Nanomaterials and their Properties
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Lecture - 18
Mechanical Properties of Nanomaterials (II)

So, students, we are going to continue our discussions on Mechanical behaviour in lecture number 18.

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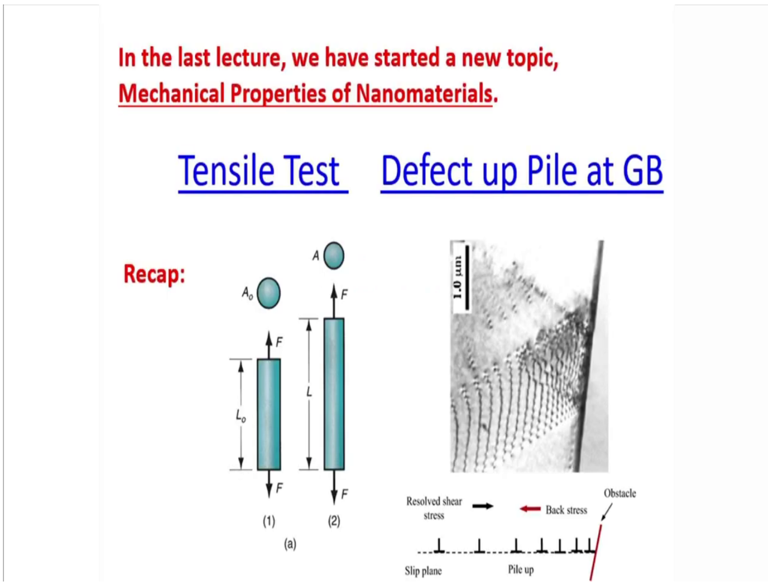

So, in the last lecture, we started this new topic. What is that? That is on Mechanical Properties of Nanomaterials. So, remember, in our syllabus, we are supposed to discuss various properties of nanomaterials and how these properties are affected by the size of nanomaterial, shape of nanomaterials. So, to give you some recap of the last lecture, we discussed about some basic aspects of mechanical behaviour right.

Mostly, on tensile test; how tensile test is to be conducted what are the separate sample specifications and the different other techniques which can augment the tensile test like hardness tests. At the end, I showed you some things about some aspects about defects in the materials because deformation majorly by plastic deformation in these cases, occurs due to generation and movement of the defects in the material and defects can be different types, we have discussed that. Mostly, we discussed about the line defect that is on the dislocation.

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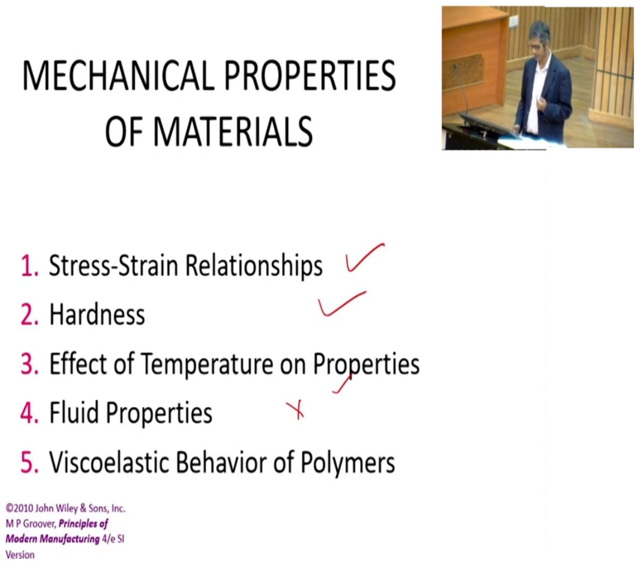

So, what all things we discussed about mechanical properties? The few things, which are important ok like stress-strain relationships that is very important, hardness provides basic mechanical properties. But we did not discuss about what is the effect of temperature, fluid properties. We have discussed little bit about the viscoelastic behavior because these are the things, which may be important.

The fluid properties and the effect of temperature on mechanical properties will not be very important in case of mechanical properties of nanomaterials. So, we have not discussed about that.

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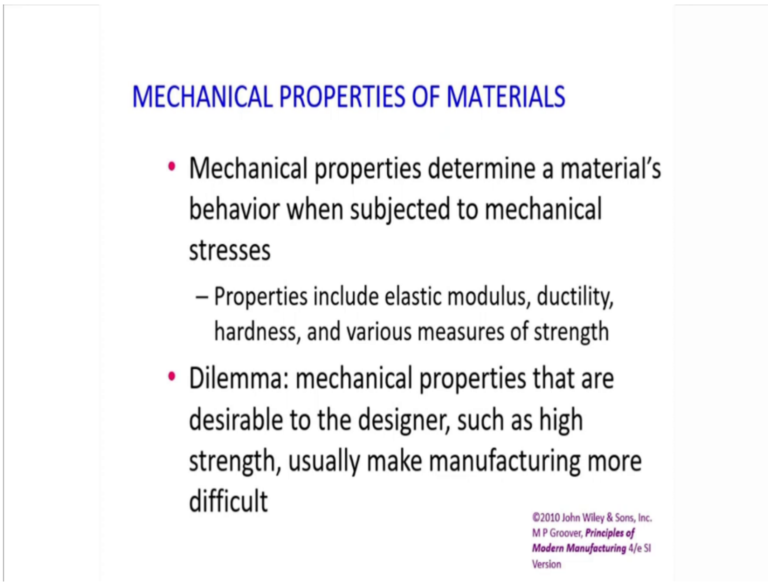

Well, and you know mechanical properties means the materials behavior, when it is subjected to some kind of mechanical stress ok. Well, we always put it mechanical stress because it can be other types of stresses also; thermal stresses. So, properties like elastic modulus, ductility, hardness and the various types of strengths as well as ductility are important for the mechanical properties and you know this is very important for designer because strength is very important for designer, but other things are also important for many other aspects.

By designer, we mean mechanical engineers or the civil engineers ok, who normally uses the material for their own basic purposes.

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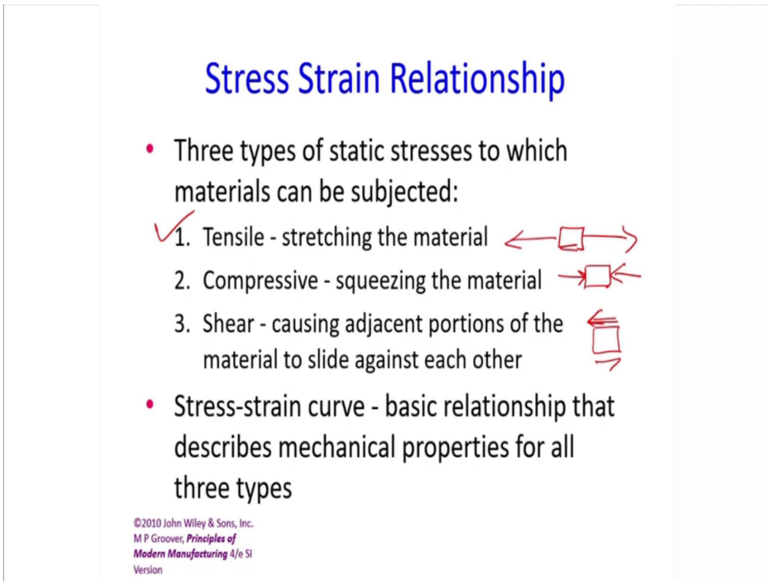

So, as far as stress strain relationship is concerned, you need to know different types of stresses; tensile, compressive, shear. In addition, of these, tensile is the most important thing. Why? Because this provides the stretching of materials; if you pull the material, if you pull this thing up, how the material will behave that is what is dictated by the tensile.

Compression means if you think of a pillar in a building and building is very tall, then the pillar is subject to compressive stress that is kind of a stressed things, which are like this. That is compressive stress ok. This is a tensile and this is the compressive by arrow, I mean the direction of loading ok.

So, this is the sample, you are pulling it up and this is your sample, if this is a sample, then you are pushing it to the sides that is compressive. So, in the building, if you have a pillar or a beam that is normally subjected to the compressive. Shear is what? When you have material like this and you are applying such a kind of stress that is called shear.

Stress strain curves are actually provides you the basic relationships, when this kind of loading is applied. Out of these three different loading as I discussed, tensile is the most important one because most of the materials fail by the tensile loading. We really find a you know failure by compressive, rarely you will find; it happens, but very rare. But compare comparatively when you pull a material breaks apart that is why the tensile is the most important way of things.

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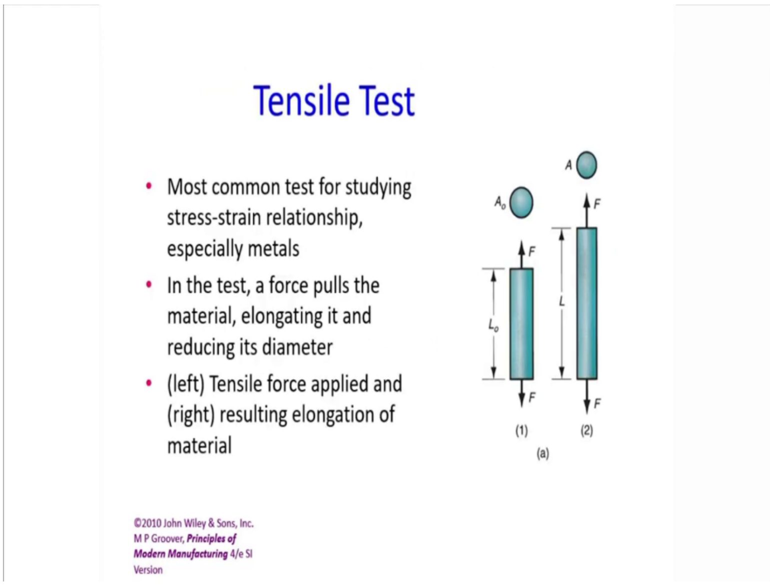

In tensile test, we use a specific sample with a particular cross section and a length and then, pull it apart by a force F and measure the elongation or the reduction of the area as well as the force per unit area, which is nothing, but stress.

