

Processing of Semiconducting Materials
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Lecture - 5
Semiconductor Statistics

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ELECTRONS OBEY FERMI-DIRAC STATISTICS

- Indistinguishability of the electrons**
- Wave nature**
- Pauli Exclusion Principle**

Distribution of electrons over a range of allowed energy levels

$$f(E) = \frac{1}{1 + e^{\frac{(E-E_F)}{kT}}}$$

f(E) : Fermi-Dirac distribution function: gives the probability that an available energy state at E will be occupied by an electron at T

E_F is the Fermi level, k = 1.38 x 10⁻²³ J/K or 8.62 x 10⁻⁵ eV/K

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Let us start the another topic the Semiconductor Statistics: You know that electrons obey fermi-dirac statistics look at the view graph; you see that the electrons obey fermi-dirac statistics right. Now this fermi-dirac statistics, what are other statistics? Bose-Einstein is applicable for photon, then Maxwell Boltzmann it is applicable for Gas Molecules, classical particles etcetera. But when we deduce something related to Fermi-Dirac distribution or say this distribution of the electron energy states, it has been derived using assuming 3 considerations; one is indistinguishability of the electrons, wave nature of electron and Pauli Exclusion Principle. So, on the basis of this thing the Fermi-Dirac statistics have been derived, which is given by F of E equals to 1 by 1 plus E to the power E minus E_F by KT . Now what is this? This is the distribution of electrons over a range of allowed energy levels. Remember that if there are allowed energy levels, there is a possibility of finding an electron there; if there are no states, then do not try to find electrons there.


So, there must be allowed energy levels. And this E_F it is given, it is known as the Fermi-Dirac Distribution function; and it gives the probability that an available energy state at E will be occupied by an electron at T so the T the temperature is very important; if you change the temperature the energy state will also change.

And in this expression, E_F is known as the Fermi level, k is the Boltzmann constant whose value is given by 1.38×10^{-23} joule per Kelvin or 8.62×10^{-5} electron volt per Kelvin. So, why we are interested to discuss about the Fermi-Dirac distribution? It is because from Fermi-Dirac distribution, you can calculate the electron or whole concentration in a semiconductor, because for us, the number of electrons in the conduction band or the number of holes in the valence band is very much, very much important parameter. When we design some material, so we have to design the material in such a manner that it is applicable for a particular purpose; say for a purpose you need 10^{17} electrons so, how do you design, how do you calculate, so what would be the other parameters; to control to get 10^{17} electrons.

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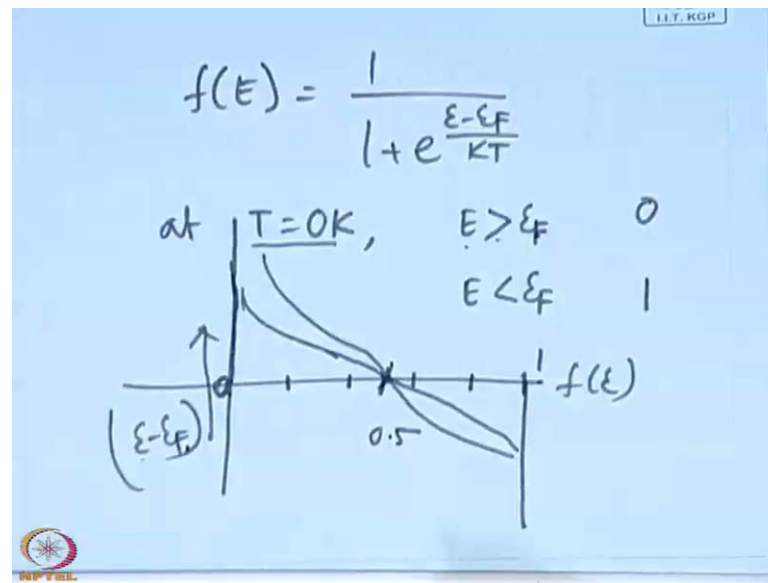
For an energy $E = E_F$,
Occupation probability: $f(E_F) = \frac{1}{1+1} = \frac{1}{2}$

Thus an energy state at the Fermi level has a probability of $\frac{1}{2}$ of being occupied by an electron.
 $f(E)$ is the probability of occupancy of an available state at E .
If there is no available state at E , there is no possibility of finding an electron there

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So, from Fermi-Dirac distribution function you will get the number of electrons or number of holes now this is the expression say.

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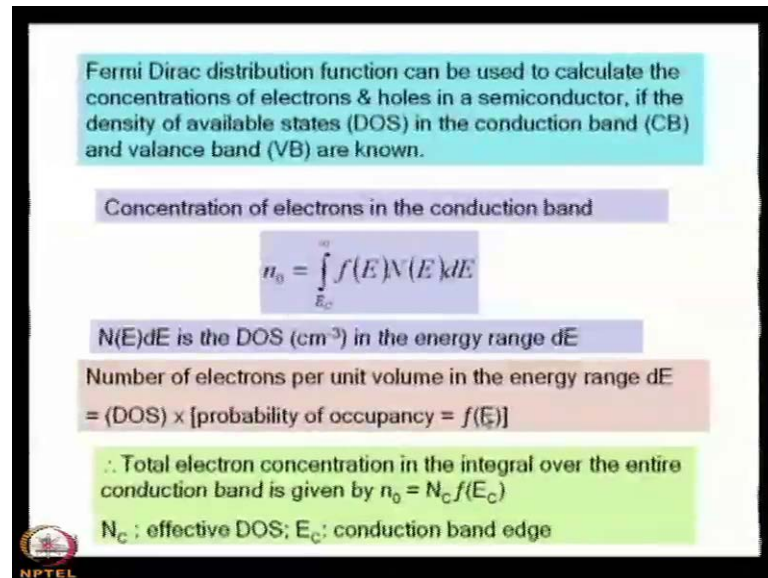


Let me write down $F(E)$ this is equals to 1 by 1 plus exponential E minus E_F by KT now for an energy E equals to E_F see for an energy E equals to E_F this is E equals to E_F so, you put E equals to E_F what you will find half for E equals to E_F you see that when E equals to E_F this becomes half so, the occupation probability occupation of what electrons in that level that is the occupation probability thus an energy state at the Fermi level has a probability of half of being occupied by an electron right half of being occupied by an electron another thing from this expression you can say that at T equals to $0K$ at absolute 0 temperature if E greater than E_F what will happen and what will happen if E less than E_F probability is zero.

