

Principles of Physical Metallurgy
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Lecture No. # 09
Plastic Deformation of Pure Metal (Contd.)

Good morning, we continue the lecture on plastic deformation of pure metal. Last class, we talked about particularly deformation elastic difference between elastic and plastic deformation, it is necessary to understand plastic deformation part of elastic deformation as well to understand plastic deformation. We looked at the difference between the two. Then, we also looked at the concept of an isotropy, why material property mechanical property of material, they are different along different crystal directions. The origin lies in their crystal structure.

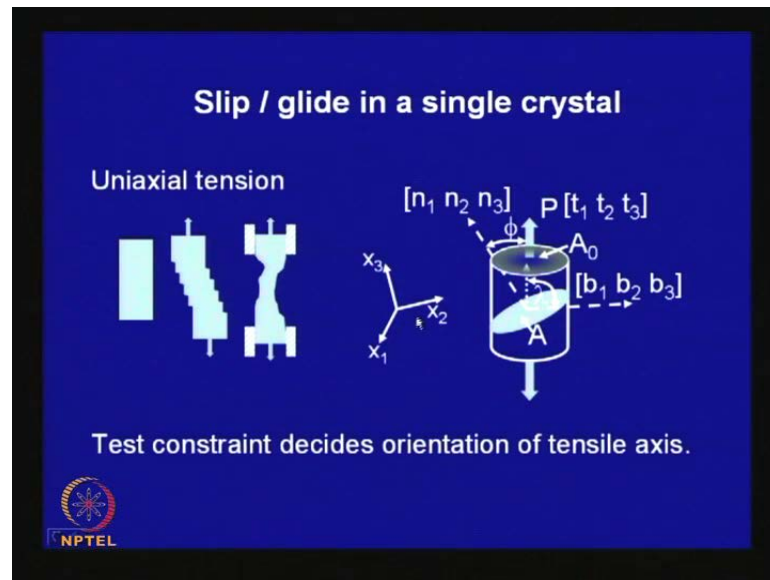
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We also looked at deformation of single crystal, how a crystal deforms and there are two primary mechanism; one is slip, another is twin. And we dealt at length to explain the difference between the two. Slip process is quite extensive and bulk of the deformation, plastic deformation in metals takes place through slip. Twin also is sometime necessary to supply additional mode of deformation, particularly in hexagonal crystal within the twin, extent of deformation is very high, and within this the magnitude of deformation within the twin is very high. But they are, they occur in isolated region of the material.

Now, we continue from there onwards. Today, we will talk about in greater detail about the mechanism of slip will introduce the concept of critical resolved shear stress; look at single crystal tensile test behavior of face centered cubic crystal. We will also look at to a rough calculation what should be the ideal strength of crystal.

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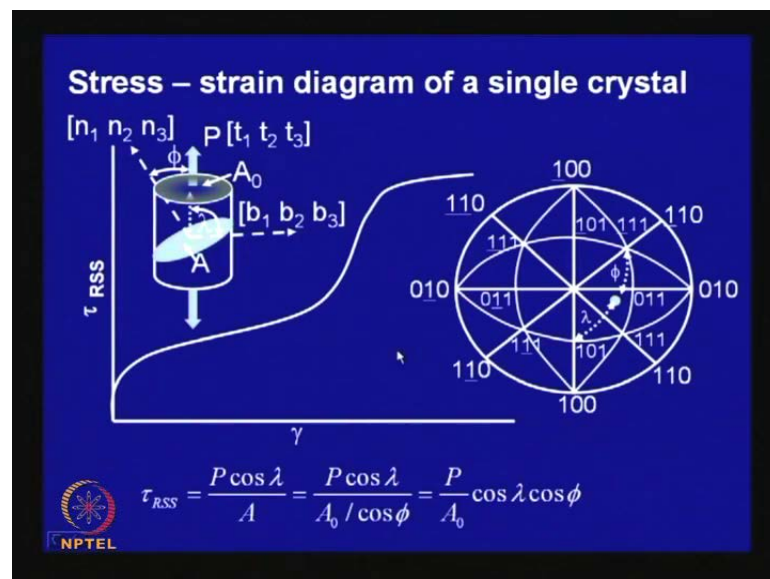
Now, large deformation can take place through the mechanism of slip. It also sometime referred to as glide. Now, the process of slip if it is allowed to take place, without any constraint is shown over here. Say, suppose this is a sample and we try to pull it in tension. When the tensile pull is large enough to cause plastic deformation, then what happens? The part of the crystal slips over another and which is pictorially shown over here. The several places, this is the slip plane; slip has taken place in this direction and in the process what happens? The direction along which, the load has been applied; they shift. But when you do an actual tensile test, you will keep try to keep these two along the same line something like this.

So, this puts a constraint so that means, this tensile axis cannot shift. So, then only way it can take place is, this part of the crystal will get distorted and on this surface, there is no restriction. So, what happens in reality is shown over here. Say, suppose this is a crystal and we have seen that slip can take place only on certain specific plane along specific direction. Now, this is a crystal here. We are applying a pull force P along a particular crystal axis say particular direction of the crystal and the crystal axis is shown over here

x_1, x_2, x_3 . Let us say, this is a slip plane and usually the plane is usually represented by its plane normal. Let us say, this is the plane normal and n_1, n_2, n_3 ; this represents actually the unit vector along, unit vector representing the normal to the slip plane A .

Let us say it is area, initial area is capital A and this cross section of this specimen is A_0 . This plane normal subtends an angle ϕ with respect to the direction of the tensile pull. The vector, this vector represents unit vector or unit slip vector. So that means, this is the slip direction and this direction also subtends an angle, λ with respect to the direction of pull. Now, since this test constraint will decide actually. Once this slip takes place, what will happen? This direction with respect to the laboratory, this direction remains unchanged. So, that means this has to that indices of the tensile axis will keep on changing. The laboratory reference system will not change; the crystal reference system will not change.

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And then, let us see what happens? Now, since the process of slip takes place, I mean mechanism of deformation is through slip. It is like a part of the crystal slipping over the other, like this. So, here even if you applying on the sample tensile pull, even if you are trying to pull the sample like this, slip will take place on a plane which is inclined to the tensile direction. And in the process basically, in that case in order to decide whether the slip will take place or not. You should try and find out what is the stress, shear stress acting on this particular plane? And this is shown over here.

Even if you apply tensile pull, you have to resolve it on to the slip plane, along the slip direction. How do you do it? Say, suppose this is the pull, P is the pull. So basically, that you try and find out its component of the force; so, one component of the force along the direction of the slip factor. So, this is P is this direction of pull and this subtends an angle λ with respect to slip vector. So, therefore the component along the slip vector is $P \cos \lambda$. So therefore, this is the shear stress along this direction and this you have to divide. So, this is the pull, along this direction. To calculate the stress, you have to divide by this cross section area. Now, what is this?