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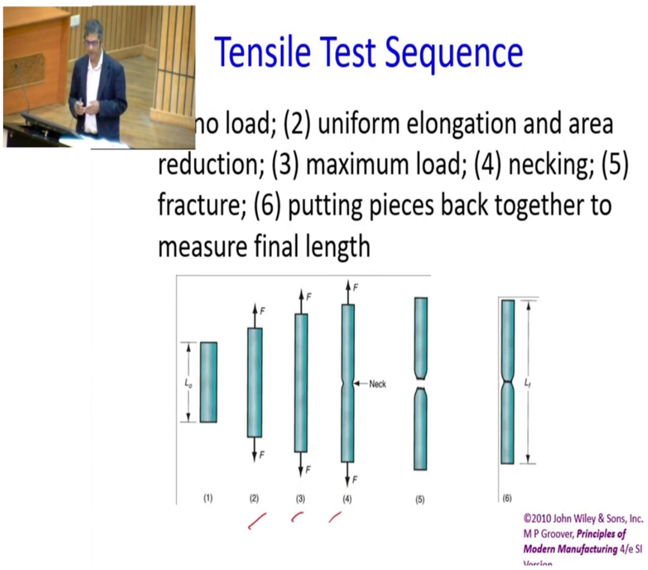

So, how do you do that? That is how it happens; you have a sample of specified length L 0. You pull it apart using a force, the length increases and initially, the deformation will be uniform. Remember by deformation, we are talking about a plastic deformation not elastic ok, that is what our basic objective is. We are no way bothered about the elastic deformation here. We are mostly bothered of plastic deformation. So, at the beginning, at you know in case of three, the deformation uniform all throughout the sample.

But after a critical load, the deformation will be concentrated in the specified regional sample that is called neck and this neck is where, the deformation is concentrated ok. And then, after some then all the subsequent deformation will be the only taking place in the neck region. Finally, the neck will grow and it will break apart ok. So, now, we can measure various properties, length and these other things also right.

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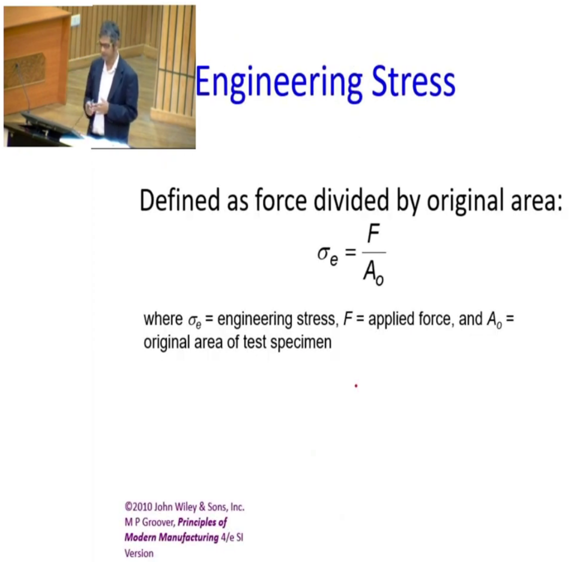

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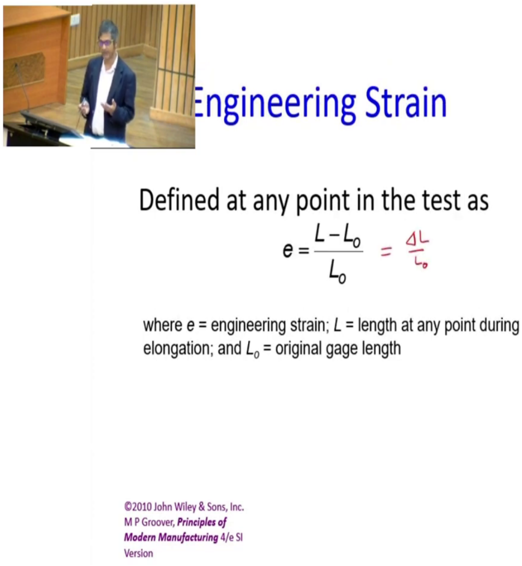

And by doing different calculations. So, this is the force divided the area that is nothing but F/A_0 that is how engineering stress is defined. Engineering strain is defined by $e = \frac{L - L_0}{L_0} = \frac{\Delta L}{L_0}$ where, L_0 is the initial length and delta L is the increase of length right.

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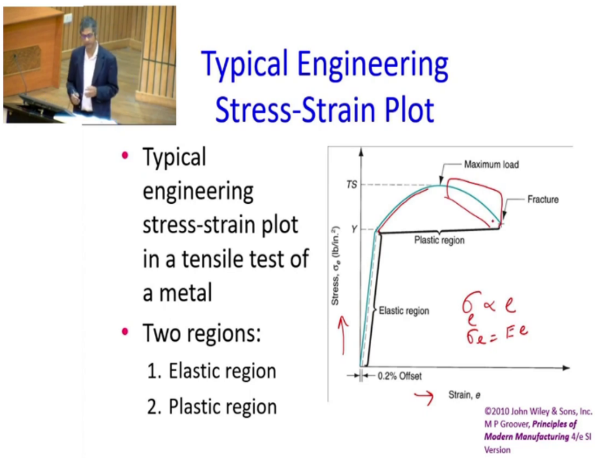

$$\sigma_e \propto e$$

$$\sigma_e = E.e$$

And then, you can have a stress strain plot which is nothing but initially of elastic part followed by a plastic part and the demarcation is given by the yield strength and then finally, you have a maximum load at which you have a ultimate tensile strength and after that, the load decreases and fracture material fractures. Load actually does not decrease; this is reflection of necking actually and I will not discuss much about details of that. So, there are two regions elastic and the plastic part.

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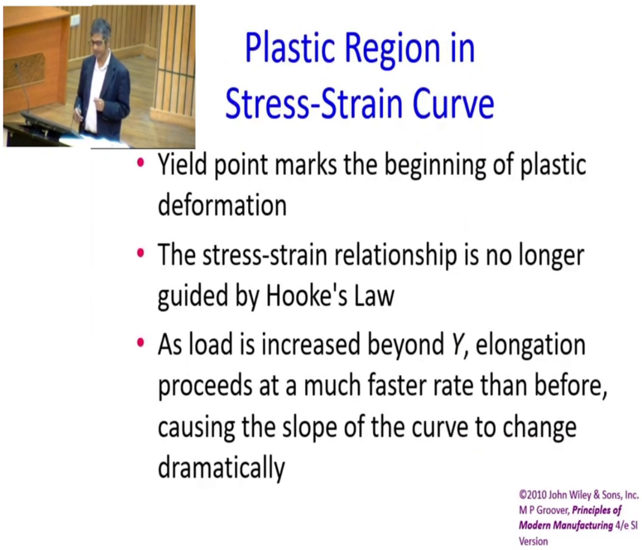

And we are mostly interested in the elastic plastic part. The elastic part is governed by Hooke's law, but the plastic part is governed by defect generation and movement right.

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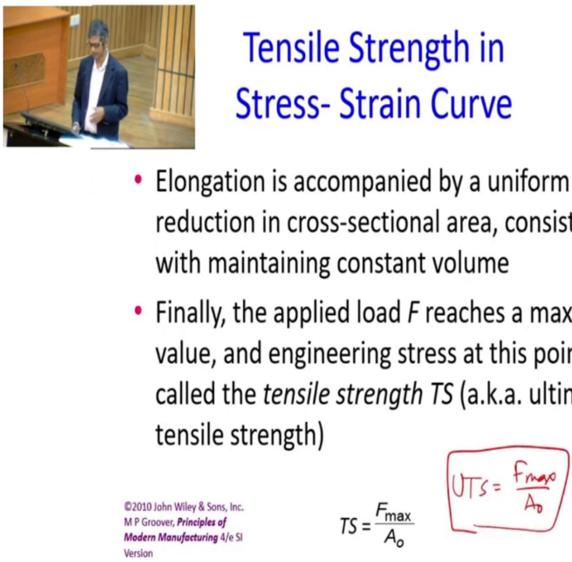

$$UTS = \frac{F_{max}}{A_0}$$

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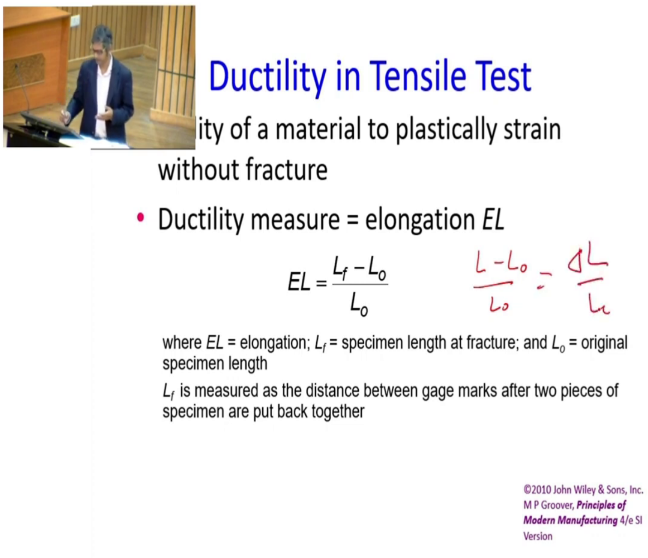

Well, that is the maximum tensile strength that is $UTS = \frac{F_{max}}{A_0}$. Ductility is always measured by change of length divided by the original length and change of length is elongation. So, delta L/L_0 that is what is the ductility right.