So, if you plot say it is E_F and it is E minus E_F in this direction it is say 0 so, what will happen at E greater than E_F means Positive so, in this direction it is basically 0 and when E less than E_F it is 1 so, it is say if it is one say it is $0.2, 0.4, 0.6, 0.8, 1$ so, you will get this type of a curve as you go on increasing the temperature as you go on increasing the temperature what you will find that this type of a curve you will get or this type, but in all the cases you will find that at E equals to E_F you will pass through 0.5 that means, the probability occupation is half so, Fermi function or say here you we are interested more on Fermi level so, what the Fermi level will give us in semiconductor.

Fermi level will give us in semi conductor some idea about the probability of electrons then as I informed you earlier that if there is no available state at E there is no possibility of finding an electron there.

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Fermi Dirac distribution function can be used to calculate the concentrations of electrons & holes in a semiconductor, if the density of available states (DOS) in the conduction band (CB) and valance band (VB) are known.

Concentration of electrons in the conduction band

$$n_0 = \int_{E_C}^{\infty} f(E)N(E)dE$$

$N(E)dE$ is the DOS (cm^{-3}) in the energy range dE

Number of electrons per unit volume in the energy range dE
 $= (\text{DOS}) \times [\text{probability of occupancy} = f(E)]$

\therefore Total electron concentration in the integral over the entire conduction band is given by $n_0 = N_C f(E_C)$

N_C : effective DOS; E_C : conduction band edge

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Now how you can calculate the number of electrons or holes using Fermi-Dirac statistics because that is required for our course in our course we are interested in calculating the number of electrons or holes in a semiconducting material and Fermi-Dirac distribution statistics can be used to calculate the concentration of electrons and holes in a semiconductor if the density of available states in the conduction band and valence band are known.

So, if you know the density of available states then only you can calculate the electrons and whole concentration in a semiconductor concentration of electrons in the conduction band can be given by n_0 equals to integration E_C to infinity $N(E)dE$ what is the $N(E)dE$ is the density of states and its unit is per cubic centimetre in the energy range d that means, between E and E plus d if the range is d then the n_0 can be given by this relation and the number of electrons per unit volume in the energy range dE is given by density of states multiplied by the probability of occupancy because you see that when we absolutely calculate the electrons one thing is that how many density of states are there and the what is the probability of occupation there.

Because probability of occupation is also important thing number of density of states is not the number of electrons when it will be multiplied by the probability then only you can say how many electrons are there. So, the probability of occupation which is given by the Fermi-Dirac statistics $F(E)$ that is important when that will be multiplied by that number of density of states that will give you the electron concentration therefore, the total electron concentration in the integral over the entire conduction band is given by n_0 equals to N_C into $F(E)$ what is N_C ? N_C is the effective density of states that means, if you integrate $N(E)dE$ over E_C to infinity then that will give you the effective density of states.

And what is $F(E_C)$ is the bottom of the conduction band as I have already introduced the term the conduction bandage from E_C to infinite what is the meaning the meaning is that from the bottom of the conduction band to infinite and now what is infinite here basically it is the limit and in that limit we can say that. So, far as the band exist the conduction band has some limit infinity is not the limit here in practical limit that is why we have used the effective density of states and E_C is the conduction bandage. So, remember this expression n_0 equals to $N_C F(E_C)$ now see it is $F(E_C)$ now we want to calculate $F(E_C)$ that means, what is the value of the distribution at the conduction bandage.

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$$f(E_C) = \frac{1}{1 + e^{\frac{(E_C - E_F)}{kT}}} \approx e^{-\frac{(E_C - E_F)}{kT}}$$

(assumed E_F lies at least several kT below CB)

$$\therefore n_0 = N_C e^{-\frac{(E_C - E_F)}{kT}}$$

where $N_C = 2 \left(\frac{2\pi m_n^* kT}{h^2} \right)^{\frac{3}{2}}$

m_n^* : electron effective mass,
 h : Planck's constant ($= 6.626 \times 10^{-34}$ J-s).

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You see this is the value of the conduction bandage just I have changed E by E_c your original expression was $1 + \frac{E - E_F}{kT}$ you see it was one by E plus E_F now I have changed that E into E_c and with some approximation we can write that it is E to that power minus E_c minus E_F by kT we have assumed that E_F lies At least several kT below conduction band what is $K T$, $K T$ is the thermal energy at room temperature at 300 K what is the value of $K T$? 26 Milli electron volt.

Now we have assumed that it lies at least several $K T$ that is 26 milli electron volts multiplied by say 3 4 5 if that is the case then we can approximate this expression by E to the power minus E_c minus E_F by KT . So, therefore, n_0 is given by N_c see this expression n_0 is given by N_c multiplied by f of E_c and f of E_c is given by E to the power minus E_c minus E_F by $K T$. So, I have put the value of $f E_c$ here. So, this is the concentration of electron in the conduction band, here there is a term N_c now what is N_c ? n_c is given by $2 \times 2\pi m_n^* kT$ by h squared to the power $3/2$ how it is deduced it is deduced from the wave mechanics and considering the Pauli's exclusion principle it is there in all the textbook you can and find in any of the semiconductor statistics textbook.

Say Banerjee and Streetman, Streetman and Banerjee or say there is a book p Bhattacharya see in all those books you will find this expression how they have calculated the effective density of states N_c in terms of this fundamental parameters of the semiconductor remember that this K is the Boltzmann constant T is the temperature h is the Planck's constant whose value is given by 6.626×10^{-34} joule second and m_n^* is the electron effective mass. We have introduced the term effective mass since we are discussing about the electrons. So, it is the electron effective mass.

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The concentration of holes in the valence band

$$p_0 = N_v [1 - f(E_v)],$$

where N_v is the effective DOS in the VB

The probability of finding an empty state at E_v is

$$1 - f(E_v) = 1 - \frac{1}{1 + e^{\frac{E_v - E_F}{kT}}} \approx e^{-\frac{E_F - E_v}{kT}}$$

(for E_F larger than E_v by several kT).

Concentration of holes in the valence band is

$$p_0 = N_v \cdot e^{-\frac{E_F - E_v}{kT}} \quad \text{where} \quad N_v = 2 \left(\frac{2\pi m_p^* kT}{h^2} \right)^{3/2}$$

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When we shall discuss the holes we shall replace this m_n star by m_p star. The concentration of holes in the valence band will be given by p_0 equals to $N_v [1 - f(E_v)]$ why it is $1 - f(E_v)$, why it is 1 have because N_v is the effective density of states in the valence band and $1 - f(E_v)$ is the probability basically this is the probability because we have to multiply the effective density of states with the probability total probability of $f(E_c)$ plus $f(E_v)$ equals to 1 . So, we have already used $f(E_c)$. So, it will be $1 - f(E_v)$.

And in this respect the non degenerate and degenerate semiconductors come into the picture and we have already discussed remember this that this type of calculation or this type of derivation is basically applicable to non degenerate semiconductor not the degenerate semiconductor remember all such statistics etcetera are related to non degenerate semiconductor.

