failure stress is sensitive to flaw size and their distribution. In ceramics failure stress is considerably scattered. How then to design critical load bearing parts with ceramics?
- Theoretically, one can fully characterize the flaws in a ceramic part : their size and orientation with respect to the applied stresses. The stress concentration at each crack tip could be calculated, then given K_{Ic} , the exact stress at which the component would fail can be determined. Such procedure is quite impractical considering the difficulty of characterizing all the flaws inside a material and the time and effort that is needed!
- More viable approach is to characterize the behaviour of a large number of samples of the same ceramics and to use a **statistical approach** to design. The analyses will identify the probability of survival of the ceramic at a given stress. Subsequently the engineer must assess an acceptable risk factor and estimate the design stress.

So, for this kind of things; some kind of probabilistic design is required. So, failure stress is sensitive to the flow size and their distribution. So, in ceramic failure stress is

considerably scattered exactly I have shown it. So, how to design the critical load bearing structure using ceramic material; how you can design it. So, it is possible at least theoretically to characterize the flaw size in the ceramic part. In each individual ceramic; what is their flaw size, what is their orientation with respect to the applied stress.

The stress concentration at each crack tip can be calculated; then you know the fracture toughness, you can measure the fracture toughness; the extract exact stress at which the component would fail, this can be in-principled determined. But this is an impractical process because it is very difficult for you to characterize each individual flaw inside the material. So, the time and afford that is required to examine for each individual, material that is stupendous; so it is not possible.

So, the more viable approach to characterize the behavior over a large number of sample is actually a statistical approach and it is a special type of statistical approach. And this analysis they eventually will determine the probability of survival of a ceramic at a given stress. So, you know that what is the stress that is required for your application and you will have to identify; what is the probability of failure of your material.

So, subsequently the engineer; he will assess that whether this kind of design stress is good for his application and then accordingly he will decide that whether the particular ceramic is often used for practical purpose.

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Weibull distribution

- The strength distribution of a ceramic can be defined by two parameter semi-empirical distribution given by $f(x) = m(x)^{m-1} \exp(-x^m)$
Where $f(x)$ is the frequency distribution of the random variable x and m is a shape factor, known as Weibull modulus.
- Plotting $f(x)$ vs x yields a bell shaped curve. The width of the curve depends on m , if m is large the distribution is narrow.
- Let us now define the random variable $(x) = \sigma/\sigma_0$
Where σ is the failure stress and σ_0 is a normalizing parameter required to make x dimensionless. The physical significance of σ_0 will be illustrated later

Design of ceramics

As m increases the distribution narrows

Truncation of Weibull distribution as a result of proof testing.

So, a two parameter Weibull kind of distribution that is proved to be quite helpful; so the strength of the distribution of ceramic, that can be defined by this two parameter semi empirical distribution function. So, the function is f ; x and m is one parameter and another parameter is this variable x right. So, if you plot this as a function of x ; then usually a bell shaped curve like this it will appear.

So, I just explain that in case of metal; there is a narrow distribution of the failures stress and in case of ceramic, there is a broad distribution. So, if it is a very broad distribution that is reflected to the modulus; so modulus will be low. If it is a narrower distribution, then the modulus will be quite large and do not confuse it that the modulus is having something to do with the strength; it is just giving you the distribution of the failure stress.

So, this m is termed as this is a shape factor related to the shape of this plot and that is termed as modulus Weibull modulus. So, this x value which is the variable in order to make it dimensionless, I can put it sigma by sigma 0. So, that will make this axis dimensionless and then there is a significance of this sigma 0; so that I will explain later.

And the type of the distribution for ceramic it is quite broad in nature and in fact, I can do a specific test at a particular stress level; whether the sample survives or it does not survive. So, I will come back to this failure analysis towards the end of the lecture to explain this plot; what is meant by this red region.

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Weibull distribution

Design of ceramics

Replacing x , the random variable with σ/σ_0 the **survival probability** (= fraction of the ceramic sample that would survive a given stress level) is given by

$$S = \int_{\sigma/\sigma_0}^{\infty} f(\sigma/\sigma_0) d(\sigma/\sigma_0)$$

$$S = \exp[-(\sigma/\sigma_0)^m] \text{ (Note } f(x) \text{ defined earlier is an interesting expression for such integration)}$$

σ_0 is the stress level at which the survival probability is equal to $1/e \approx 0.37$


$$1/S = \exp(\sigma/\sigma_0)^m$$

$$\ln(1/S) = (\sigma/\sigma_0)^m$$

$$\ln[\ln(1/S)] = m \ln(\sigma/\sigma_0)$$

$$-\ln[\ln(1/S)] = m \ln\sigma_0 - m \ln\sigma$$

Once ' m ' and σ_0 are known from a set of experiment results, the survival probability at any stress can be estimated.