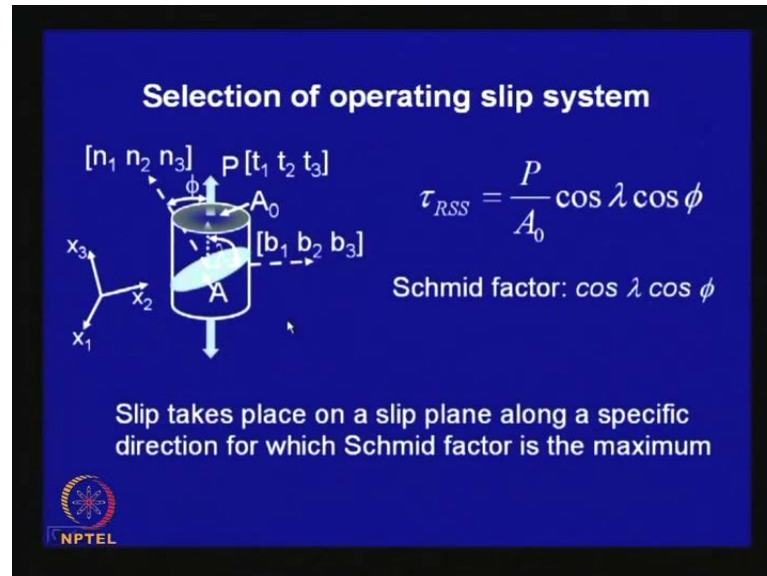
How do you represent this cross section area A ? Now, this you can see this cross section area. So, basically A_0 is this. So, direction of A_0 is this; which is coincident with the direction P . So, therefore and this angle between this plane normal, normal to plane A_0 subtends an angle ϕ with respect to the slip plane normal. So, therefore this normal this will be $A_0 \cos \phi$. This area will be $A_0 \cos \phi$. Essentially, this area which is inclined it is **it is** going to be more than this. So that and which is reflected over here. Therefore, the resolved shear stress is equal to P over original area times $\cos \phi \cos \lambda$.

This factor is called the orientation factor, orientation factor of this crystal and it is also commonly known as mean factor. Now so, this is the resolved shear stress and here is a diagram of this resolved shear stress along with against shear strain. Now, shear displacement say if you make this plot, you find the initially there is some amount of elastic deformation will be there which is recoverable. But thereafter when this stress exceeds a critical value, then the slip deformation starts and which is shown over here. And this is a period of you can possibly ignore this; even it can be straight like this going up and straight like this. Let us assume it. That means, what happens during this period **period**?

There is this slip keeps taking place and this stage is called stage 1, when slip takes place on only one plane. We have introduced the concept of standard projection. Let us say, how do we represent the same thing on the standard projection? Now, here is 001 standard projection of a cubic crystal. So, this point is 001 and this tensile axis, I have just locating it here. Suppose, this represents this tensile axis and this subtends an angle λ with respect to the slip direction in face centered cubic crystal. The face diagonal which has in this is like 110 . So, that means 101 . Let us say, this is the slip direction

and this angle is lambda and the angle with respect to the slip plane **slip plane** is 1 bar 1 1. This underscore means 1 bar; 1 bar 1 1. So, this is the slip plane.

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Now, let us see what happens? As I said, this as the deformation proceeds, the location of this will no longer be here; it will change. We will see how do we calculate or how do we estimate this? Now, how do we decide which slip system will be operating? Say, as we have seen that face centered cubic crystal has 12 possible slip systems. They are the different combination of slip plane and slip directions. In a face centered cubic crystal, there are 4 octahedral planes; that is 1 1 1 type. They are all identical and each plane you have three directions, along which slip can take place. So, there are 12 slip systems. Now, out of these which will be operating?

Obviously, the one on which the schmid factor is maximum, because if you **you** we have seen that the resolved shear stress; the deformation takes place. When this resolved shear stress reaches a critical value, which is very much similar to each stress. But this is the shear component of the each stress. So, now obviously P and A naught is constant. P keeps changing; now, this depending on its magnitude. Resolved shear stress will have different value on different slip system. One which has the maximum value for cos lambda cos phi, the schmid factor that is the one on which will slip take place first and it will continue to slip on the same slip system. So, that means this is **the** where the slip takes place only on one slip plane along a particular slip direction.

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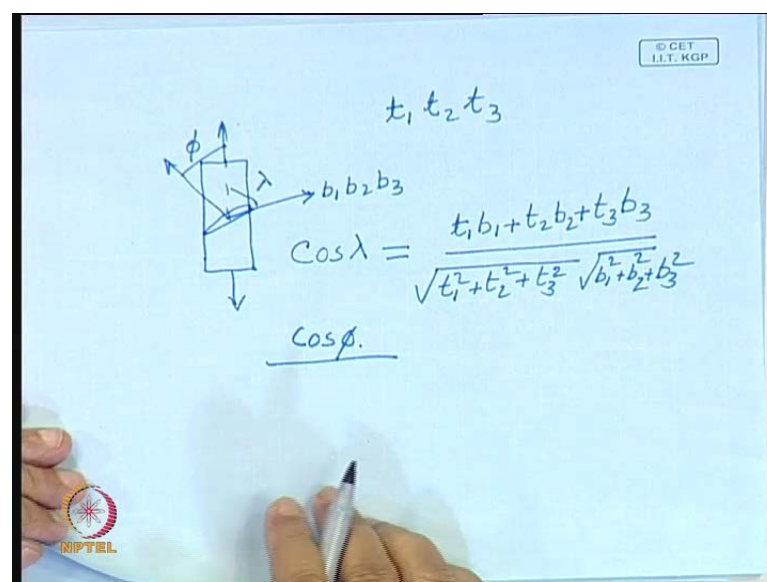
Estimation of Schmid factor for a given tensile axis for different slip system

t1	t2	t3							
1	2	3							
h	k	l	u	v	w	cos f	cos l	m	
1	1	1	-1	1	0	0.926	0.189	0.175	
1	1	1	-1	0	1	0.926	0.378	0.35	
1	1	1	0	-1	1	0.926	0.189	0.175	
-1	1	1	1	1	0	0.617	0.567	0.35	
-1	1	1	1	0	1	0.617	0.756	0.467	
-1	1	1	0	-1	1	0.617	0.189	0.117	
-1	-1	1	0	1	1	0	0.945	0	
-1	-1	1	1	0	1	0	0.756	0	
-1	-1	1	-1	1	0	0	0.189	0	
1	-1	1	1	1	0	0.309	0.567	0.175	
1	-1	1	-1	0	1	0.309	0.378	0.117	
1	-1	1	0	-1	1	0.309	0.189	0.058	

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Now, how do you calculate this, which is a last illustrated here. Obviously, what do you have to do? You have to look at different possibilities. Suppose there are 12 slip systems, which are listed here. Say, slip systems this will be a combination of plane h k l and direction u, v, w. So, what you have is say this is a crystal, you are pulling it in this direction.