Where the number of electron concentration in the conduction band or the number of hole concentration in the valence band is less than the density of states of the corresponding bands. So, that means, it is applicable till 10^{17} , 10^{18} , carrier concentration if it exceeds 10^{19} , 10^{20} , 10^{22} then you cannot use this kind of expressions or statistics fine.

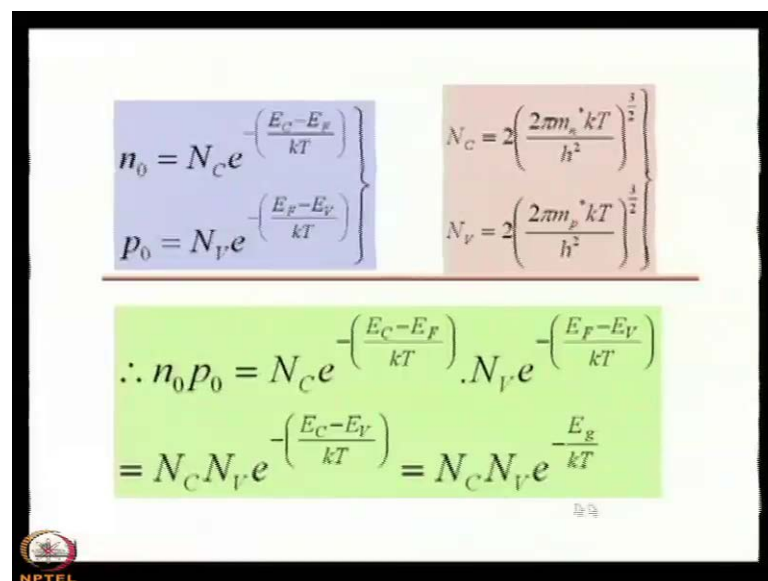
Now the probability of finding an empty state at E_v , E_v is what is E_v it the top of the valence band valence bandage it is $1 - f(E_v)$ this expression you see this

expression here we have deduced it is 1 minus 1 by this and if I assume that E_F larger than E_v by several $K T$. E_F is larger than E_v by several $K T$ thus that assumption was there for conduction band as well you see that here also we have assumed that E_F lies at least several $K T$ below conduction band here E_F is larger than E_v by several $K T$ this will be for the valence band.

So, we can approximate this expression by E to the power minus of E_F minus E_v by $K T$ now you put the value here this expression N_v into this you will find that p_0 equals to $N_v E$ to the power minus E_F minus E_v by $K T$ where N_v is the density of states which is given by $2 \times \frac{2\pi m_p^* k T}{h^2}$ to the power $3/2$.

And you find that this is same the expression is same it is like the density of states in the conduction band except that they are the effective mass was of the electrons here the effective mass was for the holes that is the only difference.

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$$\left. \begin{aligned} n_0 &= N_c e^{-\left(\frac{E_c - E_F}{kT}\right)} \\ p_0 &= N_v e^{-\left(\frac{E_F - E_v}{kT}\right)} \end{aligned} \right\} \quad \left. \begin{aligned} N_c &= 2 \left(\frac{2\pi m_n^* kT}{h^2} \right)^{\frac{3}{2}} \\ N_v &= 2 \left(\frac{2\pi m_p^* kT}{h^2} \right)^{\frac{3}{2}} \end{aligned} \right\}$$

$$\begin{aligned} \therefore n_0 p_0 &= N_c e^{-\left(\frac{E_c - E_F}{kT}\right)} \cdot N_v e^{-\left(\frac{E_F - E_v}{kT}\right)} \\ &= N_c N_v e^{-\left(\frac{E_c - E_v}{kT}\right)} = N_c N_v e^{-\frac{E_g}{kT}} \end{aligned}$$

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So, two expressions we have derived one is this n_0 equals to $N_c E$ to the power minus something and p_0 equals to $N_v E$ to the power minus something where n_c 's are given by this expression and N_v is given by this expression you can compare the expressions what you find N_c and N_v the expressions all are exactly the same except that the Effective mass are different.

Here also you see that n_0 and p_0 what are the similarities and difference here you see that it is N_c it is N_v here E_c minus E_F here E_F minus E_v . So, that is the difference now if you multiply n_0 and p_0 . So, what you will get.

Student: $N_0 (())$.


Yes. So, if you multiply n_0 and p_0 . So, the details have been shown in the slide for all the students and you can copy or you can take the copy from me also I can give you this copy it is equals to N_c into N_v E_c to the power minus E_c minus E_v by $K T$ why because as you multiply this will be added E to the power this into E to the power this. So, it will be added and you will find that E to the power minus E_c minus E_v by $K T$ what is this E_c minus E_v E_c minus E_v is known as the band gap it E_c minus E_v it is basically the band gap and this band gap is given by this E_g we. So, find that if you multiply n_0 into p_0 it will be given by N_c into N_v into E to the power minus E_g by $K T$.

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For intrinsic materials

$$\left. \begin{aligned} n_0 &= N_c e^{-\left(\frac{E_c - E_F}{kT}\right)} \\ p_0 &= N_v e^{-\left(\frac{E_F - E_v}{kT}\right)} \end{aligned} \right\} \begin{aligned} \Rightarrow n_i &= N_c e^{-\left(\frac{E_c - E_i}{kT}\right)} \\ \Rightarrow p_i &= N_v e^{-\left(\frac{E_i - E_v}{kT}\right)} \end{aligned}$$

$$\begin{aligned} \therefore n_i p_i &= N_c e^{-\left(\frac{E_c - E_i}{kT}\right)} \cdot N_v e^{-\left(\frac{E_i - E_v}{kT}\right)} \\ &= N_c N_v e^{-\left(\frac{E_c - E_v}{kT}\right)} = N_c N_v e^{-\frac{E_g}{kT}} = n_0 p_0 \end{aligned}$$

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Now, for intrinsic material. So, far we have discussed about the non intrinsic material or extrinsic material. What will happen for intrinsic material this expression these expressions we have deduced earlier and it will change to this expression. So, why the why are the changes you see that n_0 becomes N_i right and here E_F is replaced by E_i in this expression also p_0 changes to p_i and E_F changes to E_i because already we have introduced the term intrinsic level for intrinsic semiconductor we do not use Fermi level

rather we use intrinsic level that is why the Fermi level is replaced by the intrinsic level E_i stands for intrinsic. So, you see that the expressions on the left hand side they are basically for the extrinsic semiconductor part.

Extrinsic semiconductor can be degenerate can be non degenerate, but our equations are valid for non degenerate. So, this is basically the expression for the non degenerate semiconductor this has been changed into intrinsic semiconductor where we have replaced E_F by E_i and n_0 p_0 by n_i p_i why we have used 0 what is n_0 and what is p_0 here it is intrinsic, but what is 0? 0 stands for the equilibrium condition, 0 stands for equilibrium where you are not supplying any energy to the system from outside then there is a perfectly equilibrium system because you can tell me sir where the energy is supplied to the system.