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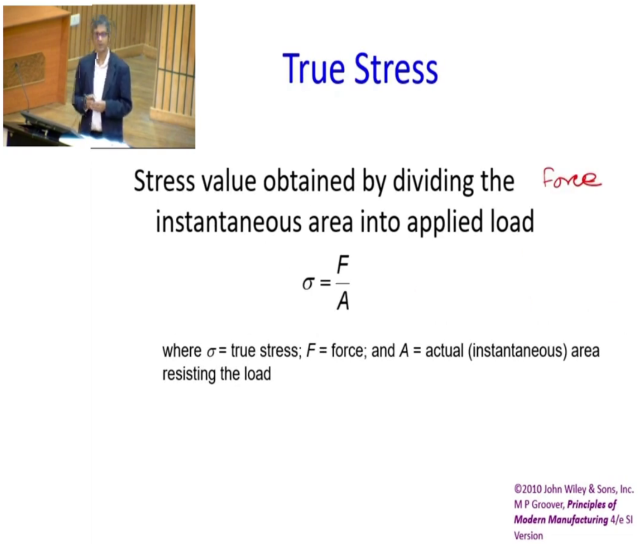

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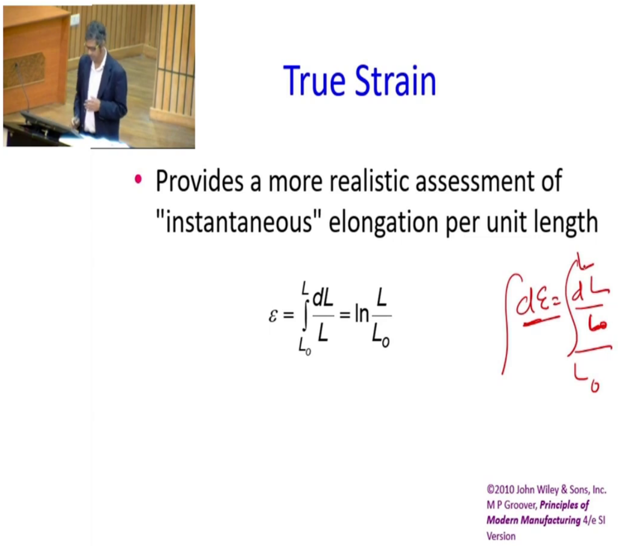

$$\varepsilon = \int_{L_0}^{L} \frac{dL}{L}$$

$$\int d\varepsilon = \int_{L_0}^{L} \frac{dL}{L} = \ln\frac{L}{L_0}$$

We also defined a true stress. True stress is defined as a load divided by the instantaneous area. As you know area is keep on changing, as a material length increases right. That is obvious because length is increasing means area is to decrease; volume is constant that is why as you increase the length, area must decrease. So, therefore, you have to reflect the change in area, we need to consider a different type of definition of the stress that is called true stress.

True stress is defined as F/A; F is the force, A is the actual area resisting the load. Similarly, you can define true strain also which is nothing but $\varepsilon = \int_{L_0}^{L} \frac{dL}{L}$ right and this is how defined? This is how these the you know instantaneous elongation per unit length divided by the actual length is integrated ok. So, if you integrate this one, you get that equation.

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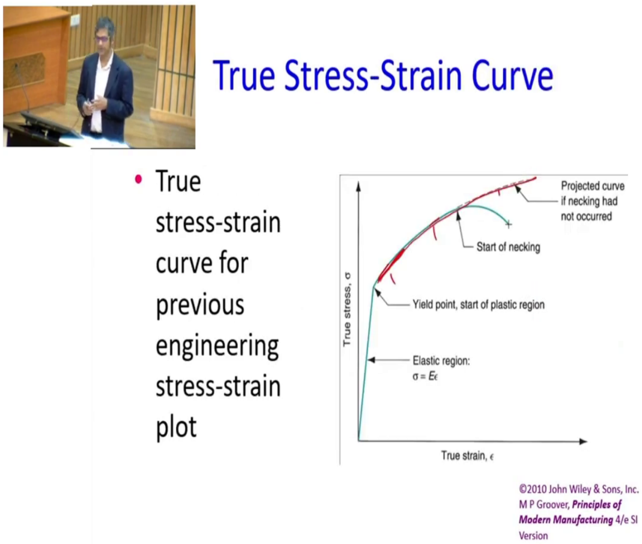

So, that is why it is $\int d\varepsilon = \int_{L_0}^{L} \frac{dL}{L} = ln\frac{L}{L_0}$ and true stress true strain curve will not show any kind of a neck thing like effect of necking on a curve that keeps on increases, you can see that and that is because you have taken care of that by using the instantaneous area.

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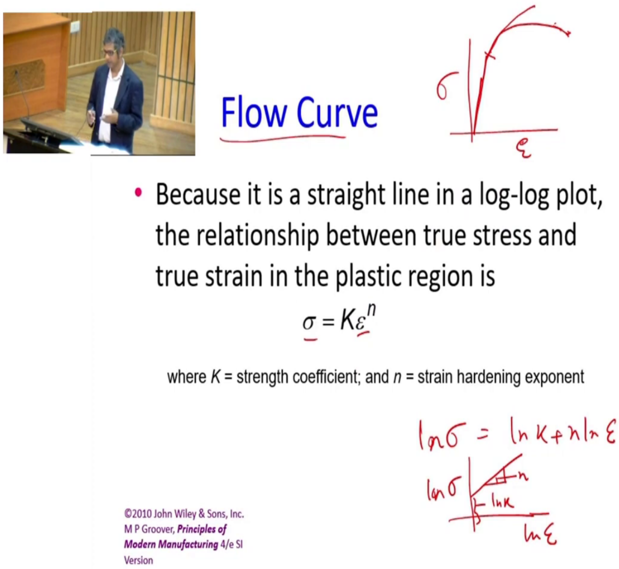

Taking logarithmic on both side

Flow curve, you know flow curve is what? Flow curve is nothing but the curve which will depict the behaviour in the plastic part and that is equation is equal to $\sigma = K\varepsilon^n$; where, sigma is a true stress, epsilon is a true strain, K is a constant known as a strength coefficient, n is the strain hardening coefficient. And if you take a log on both side, $ln\sigma = lnK + n.ln(\varepsilon)$ you can measure the slope; slope will provide you the value n, n is a strain hardening coefficient ok.

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You can always use hardness as say one of the ways to measure the materials mechanical properties because many cases, you may not have the facilities to measure the tensile strengths and you may not have the sample also to prepare a particular geometry of sample. So, in that case, is hardness is the only way to measure the mechanical properties ok.

This indicates resistance to wear plastic deformation, even scratching, you know the Mohr scale right. That is why actually scratching is can be used to measure hardness. In the Mohr scale, diamond has the highest hardness; talc has the lowest hardness. In between, you have 10 different materials present.

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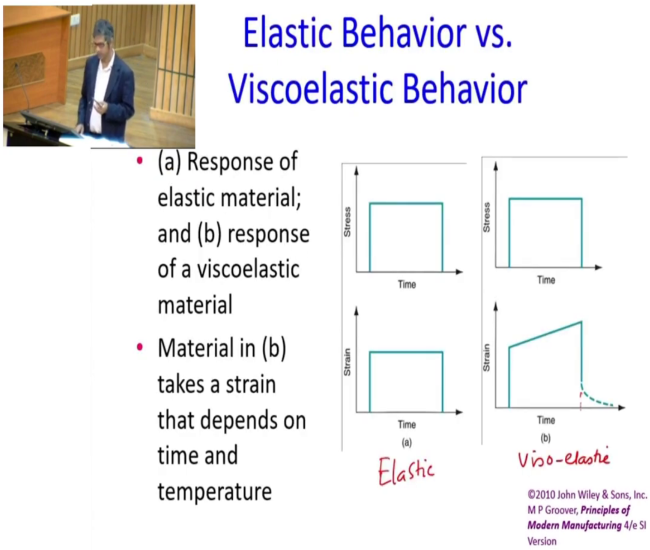

So, polymers actually so viscoelastic behavior compared to the metals. In metals, if you apply a load deform it, then keep it for some time and if you remove the load, material will come back to original shape and the size immediately, if it is deforming a plastic elastic design ok. But if it is a viscoelastic deformation, if you increase this strain, then keep it for some time and then you decrease the load, it will not come back to the original shape and size immediately.

It will never come back, it will take some time to come back to original shape and size and that is because of the viscoelastic behavior. Viscosity of the material in the polymer poly material is so high, the chains which are deform elastically. It takes some time for them to come back and that is why actually you need to know these aspects very well. So, this is a unique behavior in the polymers.