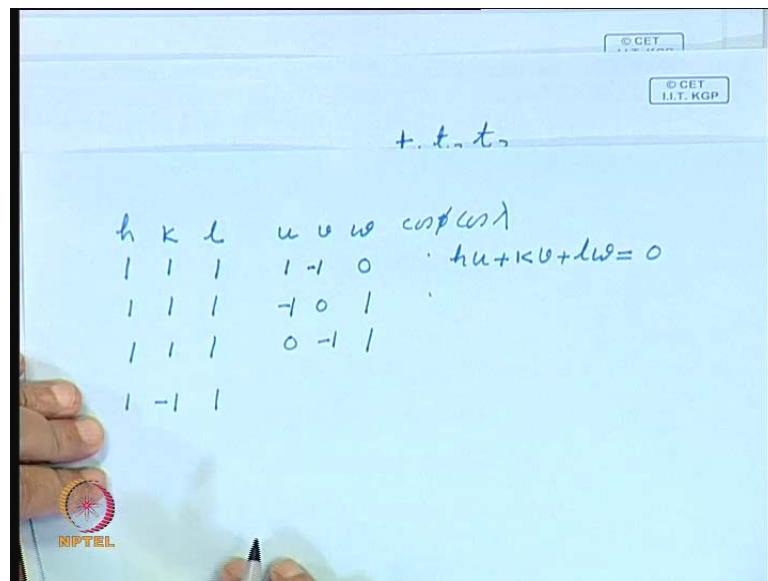
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So, this is your slip plane; this is the slip plane normal; this is angle phi; this is angle lambda. Now, what you have to do? Suppose, this axis tensile axis t 1, t 2, t 3 and this

direction, slip vector is b_1, b_2, b_3 . So, what you need to do? We have to estimate $\cos \lambda$ and $\cos \lambda$ is given by this is unit vector; this also let us say is an unit vector in this direction tensile axis. So, this will be $t_1 b_1 + t_2 b_2 + t_3 b_3$ divided by root over $t_1^2 + t_2^2 + t_3^2$. Similarly, $b_1^2 + b_2^2 + b_3^2$. Now, once you do that, see in the same way, you can also find out $\cos \phi$. So, what you need to do is, set up a table and this can easily be done on spread sheet excel. Say, suppose you see that $h k l$.

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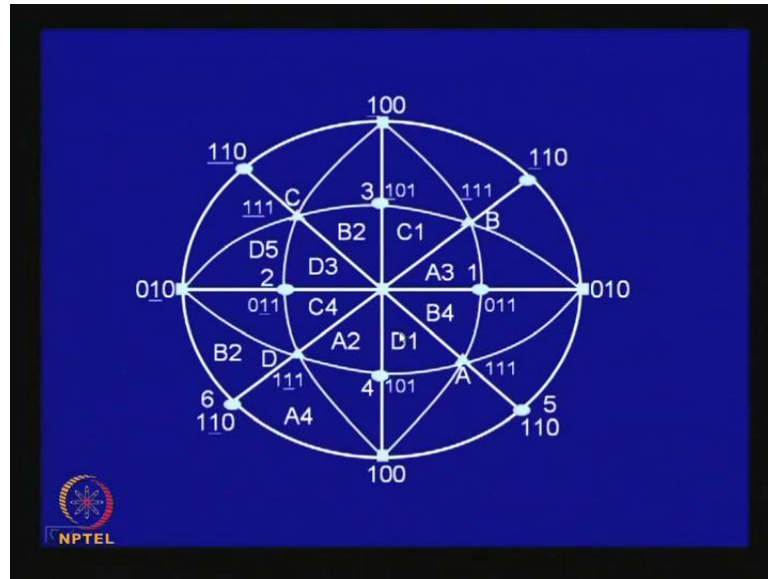


So, $h k l$ slip plane one is $1 1 1$ and you have to pick up the slip direction. Let us say the slip direction is $u v w$. Now, slip direction is $1 1 0$. Now, this must lie on the plane and to check this, it is very easy you have to do is, check whether $h u + k v + l w$ whether it is 0. Now, this is primarily because this is the plane. So, the plane normal has this direction and this is the direction lie on the plane. So, angle between the two is 90 degree. So, which is show only when, the numerator is 0. See you check this, then you find. Then, it has to be this $1 1 \bar{0}$. Then, you can easily calculate $\cos \phi \cos \lambda$ in the same way and list it for entire slip system.

Say basically $1 1 1$, this be this could be let us say $1 \bar{0} 1$; then, another $0 \bar{1} 1$. So, for slip plane $1 1 1$; there are 3 slip direction. You have 3 slip systems; repeat this for $1 \bar{1} 1$ and this is shown in the table here. Ultimately, what you find that the for this particular orientation, if tensile axis is $1 2 3$, then the plane $1 1 1$ and slip

direction $1\ 0\ 1$. This has the highest schmid factor. So, therefore slip will take place first on this slip system and it will continue to take place, until the shear stress resolved shear stress goes on increasing or until an another slip system becomes equally likely to undergo deformation. That means, until another slip plane, the schmid factor in another slip plane reaches the same value as on the previous one.

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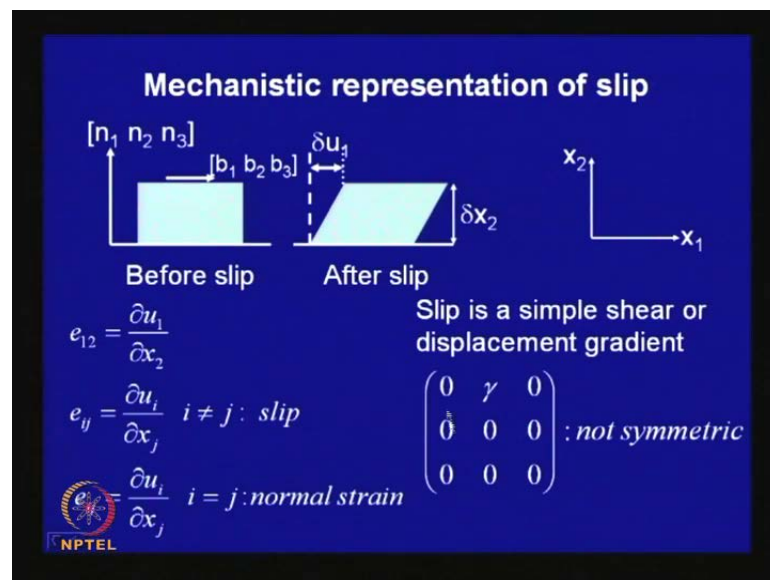
Now, this slip system it is very easy to visualize and with respect to the standard projection, it is quite easily represented. There are very simple ways of finding out which is the slip system that will operate if you know the location of the tensile axis. This is shown over here. This standard $0\ 0\ 1$ projection, now in this projection all the poles of type secured $1\ 0\ 0$, they are shown; $1\ 1\ 0$. So, these are the crystal axis $1\ 0\ 0$, $0\ 1\ 0$ and $0\ 0\ 1$; they are the crystal axis and they are axis of 4 pole symmetry. So therefore, it is represented with square and there are four $1\ 1\ 1$ type of plane which is shown here 1 2 3 4. So, they are axis of 3 poles symmetry. It is represented as a triangle.

