Properties of Material (Nature and Properties of Materials: III)
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Lecture – 05
True and Engineering Strain

So, welcome again to new lecture of the course properties of materials.

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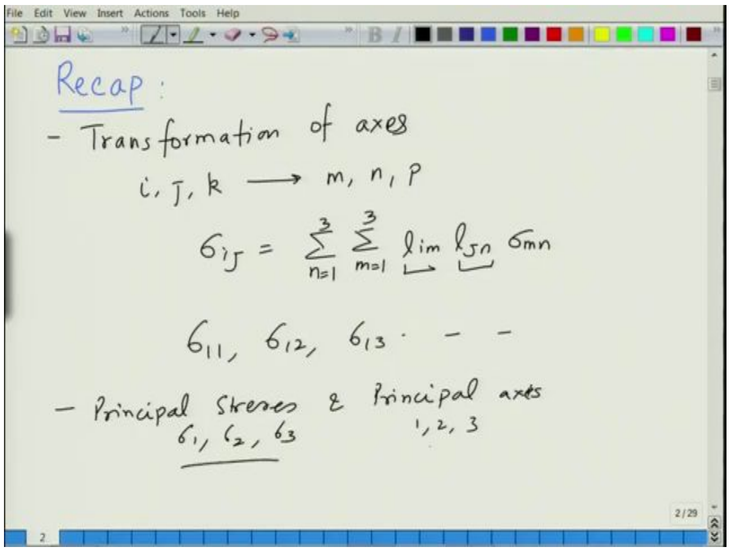

So, let us just be recap the last lecture. So, in the last lecture we looked at transformation of axes. So, basically you have one set of orthogonal axes let us say i, j, k, and you want to transform to you want to transform from let us say another axes which is m, n, p. So, and for example for stress one can use this relation sigma i j is equal to summation over n is equal to 1 to 3.

Summation over m is equal to 1 to 3 l I m l j n sigma m n. So, this is essentially and l's are the cosin of angle between i and m axes and j and n axes and sigma m n is basically depending upon the how do you add these. So, essentially we and this is also applicable for strain as you will see later on, and you can write this in general form by not writing the summation but it is employed that summation is there and so we do not, you can write any one.

So, you can write sigma 1 1 you can write expression for sigma 1 2, Sigma 1 3 and so on and so forth, and I would suggest you very strongly to do this exercise at home because, and we give certain examples of sigma x x and sigma x prime x prime, sigma x prime y prime and so on and so forth. And then you can modify them a little bit by using the symmetry of tensor, stress tensors that is sigma i j is equal to j i and then you can shorten it.

And then we were looking at the concept of principal stresses. So, principal stresses and principal axes. So, principal stress axes if we define as 1, 2, 3, then principal stresses become sigma 1, sigma 2, and sigma 3 and this what we are doing.

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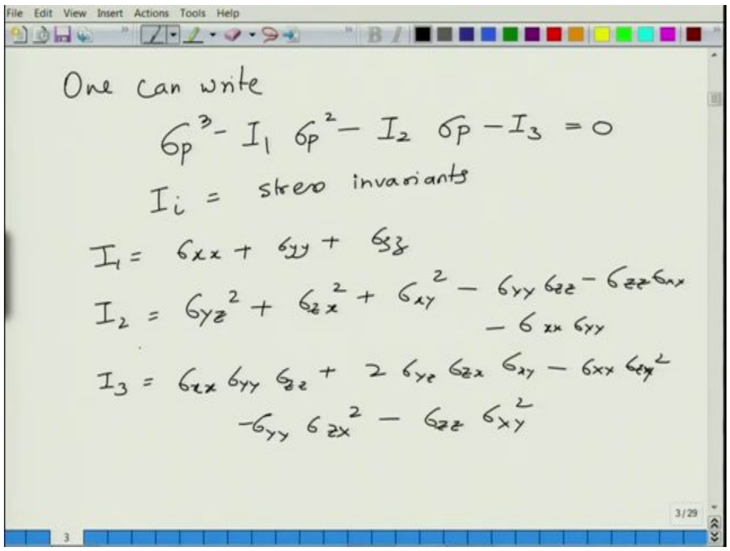

So, let us take this further. So, one can define a relation in between the principal stress values that is sigma p cube minus I1 sigma p square minus I2 sigma p minus I3 is equal to 0, and we wrote that these I are nothing but Ii are nothing but stress invariants. So, I1 for instance one can write as sigma x x plus sigma y y plus sigma z z. I2 would be sigma y z square plus sigma z x square plus sigma x y square minus sigma y y sigma z z minus sigma z z sigma x x minus sigma x x sigma y y.