Yes voltage current light anything and I shall show you that when you add some voltage or you put some light then what will happen the carriers will generate more and more electrons will come from the valence band to the conduction band more and more electrons will generate and obviously, then the situation will change earlier it was n_0 now it will be n_0 plus Δn . So, the total concentration will be n_0 plus Δn n_0 is due to the equilibrium carrier concentration and Δn is what you are supplying from outside. So, total number of electrons will be this n_0 plus your Δn and that show total number of electrons in the system non equilibrium remember it is equilibrium for non equilibrium case it will be n_0 plus Δn .

Similarly, p will be p_0 plus Δp this n_0 and p_0 you know these are the equilibrium and Δn and Δp they are created.

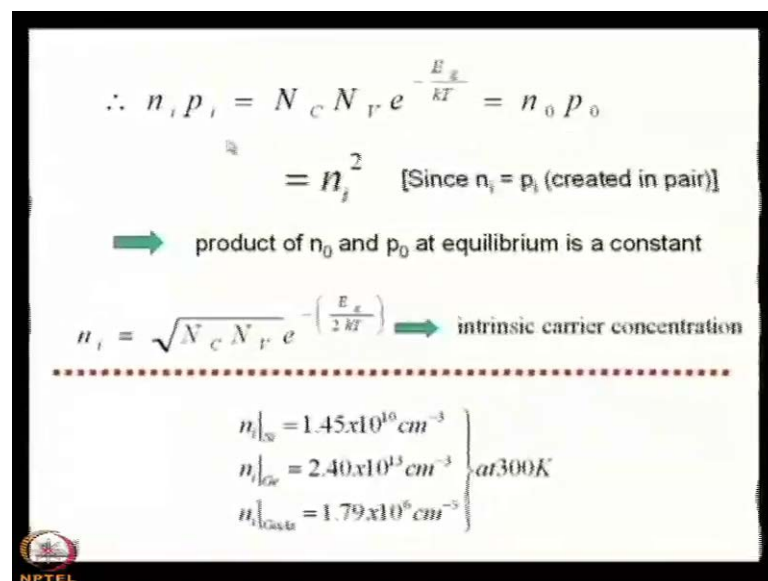
Student: Change in.

Yes change in from the outside energy then now if you multiply this n_i and p_i in in the earlier slide we have multiplied n_0 p_0 in this slide we want to multiply n_i p_i then what you find the same thing $N_c N_v e^{-\frac{E_g}{kT}}$ you see here it was $N_c N_v e^{-\frac{E_g}{kT}}$ to that power minus E_g by kT it is $n_0 p_0$ here $n_i p_i$ equals to $N_c N_v e^{-\frac{E_g}{kT}}$ that is why I have written $n_i p_i$ equals to $n_0 p_0$ that is why I have written $n_i p_i$ equals to $n_0 p_0$. So, what you find you find that $n_i p_i$ is the product of the intrinsic carrier concentrations. Concentration of electron and concentration of holes that is the product $n_i p_i$.

And $n_0 p_0$ is the equilibrium carrier concentration right show the product of the equilibrium carrier concentration is a constant because $n_i p_i$ is a constant I have shown you the value of n_i for $n_i p_i$ for silicon is 1.45×10^{10} per cm^3 .

So, that is a constant intrinsic carrier concentration is constant for all material be it germanium, gallium, arsenide, indium phosphide whatever be the material it is constant. So, $n_i p_i$ is constant and $n_0 p_0$ is the equilibrium carrier concentration. So, the product of the equilibrium carrier concentration is constant is very important relation I can ask you to deduce relation in the exam also. So, that the product of the equilibrium carrier concentration in a semiconducting material is constant. So, you have to start from the Fermi-Dirac statistics then you can use the density of states and the probability some mathematical manipulation and you will arrive at this station $n_i p_i$ equals to $n_0 p_0$.

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$$\therefore n_i p_i = N_c N_v e^{-\frac{E_g}{kT}} = n_0 p_0$$

$$= n_i^2 \quad [\text{Since } n_i = p_i \text{ (created in pair)}]$$

→ product of n_0 and p_0 at equilibrium is a constant

$$n_i = \sqrt{N_c N_v} e^{-\left(\frac{E_g}{2kT}\right)} \rightarrow \text{intrinsic carrier concentration}$$

$n_i _{\text{Si}} = 1.45 \times 10^{10} \text{ cm}^{-3}$ $n_i _{\text{Ge}} = 2.40 \times 10^{13} \text{ cm}^{-3}$ $n_i _{\text{GaAs}} = 1.79 \times 10^6 \text{ cm}^{-3}$	}	at 300K
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Therefore $n_i p_i$ equals to $n_0 p_0$ now $n_i p_i$ is what is $n_i p_i$? $n_i p_i$ is the intrinsic carrier concentration they are same. In intrinsic material the number of electrons and number of holes are equal.

So, it is n_i square since n_i and p_i created in pair are also annihilated in pair. So, it is equals to n_i square right therefore, we can write that n_i equals to root over of $N_c N_v$ into E to the power of minus E_g by twice $K T$. Why how that two comes.

Yes power 1 by 2 right. So, the intrinsic carrier concentration of a material can be written as n_i equals to this expression. So, that is also important thing that if you know the density of states in the conduction band and valence band and if you know the band gaps. So, for a particular temperature you can calculate what is the intrinsic carrier concentration of the material right and you see that I have shown you some values of the intrinsic carrier concentration in silicon it is 1.45×10^{10} per centimetre cube inverse for germanium it is 2.4×10^{13} per centimetre cube inverse.

For gallium arsenide it is 1.79×10^6 per centimetre cube inverse and all are at room temperature of 300 K because it is very much dependent on the temperature it is very much dependent on the temperature.