And these are the slip plane and the slip direction of these and what I have listed is, we have name this slip plane as A, B, C, D and we have note this slip direction as 1 2 3; like this is 1; this is 2; this is 3. If the tensile axis lies within a triangle somewhere here, in that case B 4 is the slip system; that means, B is the slip plane. So, slip plane is this and slip direction is this. Now, I have listed following the same logics in some of the slip

systems. I leave it to you as an exercise to fill up, which are the possible slip system here. And this can be done, without going through the detail calculation.

But it will be a good exercise for you to find out whether say suppose pick up an orientation in this triangle and try and find out which is the operating slip system. Now, there are certain places here; say suppose here. Now, it is it can slip can take place here as well as here. So, that means when the orientation because assumption has been said that, tensile axis orientation will keep on changing with deformation. So, when it reaches here, then two slip systems will start operating, A 3 and C 1. Similarly, if the orientation is somewhere here, then all these 8 systems slip systems; they are equally likely.

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Now, let us look at some of the mechanistic representation of the process of slip. And this basic understanding is necessary to appreciate the deformation of a poly crystal as well. Now, the process of slip is shown over here. Say, suppose this is a 2 D crystal; the process of slip, it takes up the shape like this. So, a rectangle becomes a parallelogram; the area is same; there is no change in area. So, process of slip of also in 3 D, there is no change in volume. Now, this point is displaced by an amount delta u and this one represents this axis, the subscript one represents this axis.

So, along x 1 axis, this is the displacement. Now, usually shear is represented by the ratio tan of this angle. So, that makes this is this over this distance of this plane from this and this is called a displacement gradient and it is represented as e 1 2. So that means,

displacement; e_{ij} represents displacement along direction i on plane j . So, plane j you can say normal to this is plane j . So, we follow the same convention. This second suffix, if it is i, j , the second suffix represents plane; the first represents the direction. So in general, you can represent this displacement gradient in general, like this $\frac{\partial u_i}{\partial x_j}$.

Now, if i is not equal to j like over here; if it is e_{13} or if it is e_{23} or e_{12} , they all represent shear deformation. But when they are equal, i equal to j ; so that means, e_{11} and e_{22} , they represent normal strength tensile or normal compressive strength. Now, the slip is a simple shear process or you can say it has basically, let us say, it will have a displacement gradient which will be represented like this. So, this is slip take has taken place on the $1-1$ slip plane. So only one shear deformation, and the magnitude of this is γ . So, e_{12} is γ and rest of the component are 0. So, essentially this is not symmetric, but pure shear is symmetric.

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Relation between strain & displacement gradient

$$e_{ij} = \varepsilon_{ij} + \omega_{ij}$$

$$\varepsilon_{ij} = \frac{(e_{ij} + e_{ji})}{2}$$

$$\omega_{ij} = \frac{(e_{ij} - e_{ji})}{2}$$

$$\begin{pmatrix} 0 & \gamma & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \gamma/2 & 0 \\ \gamma/2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & \gamma/2 & 0 \\ -\gamma/2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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And this distinction is usually made in $((\))$ mechanics and which is shown over here. So, how do you make a distinction between displacement gradient and pure shear, which is shown over here. So, what you imagine is, suppose you rotate this by about this axis normal to this plane. So that means, that rotation axis is x_3 ; if you rotate by an amount of angles γ over 2. So, this is. So, in that case what will happen in every what we are doing; we are splitting this into two parts; this e_{ij} is sum of ε_{ij} plus ω_{ij} .

So that means, what we are adding to this strain; we are adding a rotation. We are adding to this strain, a rotation which is like this and if you add the two, you will get this. So, this any matrix, say 3 by 3 matrix, where these off diagonal element; if one is the negative of the other. This represents, you can say a rigid body rotation. So, as you fix a rotation matrix, this does not contribute to strain. It is like a rigid body simple rotation whereas, this generates strain. So, if you have to calculate strain energy, you have to find out this strain matrix.

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Tensile deformation with & without constraint

$$\bar{L} = \bar{I} + \gamma (\bar{n} \bar{I}) \bar{b}$$

$$L^2 = \bar{L} \cdot \bar{L} = \bar{I} \cdot \bar{I} + \gamma^2 (\bar{n} \bar{I})^2 \bar{b} \cdot \bar{b} + 2\gamma (\bar{n} \bar{I}) \bar{b} \cdot \bar{I}$$

$$L^2 = l_0^2 + \gamma^2 l_0^2 \cos^2 \phi_0 + 2\gamma l_0^2 \cos \phi_0 \cos \lambda_0$$

$$\gamma = \left[\left\{ \left(\frac{L}{l_0} \right)^2 - \sin^2 \lambda_0 \right\}^{\frac{1}{2}} - \cos \lambda_0 \right] / \cos \phi_0$$

Tensile axis rotates w r t crystal / lab axis

So, this it is, that physical concept is quite important in understanding the subsequent part or particularly, them to understand deformation of poly crystal. Now, let us try to find out, is it possible? Can we find out that magnitude of tensile deformation with and without constraints? Let us say. Now, how do you represent? So, basically as a result of slip or tensile deformation, this length; direction this changes. Now, here two things are happening. This magnitude is also changing and the direction of the tensile axis is also changing. So, now this is So, which is shown over here. Let us say, as a result of this deformation, **as a result of this deformation** suppose slip, the sample takes of this orientation and the constraint of testing will force this tensile axis to rotate to this position.

So, now let us try and find out if there is a small amount of slip, what is the relationship between the original length l and the final length capital L , which is shown pictorially

here. So basically, this is the final length is equal to original length plus that amount of slip. How do you find out the amount of slip? The magnitude is written here; gamma n is basically, n dot l times b. So, basically how do you find out what will be the magnitude? So, l is this and so component, the dot product l dot n times b. This is how basically if you go back here; say how do you find out this shear strain? This is the plane you can see, this is the slip plane normal is this and basically this dot product will give you; this is the slip vector and this is the plane normal.