And I3 will be equal to sigma x x sigma y y sigma z z plus 2 sigma y z sigma z x sigma x y minus sigma x x sigma z x square, sorry sigma x x sigma z y square minus sigma y y into sigma z x square minus sigma z z sigma x y square. And this is where we were and so this basically I1 is nothing but essentially some of the three principal stress some of these three normal stress sigma x x sigma y y and sigma 3 3 and I2 is this and I3 is this.

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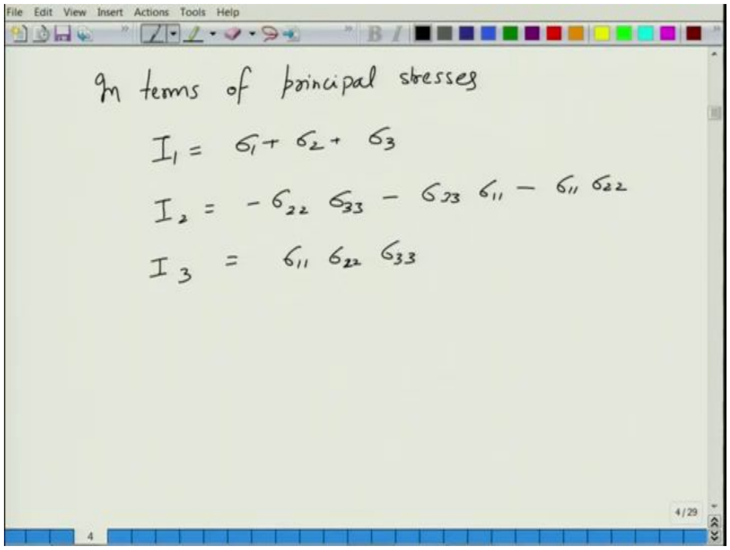

And in terms of principal stresses hence one can write these as, so we can write I1 as sigma 1 plus sigma 2 plus sigma 3. And I2 can be written as minus of sigma 2 2 sigma 3 3 minus of sigma 3 3 into sigma 1 1 minus of sigma 1 1 into sigma 2 2. And I3 can be written as sigma 1 1 into sigma 2 2 into sigma 3 3 is the expression in terms of principal stresses. So, this is a way to write the principal stresses.

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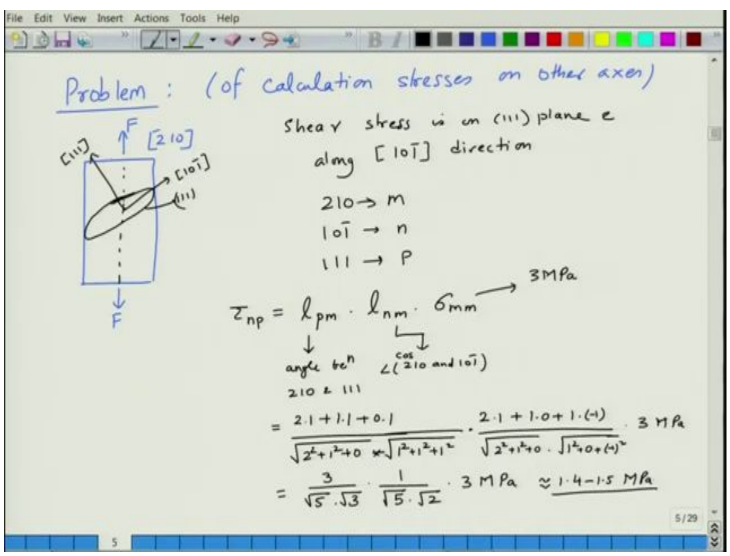