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$$\begin{aligned}
 n_0 &= N_c e^{-\left(\frac{E_c - E_F}{kT}\right)} & n_i &= N_c e^{-\left(\frac{E_c - E_i}{kT}\right)} \\
 p_0 &= N_v e^{-\left(\frac{E_F - E_v}{kT}\right)} & p_i &= N_v e^{-\left(\frac{E_i - E_v}{kT}\right)}
 \end{aligned}$$

$$\begin{aligned}
 n_0 &= n_i e^{\left(\frac{E_F - E_i}{kT}\right)} \\
 p_0 &= n_i e^{\left(\frac{E_i - E_F}{kT}\right)}
 \end{aligned}$$

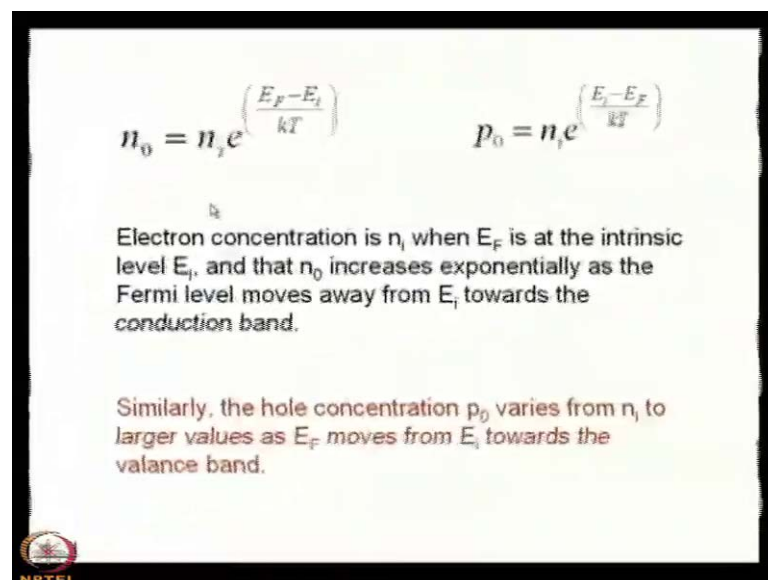
Now you see that for thermal equilibrium n_0 equals to this and p_0 equals to $N_v e$ to the power E_F minus E_v by $K T$ there is a minus sign and for intrinsic carrier concentration n_i equals to this and p_i equals to this only change is that we have replaced E_F by E_i if you put the value of N_c from this expression to here what is the value of N_c .

Yes plus E_c minus E_i by $K T$. So, you put here similarly, you put the value of N_v from here to there. So, you will find that n_0 equals to $n_i e$ to the power E_F minus E_i by $K T$ and p_0 equals to $n_i e$ to the power E_i minus E_F by $K T$ right. So, this is the thermal equilibrium carrier concentration in terms of the intrinsic carrier concentration.

And intrinsic level why it is very why we are manipulated to such extent it is because intrinsic carrier concentration is known to us that is the standard value and you will find the intrinsic carrier concentration in the literature also you need not to calculate if I give you the numerical example to solve in the exam I shall supply you the value of n_i because it is a constant like Planck's constant like Boltzmann constant it is a constant value for us.

So, only thing is that if you know E_F you can calculate n_0 or if you know n_0 you can calculate E_F . So, that is why these expressions are very important expressions for semiconductor statistics in semiconductor physics we shall use this expression very widely.

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$$n_0 = n_i e^{\left(\frac{E_F - E_i}{kT}\right)} \quad p_0 = n_i e^{\left(\frac{E_i - E_F}{kT}\right)}$$

Electron concentration is n_i when E_F is at the intrinsic level E_i , and that n_0 increases exponentially as the Fermi level moves away from E_i towards the conduction band.

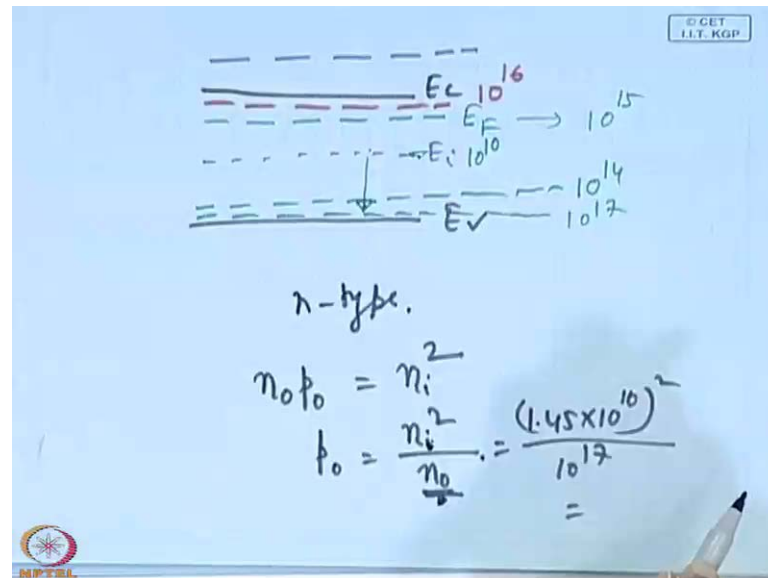
Similarly, the hole concentration p_0 varies from n_i to larger values as E_F moves from E_i towards the valence band.

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So, n_0 equals to $n_i e$ to the power E_F minus E_i by kT , you see that this is the expression for n_0 and this is the expression for p_0 here, I have again written those two values. So, electron concentration is n_i ; when E_F is at the intrinsic level E_i ; that means, if the value of E_F is E_i if you replace E_F by E_i then what will happen E to the power 0; that means, this term will be 1.

So, n_0 and n_i will be equal; that means, the electron concentration is n_i ; when E_F is at the intrinsic level E_i and that n_0 increases exponentially as the Fermi level moves away from E_i towards the conduction band.

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Let me explain this thing this is your conduction bandage, this is your valence bandage, and this is our intrinsic level E_i , E_i is at almost at the midway between E_c and E_v when this E_F equals to E_i when E_F is E_i then you see that n_0 equals to n_i . Now, n_0 increases exponentially as the Fermi level moves away from E_i towards the conduction band; that means, if it moves from E_i to the conduction band, conduction band means towards the E_c suppose the Fermi level is now here it is Fermi level so, that means, Fermi level is moving towards E_c the value will be how much? n_0 will be given by n_i into e to the power this; that means, exponentially increase it is plus e to the power plus something; that means, it will exponentially increase from n_i value. So, that is very important thing. So, more and more electron will be there in the system E_F will moves towards more and more close to the conduction band. So, how much n is there? How much doping has been done? What is the electron concentration? what is the hole concentration from the position of the Fermi level? We can physically think over the situation.

If Fermi level is at E_i ; that means, it is the intrinsic carrier concentration, if Fermi level moves from E_i towards E_c ; that means, number of electron increases here number of electron is say 10 to the power 15 . Example, if Fermi level moves to this position;

obviously, the concentration will be 10^{10} to the power more than 10^{15} it may be 10^{16} , 10^{17} and you if you go on increasing the concentration then what will happen non degenerate semiconductor will be changed into degenerate semiconductor. Remember up to 10^{18} we have seen for silicon then up to 10^{18} it is ok, but if it exceeds it is degenerate semiconductor and Fermi level will move inside the conduction band Fermi level will move inside the conduction band there the Fermi level will be for degenerate semiconductor Fermi level can move here also.