So, this dot product will give you that magnitude of that basically, the direction and this is the direction; this is the unit vector along b. Now, if you take that l dot l; so, then this is the magnitude; l dot l is the magnitude and which is shown here and then, rest is a simple substitution. So, capital L dot L, this is L square; the magnitude L square; l small l dot l is the original length square. And this l dot n, so this is the slip plane normal. So, basically this is cos phi. **cos** So, this is the cos term. So, these are worked out here. So, what it shows that this will be the magnitude of shear strain and this tensile axis will also rotate with respect to the laboratory axis.

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Shear stress estimation

Slip plane remains undistorted. Its area is the same as the original area.

$$\tau = \frac{P \cos \lambda}{A_0 / \cos \phi_0} = \frac{P}{A_0} \cos \lambda \cos \phi_0$$

$$\cos \lambda = \frac{\bar{L} \bar{b}}{|\bar{L}|} = \frac{\bar{L} \bar{b} + \gamma (\bar{L} \bar{n}) \bar{b} \bar{b}}{L}$$

$$\cos \lambda = (\cos \lambda_0 + \gamma \cos \phi_0) \frac{l_0}{L}$$

$$\tau = \left(\frac{P}{A_0} \right) \cos \phi_0 \left\{ 1 - \left(\frac{l_0 \sin \lambda_0}{L} \right)^2 \right\}^{\frac{1}{2}}$$


Now, how do you calculate shear stress? Now, shear stress will be defined as this force along the slip direction which is P cos lambda divided by this area. So, this is that. Now, it is important how do you find out this and what important thing is you must remember that slip plane. When the slip takes place, the slip plane does not get distorted. Its area

remains exactly same as that before slip. So, that is why, when you calculate that final area, it is $A_0 \cos \phi_0$ and now $\cos \lambda$ you can easily find out $L \cdot b$ over magnitude L . So, once you go through this, you can get an estimate of shear stress. This shear stress is also a function of a part from that initial orientation. It is a function of the length of the specimen at any instant. So, as L increases, you can see there is possibility; see initially now when as L increases, this term increases. So, that means τ goes on increasing.



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Geometrical softening

$$\tau = \left(\frac{P}{A_0} \right) \cos \phi_0 \left\{ 1 - \left(\frac{l_0 \sin \lambda_0}{L} \right)^2 \right\}^{\frac{1}{2}}$$

$$\frac{d \ln \tau}{dL} = \frac{l_0^2 \sin^2 \lambda_0}{L^2 - l_0^2 \sin^2 \lambda_0}$$

As L increases shear stress increases. If there is no work hardening material would exhibit softening.

So, this means that, as the strain accumulates in the specimen; there is a possibility of some amount of softening taking place. If there is no hardening, if the material does not harden in that case, this softening will be extremely; it will be quite significant and which is shown over here. You can easily show this, if this is the expression, you take a logarithm and differential and you will find and it is possible to show the τ .

If you plot τ against strain, say suppose for different λ for λ_0 ; that means for differently oriented crystal, for certain particular orientation, you will find that this drop that softening is quite significant. And this is and this will become particularly prominent **prominent**, if you are working with a material; if you are testing a material which does not work harden. We will see later that most material, they get work harden. So, in real material you may not see this, but this is a possibility and this type of softening is called geometrical softening.

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Crystal rotation

Rotation: $r = L \times l$

$$\bar{L} = \bar{l} + \gamma (\bar{n} \cdot \bar{l}) \bar{b}$$
$$\bar{L} \times \bar{l} = \gamma l \cos \phi_0 (\bar{b} \times \bar{l})$$

Rotation axis is perpendicular to both tensile axis & slip direction

Now, we have mentioned that the crystal during deformation, it rotates. So basically, how do you find out that rotation axis?

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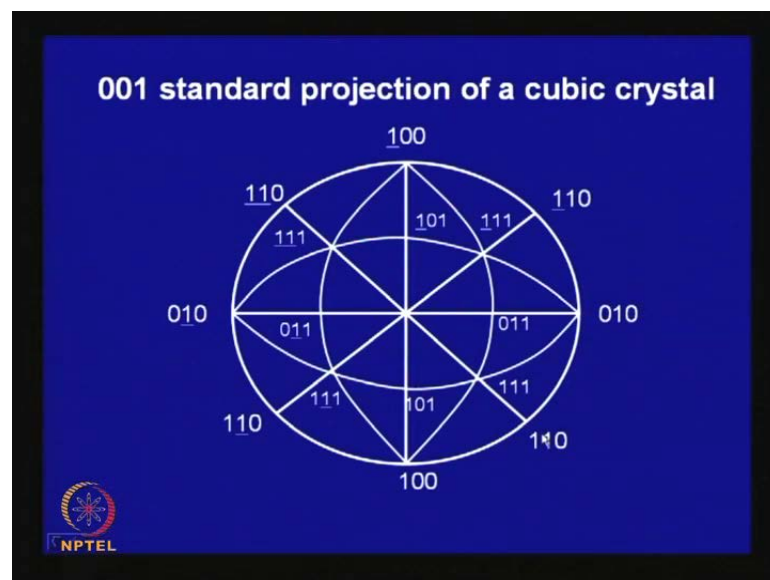
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So, if this is your initial tensile axis and this is with a little step deformation, say suppose this is the vector representing say this gets; with a slip, this get transform to this. So that means, basically this has to rotate like this to keep the alignment of the loading direction perfect. This will be forced to rotate like this; but the crystal axis will not change. The lattice axis say x_1 , x_2 , x_3 will still remain exactly same. Now, how do you find out this

rotation? Now, rotation vector is the cross product. So, basically $L \times l$. So, this is the rotation vector which is shown here. If you go through this, what you find is the rotation axis; you can say $b \times l$.

