

**Advanced Materials and Processes**  
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**Lecture – 09**  
**Bulk Metallic Glass, Glassy and Amorphous Materials (Contd.)**




Welcome to NPTEL, myself Dr. Jayanta Das from Department of Metallurgical and Materials Engineering IIT Kharagpur. I will be teaching you Advanced Materials and Processes.

In last classes, we have discussed many of the different aspects of advanced material and we were discussing about platonic or regular solids, which is one of the basis of making any of the materials, metals, alloys, ceramics and polymer. Now, in case of these regular solid, we have talked about 5 different solid. And these solids are tetrahedron, cube or octahedron, dodecahedron or icosahedron.

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**Solid Geometry: Regular solids**

Polyhedron	Vertices	Edges	Faces	Schläfli symbol	$1/s + 1/m > 1/2$	at each vertex	Angles at Vertex (Less than 360°)
tetrahedron	4	6	4	{3, 3}	0.666... ✓	3 triangles * 60°	180°
cube	8	12	6	{4, 3}	0.583... ✓	3 squares * 90°	270°
octahedron	6	12	8	{3, 4}	0.583... ✓	4 triangles * 60°	240°
dodecahedron	20	30	12	{5, 3}	0.533... ✓	3 pentagons * 108°	324°
icosahedron	12	30	20	{3, 5}	0.533... ✓	5 triangles * 60°	300°

In case of tetrahedron there are basically 4 vertices means here 1 2 3 and 4 8 vertices in case of cube, 6 vertices exists in octahedron and in case of dodecahedron it is 20 like here, and icosahedron there are total 12.

However, we can also count the number of edges, which are 6 and total 4 number of such members or let say equilateral triangles exist in case of a tetrahedron.

And, according to Schläfli symbol, we represent a tetrahedron with 3,3 type of symmetry. Now, here this 3 came because of the member or the sides of each member, like here, like equilateral triangle has 3 sides. And here at every vertices you can see 3, 2 and 3. So, there are basically 3 number of members that are coming close together and present in this vertices. Very similar way, in case of a cube, we have a square, which has 4 regular sides and 3 of such square is meeting at a corner, where corner means basically the vertices.

Similarly, here there are equilateral triangle which is one of the face of the octahedron and at each of these corner, there 4 such triangle at joining together. Now, in case of dodecahedron where we have basically regular pentagons a regular pentagon has all the sides, which are equal in length and so, we have 5 such pentagon edges and here, there are 3 of such pentagon are meeting at the corner.

Now in case of an icosahedron, here we have total 3 number of this edges and which are meeting at this corner or vertices 5 number of such equilateral triangle. Now, why have we have chosen only this 5 regular solid, the questions always come to our mind is there any possibility that we can imagine about any different solid, besides this 5 and here there is a rule of Euler, that has been discussed in the last class. And Euler said that  $\frac{1}{s} + \frac{1}{m} \geq \frac{1}{2}$ . So, you take any of these s and m value and put it together,

You will see in case of a tetrahedron it is 0.666, it is 0.583 in case of a cube, the same as for octahedron and 0.5333 is for dodecahedron and for icosahedron. You can try with any of this Schläfli symbol with very different number of edges; it is not at all possible ok.

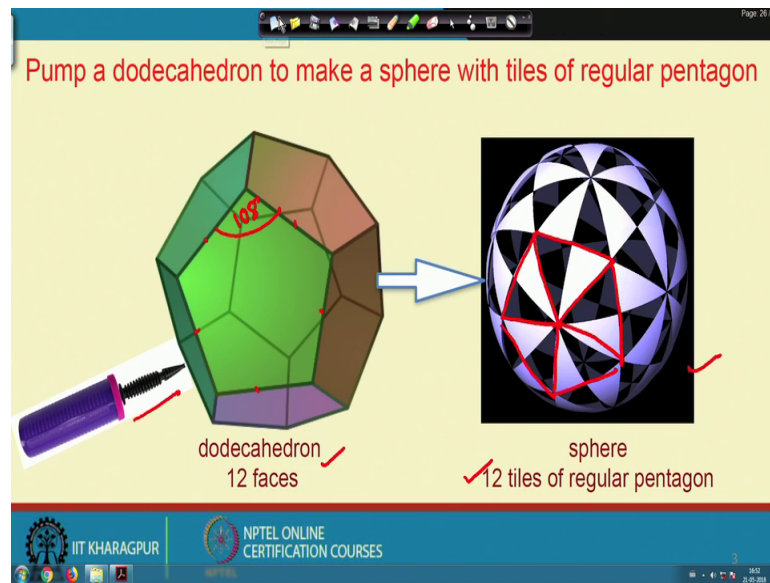
And, now if we extend this discussion little bit further there are something more interesting, because at each corner in case of a tetrahedron there are 3 triangles that are meeting and that is why we call it as 3 here, because 3 members and each of these triangles since these are equilateral triangle, regular triangle, so, it has a angle of 60 degree. So, if you multiply it is  $180^\circ$ . Similarly in case of a cube it is  $270^\circ$ .

