Properties of Materials (Nature and Properties of Materials: III)
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Lecture 08
Introduction of elasticity and elastic properties

So welcome again to the new lecture of this course Properties of Materials. So let us just briefly recap what we did in the last lecture.

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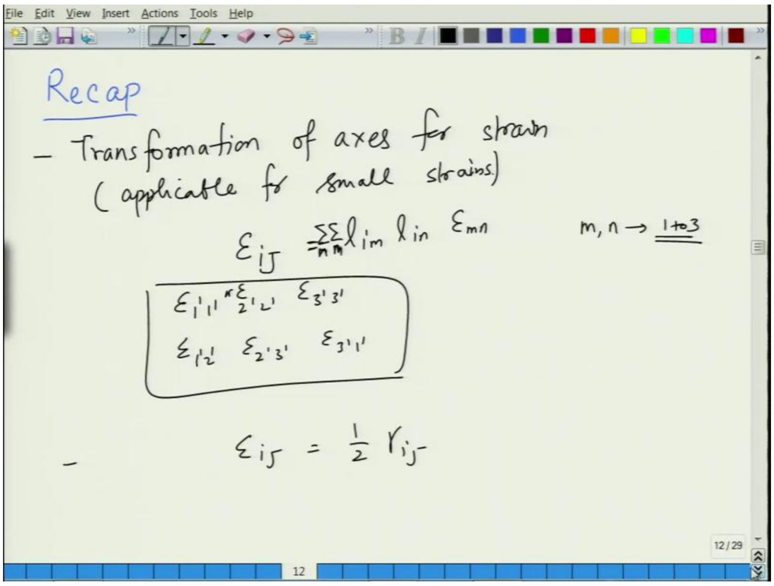

In the last lecture we looked at transformation of axes for strain with the caveat that this is applicable only for very small strains. And the relation was you can calculate it is epsilon ij is equal to, epsilon ij is equal to lim lin into epsilon of mn. So you can include summation over n, summation over m, m and n and m and, m and n will vary from 1 to 3.

So this is how you can calculate the stresses like it is, strains like epsilon 1 prime 1 prime, 2 prime 2 prime or epsilon 3 prime 3 prime. You can also calculate epsilon 1 prime 2 prime, epsilon 2 prime 3 prime and epsilon 3 prime 1 prime. So this is all possible to calculate from this. And then we also saw that shear stress is related to, engineering shear stresses related to mathematical shear stress using this relation. So epsilon ij is equal to half of gamma ij.

So now let us move on to the next topic, having introduce the mathematical framework for the relations of stress and strain and transformations of axes. So the transformation of axes that we are doing for true stress can also be applied for engineering stress or engineering strain.

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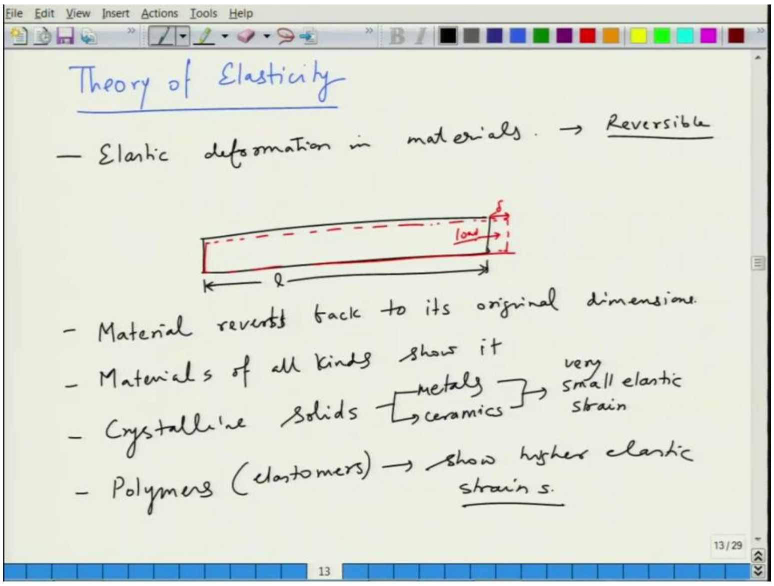

And now what we will do is that we will looked at, look at what is theory of elasticity. So we will just introduce the concepts related to theory of elasticity in this lecture without getting into too many details about it.