Now, as I said the random variable; if you replace with σ by σ_0 , then the survival probability that is defined by the fraction of the ceramic sample that would survive a given stress level; that is given by this relation. So, we integrate from this stress level to infinity; the function of x and x is replaced by σ by σ_0 ; $d x$.

So, if you consider the earlier relation; it is quite interesting relation and if you just integrate that; it is nothing, but exponential of minus σ by σ_0 to the power m . So, you can just work it out; you will get that after this integration, you will get the survival probability by this right.

Now, you can define σ_0 ; so σ if it is equal to σ_0 , then this probability is $1/e$; that is roughly 37 percent. So, σ_0 is a stress level where about 37 percent of the sample, whatever you are testing that will survive. So, now, we will do some algebraic relationship. So, just inverse this thing first $1/S$, then take a log \ln in both the sides; so this exponential will go.

Then take another log; so \ln of \ln of $1/S$ that is given by m into \ln of σ by σ_0 . Then, you take this minus term here; minus \ln of \ln of $1/S$; so that is given as this was negative. So, it was it will become positive because you are multiplying by minus; so this minus m ; $\ln \sigma$.


So, once this m and σ_0 are known from a set of experiment results; eventually you can put it back here and then you can calculate the survival probability at any stress; any stress level you can estimate it. So, this is the theoretical foundation of two parameter Weibull distribution.

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The strengths of 10 nominally identical ceramic bars were measured and found to be 387, 350, 300, 420, 400, 367, 410, 340, 345, and 310 MPa. (a) determine 'm' and σ_0 for this material; (b) calculate the **design stress** that would ensure a survival probability higher than 0.999.

Rank j	$S_j = [1-j/(N+1)]$	σ_j	$\ln(\sigma_j)$	$-\ln \ln(1/S_j)$
1	0.932	300	5.700	2.6532
2	0.837	310	5.734	1.7260
3	0.740	340	5.823	1.200
4	0.644	345	5.840	0.8200
5	0.548	350	5.860	0.5080
6	0.452	367	5.905	0.2310
7	0.356	387	5.960	-0.0320
8	0.260	400	5.990	-0.2980
9	0.160	410	6.016	-0.6060
10	0.070	420	6.040	-0.9780

$S_j = [1 - (j - 0.3)/(N + 0.4)]$
more accurate probability estimation



And I will site one example to illustrate this phenomena in a better way. So, for example, you have made 10 identical ceramic bar; it could be silicon nitride bar it could be silicon carbide or any other ceramic which is prepared by conventional ceramic processing root. And you estimate the value of the fracture stress for all these samples like this; 387, 350, 300, 420; down to 310 mega Pascal.

So, you can see that there is a scattering and you will have to have a design stress so that you can tell customer that say 99 percent survival probability will be having this stress value. So, if you use below the stress value; then 99 percent case; my material will not fail; so that is the design stress concept. So, there is a procedure to identify that how you can calculate the design stress. So, the probability you are fixing 99.9; so you will have to calculate the design stress.

So, the first measurement you get this number of value; so you will have to calculate the Weibull modulus and you will have to calculate the sigma 0 value; for this particular material. And then finally, you will have to calculate the design stress that would ensure a survival probability which may be higher than 99.9 percent.

So, first I will have to rank this stress; the low value of the stress that will come first and the higher value of the stress, that will come last and here I have taken 10 samples. So, I have organized this stress value 300, 310 up to 420.

Then, I can calculate the probability by this simple relation where this number of observation here is 10. So, the first case; it will be 1 minus 1 divided by 10 plus 1; so, that will give me this value. If you want to calculate this type of probability in a better way, then you can use this relation where it is slightly modified; instead of j, it is j minus 0.3 and instead of simple N plus 1; it is N plus 0.4.

So, this value you can calculate; you can calculate yourself. Sigma j; I have already rank, it this is from my experiment and I will estimate ln of sigma j and also the minus ln of ln of 1 by S j. So, S j is calculated from here; so, I will also estimate this; this value progressively for all the samples. So, I have gotten the y axis of my Weibull distribution plot and the x axis right. So, as I have already described eventually that should give me a straight line.