In case of a octahedron it is  $240^\circ$ , in case of a dodecahedron it is actually 324, because 108 is the angle here and there are 3 pentagon, that are meeting at the corner. Now, similarly, here 5 triangles equilateral triangles are meeting. So, it is basically 300. So, we

classified all different of the parameters that are available for this regular polyhedrons, which is considered as the units of all different solids.

Now, we can also think about taking this solid and make a plane. So, means what I am talking about I can take this triangles and then try to make a plane out of it. And very similarly I have discussed these things in case of a cube and tetrahedron, those things also discussed in the last class.

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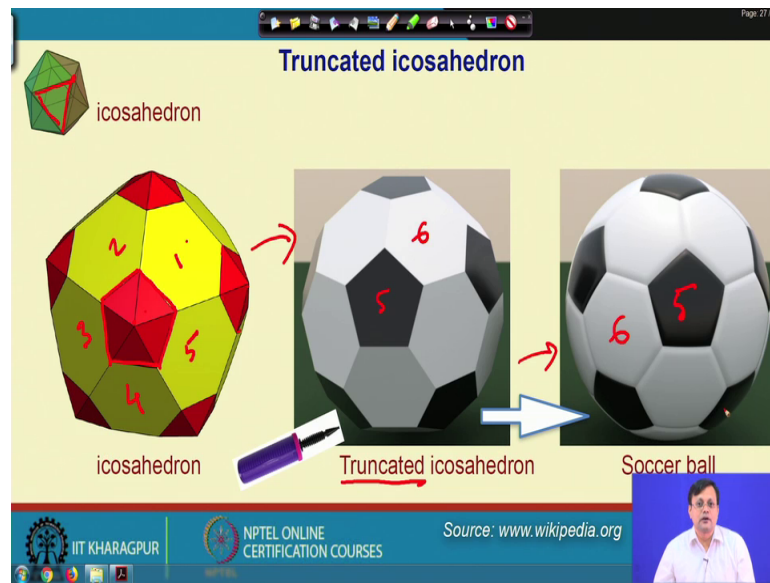


And so, we can see some of the object that are available around us, which has some of the symmetry and that has very much interesting let us have a look, let us take a dodecahedron ok. A dodecahedron is consisting of the regular pentagon, which has 5 of this sides or edges and here these are equal and here this is 180°ok.

Now, all these faces are regular pentagon, if we simply pump it then we can make a bowl out of it. So, we can make a ball out of a dodecahedron where each of the sides are pentagon ok. Here just to see this I am making this drawing and to understand this matter, we basically divided into 5 small triangles, but these are not equilateral triangles actually this is just for to see that it is really a pentagon ok.

So, here we will get basically 12 tiles of a regular pentagon. Now, let us have a look at some other.