So the first thing as we know, elastic deformation in materials is basically a reversible deformation. So when you stretch something, let us say, of length l and when you stretch it to, let us say, to let us say, l plus delta. So this is upon application of load. So this is where you apply the load.

So when you release the load, the material gets back to its original dimension. So basically essentially what happens is that material reverts back to its original dimensions. This is what basically reversible deformation is. And most, and all materials will show it. Materials of all kinds show it, but to a different extent.

So if you look at crystalline solids such as metals, ceramics, they show small elastic strains or rather very small elastic strains. Whereas materials which are polymers, especially elastomers show higher elastic strains. So normally they show higher elastic strains, it is also that their behavior can also be different.

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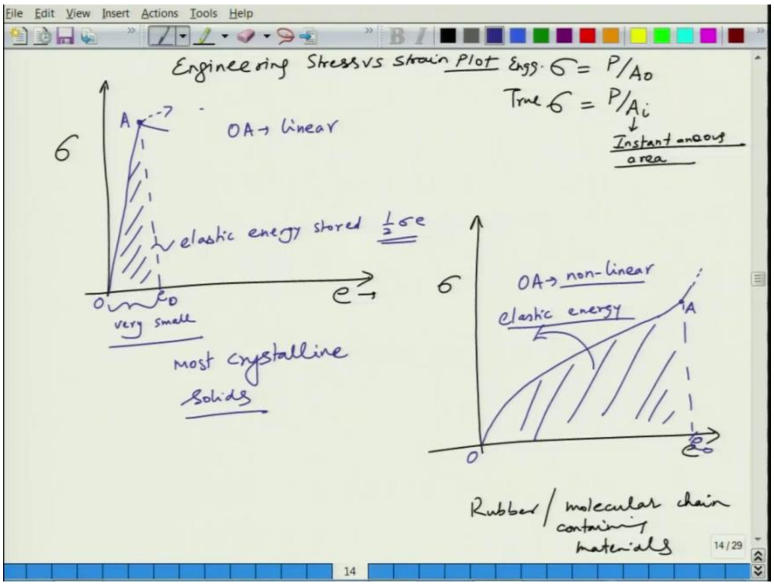

So, for example, if you make a plot of elastic strain of, let us say, crystalline solid sigma versus e, so this is basically a engineering stress strain plot, engineering stress versus strain plot. So we have seen what engineering strain is. And if you recall engineering stress is P divided by A naught and engineering, and true stress is, so this is engineering, and the true stress is P divided by Ai, where this is instantaneous area. So this is the difference.

So generally, for engineering purposes, we use engineering strain. So in solids, which are crystalline in nature, generally this region, the stress strain plot is like this and it continues to, so let us say, it continues. So it is linear up to a point A and after that non-linearity sets in and we will see the behavior of this plot later on. So this region is basically, so you can say from O to A it is linear and this is the, and the product area under the curve is basically you can say elastic energy stored, which is half of sigma e essentially.

How about the magnitude of this elastic strain up to this point, let us say, e1 or e naught is very small. This strain is less than 1% and can be very, very small, extremely small. And so this is shown by most crystalline solids.

On the other hand, if you look for materials which are rubbery in nature. So sigma versus example, so let us say, this is for rubber or you can say molecular solids. Molecular chain containing materials, like plastics, polymers, rubbers et cetera. In these materials generally the behavior is slight different. So there is a, so let us say this is point A, this is O. So OA is non-linear. So the behavior is elastic, up to this point you have elastic strain. So let us say, this is e naught and this is the area under the curve, which is the elastic energy, but this is a non-linear region. So they show elastic behavior, but it is non-linear in nature and this is recoverable.