So, the position of Fermi level is very much important for us where it is, if it is at E_i it is intrinsic, if it moves from E_i towards the bottom of the conduction band gradually; that means, the carrier concentration increases gradually and if at any instant of time it moves inside the conduction band; that means, the material is degenerate. So, that is the concept similarly, the hole concentration p_0 varies from n_i to larger values as E_F moves from E_i towards the valence band for holes you see that at E_i it is, but if the hole concentration increases; that means, it will come towards the E_v , E_v bottom of the conduction band it will move towards the conduction bandage.

What happens for the conduction band just the reverse will happen for the valence band. We have seen that the Fermi level move from E_i towards the bottom of the conduction band here we see the Fermi level moves from E_i towards the top of the valence band. valence bandage right it moves here. So, if the electron concentration here is 10^{10} to the power 10 what is hole concentration what should be the hole concentration here.

Increases hole concentration increases here it is say 10^{10} to the power 10 here; obviously, it will be 10^{14} , 10^{15} , 10^{16} . If it comes closer it is say 10^{17} , 10^{18} etcetera. If you go on adding the impurity the Fermi level will go down. So, it should reach inside the valence band for degenerate in that case the materials become degenerate.

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$$n_i = \sqrt{N_c N_v} \cdot e^{-\left(\frac{E_g}{2kT}\right)}$$

$$N_c = 2 \left(\frac{2\pi m_n^* kT}{h^2} \right)^{\frac{3}{2}}$$

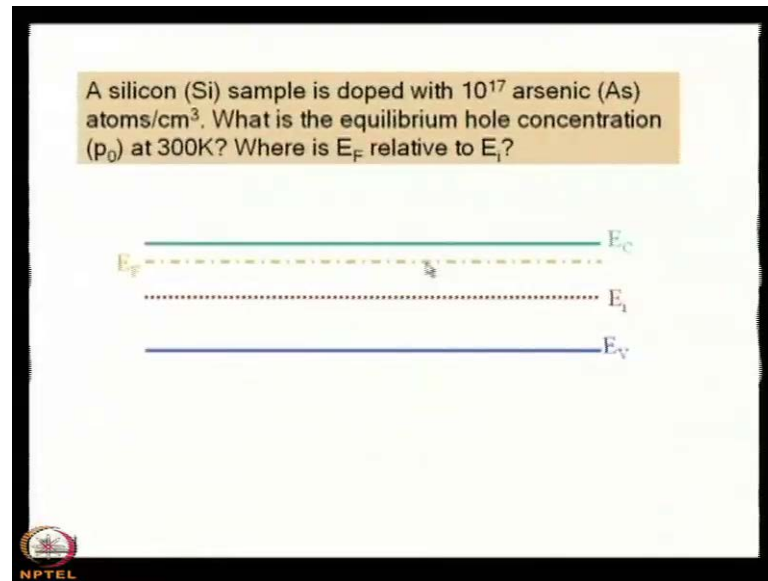
$$N_v = 2 \left(\frac{2\pi m_p^* kT}{h^2} \right)^{\frac{3}{2}}$$

$$n_i(T) = 2 \left(\frac{2\pi kT}{h^2} \right)^{\frac{3}{2}} (m_n^* m_p^*)^{\frac{3}{4}} e^{-\left(\frac{E_g}{2kT}\right)}$$

So, that is very important thing now another expression we have deduced n_i , n_i equals to root of N_c , N_v , E to the power minus E_g by twice kT that we have deduced and N_c , N_v you know if you put the value of N_c and N_v here if you put the value of N_c and N_v here then what you find you will find that n_i the intrinsic carrier concentration is given by this expression $2 \left(\frac{2\pi kT}{h^2} \right)^{\frac{3}{2}} (m_n^* m_p^*)^{\frac{3}{4}} e^{-\left(\frac{E_g}{2kT}\right)}$. So, it will 3 by 2 or 3 by 4 , 3 by 2 .

So, the expression is then you see that the intrinsic carrier concentration can be calculated from this expression also if you know the effective mass of the electrons and holes this is also very important relation and the second batch of students who have been performing the semiconductor experiments. Friday afternoon who are they raise your hands yes. So, for you this expression I shall show in the next day the; that means, then coming Friday that from these expression you can calculate that band gap from the temperature variation of the resistivity during the time I mentioned that I shall deduce in the theory class the expressions required that this expression n_i it is a function of temperature; obviously, this using this thing I shall show you the calculation of the band gap as a function of the temperature dependent of the resistivity.

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Now this is a small numerical example you please ready be ready with your pen and paper a silicon sample is doped with 10^{17} arsenic atoms per centimetre cube what is the equilibrium hole concentration it is denoted by p_0 at 300 k where is E_F relative to E_i where is E_F relative to E_i ; that means, 2 questions are asked, one is what is the equilibrium hole concentration? that means, I want to know p_0 at 300k and where is this E_F relative to E_i , I have mentioned this is E_C , this is E_V ; this is E_i ; and this is E_F .

Dot dash line is the E_F , dotted line is E_i , and the solid lines are E_C and E_V you tell me what is the equilibrium hole concentration and where the E_F will lie relative to E_i see this thing that means, this numerical example it is basically you see that the material is n type why the material is n type, yes because we have doped the material with 10^{17} arsenic atom. So, as soon as you dope the material with 10^{17} arsenic atom it becomes n type material, but what is the concentration of holes in that n type material you calculate n_0 , p_0 equals to n_i^2/n_0 , p_0 equals to n_i^2/n_0 . What is the value of n_0 ? It is 1.25×10^{17} .

Why because 10^{17} arsenic atoms have been used for the doping and we consider that all the atoms are ionized if all the atoms are ionized then what will happen you will get 10^{17} electrons out of 10^{17} arsenic atom. So, your carrier concentration n_0 will be 10^{17} . So, put n_0 equals to 10^{17} you see put n_0 in 10^{17} and n_i , what is the value of n_i ? This is

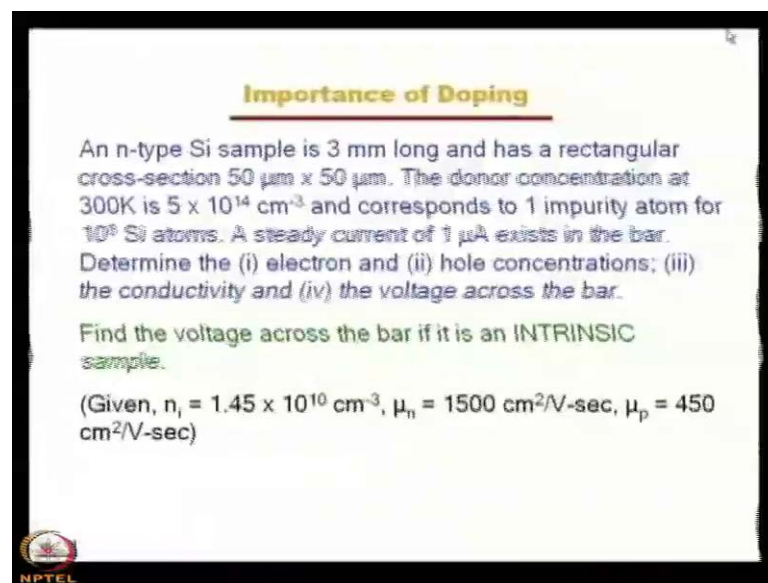
silicon the n_i value is 1.45×10^{10} to the power 10 centimetre cube inverse it will be given in the exam do not worry it will be given in the exam.