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Design stress estimation

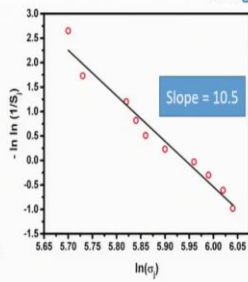
$$0.999 = \exp \left[- \left(\frac{\sigma}{387} \right)^{10.5} \right]$$

$$\sigma_{\text{design}} = 200 \text{ MPa}$$

Weibull plots for design purpose has to be handled with care:

1. As with all extrapolations, a small uncertainty in the slope can result in large uncertainties. Hence $N > 100$ is preferable.
2. Single flaw size that does not change with time, defects are randomly distributed and are small relative to the sample size.
3. Uniformity of microstructure (including flaw size, grain size and inclusions) are critical for large m
4. Toughening of a solid may not be reflected in Weibull modulus.

Design of ceramics



The graph shows a series of data points forming a straight line with a slope of 10.5. The y-axis is labeled $-\ln \ln(t/S)$ and ranges from -1.5 to 3.0. The x-axis is labeled $\ln(\sigma)$ and ranges from 5.65 to 6.05.

So, I have just plotted it. So, you know that these are all experimental point; so you will have to basically linear (Refer Time: 21:15). So, number of points most; large number of points we are having, it is good for you because you can see the value of N that depends on the slope; here it is 10.5. So, slight change in the slope that will change the value of the survival probability; so one should be very cautious.

So, number 10; 10 numbers of sample may not be sufficient, you may have to do the experiment for more than 100 numbers of cycles. So, once you get the value of; value of N and from the intercept, you get the value of sigma 0; then this sigma 0 is known and N

is known. So, here survival probability; let us assume that I want 99.9 percent of the sample will not fail at this stress level. So, I can just put it in this equation; here your σ_0 is estimated from the intercept is 387 and from this linearity; linear plot m value is 10.5. So, my design stress is coming about 200 mega Pascal.

So, out of the data that whatever you have gotten; you can safely take this 200 mega Pascal, as your design stress with a probability of 99 percent of the sample will not fail below this stress limit. So, that is the beauty of this Weibull distribution function. Now, you will have to use this Weibull plot with care; for any extrapolation, as I said small uncertainty in slope can result a big difference in the design stress; so, number of samples should be much larger.

Single flaw size that should not change with time; so defects are randomly distributed and they are relatively small, as compared to the sample size. So, centering is important; that is why I spend so much of time to make you understand about the centering. The defects like (Refer Time: 23:30) square, void, inside the material, their distribution; they are very important for their strength and for the reliability of the ceramic material.

So, microstructure should be uniform including the flaw size, grain size, inclusion; they are all critical for larger value of m . You can work with tough end ceramic material, but Weibull modulus; if it is a tough end material, maybe you can expect the value of m ; it will be sharper, but that is not really guaranteed that all the time you will be getting the higher value of m .

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Design of ceramics

Effect of size and test geometry on strength

- Larger specimens will have a higher probability of containing a larger defect and as a result its strength will be low. 'm' should not be confused with strength. Weak solid may have high m value. A solid with large defects that are all identical in size would be weak, but it would exhibit large m.
- If the strength is controlled by defects randomly distributed within the volume then the strength becomes a function of volume, with S decreasing with increasing volume. If strength is controlled by surface defects S will scale with area instead.
- In proof testing, a component is loaded to a stress level which is higher than the service stress. It eliminates the weak samples. Truncating the abnormal stress values establishes a reasonable scattering to obtain meaningful m and σ_0 to estimate the design stress.

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So, size and test geometry on strength is another factor; usually we do it for a small sample. But if it is a large sample that; whether it is correctly reflected, usually it does because of the uniformity if it is there; then of course, it will reflect for large sample also you get the same kind of reliability.

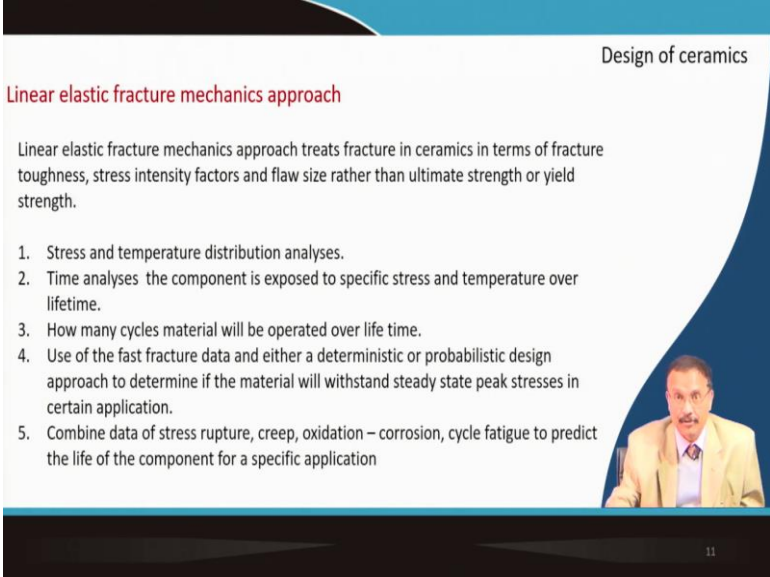
But the volume factor always remain and this is very peculiar that as I told that m should not be confused with the strength; weak solid they may have high value of m. A solid with large defects that are all identical in size they will be weak, but it would exhibit actually large value of m.