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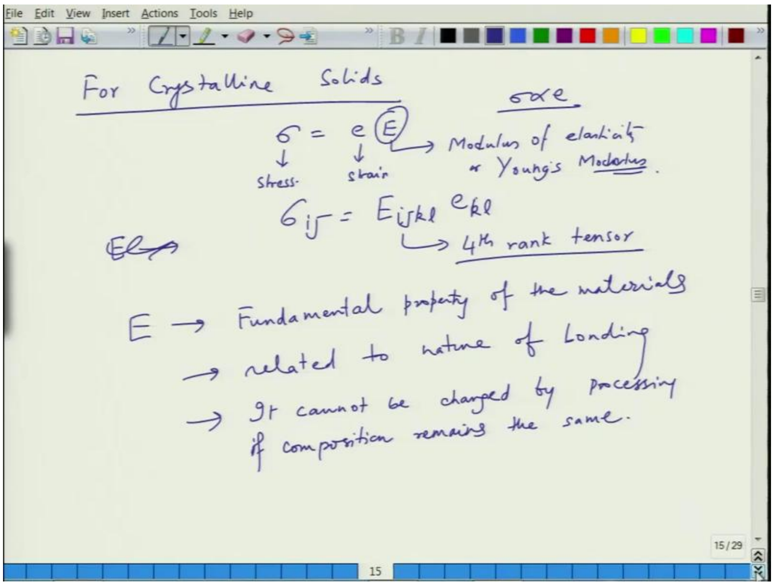

So for the linear region, in terms of crystalline solids, let us say first, for the linear region, we define sigma as e into E, where sigma is the stress, engineering stress, this is strain, engineering strain, and this proportionality constant, you can say sigma is proportional to e and the proportionality constant is called as modulus of elasticity or in exact term it is called as Young's modulus. This is what it is.

The modulus of elasticity is a fundamental property of material. So this E, if you write this in tensorial notation, of course, it becomes sigma ij becomes Eijkl ekl. So essentially it is a 4th rank tensor, but for lot of practical purposes we just write it in the scalar form. And this E is a fundamental property of the, and is related to nature of bonding. And we will come to that a little later. It cannot be changed by processing if composition remains the same, in general.

So you have to make a change in the composition to achieve a change in the modulus of elasticity. It is a fundamental property. In general, this is true that for a give material if you change its heat treatment schedule, you might change its strength, but you may not change its modulus.

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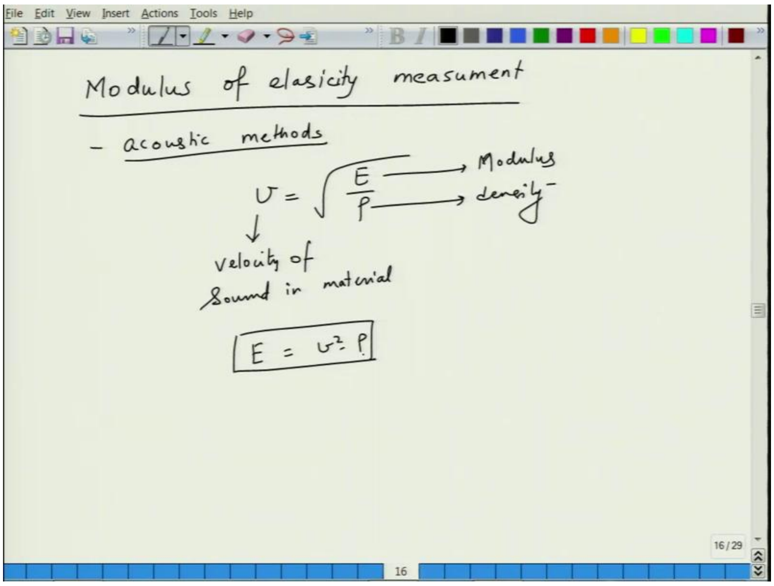

So modulus of elasticity can also be determined by, so not only you can determine that by carrying out a stress strain measurement, so if you carry a, basically a stress strain measurement from the slope of liner region, you can determine what the modulus of elasticity is. But you can also determine it by acoustic methods, because velocity of sound v in a material is related to modulus in such manner.

So this is velocity of sound, this is density and this is modulus. So essentially the dense the material is, higher the density of a material is, lesser is the velocity of sound for a give modulus or alternatively you can write this as E is equal to v square into rho. So if you can measure the velocity of sound in materials, you can sort of determine the modulus of elasticity by acoustic measurements.