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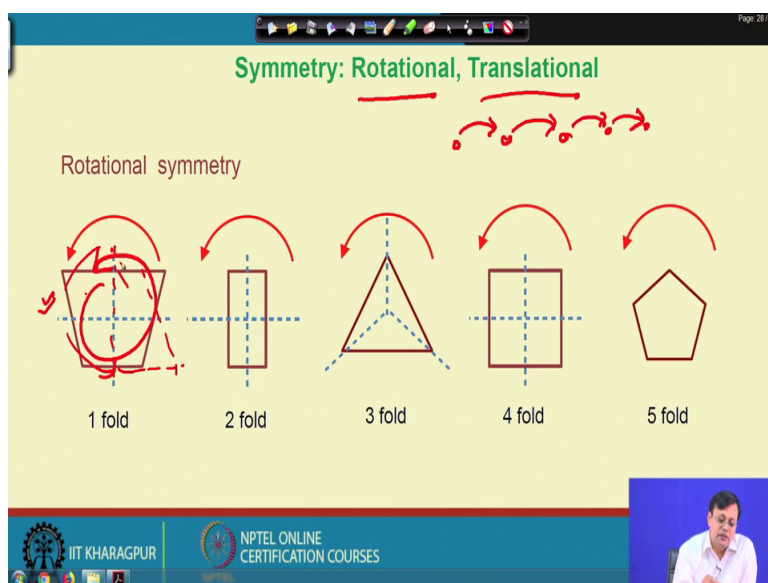
Let us take an icosahedron. In case of an icosahedron, here we see these are the regular triangles and at every corner there are 5 triangles that are meeting at a vertex. So, here this is 1 vertex so, 1, 2, 3, 4 and 5 ok. And this is very much interesting to see.

So, now what I did, I simply cut or make a slice of one of this corner. So, here I can create actually one pentagon, if I take out this slice it will be a pentagon and here also the other vertices I cut it very similar way. So, here this will be a hexagon and this will be a pentagon ok. So, if you look here this is the case where we have a pentagon and here is a hexagon and simply if we pump such kind of icosahedron, which is a truncated icosahedron, we call it as a truncated because we have truncated this corner and then if we pump it we will get a football.

Usually, if you have a look at any of this football it has always the tiles or a soccer ball it has the tiles of a regular pentagon and we have a regular hexagon ok.

So, this is very much interesting that even in our normal life we can see all these different kinds of geometry; however, there are only 5 possible ways to get these regular solids. Otherwise all these different solids are sum of derivative of these 5 regular solids.

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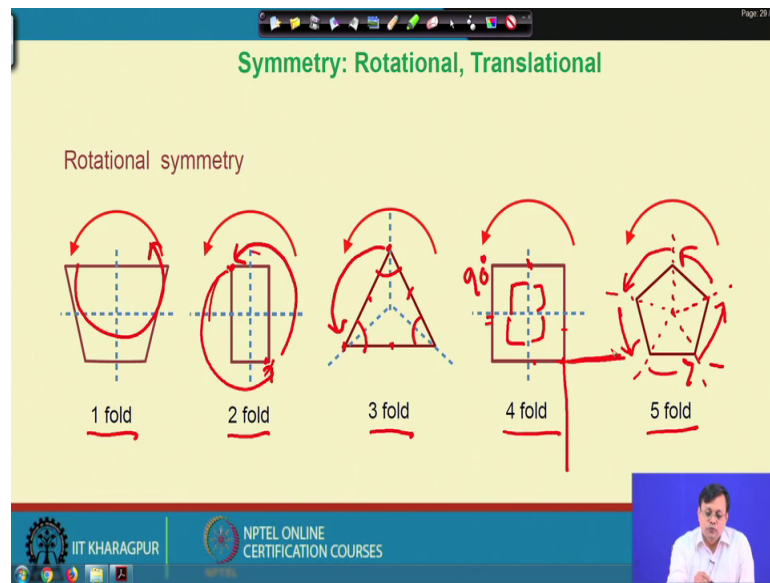
And therefore, we need to learn a bit about the rotational or let us say the symmetry of this objects. So, before we begin with this symmetry we must make or recapitulate what we really mean by this symmetry.

Usually, when we talk about symmetry, we think about a rotational symmetry or let us say translational symmetry ok. You can take about your hand and try to rotate that how much a hand impression should be rotated. So that it will come back to it is original position ok. And, if we think about some of the atomic arrangement like this way and then we need to see that these items would be positions can be translated along certain direction or not ok.

And therefore, this symmetry terminology comes actually and here let us have a look at this is a geometry ok. So, in this case definitely you may think about that there is a mirror plane where both the sides look similar, but when you talk about a rotational symmetry, then we can rotate this  $180^\circ$ , but here this will be like this.

So, it will not come back to it is original shape and only it will come back when we will rotate it complete 360 degree ok.