So, all the material will fail in a very short range; short range of fracture stress value. So, the m value can be large, but your sample may be weak. So, if the strength is controlled by defects which are randomly distributed within the volume; then the strength becomes a function of the volume. So, with S decreasing with increasing volume; if the strength is controlled by surface, then the survival probability will scale with the area instead.

So, as I mentioned in the very first slide that we do really a proof testing; a component is load to a stress level which is higher than the service stress value. And if the sample readily fails, then we usually discard it and if a sample exhibits exorbitant fracture stress value; that also get eliminated so that you know that the scattering if it is there; the way I have shown it.

The scattering also is having a limit; if suddenly a material fails at a very low level of stress maybe that particular sample had not being centered well or something is showing a very extra ordinary strength value; fracture stress value. So, usually that should be removed in order to get a reliable design stress value. So, that is done for the analysis point of view.

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Design of ceramics

Linear elastic fracture mechanics approach

Linear elastic fracture mechanics approach treats fracture in ceramics in terms of fracture toughness, stress intensity factors and flaw size rather than ultimate strength or yield strength.

1. Stress and temperature distribution analyses.
2. Time analyses the component is exposed to specific stress and temperature over lifetime.
3. How many cycles material will be operated over life time.
4. Use of the fast fracture data and either a deterministic or probabilistic design approach to determine if the material will withstand steady state peak stresses in certain application.
5. Combine data of stress rupture, creep, oxidation – corrosion, cycle fatigue to predict the life of the component for a specific application

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The final thing is what is done is related to mechanical engineering and this is a linear elastic fracture mechanics approach. So, linear elastic fracture mechanics treats the fracture in ceramics in terms of fracture toughness, in terms of stress intensity factor; in terms of the flaw size, rather than the ultimate strength which so far we were talking about.

So, several analysis is done and it is a computational kind of work; that is usually needed to have a much better idea rather than this two parameter Weibull distribution and stuff like that. So, stress and temperature distribution analysis is done; time analysis of the component is exposed to a specific stress and temperature over their life time.

Then, it is also considered that how many cycles the material will be operated over the life time. So, use of the first fracture data; either the way I determine the deterministic data all probabilistic design approach to determine if the material will withstand the steady state peak stress in certain application; that is also utilized in this kind of analysis which is much global in nature.

And combine data of stress rupture, creep oxidation, corrosion, cyclic failure to predict the life time of the component for a specific application that is also done.

So, these are all done in a fracture mechanics approach which is beyond the scope of this lecture. But this is a separate field altogether which make use of finite element analysis and this is a reasonably good approach to tackle the reliability issue of the ceramic material, in particular where the fracture strength value is quite scattered.

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REFERENCES

- **David W Richerson** Modern Ceramic Engineering: Properties, Processing, and Use in Design Marcel Dekker Inc, Chapter – 12 Final Machining page 596 – 619, 1992 (Study material)
- **M.W. Barsoum** Fundamentals of ceramics, 2nd Edn. Chapter – 11 Mechanical Properties : Fast fracture page 399 – 413, CRC Press (Study material)
- **R.W. Davidge**, Mechanical behavior of ceramics, Cambridge University Press, New York 1979

The slide features a dark blue header with the word 'REFERENCES' in yellow. Below the header is a white area with a blue decorative shape on the right. A small video inset in the bottom right corner shows a man with glasses and a mustache, wearing a yellow jacket, speaking. At the bottom of the slide, there are logos for NPTEL and IIT Kharagpur, along with the number 12.

So, the study material is the chapter from Richerson chapter number 12 and also Barsoum; chapter number 11 and also a good book by Davidge on Mechanical behavior of ceramic material.

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CONCLUSION

- Reliability requirement
- 2 parameter Weibull distribution
- Estimation of design stress

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So, in this particular slide lecture; we talked about the reliability requirement and we explained the two parameter Weibull distribution and finally, how to calculate the design stress and what is their implication that has been illustrated.

Thank you for your attention.