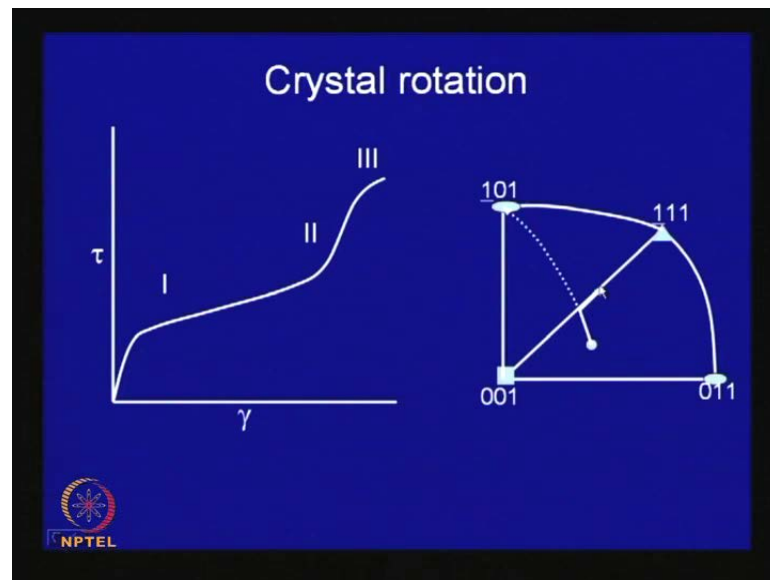
So, that means rotation axis is perpendicular to slip vector and the tensile direction. Now, suppose here in this standard projection, this is the tensile direction. Now, in this case what is your slip plane which is operating? Obviously, this is the slip direction and this is the slip plane; now for this, slip direction is this. Now, the rotation axis is perpendicular to this as well as this. Now, how do you determine this? So, you draw a great circle over here; So, that means a longitude and find out a pole, which is at right angle to this great circle. So, may be somewhere here and that will be normal to this, as well as this.

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We will come back to it, if we have time.

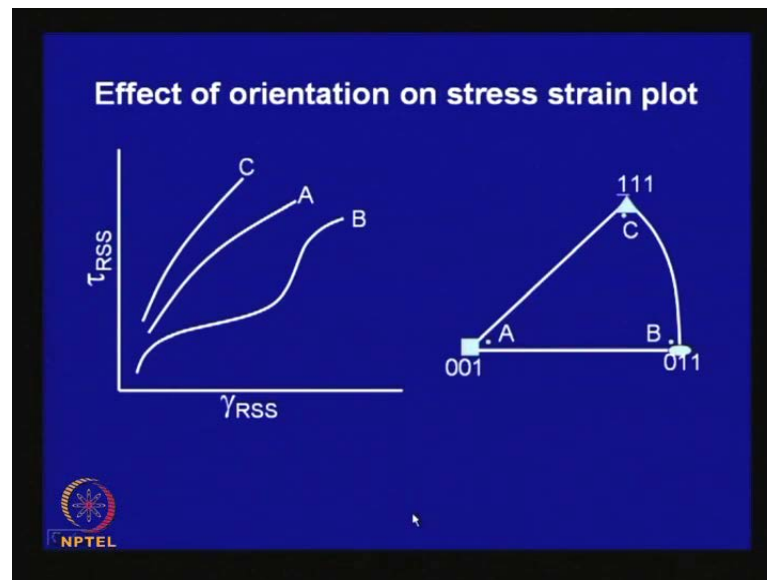
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So, now let us come back to the crystal rotation. This is the shape of stress strain diagram. This part is negligible, I mean this part is often quite small and this is the time. This is called the stage 1 deformation, when slip takes place only on one slip system and this is also called the period of easy glide. So, here this is case, where slip takes place only one slip system. The period of easy glide refers to that and this is a place, where you can say that another slip system becomes operating. So, you can say this is a 2 plane slip; another slip system is operating. So, therefore when two slip systems are operating, there is large departure.

There are suddenly, that work hardening is much more and again after certain time, if more number of slip systems are operating; in that case again, there is a slight amount of softening. We will learn about the mechanism micro mechanism of this later in detail and which is shown here. This rotation, so if this is located here, this is the slip direction. So, basically this will move on the plane containing this and this. So, this is a great circle. This will move along this great circle and when it reaches here; this is the place, when that slip will occur on the slip system, which is a mark for this. So, that means here the slip direction is this. So, this is trying to rotate like this; this is trying to rotate like this. So, this average rotation will be along this direction.

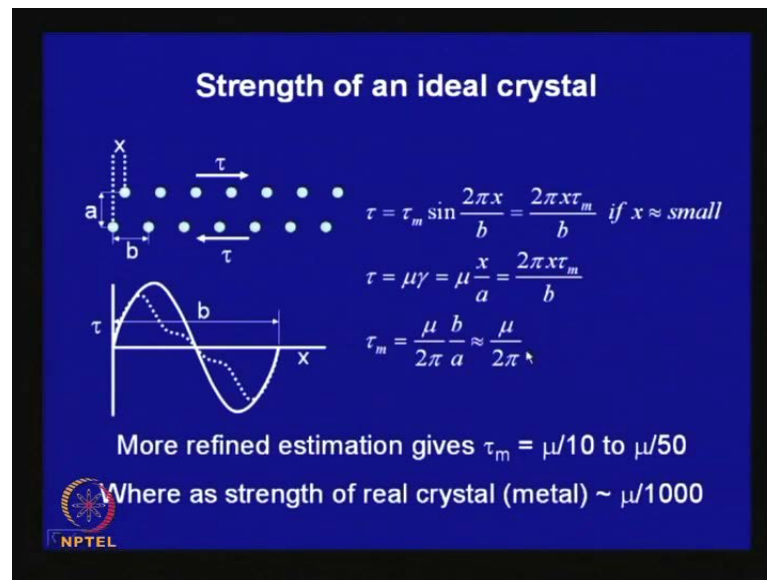
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Now, let us look at, what is the effect of orientation on the nature of stress strain, resolved shear stress strain plot which is shown over here. So, let us consider for a face centered cubic crystal, three orientations. One is close to this point, which is near cube orientation; another is somewhere here, which is close to 0 1 1 and another which is close to 1 1 1; not exactly 1 1 1. Now, if it is here, then this will try to rotate your slip system; it will try to rotate this direction towards the slip vector which is somewhere here. So, this is tried to rotate.

So, there will be obviously a large period during which a large extent of strain; during which, slip will occur only on single slip systems. So, B has a prominent period of easy glide and then, once it reaches here; then slip starts operating on two slip systems. So, here you get three distinct stages; whereas, if it close to here with a little rotation it comes here, then it goes like this. So, similarly this is even a harder **constraint** area. So basically, the anything which is located here, that will be the strongest; that will have the maximum strength. You can say the crystal say, 1 1 1 crystal will have the higher strength. So, basically and which is reflected over here.

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Now, let us try and find out, what is the strength of an ideal crystal? The crystal we have seen; it is a periodic array of points; periodic array of atoms. That means, an each atom will have some potential field. So, when you have to move a part of the crystal over the other across the slip plane, the movement will be in a periodic stress field. So, which is shown schematically over here, suppose this is one atom here and this, it has been periodically this has been shifted like this; slightly a small amount. So, originally towards here, it has been moved here. So, this is the magnitude of the shear displacement.

x is the magnitude of the shear displacement. As a result, when you apply a stress τ and what happens; if you move it a little more some may be go beyond this half? Then automatically, it will come back; come to it will be much easier for it; to come to this position than come back. So, **so** that means that shear stress will have some kind of an approximate representation. It can be represented by a periodic function which is shown here. So, this is the maximum value of stress that you need. So, this you can say this is the yield stress or critical resolved shear stress. So, τ_m is the critical resolved shear stress. So, this is as a periodic nature, sine twice pi x over b .