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So, 360 degree rotation it require and therefore, we call it as a 1 fold symmetry. Now in this case let us say a rectangle let us say, we can see these are some of the symmetry axes and if we rotate it  $180^\circ$  then this point will come here and definitely we will get a 2 fold. So, means this point will come from here to here and then we make another fold. So, that this point will come back to it is original position ok.

Now, in case of an equilateral triangle, we can rotate about  $60^\circ$  and then this point will come here, because all these angles are 60 degree angle and the length of these members are equal and therefore, we call it as a 3 fold symmetry. Now, if we take a square, since square has each and all these members are equal and therefore, we can rotate only at ninety degree and we will get that symmetry and therefore, this 360 divided by 90 which is basically the 4 fold.

Now, in case of a pentagon it is definitely fivefold that we really understand by comparing all this different images ok. So, here we can take the center and we can simply rotate around that center and we can rotate it 5 times ok. However, in case of mathematics and the people who were working with this geometry of solids there it is told that this 5 fold is a forbidden symmetry, because you cannot make a symmetry and you cannot fill the space with this 5 fold ok. So, let us you can take this 4 fold and then extend it and try to try to fill this ok. So, like here.

So, here I can take a square, here I can take another square, here I can take another square. So, I can fill the space, but in case of 5 fold which is the forbidden symmetry you cannot really fill the space and that is one of the major major issues.

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**Symmetry and Spin**

A spin of 2 of an object like a playing card which look similar after half (180°) of a full 360° rotation.

An Electron/ or Fermions has spin of 1/2 means that an electron comes to original state after 2 times (720° = 360° X 2) a of complete 360° rotation.

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And therefore, let us continue this discussion let say before going to the forbidden symmetry we can think about the spin which is often known to you also that a spin of a object of 2, which basically means a spin of 2 of an object like playing card ok, which is like a rectangle, which looks similar after half of the full 360° rotation ok.

Now, we already know about electron and if we recall our old textbooks there it is written that it has a spin of half ok. This is very interesting, this means an electron should rotate it is around a nucleus 2 times. So, that it will come back to it is original position. So, they need electron need 720° to rotate to come back to it is original state. So, this symmetry and spin are these are very very closely related terms.

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The slide is titled "Solid Geometry: Forbidden Symmetry". It features several diagrams and a text box. On the left, a honeycomb lattice of hexagons is shown with a red arrow and the label "6-fold" indicating its symmetry. Below it, a cluster of three pentagons is shown with a red arrow and the label "5-fold". To the right, a text box with a red underline states: "5-fold, 7-fold, 8-fold, or higher-fold rotation axes, these are not possible in crystals". Below this text, two diagrams show attempts to tile a plane with pentagons. The first diagram shows four pentagons labeled 1, 2, 3, and 4, with a red hatched area indicating an overlap region. The second diagram shows four pentagons labeled 1, 2, 3, and 4, with a red hatched area indicating a gap between them. The labels "5-fold" and "8-fold" are written below these diagrams. The slide footer includes the IIT Kharagpur logo and the text "NPTEL ONLINE CERTIFICATION COURSES".

Now, using those concept, let us try to think about a filling of a space or let us say making a plane, we have talked about let us a hexagon and we can take such hexagon and easily put together to fill the space. This is like a honeycomb structure, and there is no problem at all to fill the space using a 6 fold symmetry ok.

Now, if I have a 5 fold symmetry which is a pentagon actually. So, this is a pentagon actually and we can think about taking 3 pentagon and join together, but it will be ultimately like a dodecahedron ok.

So, it is no longer a dodecahedron here ok. So, it is no longer a plane. And, now here you see if I take 1, 2, 3 and 4 pentagon together then there is a overlap region or let us say if I do not want to overlap, then here I have taken 1 pentagon here I have taken the second this is third and this is a fourth and then if I want then there will be always a empty space.