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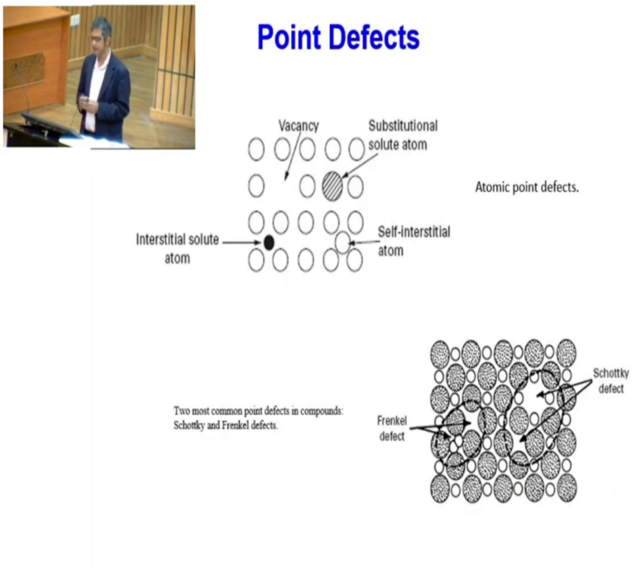

Then, we will discuss about something of the point defects, you know defects actually defects can be point defects can be vacancy or interstitial or maybe substitutions solid atoms or maybe say to self-interstitial in case of normal metallic materials. In case of ceramic or the kind of ionic material, you can have Frenkel or Schottky defects. All of you have probably read about it ok.

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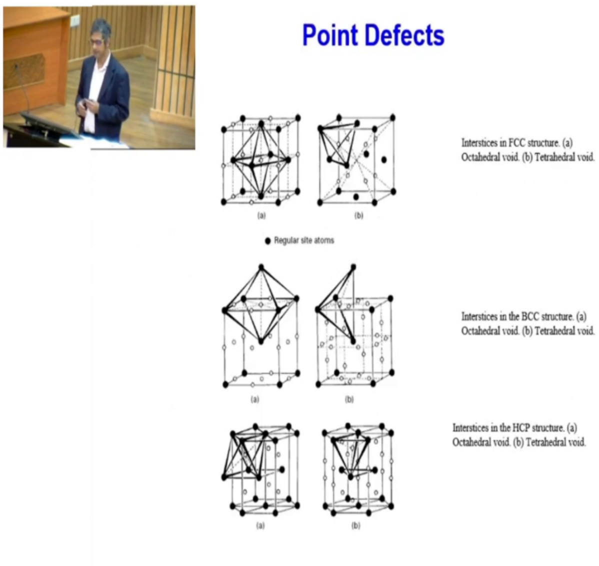

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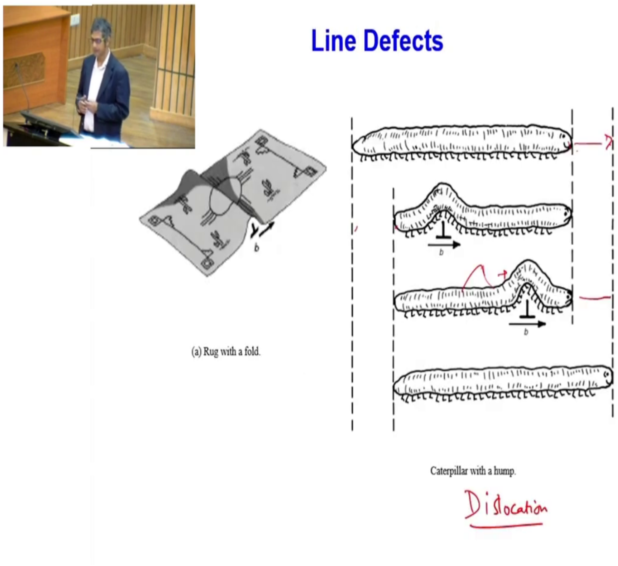

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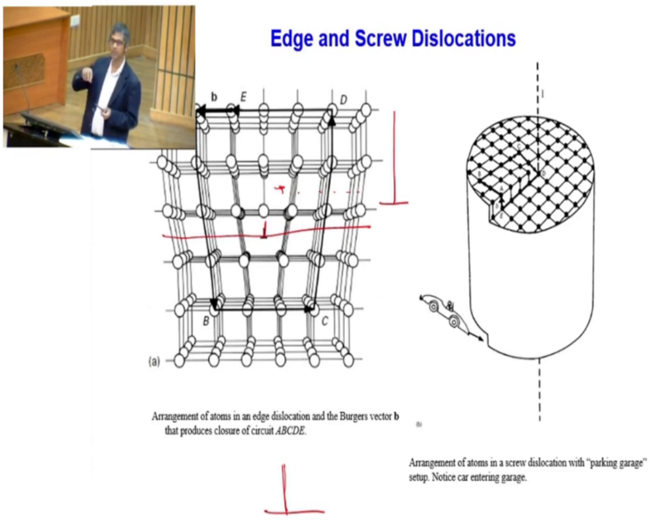

There are different types of voids like you know octahedral tetrahedral voids, you know they are actually also point defects. Then, most important defects, will line defect which we have discussed the large. They behave like a caterpillar moving from one end to the other end and dislocations do move like that way. So, dislocations can be edge or screw. Edge dislocation is nothing but extra half plane and that is basically depicted by this kind of symbol, where this is the slip plane and that is the dislocation line ok.

So, they are perpendicular to each other. The screw this is parallel. The slip plane and the screw, the direction of the dislocation line, they are parallel to each other that is the difference. Screw atomic arrangement in the screw dislocation looks like a screw from the top or bottom, if you look at it that is why it is called screw dislocations.

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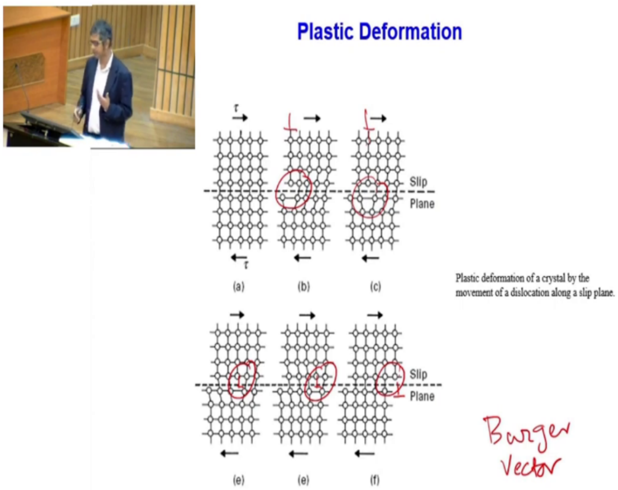

So, once you move a dislocation from one end to the crystal to other crystal, it creates a shape or distance of slip ok and this dislocation movement is characterized by burger vector. All of you know that. In every crystalline structure, there is a specific burger vector.

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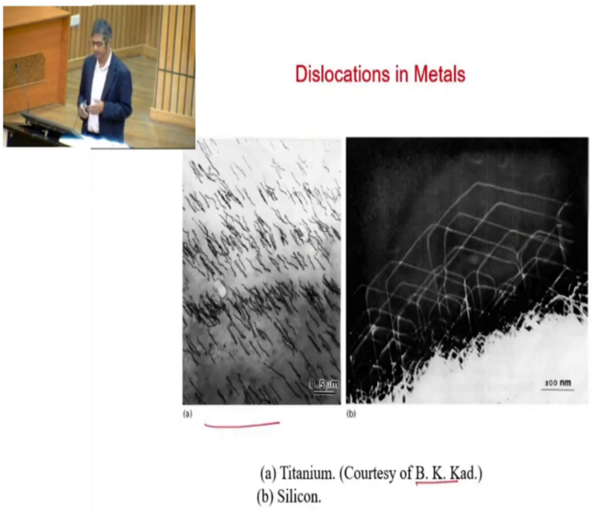

Dislocation in material looks like this. This is on the left side this is titanium, right side is a silicon and dislocation looks like a line sequence or they may look like a curved one.

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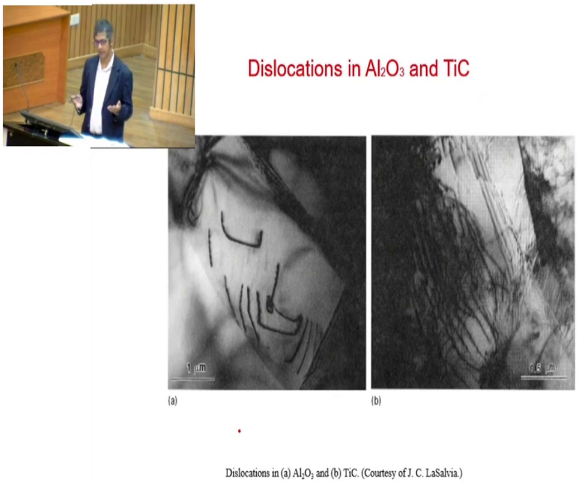

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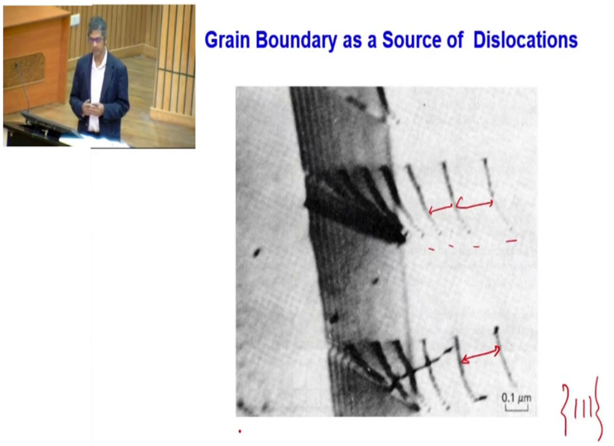

Emission of dislocations from ledges in grain boundary, as observed in transmission electron microscopy during heating by electron beam. (Courtesy of L. E. Murr.)