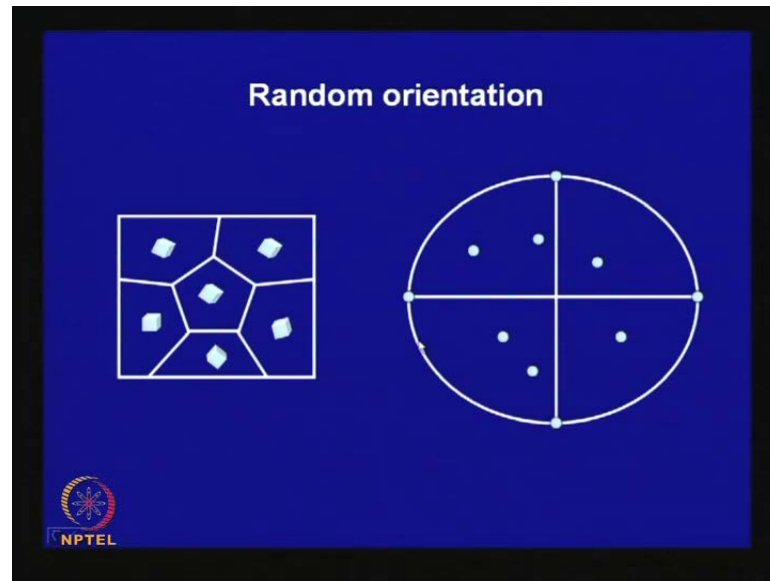
Now, for a shear displacement make a $\sin \theta$ is equal to θ . So, if x is small, we can say approximate, that τ is this and you can also calculate τ from the shear modulus. If γ is the shear strain, shear strain is x over this lattice spacing; x over a is the shear

strain. So, you can also say the tau equal to mu time x over a. Now, both represent the same thing. If you equate the two, the algebraic simplification; you get that shear stress or shear strength of the crystals is of the order of this mu over twice pi times b over a. Now, these are the lattice spacing, b is the spacing along the slip direction; a is the spacing normal to the slip plane.

And usually, if we assume they are nearly same; what we can say, the roughly that shear strength of a real crystal **of a** of an ideal crystal is mu over twice pi. But in most cases, we do not get I mean normal crystal, that we know is much softer. And people have even than more refined calculation by more realistic representation of this periodic stress field; Say, something like this. But even then also, at first you can say that the more refined calculation may give after this. So that means there is a two order of mismatch. So, strength of real metal crystals are of the order of shear modulus, over may be 2000 3000 may be it that means two order of magnitude lower than this.

Now, later on we will see why it is so? In fact, this was first shown I mean normal real crystals; they are not perfect. What we have assume that there is a perfect lattice arrangement is there; each sight is occupied. There is no internal stress field. Later on, we will see how it is I mean it has been necessary to visualize presence of crystal defects. And we will talk about the nature and types of crystal defects and with this, it is possible to explain why strength of real metal is two order of magnitude lower than the ideal estimate of strength estimated for ideal crystal.

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Now, materials that we use, they are not made up of one crystal. We have, they are made up of several crystals. And in actual in most cases, they will be oriented at random and that is why, I think in the initial part; last class also it was mentioned. If the crystals are arranged at random, in that case that if you try and calculate tensile modulus or any crystal modulus, elastic modulus and it will be independent of the direction. So that means if the crystals are randomly oriented, there is great possibility that elastic properties will be identical along all direction. Same way, if this orientation is random, the plastic behavior or deformation behavior also will be identical.

But deformation in one crystal will always take place through the process of slip at (()) and this process, a restriction. And as we have seen that to get any arbitrary strain or arbitrary change in shape of the crystal, the crystal must have 5 independent slip systems. In face centered cubic crystals, they are readily available. So, that is why, face centered cubic crystals, they are most ductile. When this poly crystal material deform, you do not expect, I mean any discontinuity forming along the grain boundary; that means you do not want any cavity to nucleate or any crack to nucleate at grain boundary. How can this happen?


Now, let us see this is a pictorial representation of random orientation. So look at piece of metal. They are all, these are different grains and each grain, they are differently oriented and it is (()) say say suppose, this crystal axis what how this crystal axis

oriented here for this particular which is shown diagrammatically here. Now, say for a case like this, say suppose in this particular case which is quite familiar; what we see that this, one of this the top planes say suppose is matching with this. So, in that case here, on the standard projection, its cube axis will be represented as this, this, this, this points; but for this, the cube axis they are tilted.


So now, for this crystal this will be arrange; they may be one here, one here, one here. There are three mutually perpendicular directions. Similarly, for another crystal let us say, this is this. Now, in a real metal, you will have innumerable number small piece; you will have innumerable number of this crystals. And if you try to plot orientation of all of these, then you will find that this entire projection is filled up by this part and which is uniformly distributed all over. So, this is means this type of arrangement of crystals is known as random orientation.

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Plastic deformation of poly crystal




Before deformation



After deformation

$$e_{ij}^{PC} = e_{ij}^{G1} = e_{ij}^{G2} = e_{ij}^{G3} = \dots$$

$$\epsilon_{ij}^{Gk} = \epsilon_{ij}^{G1} \quad \& \quad \omega_{ij}^{Gk} = \omega_{ij}^{G1}$$

$$\omega_{ij} = (\omega_{ij})_{slip} + (\omega_{ij})_{lattice}$$


Now, let us look at that, what happens during plastic deformation? So now, this is the crystal structure let us say, before deformation. Now, when you deform each crystal has different orientation. And after deformation, this takes of a shape here and knowingly there is no crack has nucleated and how can this happen? So that means, each crystals they had different orientation; but there met that displacement vector. That means, displacement vector which represents the change in shape that must be same for all crystal and which is shown mathematically here. This is the displacement vector for the

overall material; that is the poly crystal and this is the displacement vector for grain 1. So, let us say, this is grain 1 and this is the displacement vector for that is say, grain 2; they must be equal and how can this happen? Now, this displacement vector we have seen is made up of two parts.