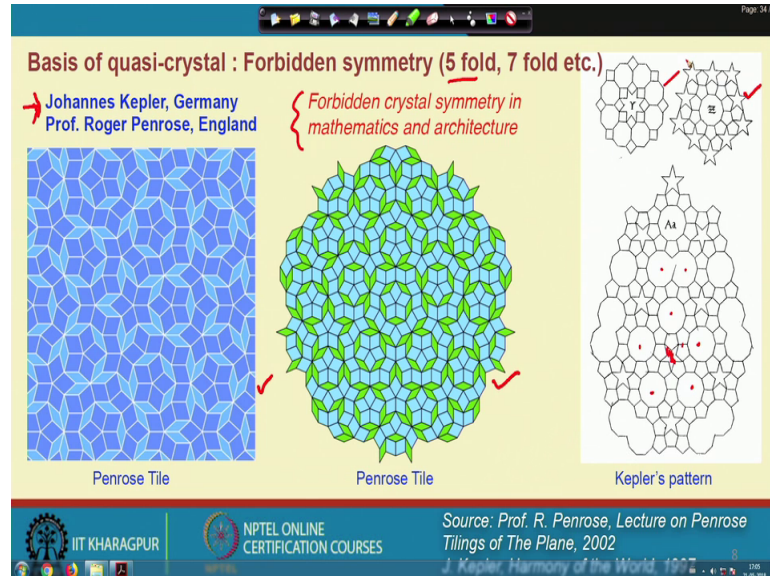
So, using a pentagon we cannot fill up the space on a on a plane. We cannot make a plane out of it and therefore, there are many mathematicians who are always thinking about these which are the forbidden symmetry we have.

So far, we understood that this 5 fold, 7 fold, 8 fold or let say the higher order rotation axis, these are not possible for in case of a crystal, because we cannot fill up the space there will be always a gap inside. Let us take a 8 fold symmetry here 1, 2, 3 and 4 you



see, if I join together there is empty space and therefore, this is called as a forbidden symmetry and we need to continue a little bit discussion along this direction .

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So, I already told you about the quasicrystal and the forbidden symmetry is very much linked with this quasicrystal and Kepler who is a philosopher and mathematician and professor Rodger Penrose from England, they were thinking about this forbidden symmetry in mathematics in architecture.

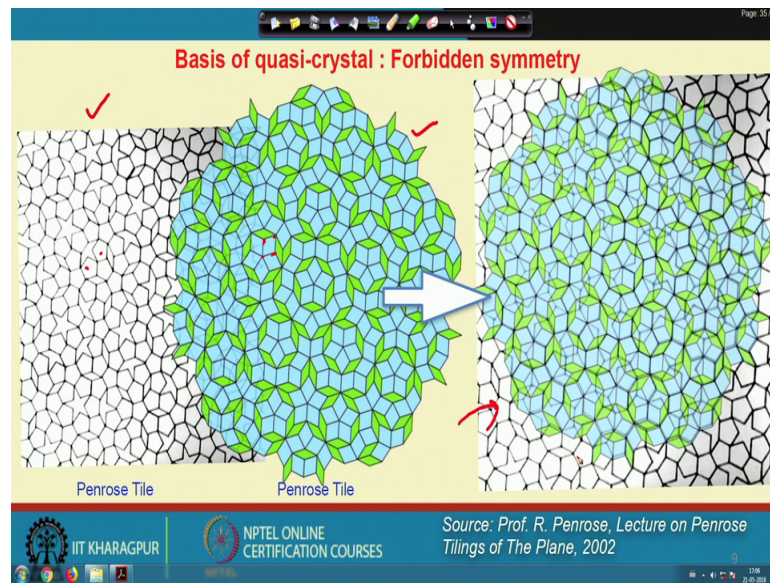
So, you may have visited many of the churches or let us say many of the mosque or temples and several places, you may find some of the tiles ok. And this tiles means it is basically a plane which is filled up by some sort of geometry I am talking about. So, this tiles could be various different types, but there are many different objects or tiles or their units are very complicated. And so, Penrose's tiling is very much interesting because they have these 5 fold type of symmetry and so on and we are trying to look at what are the those different possible ways to fill up a plane.

So, here I show you one type of Penrose tile and this is also a little bit different looking Penrose tile and quite a long time ago Kepler who was also dealing with many of the mathematics and so on. So, here he has also shown some sort of pattern and you can see here some starlight pattern and the starlight pattern is lying around some regular pentagon ok. And there is such kind of spaces field and where is this is taken from his

book actually and here, he could not find out some way to fit these 2 different 2 different geometries actually.