Dislocation in ceramics, they also look like this alumina and titanium carbon, they looks like also line. So, they also deform by plastic deformation. Well, dislocation generated from these sources like Frank-Read sources or maybe from the grain boundaries and then, they move. When they move, they keep this minimum distance from each other so that they do not overlap the field associated with themselves. In FCC, dislocation plane is {111} and direction is <110>.

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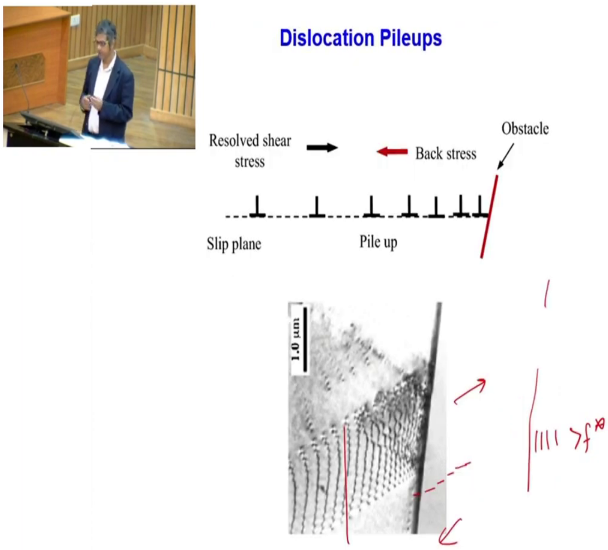

So, when the dislocations actually move from one side of the grain to the other side of the grain, they come in front of an grain boundary and grain boundaries can act as an obstacle for the movement of dislocations and because of that dislocations, we can get pileup in the grain boundaries. So, in a multi crystalline material like in a large number of grains are you know, then these effects is very significant ok. So, grain boundary can act as a obstacle.

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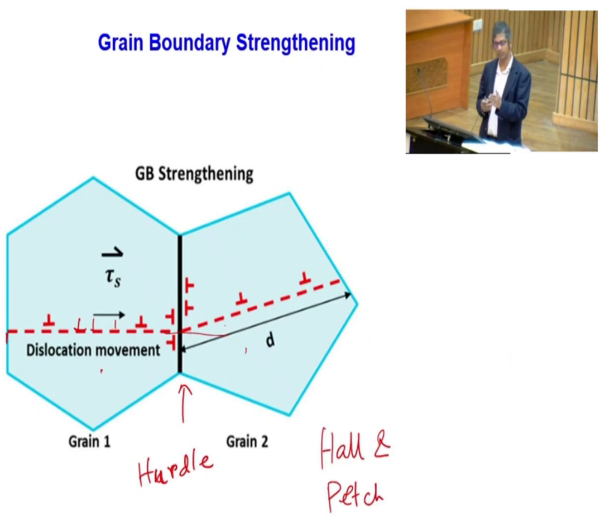

That is what it leads to grain boundary strengthening and the Hall petch relationship, which we are going to discuss today. So, you know as you know this is the two grains you can see there. So, this is a normal way of showing that. Dislocation generated from one grain boundary moves and reaches the other grain boundary.

Now, those two grains may not be on the co-planar that is the slip plane for this grain and this grain may not be on the same plane. So, therefore, even if dislocations want to move from this grain to this grain, it cannot move because these planes are not continuous. That is why there are two separate grains right.

Not only that, even if they want to start the dislocation process on this grain, the force required for that process may be different on this grain than this grain. So, therefore, this means that grain boundaries can act as an obstacle for dislocation motion and that is why they can provide strength.

Because any kind of obstacle to the path of dislocation motion is going to make the material stronger right, because dislocation needs to cross over that obstacle or dislocation need to break that obstacle. That is what it is. All of you know the hurdle race ok.

And hurdle race, the runner moves from one hurdle to other hurdle right. That is why they can cross the hurdles. Similarly, here, the dislocations are facing an hurdle; big hurdle at the grain boundaries ok. This is a kind of a big hurdle right; hurdle. This hurdle needs to be crossed over. So, they can actually force this, this dislocation generation movement in the other grain or they can actually cross over this grain boundary.

But the second one is difficult. First one is easier ok. Nonetheless, whatever it is, they need to put lot of force on this boundary. That is why they can actually allow the grain boundary or the dislocation to move from one grain to other grain. So, such a kind of things leads to pileup dislocation in the grain right; this in this grain 1 and this pile up leads to stress built up on the grain boundary correct.

This is nothing like a you know people standing in a rail you know railway ticket counter as you have seen probably You know so, they may they are pileup, many passengers are buying the tickets. So, there is a pile up created at the front of the ticket counter and then, if the first person, who is at the front person, a person sitting standing at the front, if that person is moving little bit. So, the line will move on the backward directions that is what is known as a back stress right.

That means, the first person is little bit pushing the back that gives a back stress to the person behind him and same thing gets per correlated down to the end person right, since the person standing at the end of the queue. The same thing will happen here also.

So, dislocations getting pile up in the grain will apply stress on the grain boundary. So, only when this stress will reach a critical value, the dislocation will; well this dislocation movement will start in the other grain, a generational dislocation movement will start in the other grain.

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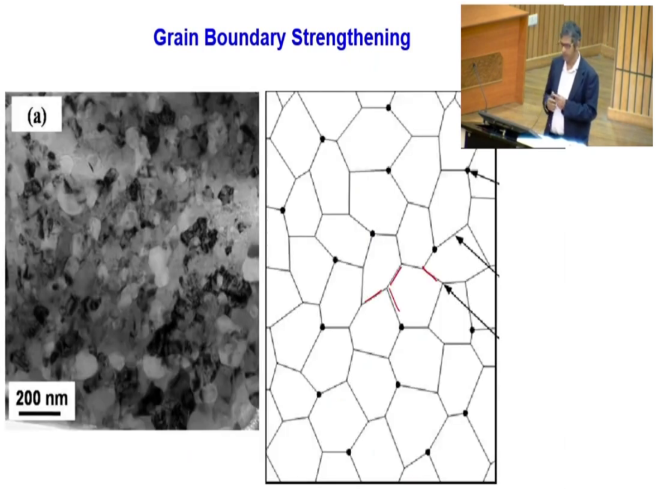

As you know grain boundaries actually in a multi crystalline material or a bulk nanostructure material, you have large number of grain boundaries presents. So, therefore, these aspects is augmented. This aspect is felt the large number of grains actually; every grain boundary will act less in obstacle and this will lead to huge strengthening effect. You will be surprised to note, if I have a copper, the grain size of 10 micron and grain size of a of 15 nanometres, the strength or yield strength of that copper will be almost 4 times ok. That is what it is.

So, grain boundaries can act as a obstacle to the dislocation motion and that is provides the strengthening effect to the material. That is something, which you need to understand very well. So, I will go back here ok, that is mainly because of these aspects, you know grain boundaries provides obstacle to the movement of the dislocations.

Remember, plasticity or the plastic deformation is dictated by the movement of the defects; generation and movement of the defects. The more is the movement of the defects, more is the slip steps created at the surface, the more is the plastic deformation.

So, if grain boundaries are not allowing the dislocation to move, material will not deform. So, material will show strength higher and higher ok. That is something which is very unique because of the grain boundaries.

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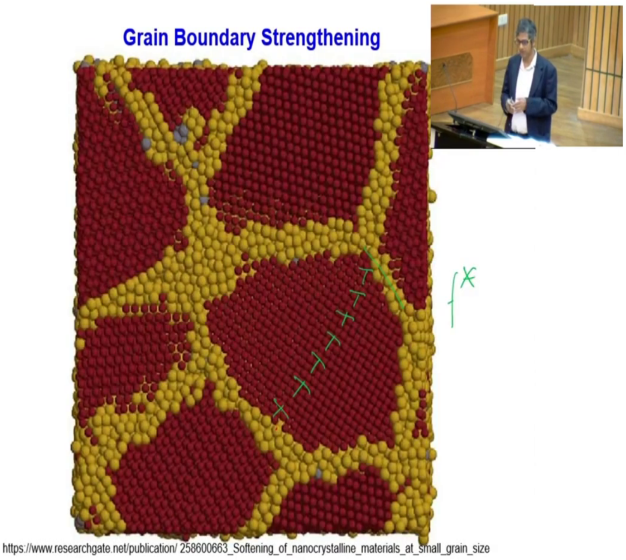

Well, this is shown here also in a atomistic structure, grains can be dislocation starting from this grain it slowly move to the other grain and then, get stopped because of the grain boundary.