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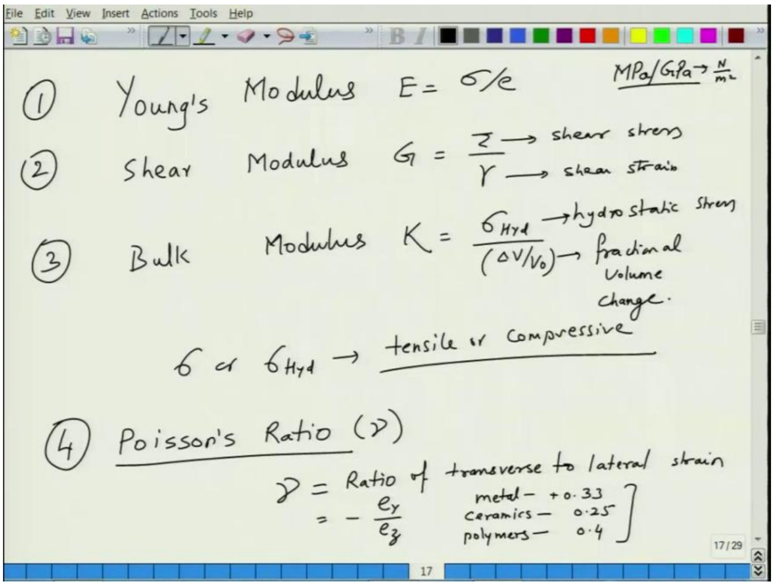

There are a few more quantities, so we can write, first of all, elastic modulus as Young's modulus and E is equal to sigma divided by e. In another quantity called as shear modulus, which is defined as G and which is tau divided by gamma, so this is shear stress, this is shear, engineering shear strain or shear strain. In another quantity called as bulk modulus, which is essentially K and this is sigma hydrostatic, so this is hydrostatic stress divided by fractional volume change.

So this is hydrostatic stress and this is fractional volume change. And this stress could be tensile or compressive in case of Young's modulus, in case of shear stress and again the hydrostatic could be tensile or compressive. So sigma or sigma hydrostatic could be either tensile or compressive.

And there is another quantity, so this is first quantity of importance, Young's modulus, second quantity of importance is shear modulus, third quantity of importance is bulk modulus and there is fourth quantity of importance, which is for elastic behavior, so this is defined as nu, and this nu is equal to ratio of transverse to lateral strain which is minus of ey divided by minus of ez.

So this is what, these are the four properties that we have. And this is, so we say that this is sigma y, ey divided by ez minus and poisson's ratio value for, generally for metals, it is minus of 0.33, because one of the strains is going to be negative.

So negative, negative it is cancel each other. So metals it is about 0.33, ceramics is about 0.25, and polymers have a value of nearly 0.4. So these are sort of benchmarks. And you can see that the value of modulus is also in MPa or GPa, which is basically Newton per meter square, mega newton or giga Newton's per meter square. So basically they follow the same unit as of stress. So these are 4 fundamental quantities as far as elastic behavior is concerned the Young's modulus, the shear modulus, the bulk modulus and the poisson's ratio.

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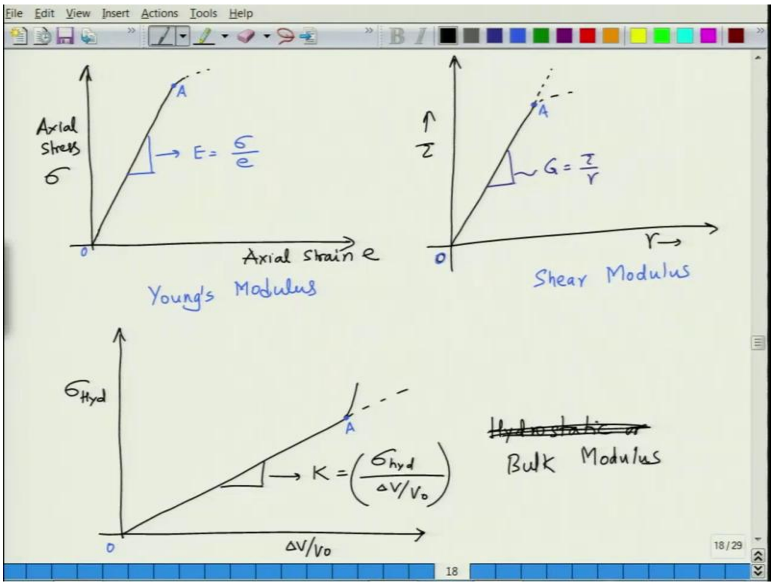