So, here also there is some 2 of these thing, but ultimately we can see that it looks like a regular pentagon symmetry, here also you can see this is very similar like here. And so, what we can do, we can think about this patterns and try to see what is the update on along this direction.

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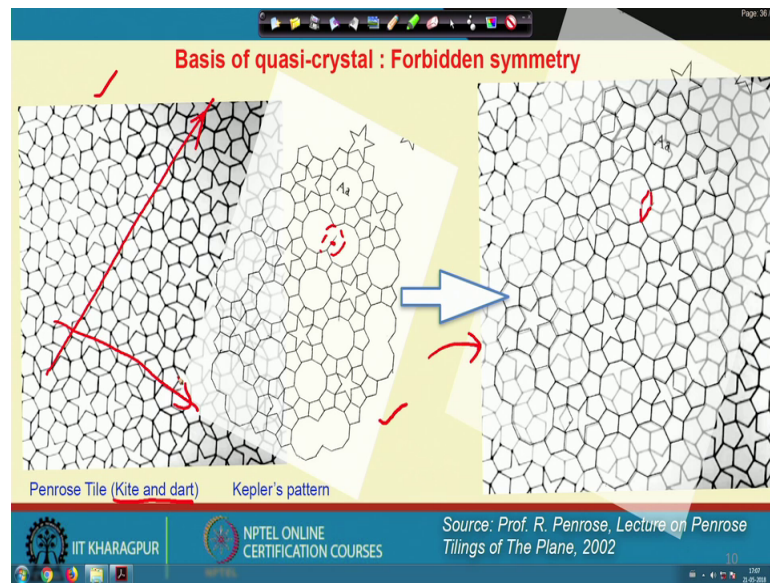


So, along this direction Professor Penrose has given the idea of his tile, which is often called as a kites and darts.

So, so, this kites and darts here I have taken and I have taken another one of his Penrose tile actually. And try to overlap of it and you see basically, that these both of these pattern even though they look very different. So, here you see it looks like a rhombus and so on and here it looks like a pentagon yes.

So, here if we overlap this 2 then you can see that they completely matching with each other. So, there is no problem at all. So, both these pattern are basically similar.

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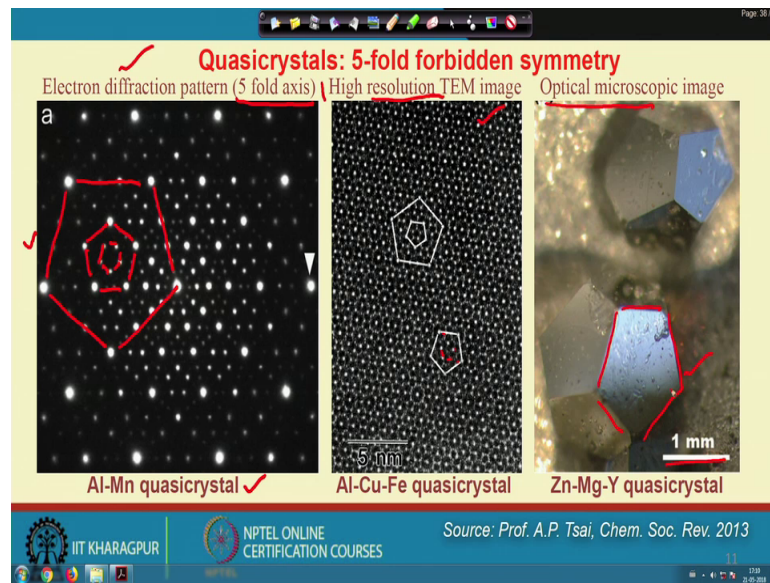


Now, let us take another one of professor Penrose tile which is he explained as a kite and dart model. So, here I have taken and I have also taken Keplers pattern, which he has also unexplained type of thing here. And I have basically super imposed this 2 pattern and merge it together and you can see here, this was the line which is this one. And now it could be explained using Penrose tiling, which is a kite and dart type of 5 fold symmetry.