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$$e_{ij} = \overset{\text{stm}}{\epsilon_{ij}} + \underbrace{w_{ij}}_{\text{rotation}}$$

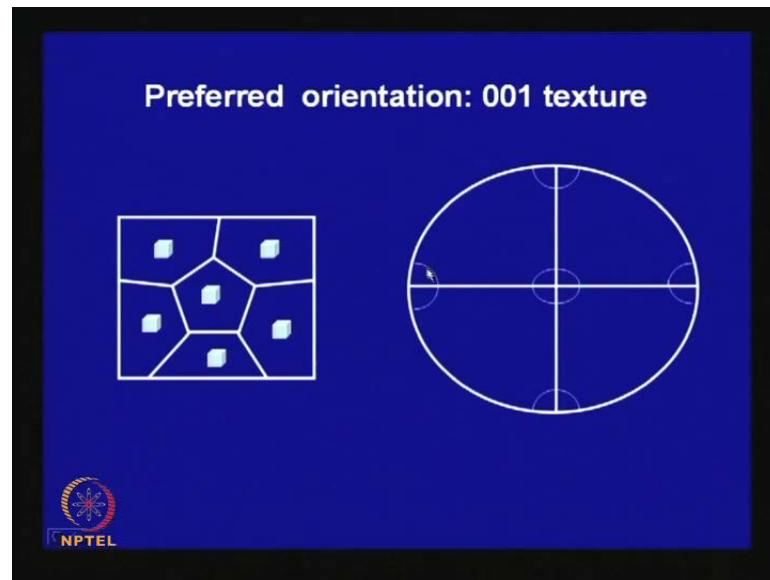
$$w_{ij} = (w_{ij})_{\text{slip}} + (w_{ij})_{\text{rot. Lattice.}}$$

Say, this displacement vector is made up of two parts. One is this is the strain; this represents change in shape plus there is a rotation vector. Now, this rotation tends this does not represent any change in shape. It is like say, this is here; this length is here; this is getting rotated; there is no change in dimension; whereas, this refers to change in dimension; this is rotation; whereas, this represents strain. So, two changes, to change the shape, this is most important. And to get any arbitrary change in shape, you can have say for the if this particular grain will pick up one set of slip system, which gives you that identical same strain as this. So, which in reality, what it shows for each grain? This strain matrix, they should be same.

Say this is for grain; say grain k and grain l; they should be exactly same this. Now, if these are same, then this also should be same for each grain. So, actually there are two things are happening. This rotation, it is made up of two components. One is the rotation because of slip; the deformation takes place because of slip and the second is rotation. This rotation is called lattice rotation. So now, this is the constraint. This rotation is rotation of the slip vector or the direction in which the stresses are applied. So, that

change in the crystal direction of those stress direction, they change; but this does not include that lattice rotation. Now, to remind that the grains are that continuous. There is no crack; there must each grain must have different lattice rotation and to get the will give a combined rotation and which is same for all grain. So that, what will happen?

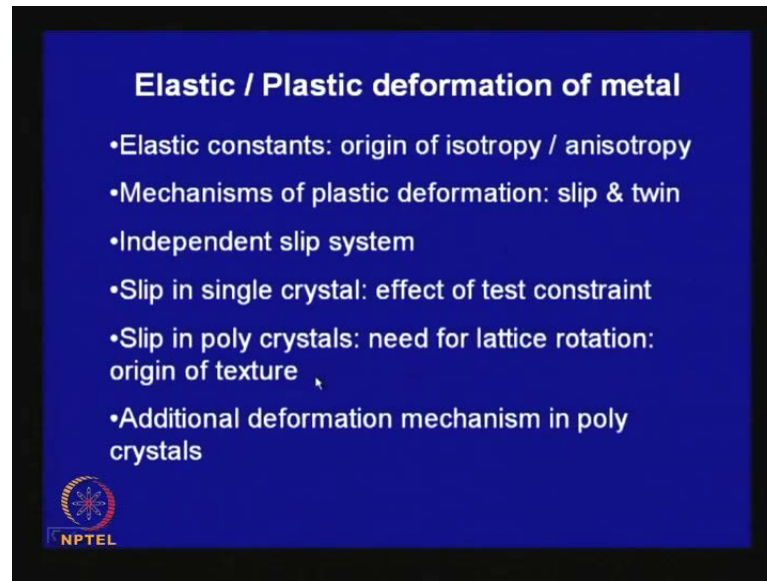
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So, this what will happen if we go back here. So, each grain this will have a lattice rotation which is different from this; an lattice rotation of this will be different from this. So that this continuity is maintained across the boundary, so this will result in development in the origin of some preferred orientation, due to deformation process and which is pictorially shown here. Now, here you see in each grain, what I have done here. All grains, all these crystals axis; they are identically oriented, which is shown like this. So, in reality, they may not be as perfect as this; but what will happen the most of this cube planes. So, they cube poles would be oriented in this particular ranging.


So this type of; so that means, plastic deformation will result in change in orientation of that grain and the deformed crystal will have a definite a particular texture characteristic of its crystal structure state of stress how you deform.

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Elastic / Plastic deformation of metal

- Elastic constants: origin of isotropy / anisotropy
- Mechanisms of plastic deformation: slip & twin
- Independent slip system
- Slip in single crystal: effect of test constraint
- Slip in poly crystals: need for lattice rotation: origin of texture
- Additional deformation mechanism in poly crystals

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Now apart from this, there can be other mode of deformation as well. Like, to maintain this continuity, it is quite possible that, grain boundaries can also move over another. But this will be determined by the characteristic nature of that grain structure; whether the grains are smooth or whether the grains are smooth means, grain boundary if it is smooth like this or whether the grain boundaries like this. In that case, if this is kind of a selected kind of thing, then the sliding will be more difficult if it is smooth. So, it depends on the nature of the grain boundary or interface and also we will see later, there is a bulk diffusion; but depending on the temperature.

If you increase the temperature, critical resolved shear stress will go down. There will be additional; there can be different another mode of deformation, which is associated with the movement of bulk movement of atoms and they consists **they consist** rather. With this, we cover the topic on plastic deformation of metals. In this, we have looked at nature does elastic constants and we tried to explain the origin of anisotropy. Why crystals are anisotropy? Is it that all crystals are anisotropy or not? We gave an example, the tungsten; even if it is a single crystal, it is isotropic.

We looked at mechanism of plastic deformation, particularly slip and twin. We looked at the difference; critically this is an extensive process. Here this is local but within that local region, there is a large amount of deformation. We also looked at independent slip system for arbitrary deformation. It is necessary that each crystal they have sufficient 5 at

least 5 number of independent slip system. Then, we looked at today slip when single crystal, we introduced the concept of critical resolved shear stress and we also have seen, how slip in poly crystal is possible?

Here lattice rotation is a must, different grains. They rotate in a two different extent and this is causes texture or this gives a deformation texture in the deform vector. And additional deformation mechanism also is possible in poly crystal. And we also looked, we got an estimate of strength of an ideal crystal and which we found is much higher than what we really get. In subsequent lecture, we will see why it is so? With this, we finish this chapter and next class, we will begin a new chapter. **Thank you.**