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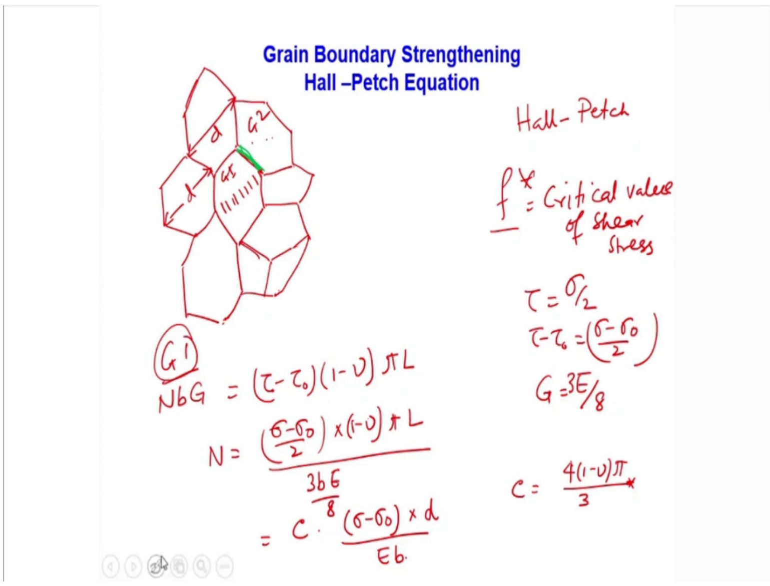

Hall-Petch relationship

f^* =Critical value of shear stress

$$\tau = \frac{\sigma}{2}$$

$$(\tau - \tau_0) = \frac{(\sigma - \sigma_{0)}}{2}$$

$$NbG = (\tau - \tau_0)(1\text{-}\vartheta)\pi L \quad \text{where} \quad G = \frac{3E}{8}$$

$$N = \frac{(\frac{(\sigma-\sigma_{0)}}{2})(1-\vartheta)\pi L}{\frac{3bE}{8}}$$

$$N = c\frac{(\sigma-\sigma_0)^{*}d}{Eb} \qquad \text{where } c = \frac{4(1-\vartheta)\pi}{3}$$

Well, this aspects was first conceived by Hall and Petch right. You should know that Hall and Petch, they first time discussed about these aspects ok. That is something, which you should know very well. So, what they told is very simple, very simple aspects. So, if you have grains like this right, these are the grains as you see.

This is the dislocation pile up at the boundary ok; let me put the colour different. So, this is suppose boundary on which the pile up is happening ok and you can always draw another grain here, correct? That is how it looks like. These are the several grains. Now, as I said, they pile up which is happening at the boundaries, once it reaches a critical value f^{*}=Critical value of shear stress.

Let us suppose the critical value is f^{*}. Critical volume means critical value of stress of shear stress. Once I retest this value, the dislocation motion in this dislocation motion in this one grain, this is grain 1, this is grain 2; in this motion, we will start that is what you discussed. So, now, suppose number of dislocation present in this grain 1, G1, the number of dislocation is N right.

So, as you know the if the burger vector is b, then this is what is total displacement. Each dislocation leads to displacement by 1 burger vector right. In FCC crystal, it is a/2{1 1 0}; where a is the lattice parameter. So, if each one is giving such amount of displacement, N number of dislocation will give you N into b amount of displacement and that if you multiply with this shear modulus, it gives you the modulus process distance gives you this the stress per unit length right.

Stress per unit length is newton per meter; not newton per meter square, stress per unit length. Remember modulus and b is in terms of nanometres ok. So, that is the unit of meter and modulus is always newton per meter square. So, therefore, the multiplication of these two will provide you force per unit length.

And this must be equal to the shear stress which is required $NbG = (\tau - \tau_0)(1- \vartheta)\pi L$; where, L is the length of pile up. What is the πL thing? You know in a circle, if L is the radius, the πL gives you half of the circumference, correct and if L is the diameter, then what happens?

πL gives you the total circumference. Same thing here, correct? If you know the grain size to be given by L ok, then that is what it is; πL is the periphery of the grain that is something related to the grain size actually, correct? As the grain size decreases, πL also decreases.

So, then what is $(\tau - \tau_0)$, let us explain. τ_0 is the comp or the value of the shear stress which is used for other strengthening mechanisms. There will be other strengthening mechanism like there will be solid solution strengthening, there will be British precipitation strengthening, there will be dispersion strengthening, many other things are there correct or there will be strengthening because of you know aspects of you know what Suzuki locking, many other things are there.

So, those things if you take out from the total shear stress, that gives you the shear stress which is operating because of the because of what? Because of the dislocation pile up. So, that is tells you the shear stress for this dislocation pile up only ok, which you are discussing and remember, this is happening in grain 1 right that is why and this 1 minus nu is because of Poisson ratio; nu is a Poisson ratio ok.

When do you multiply)$(1- \vartheta)$? Whenever you know Poisson ratio comes when you have a lateral and the longitudinal strain that ratio is given as the Poisson ratio and so, therefore, that is gives you the 3d picture of the deformation. So, that is what it is. So, these $NbG = (\tau - \tau_0)(1\text{-}\vartheta)\pi L$ right. So, ϑ is this thing.

Now, as I said τ_0 is the shear stress a contribution of the all strengthening mechanism except this grain boundary. So, therefore, $(\tau - \tau_0)$ gives you the grain boundary strengthening, where tau is the total strengthening correct and you know shear stress tau caused by any tensile and compression stress, shear stress tau caused by any tensile and compression stress that is going to happen with the compression tensile is given by $\frac{\sigma}{2}$ ok.

This is standard solid mechanics. All of you have read about it. So, therefore, I can write down $(\tau - \tau_0)=\frac{(\sigma-\sigma_0)}{2}$ right, very easily. Am I clear? And also, you know that shear modulus G is can be related to E/8 in a metallic material ok. It is all given, this is not G= E/8, its $\frac{3E}{8}$ ok that we know.

So, therefore, we can write down $N=\frac{(\frac{(\sigma-\sigma_0)}{2})(1-\vartheta)\pi L}{\frac{3bE}{8}}$ right and we can always write down correct, we can always write down one thing very clearly, what is that? The very easily, we can write down this one that is $N=c\frac{(\sigma-\sigma_0)*d}{Eb}$. I will come back to you what is that multiplied by E×b.

What is d? Well, so C is equal to as you know what? where $c=\frac{4(1-\vartheta)\pi}{3}$ correct; that is it. Not only that, L and d are related. So, that is it actually because L and d, d is the grain size ok. You can always talk about grain size for d like that. Am I clear? That is the grain size.

So, that is equal to N; N is the number of dislocation, that is proportional to this shear stress or tensile or compressive stress grain size; obviously, grain size increases the thermal, dislocation will increase because you have much longer length and also, inversely proportional to elastic modulus and the burger vector that is very clear right. Now, we can expand that thing ok.

Now, very simply as you know as the grain boundaries are acting as an obstacle, so dislocation loads motions are heavily affected by the grain boundaries. The obstacle strength;

the obstacle is defined by the obstacle strength right. So, obstacle strength, we have defined as a f star that is the critical value of shear stress which is required to be applied to start the dislocation activity in the grain 2 that is what is we have defined as the obstacle strength.

Now, so the force, this dislocation of you know exact on the obstacle is magnified by their number. So, the obstacle can be overcome obstacle can be overcome, when? Let me just erase this part, obstacle can be overcome when? You can imagine what, when the obstacle can be overcome.

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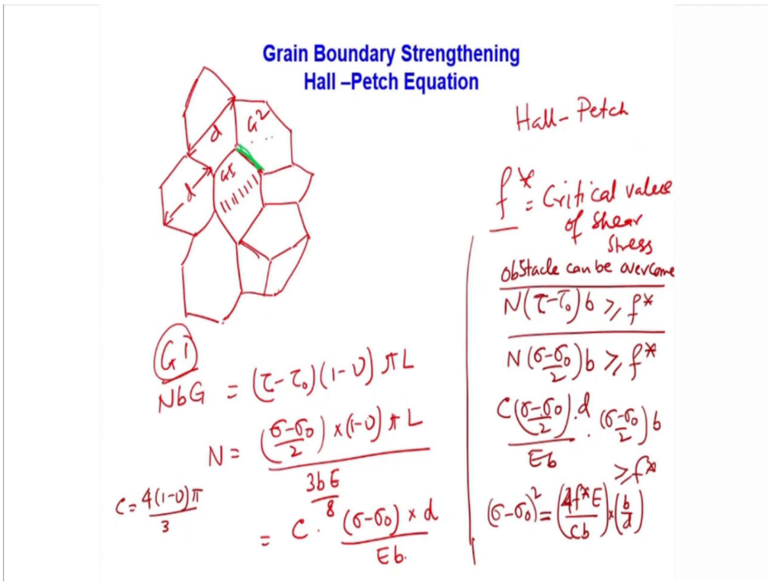

$$NbG = (\tau - \tau_0)(1\text{-}\,\vartheta)\pi L$$

$$Where \quad G = \frac{3E}{8}$$

$$N = \frac{\left(\frac{(\sigma - \sigma_0)}{2}\right)(1 - \vartheta)\pi L}{\frac{3bE}{8}} \qquad where\ c = \frac{4(1-\vartheta)\pi}{3}$$

$$N = c\frac{(\sigma - \sigma_0)^{*} d}{Eb}$$

f^{*} =Critical value of shear stress

Obstacle can be overcome

$$N(\tau - \tau_0)b \geq f^*$$

$$N\frac{(\sigma - \sigma_{0)}}{2}b \geq f^*$$

$$\frac{C\frac{(\sigma-\sigma_{0)}}{2}*d}{Eb}\left(\frac{(\sigma-\sigma_{0)}}{2}\right)b \geq f^*$$

$$(\sigma - \sigma_0)^2 = \left(\frac{4f^*E}{cb}\right)*\left(\frac{b}{d}\right)$$

Well, obstacle can be overcome at the time when $N(\tau - \tau_0)b \geq f^*$, then only the obstacle can be overcome. Remember this is τ. So, we can write down $N\frac{(\sigma-\sigma_{0)}}{2}b \geq f^*$ when reaches a critical more than a critical value f^*, then only the obstacle can be this obstacle can be you know overcome; then only it can be overcome right. So, you understood right?