And the plots that we use to measure them are like this, the way they are represented in measurements. So if you apply axial stress sigma and measure axial strain e, then the behavior will be like this before it turns non-linear and up to this point this is, so let us say, A, OA, this is linear region, within linear region the slope of this part will give you E is equal to sigma divided by e. So this is for Young's modulus.

The second quantity is shear modulus. So here you apply shear stress before it turns into non-linear, non-linearity, this is let us say again, OA. On the y axis we have shear stress tau, on the x axis we have shear strain gamma, engineering shear strain and the slope of these will be, G will be equal to tau divided by gamma.

And similarly, for hydrostatic stress, this will be sigma hydrostatic and this is fractional volume change and if you measure the plot, the plot would be something like. So this would be, hydrostatic things would be something like under very heavy pressure, let us say, pressure vessels or objects that are residing under sea and things like that. So this would be the point of non-linearity OA. And the slope of this, so this will be again be non-linear region and this will give you a slope of sigma hydrostatic divided by delta v divided by v naught.

So this is sort of the relation between, this is, these are the three, this is hydrostatic or bulk, let us not write hydrostatic, just write the bulk modulus. So these are the sort of three things.

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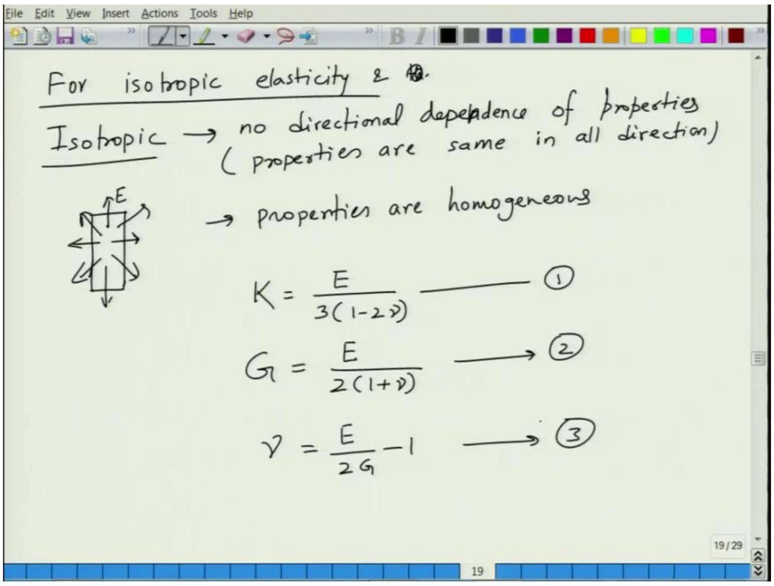

Now for a elastic material for isotropic elasticity. We will come to isotropic in a minute. So for isotropic elasticity, let us first see what isotropic is. Basically there is no directional dependence of properties. That is, properties are same in all directions. So here what we mean is that the modulus of material is similar in all directions.

So if you take a cube, let us say, if you take this bar, the E is in this direction, E in this direction, E in this direction, E in this direction, this direction, that direction, in all the directions the E is same. This could be fair approximation as far as polycrystalline materials are concerned, but it is a, it is assume to make things simpler, but it could, life would be a little harder than that.

So assume that material does not have any dependence of properties on the direction, which means properties are similar in all the directions. So if you assume that elasticity is isotropic, which means properties do not vary from point to point and they are identic in all the directions. So homogeneous and they are, another condition is properties are homogeneous, which means they do not vary from point to point.