So, usually professor Penrose try to explain, that even though you cannot really fill up a plane using a pentagon. However, there is some sort of symmetry exist along this type of direction and along this direction ok. So, so he was trying to find it out and since these are all discussed in mathematics and architecture and so on. There was a need of material where the structural unit supposed to be like that and that was basically proved by the quasicrystal community.

So, in case of quasi crystal actually this pattern of or the atomic arrangement is similar like this Penrose pattern, which is forbidden. And, because of that professor Dan Schechtman get the Nobel Prize.

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So, I must show you some of this pattern electron diffraction pattern of quasicrystal and the in the left hand side this is an image that is taken from a 5 fold, and this is an aluminum magnesium quasicrystal and you can see here very interesting thing. So, I show you here 1 regular pentagon here you can see and now here is another pentagon.

Now, let us have a look again, here is another pentagon and if you keep on making, this will be bigger and bigger. So, inside a pentagon there is another pentagon there is another pentagon just in the opposite sides actually, and this is the interesting phenomenon and this was taken a as a electron diffraction pattern which is often known as a selected area refraction pattern. Now the same crystal quasicrystal, if you look at high resolution TEM image these are let us say almost showing the nuclear positions of the atoms and they look very similar. So, here is also a pentagon and here is also a pentagon.

However, they are filling up the space or they are filling up the plane. And here this is which is often known as the forbidden symmetry by the mathematicians, and very interestingly even though these are electron diffraction pattern and this is a high-resolution transmission image, however, in case of optical microscopic image of those crystals appeared to be very similar. So, you can see these are the very big crystals of 1 millimeter of each side and these are also a big crystals with having faces, that are that are pentagon. And along this direction Professor Tsai has done a very great work to

develop quasicrystal along different directions, which has a very large range of applications.

And so, we understood that the regular solids are very much important to explain the basis of the crystalline structure.

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The slide is titled "Quasicrystals: The Fibonacci sequence". On the left, there is a dark image of an Al-Mn quasicrystal with a red rectangular box highlighting a specific region. Below it is the text "Al-Mn quasicrystal". On the right, there is a diagram of a Fibonacci spiral on a grid. The spiral is composed of quarter-circles with radii corresponding to Fibonacci numbers. The squares in the spiral are labeled with their side lengths: 5x5, 8x8, 13x13, 21x21, and 34x34. Below the diagram is the equation  $X_n = X_{n-1} + X_{n-2}$ . At the bottom, the Fibonacci series is listed as  $0, 1, 1, 2, 3, 5, 8, 13, 21$  with red arrows indicating the addition of the two previous terms to get the next term. The slide also features logos for IIT KHARAGPUR and NPTEL ONLINE CERTIFICATION COURSES, and a small video inset of a speaker in the bottom right corner.

Now let us have a look to some of this quasicrystalline diffraction pattern, which is here shown as an aluminium magnesium quasicrystals. And these are the diffraction pattern of the reciprocal lattice point and in between they are also points and this cannot be translated. And that is why this pattern usually called as a forbidden symmetry; however, people find out that the ratio of the distances follow a Fibonacci series. A Fibonacci series means that  $X_n = X_{n-1} + X_{n-2}$ . So, if I start from 0, then the next number will be 1; (0 + 1) 1; (1+1) 2; (1+2) 3; (2+ 3) 5; and (3+5) 8; this is the Fibonacci series. So, the position, atomic position if you see the ratios or they directly matches almost like a Fibonacci series.

So, even though we think about some forbidden symmetry in crystals in case of quasicrystals actually such kind of forbidden symmetry exist and the atomic positions could be explained using the Fibonacci sequence. So, so far we try to understand the symmetry in crystalline structure the regular solid and not only in case of the crystalline or Bravais lattice of 7 different crystal systems.

However, they have a very long idea that has been developed by the mathematician's long time ago to explain the crystalline lattice. And quasicrystal, which has been discovered very recent days, which is also explained using such kind of 5 fold symmetry or forbidden symmetry.

However, in case of a glassy or amorphous structure the short range order is also often find out to exhibit such kind of forbidden symmetry or let us say like a like a quasi-periodic type of or let say type of order. And therefore, such kind of symmetry even in case of an electron spin, these are very much important aspect. And, we will restrict our discussion within this and we will again continue with the glass forming ability or glass transition related issues in the next classes.

Thank you very much.