So, there will be no difficulty for all the students to answer we expect that value to be remembered by the students of with semiconductor specialization or semiconductor background we expect, but, for other students we shall supply the value right. So, n_i is equals to one point four five into 10 to the power 10 square divided by 10 to the power 17 10 to the power 17 then what is the value. 2.1×10^{32} .

Student: 2.22×10^3 into 10 to the power of 3.

2.1×10^3 . So, that is the value. So, you see that even if the sample is doped with n type there is hole concentration there is hole concentration right.

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Importance of Doping

An n-type Si sample is 3 mm long and has a rectangular cross-section $50 \mu\text{m} \times 50 \mu\text{m}$. The donor concentration at 300K is $5 \times 10^{14} \text{ cm}^{-3}$ and corresponds to 1 impurity atom for 10^8 Si atoms. A steady current of $1 \mu\text{A}$ exists in the bar. Determine the (i) electron and (ii) hole concentrations; (iii) the conductivity and (iv) the voltage across the bar.

Find the voltage across the bar if it is an INTRINSIC sample.

(Given, $n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$, $\mu_n = 1500 \text{ cm}^2/\text{V-sec}$, $\mu_p = 450 \text{ cm}^2/\text{V-sec}$)

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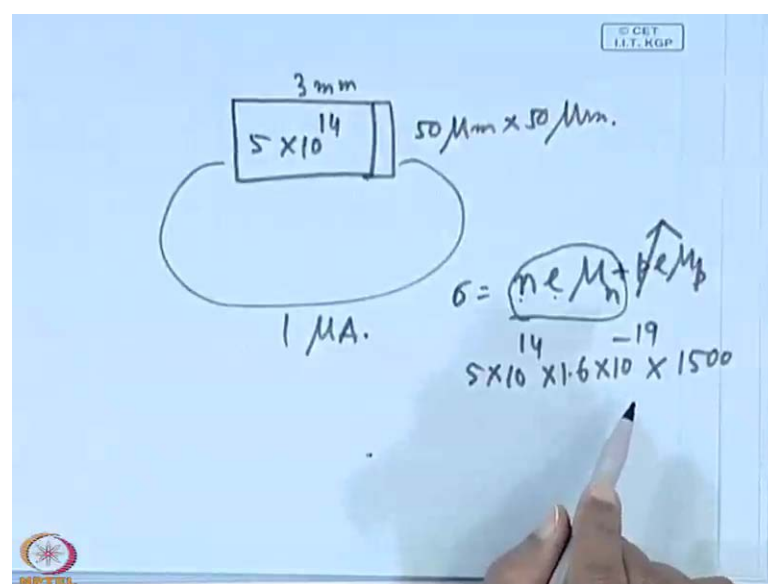
And the last topic for today's discussion is the importance of doping oh there is a problem in the screen. So, you see that with this example I shall show you the importance of doping and n type silicon sample is 3 millimetre long look at the view graph an n type silicon sample is 3 millimetre long and has a rectangular cross section 50 micron by 50 micron the donor concentration at 300k is 5 into 10 to the power 14 centimetre cube inverse and corresponds to one impurity atom for 10 to the power 8 silicon atoms.

A steady current of one micron exists in the bar determine the electron concentration, hole concentration conductivity, and voltage across the bar and find the voltage across the bar if it is an intrinsic sample, if it is an intrinsic sample find the voltage across the bar and these are given n_i is given, μ_n is given, and μ_p is given. So, there are basically 8 answers to be given at least five answers one is for the first case the sample is n type. So, you we need electron concentration hole concentration, conductivity and voltage 4.

In second case only the voltage for if the sample is intrinsic type we have given you n type. So, you just calculate and tell me what are the values, what are the electron concentration, and what is the hole concentration, who can tell, what is the electron concentration, and what is the hole concentration who can tell you tell what is the electron and what is the hole concentration.

Question is an n type silicon sample is 3 millimetre long an n type, silicon sample is 3 millimetre long and has a rectangular cross section 50 micron by 50 micron just you do not write the all those things just you write down the values length is three millimetre and cross section is length into breadth is 50 by 50 micron and carrier donor concentration is given 5×10^{14} and impurity one micro ampere currents we want.

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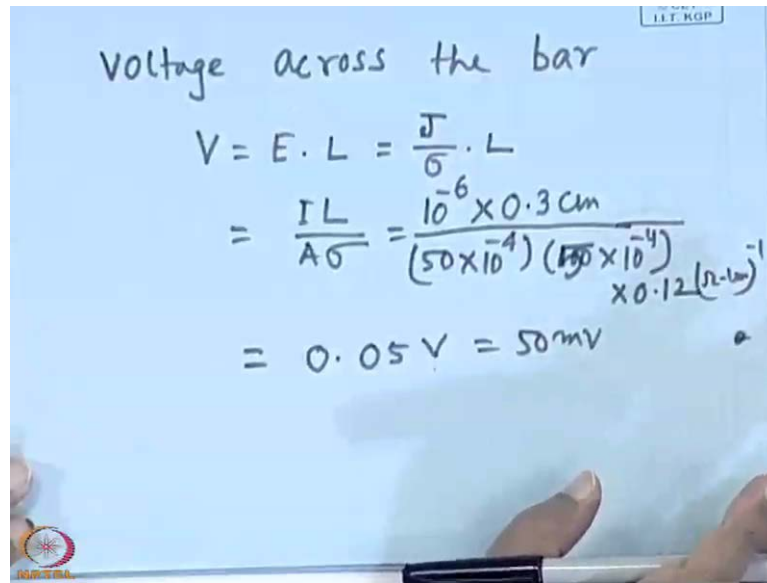


So, that is the silicon sample say this is a silicon sample its length is three millimetre and the area of cross section here you see it is length into breadth is 50 micron by 50 micron it is 5×10^{14} is the carrier concentration and 1 micron current we want from it.

One micro amp current what is the electron concentration the electron concentration is 5×10^{14} because you have doped with 5×10^{14} you have doped with 5×10^{14} . So, the electron concentration is; obviously, 5×10^{14} , remember you have doped with 5×10^{14} centimetre cube.