In such a situation, I can write that if you know these, out of these four, if you know two properties, rest can be worked out. And the relations are K is equal to E divided by 3 into 1 minus 2 nu. So if you know poisson's ratio, if you know the elastic modulus, you can work out what the bulk modulus is.

Then if you can determine G, G is equal to E divided by 2 into 1 plus nu, this is second relation. So if you know elastic modulus and poisson's ratio, you can work out K and G and of course, if you know this you can replace E, if you combine the two equations, rearrange it, you can find out what nu is in terms of elastic modulus and shear modulus. This is the third relations. So these are three fundamental relations that we obtain for different materials.

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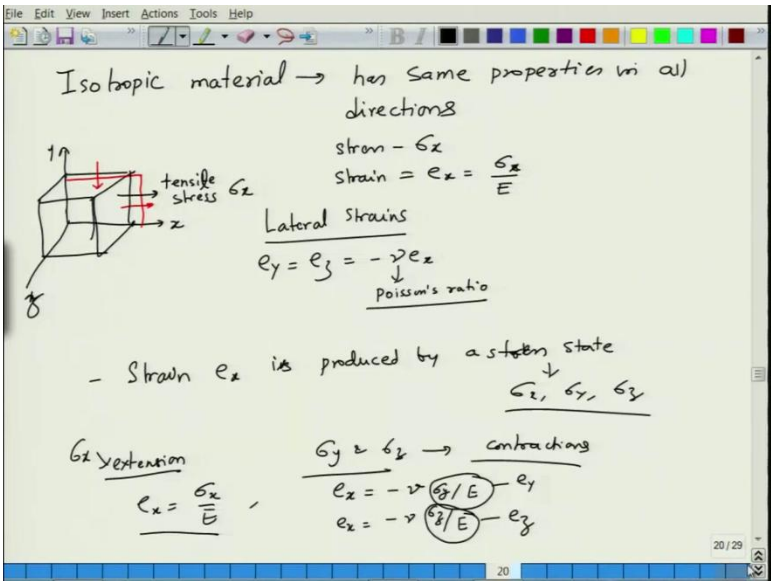

So now, now what we do is that, we just go a little bit further into isotropic elasticity and close it. So basically what we said that isotropic material has same properties in all directions. Now let us say, you take a piece of material, this is x direction, this is y direction, and you apply a uniaxial, so let us take a bar in this case and you apply let us say a stress in x direction, so let us say a uniaxial tension, tensile stress, tensile stress.

So if you apply tensile stress as sigma x, then strain can be given as ex, which can be written as sigma x divided by E. So here E is the Young's modulus, sigma x is the tensile strain in x direction, sigma x.

Now if you apply a uniaxial stress, what will happen is that the material will deform, which means you will have the strain in this direction. You will have lateral strain. So not only you will have strain in this direction, you will have lateral strain. So let us see what lateral strains are.

So, lateral strains will be in y direction as well as in z direction. So if you have a three dimensional piece of material like this. So let us say this is z. If that is the case, then you will have strain in all the three directions. You will have ey which is equal to ez will be equal to minus of nu ex. So this nu is basically poisson's ratio.

So assume that now suppose, supposing that strain ex is produced by astres state and astres state is defined as three components, sigma x, sigma y and sigma z. So contribution of sigma x is essentially ex, which is equal to sigma x divided by E. Similarly, we have these contributions of. So we saw that ex is equal to, so let us just rub it out, just one second.

So when you have this ex we said is equal to sigma x divided by E, so ey will be equal to minus of nu into ey will be equal to sigma y divided by E and ez will be equal to minus of nu sigma z divided by, sorry, ey is equal to minus ex, which is minus of ex, sigma x divided by E and ez will be equal to sigma x divided by E.

So you can also correlate this to sigma, you can also write it in this fashion that ex is equal to, we have said that, ey and ez is equal to minus of nu ex. You can write a little differently. So if you have these relations, ex is equal to sigma x by E, ey is equal to, sorry, let me just rephrase this. So when you have sigma x causing a strain of ex, which is equal to sigma x divided by E, the corresponding stress is sigma y and sigma z. So we have sigma y and sigma z. They will give rise to contractions. So this sigma x causes extension.