First is that we collected, connected what? Connecting the dislocation density in the grain or number of dislocations with this shear stress. So, what did you say? If number of dislocation is N and each dislocation is having displacement given by burger vector b and then, N multiplied b is total displacement. That once you multiply with the shear modulus, that gives you the force per unit length correct.

On the dislocation, on this plane, for this dislocation pile up and that can be written as $\frac{C\frac{(\sigma-\sigma_{0)}}{2}*d}{Eb}\left(\frac{(\sigma-\sigma_{0)}}{2}\right)b \geq f^*$; where, d is the grain size, sigma is this tensile or compressive stress, E is the elastic modulus and b is the burger vector and C is a constant and I told you what is C.

So, you can figure out from this equation also. So, therefore, this obstacle which is there in the form of grain boundary can be overcome only when this equation is satisfied $N(\tau - \tau_0)b$ is again as you know that is the total displacement multiplied by $(\tau - \tau_0)$, τ is the applied shear stress and τ_0 is the shear stress which is used for other strengthening.

So, $(\tau - \tau_0)$, is basically shear stress applied because of dislocation pile up on the grain boundary. Once that reaches more than a critical value f^*, then this pile up will lead to start of dislocation motion generation and the movement in the grain 2 right that is what is the thing we discussed.

So, now, we can put the value of N; $N = c\frac{(\sigma-\sigma_0)*d}{Eb}$ correct, that is what is N, N is this and this factor is this$N\frac{(\sigma-\sigma_{0)}}{2}b\geq f^*$ right. So, then what we can write down? If you make the equations very clearly, you get $(\sigma - \sigma_0)^2$ right.

So, that is $(\sigma - \sigma_0)^2 = (\frac{4f^*E}{cb})*(\frac{b}{d})$.

So, we can see $\sigma - \sigma_{0)}$, these two become square ok and this, this will be 4, 2 into 2 become 4 and f^* is remaining f^* ok and remember, this is the limit that is why you put the equal to sign. So, that is we can take it down to the other slide.

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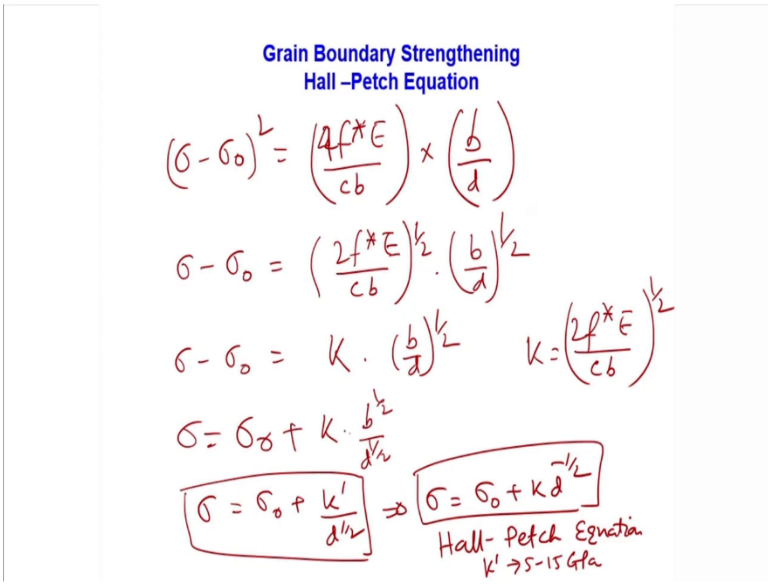

$$(\sigma - \sigma_0)^2 = (\frac{4f^*E}{cb})*(\frac{b}{d})$$

$$(\sigma - \sigma_0) = [(\frac{4f^*E}{cb})^{\frac{1}{2}}]*(\frac{b}{d})^{\frac{1}{2}}$$

$$(\sigma - \sigma_0) = K.\left(\frac{b}{d}\right)^{\frac{1}{2}} \qquad K = \left(\frac{2f^* E}{cb}\right)^{\frac{1}{2}}$$

$$\sigma = \sigma_0 + \frac{K'}{(d)^{\frac{1}{2}}}$$

$$\sigma = \sigma_0 + Kd^{-\frac{1}{2}} \quad \text{Hall-Petch equation}$$

$$K' = 5\text{-}15 \text{ GPa}$$

So, let me write down, the $(\sigma - \sigma_0) = [(\frac{4f^* E}{cb})^{\frac{1}{2}}] * (\frac{b}{d})^{\frac{1}{2}}$ correct. That is what you can write down. This is what its you get it ok. You do, if you do the mathematics properly here correct. So, I think you should try and do it as much as possible. So, what does you do?

. So, that we can write down $(\sigma - \sigma_0) = K.(\frac{b}{d})^{\frac{1}{2}}$; where, K is a constant which is given by this $K = (\frac{2f^* E}{cb})^{\frac{1}{2}}$. Remember this constant depends on the critical or the obstacle strength multiplied by elastic modulus in divided by b, C into b ok or $\frac{f^* E}{cb}$.

So, that is basically depends on particular material. Obviously, grain boundary for a particular material, dictate the f^* and E also. Burger vector is given by crystalline structure and see, we have seen what is a C. C is a constant, which depends on many factors right that we have discussed; C can be easily obtained from this.

If you see that C is equal to C is what? $c = \frac{4(1-\vartheta)\pi}{3}$ correct that is what it is, that is really purely constant in the material because nu is a constant in most of metallic material is 0.33 and pi is a constant. Therefore, it is a constant. So, then, I can write down sigma is equal to ($\sigma - \sigma_0) = K.(\frac{b}{d})^{\frac{1}{2}}$, $\sigma = \sigma_0 + \frac{K'}{(d)^{\frac{1}{2}}}$ correct. So, many cases, people write sigma is equal to constant K. This is what it is and that is the Hall-Petch equation or you can write down $\sigma = \sigma_0 + Kd^{-\frac{1}{2}}$ Hall-Petch equation, you must do yourself that is the Hall-Petch equation.

So, you understood how to derive the Hall-Petch equation right very well. So, now, the question is this. The quantity K' here basically is the dimensions of stress as you can clearly sees, it is the characteristics characterizes the strength of the obstacle or the boundary; value normally typically ranges K'=5-15 GPa.

That is what it is. It is about equal to the ideal strength. This in very important that as the grain size decreases, the strength increases that is what it tells you. So, that means, what? As you go down the nano crystalline grain, the strength will increase drastically that is why we can come back to the nano size. So, as you know d is basically one-half minus.

So, therefore, effect is more significant. This is not a linear; effect is more significant because of the factor half and this is inversely proportional. But you know this can happen very easily till a critical grain size below which no longer this is possible, we will come back and discuss about that aspect also.

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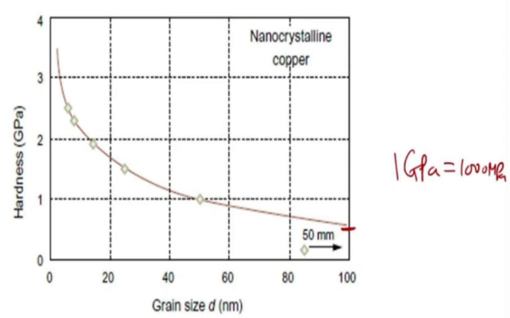

So, first of all, you understood that if you plot grain size hardness versus grain size here for nano crystalline copper, you can see here its strength significantly increases. We look at the value of hardness at 100 nanometres that is about 400 MPa and look at the value of hardness at 10, I mean 5 nanometres that is almost 2500 MPa that is a 2.5 GPa right and 1 GPa is 1000 MPa right.

So, therefore, very clear that as you decrease the grain size, the strength increases rapidly; very very rapidly and it has increased 4 order magnitudes here. That is the advantage of making nano crystalline grains. You can make the material strong and strong and strong, but is it possible to make the material strong and strong and strong by decreasing the grain size from micrometre to about 2 nanometres? Answer is no.

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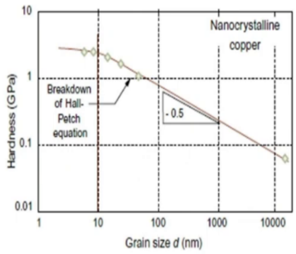

FIGURE 7.5
The increase in strength of copper as the grain size is reduced to nanodimensions (top); the same data plotted on logarithmic scales (below). (Data from Goldstein, 1997.)

Because as you clearly see here this is nano crystalline copper, you see here the strength increases drastically, then strength remain constant and (Refer Time: 38:30) decreases also. There are many materials, it has been found the strength decreases. So, that means, the Hall-Petch equation breaks down below a grain size. You must be thinking why does it happen; that is not very difficult to understand. Let me first show you all these slides, then I will come back to it ok.