So, electron concentration is this what is the hole concentration what is the hole concentration 5×10^{14} and n_i^2 is given by 1.45×10^{10} square. So, you can easily calculate the value of the hole concentration, then conductivity is $\sigma = n e \mu_n + p e \mu_p$ it is μ_n , but we can neglect p why this term can be neglected this term can be neglected because, It is n type and you will find that the carrier concentration of the n type is very high compared to the p type. So, just use this expression $\sigma = n e \mu_n$ what is n ? n is 5×10^{14} e is 1.6×10^{-19} ; and μ_n is 1500. So, with this you will find that the conductivity, conductivity what is the unit of conductivity? what is the unit of conductivity? Ohm centimetre is the unit of resistivity and conductivity is a reciprocal of it is ohm centimetre inverse.

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Handwritten calculation on a whiteboard:

$$\begin{aligned}
 &\text{Voltage across the bar} \\
 &V = E \cdot L = \frac{j}{\sigma} \cdot L \\
 &= \frac{IL}{A\sigma} = \frac{10^{-6} \times 0.3 \text{ cm}}{(50 \times 10^{-4}) (100 \times 10^{-4}) \times 0.12 (\text{cm}^{-1})} \\
 &= 0.05 \text{ V} = 50 \text{ mV}
 \end{aligned}$$

Then the voltage across the bar you calculate the voltage across the bar you calculate the voltage across the bar how you will calculate? You calculate the voltage across the bar and tell me what the voltage is?

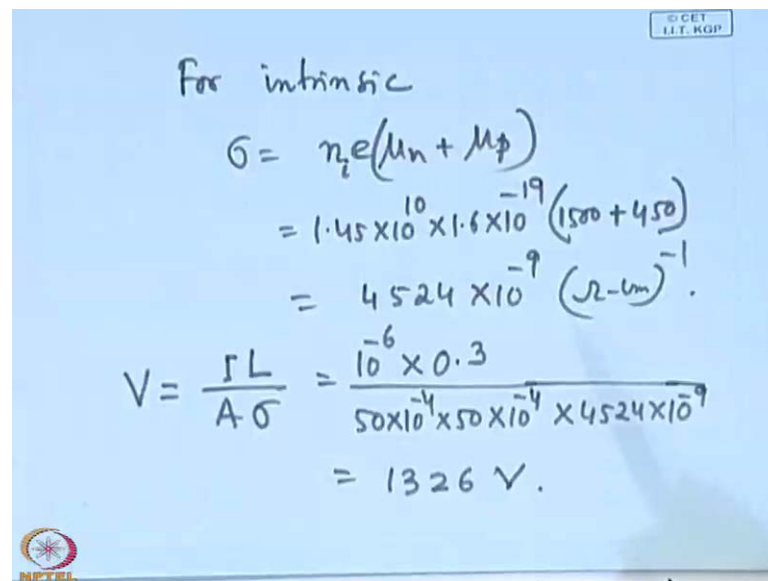
No, you calculate you calculate and tell me what is the voltage across the bar because that is very important and I shall show you the importance of doping in this case determine the voltage across the bar thus see voltage across the bar voltage across the bar v equals to E into l and E is j by σ into l . So, that is equals to Il by σ why because j is I by A now what is the value of I one micro amp. So, 10 to the power minus 6 what is l is 0.3 centimetres why it is 3 millimetre you change it into cm then what is the area of cross section 50 micro ampere. So, 50 into 10 to the power minus 4 centimetres in centimetre micron to centimetre and then 100 micro ampere 100 into 10 to the power minus 4 .

And what is the value of σ you have not calculated, but you know how to calculate the value is please write down 0.12 into 0.12 centimetre inverse centimetre inverse. So, all the values are in cgs now if you calculate this value if you calculate you will find that the value is 0.05 volt.

Student: Sir there would be 50 into 5050 units 50 into 50 why we were why it will be 50 into 50 .

Because there is (()) cross signal (()) sorry you'll find that it will be 0.05 volt or some value because I do not know you have calculated by 50 or hundred I could not remember, but, it will be 0.05 volt; that means, how much it is 50 mill volt point 0 five volt means right 50 mill volt very small, but, if it is an intrinsic material if it is an intrinsic material then what would be the voltage everything will same except the conductivity.

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For intrinsic

$$\sigma = n_i e (\mu_n + \mu_p)$$

$$= 1.54 \times 10^{10} \times 1.6 \times 10^{-19} (1500 + 450)$$

$$= 4524 \times 10^{-9} (\Omega\text{-cm})^{-1}$$

$$V = \frac{IL}{A\sigma} = \frac{10^{-6} \times 0.3}{50 \times 10^{-4} \times 50 \times 10^{-4} \times 4524 \times 10^{-9}}$$

$$= 1326 \text{ V.}$$

Everything will be same except the conductivity for intrinsic very quickly for intrinsic sigma is equals to n E mu n plus mu p it is n i value of n i is 1.54 into 10 to the power 10 E is 1.6 into 10 to the power minus 19 and mu n plus mu p is 1500 is mu n given 450 is mu p given.

So, if you calculate you will get 4524 into 10 to the power minus nine ohm centimetre inverse. So, conductivity is very low earlier it was 0.12 now it is 4524 into 10 to the power minus 9. Then what should be the voltage, voltage will be I l by a sigma I is 10 to the power minus 6, one is 0.3 a is 15 into 10 to the power minus 4 multiplied by 50 into 10 to the power minus 4 and then sigma will be 4524 into 10 to the power minus 9 you will find that it is almost 1326 volt 1326 volt. So, what you find?

Only 50 milli volt was required to get one micro amp current now you need 1326 volt to get one micro amp current what is the difference the difference was this was intrinsic the second case was the intrinsic and the first case was the doped material we have doped

with arsenic say with 10 to the power 5 into 10 to the power 14 the carrier concentration is 5 into 10 the power 14 , but, the intrinsic carrier concentration is 10 to the power 10 1.45 into 10 to the power 10 . So, you see the importance of doping is that doping can cause havoc to change the electrical conductivity doping of various minute amount is sufficient and to support 1 micro ampere of current only you need 50 millivolt for a doped semiconductor and some thirteen hundred volt for a intrinsic semiconductor of same nature. So, this is the implication of doping.

We have started from the statistics right we have derived some of the equations and after derivation we have used some expressions to obtain some of the parametric values like E_F , n_0 , p_0 , etcetera. And now you are in a comfortable position to say what should be the doping level what should be the Fermi level for a particular type of semiconductor if you know the carrier concentration etcetera. So, those things you will feel comfortable now because only doped semiconductors are used for device fabrication.

Thank you.