So corresponding contributions from (sigma i), sigma y and sigma z will be the contractions. So in that case we can write ex as minus of v, nu sigma y divided by E and we can write ex again as minus of nu sigma z divided by E. So contribution of sigma y and sigma z. So sigma y, sigma x is causing extension, whereas sigma y and sigma z are causing contractions and these contractions can be represented as ex being equal to minus of nu into ey. So basically this is sigma y divided by E, this is ey. And ex will be equal to minus correspondingly this will be ez. This is correct now.

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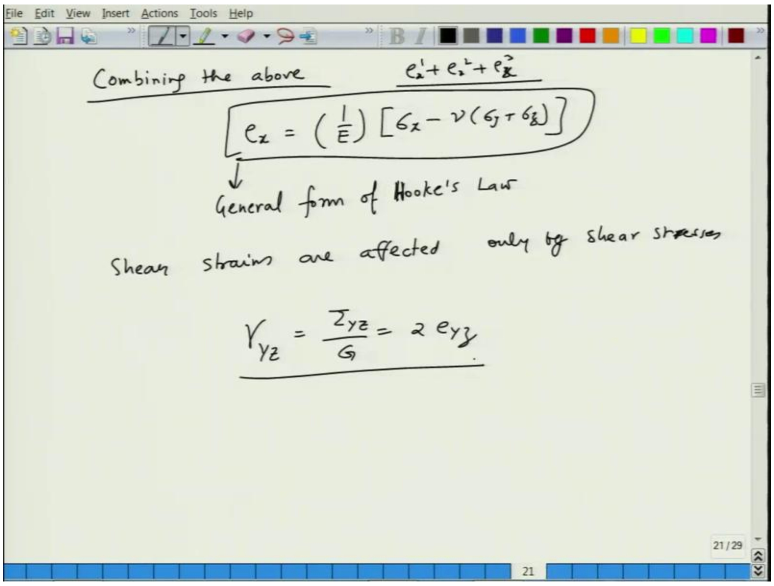

So now, if this is true, so if you combine all these contractions, we get an equation which is ex is equal to 1 over E into sigma x minus nu into sigma y plus sigma z. So let me say that this is let us say the extension, this is the contraction. So the net deformation or strain will be equal to, so we can say that this is 1, this is 2, this is 3. So basically we are saying that it is ex is equal to ex1 plus ex2 plus ex3. And when you sum these together this is what you get. So the net ex is equal to 1 over E into sigma x minus nu into sigma y plus sigma z.

So now shear strains are, so this is, this form is basically called as general form of Hooke's Law. And since shear strains are affected only by shear stresses, we can write gamma yz equal to tau yz divided by G, which is nothing but 2eyz. And if we apply this to all directions, we can write this, the above equation, so we have this relation, we have this relation, if we apply it to all the directions, we can create a framework for all the three directions, which we will do in the next class.

So basically what we have done in this lecture is we have understood, we have first went into defining the elastic properties. So we saw that elastic behavior of metals or ceramics, crystalline solids is generally up to very small strain showing a linear region, from which you can calculate what the modulus of elasticity is, because the slope is modulus of elasticity, slope of stress versus strain, but there is a non-linear region shown by a molecular material such as polymers and rubbers.

The relation between stress and strain in elastic region, linear region can be represented by Hooke's Law, which is sigma is equal to, so this is Hooke's Law, which is sigma is equal to eE. You can write this in tensorial form, modulus of elasticity being the fundamental property of material. Then we looked at four more properties which are fundamental properties, Young’s modulus, shear modulus, bulk modulus and poisson’s ratio. And then we saw the differences between the three properties, so all the four properties.

We are trying to deal a little bit into details of isotropic elasticity and we are on our way to define relations for the relations that we have seen in previous pages. So what we have come up with when you apply it in sinus stress correspondingly you have compressive stresses in other directions. So there is a net one strain, one stress cause extension, other stresses cause contraction. As a result, we have a net deformation which is given as the 1 over E into sigma x minus of nu sigma y plus sigma z.

So you can see that this is the net stress that material experiences, giving rise to net elongation Similarly, you can write expression for shear stresses. We will do this in detail in the next lecture. Thank you.