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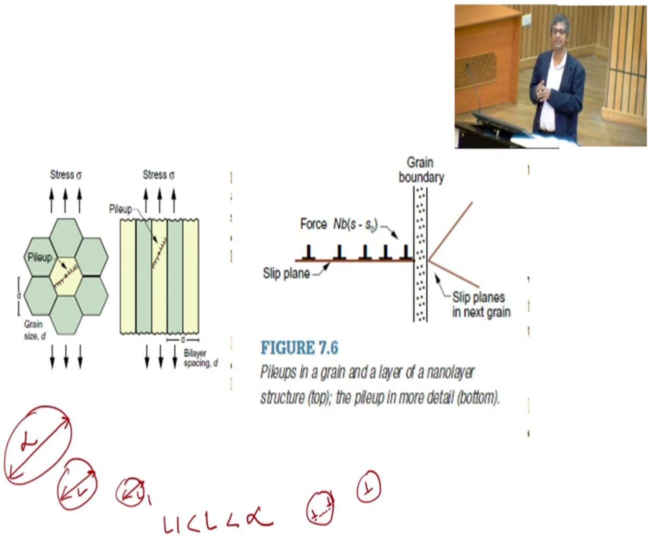

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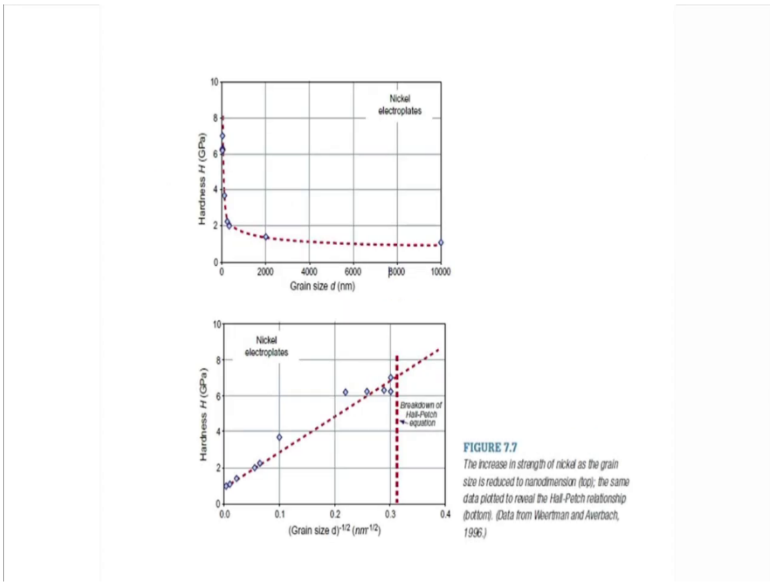

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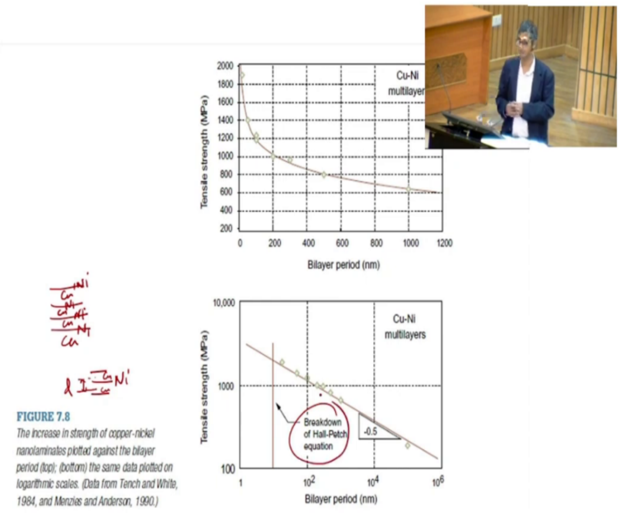

We will come back to it. There are few more things to show this is nickel electro deposited and as you can clearly see the grain size still about 2000 nanometres, does not change much; but then something about 100 nanometres, so grain sizes shoots up from 2 GPa to hardness to about 8 GPa that is about 4 times increase of hardness correct.

And this can be also plotted as function of d to the power minus half, you have to see whether the Hall-Petch equation is valid or not and it is found the Hall-Petch equation is really valid till a value of d to the power minus half of 0.3; below which or above which actually not below which the Hall-Petch equation breaks down.

So, Hall-Petch equation is has been observed in case of nano crystalline copper breaking down below a certain grain size. Similarly, it has been observed in case of nickel also, this is breaking down below certain grain size; not only that, if even if you creates copper nickel multi layers, multi layers means what? You can deposit thin flames layers copper, then nickel, copper, nickel like this ok; copper, nickel, copper, nickel. So, this is copper, nickel, copper, nickel, copper, nickel, copper, nickel ok. So, this is nickel actually.

So, if you deposit those kind of multi layers and if their bilayer periods vary, bilayer periods means what ok? Bilayer period means this distance, this is d half center of these copper to the center of this copper in between the cynical layer. So, that is the bilayer period, with this bilayer period is plotted as a you know tensile stress plotted as a function of bilayer period, you can clearly see that there also this equation shows a tremendous increase of strength.

But it has been observed this equation breaks down below a certain value of bilayer period. So, this is universal. Nano crystalline copper, nano crystalline nickel, bilayer copper nickel and many many other materials this equation has been found to break down.

So, the question is that then, why does it break down? Why does it so you know easy to so much happens; so, such a thing happens at all in nano crystalline material. That is very important. So, you can see the Mother Nature right. Initially, we thought when you derive the Hall-Petch equation here, we thought that let us keep on decrease the grain size.

We will have tremendously strong material. Unfortunately, that does not happened. This is a limit to which the strength increases and then, if you still decrease the grain size, strength decreases and this is known as the grain boundaries softening ok. Remember this word.

Mostly, the Hall-Petch equation is derived because of the grain boundary strengthening; grain boundary is acting as an obstacle for the motion of dislocations and that is why they provide the strengthening. Now, the question is here below a critical grain size instead of having strength, then you have a decrease of the strength or rather grain boundaries act as a decrease of strength.

As you know smaller the grain size, more is the grain boundary area. So, more is the grain boundary area, more it should be the strength; but we are saying opposite, we are seeing the strength going down that is why it is called grain boundary soften. Softening means material is becoming softer and softer as the grain size decreases.

Now, the question is why does it happen? Let us set some light on to which and then, we will stop this lecture. Well, that is very easy to understand. The grain boundary strengthening is because of the pile up of dislocation at the grain boundaries.

So, this pile up of dislocation means you need many many dislocations to be accommodate inside the grain, then only the pile up is possible right. If you have 2 passengers standing in front of a railway counter to buy a ticket, there is no pile up. Only when you have a 10s of passengers or 100s of passengers, then you have pile up. But think about it.

If you want to create a 10s of passengers in front of a ticket counter, you need space, you need critical space. They cannot you know stand you know just close as close as possible so that they can touch each other, they can force each other. This will be uncomfortable situation right. So, they will be standing, the passengers will be standing in front of the railway passage counter, ticket counter in a safe distance.

So, that they do not touch each other, they do not force on each other, they are comfortable right. Same thing is valid for dislocations. When dislocations are lying up inside the grain, they must be sitting at a distance from each other which will make them comfortable; otherwise, the stress field as said to each dislocation will interact will fall on each other and because of this stress field falling on each other, they will be uncountable, they will feel jittery just like if you ask the human standing in the queue to come and you know just stand just right behind each other.

They will not feel each other good actually, they will feel that they are crashing onto each other. That is not good. Same thing is true for dislocations. So, now, if you keep on decrease the grain size, what is happening? The length on which this dislocation can stand is decreasing that is what it is.

So, you have a big grain size and you have a small grain size, this length L on a dislocation can stand L and L 1. So, L 1 is smaller than L 1, L right and you can have big grain, this is say this L and this is again smaller than this L right. So, what do you see? As the grain size decreases, length only dislocation can pile up with decreasing.

So, there will be critical length, where only two dislocations can pile up and when two persons are standing in front of a ticket counter, they can peacefully sit far apart from each other, without applying force on each other. There will be no pile up, there will be no stress acting on the grain boundaries.

If no stress is acting on the grain boundaries, grain boundaries action as an obstacle does not make any sense ok because amount of stress applied on the grain boundaries will be much small. Even you can have grain size which is much smaller where you can only have one dislocation. There this dislocation is comfortably sitting inside the grain without even interacting with other dislocations.

So, therefore, there will be no force acting on the grain boundaries. So, now, you understand right that is what is happening as you decrease the grain size. As you keep on decreasing the grain size, this is exactly what is happening; dislocations the number of dislocation inside the grain is decreasing, decreasing, decreasing till it reaches 1 or 2 and the grain, once it reaches that value of the grain size, when only 1 or 2 dislocation can be accommodated inside the grain, Hall-Petch equation loses its meaning.

Because then the concept of pileup, concept of pileup and concept of this stress acting on the grain boundaries no longer happening that particular aspect of the strengthening because of grain boundary is no longer valid. So, that is why Hall-Petch equation breaks down below a certain grain size, remember these aspects. This is a very critical thing which you should understand.

The breaking down of Hall-Petch relationship is also same thing, as same thing is true for bilayers, if you decrease the bilayer thickness, the length on a dislocation pile up can happen also decreasing. So, I would rather say that these aspects of pile up and the force acting on the obstacle is only can happen whenever critical size of grain and that size is what is where the this kind of break up or this kind of breakdown of Hall-Petch relationships or equation can happen ok.

You can see here it is happening here, see it is happening there also, it is happening here also; so everywhere it is happening correct. So, what did I discuss in this today's lecture, let us you know kind of compile. So, I told you the tensile test is the most important test to determine the mechanical behaviour of material correct. Then, I talked about multi grain materials, how defects can pile up at the grain boundaries and grain boundary can act as a obstacle correct. Then, I derived this Hall-Petch equation and finally, I showed you why the equation breaks down.

So, that is for today. We will come back in the next lecture and discuss some more aspect of mechanical behaviour and move on to the other aspects.

Thank you.