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One can write

$$\sigma_p^3 - I_1 \sigma_p^2 - I_2 \sigma_p - I_3 = 0$$

I_i = stress invariants

$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$$

$$I_2 = \sigma_{yz}^2 + \sigma_{zx}^2 + \sigma_{xy}^2 - \sigma_{yy}\sigma_{zz} - \sigma_{zz}\sigma_{xx} - \sigma_{xx}\sigma_{yy}$$

$$I_3 = \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\sigma_{yz}\sigma_{zx}\sigma_{xy} - \sigma_{xx}\sigma_{yz}^2 - \sigma_{yy}\sigma_{zx}^2 - \sigma_{zz}\sigma_{xy}^2$$

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Recap:

- Transformation of axes

$i, j, k \longrightarrow m, n, p$

$$\sigma_{ij} = \sum_{n=1}^{3} \sum_{m=1}^{3} l_{im}\, l_{jn}\, \sigma_{mn}$$

$\sigma_{11}, \sigma_{12}, \sigma_{13} \cdot - -$

- Principal stresses & Principal axes

$\sigma_1, \sigma_2, \sigma_3$ $1, 2, 3$

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In terms of principal stresses

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2 = -\sigma_{22}\sigma_{33} - \sigma_{33}\sigma_{11} - \sigma_{11}\sigma_{22}$$

$$I_3 = \sigma_{11}\sigma_{22}\sigma_{33}$$

So, let us do solved example for instance. So, if you take a problem, so this would become little clearer I have a new one to, so let us first take the problem of the calculation of shear stress, stresses on other axes. So, what we have looked at basically we first looked at the transformation and then we looked at the expression for principle stresses, stress variants inbuilt in them and then from these stress variants one can calculate what are the values of principle stresses and that is what we have to do now.

So, let us assume there is a bar which is under of force along this axes and let us say this axes is 2 1 0 Crystallographic axes. The materials deformed by is a process as we will see later on called a slip on plains where. So, let us not invoke the phenomena (())(06:43) stress. But on certain plains, on certain place on which atoms can move easily. So, this direction along which the plains are let us say it is a 1 1 1 plain.

So, 1 1 1 plain in a cubic system will have a normal which is 1 1 1 perpendicular to the plane and the direction along which the materials will atoms will move during the formation is that direction that lies within the plain and this is 1 0 1 type of direction and since it lies, it lies in 1 1 1 plain the dark product of this direction with respect to plain should be equal to 0. As a result this direction happens to be 10 bar 1 direction.

So, basically we can say there is a, so although the normal stresses applied along to 1 0 axes the shear stress is on 1 1 1 plain and along 1 0 bar 1 direction. So, let us say we define 2 1 0 as M axes 1 0 bar 1 as n and 1 1 1 as p. If these are three convention we follow then we can write the expression for tau n p. Tau n p is the shear stress within along this direction 1 0 1 and in 1 1 1 plain. So, if we now write the expression for this, this happens to be lpm multiplied by l nm into sigma m m.

l pm is basically angle between, pnm that is angle between 2 1 0 and 1 1 1 and this is the angle between n and m that is the angle between 2 1 0 and cos of angle between 2 1 0 and 1 0 bar 1. And sigma m m is the stress that is applied. Let us say this stress works to be a roughly let us say 3 mega Pascal. So, this is the 3 mega Pascal stress that is along the m direction and, so what is the angle between 2 1 0 and 1 1 1.

So, that is nothing but 2 into 1 plus 1 into 1 plus 0 into 1 divided by square root of 2 square plus 1 square plus 0 multiplied square root of 1 square plus 1 square plus 1square this is the cos theta with the angle co sin of the angle between 2 1 0 and 1 1 0. Now the angle between these two is 2 into 1 plus 1 into 0 plus 1 into minus 1 divided by square root of 2 square plus 1 square plus 0 into square root of 1 square plus 0 plus minus 1 square into 3 MPa.

And this will turn out to be 2 plus 1, 3 divided by square root of 5 into square root of 3 this will turn out to be 1 divided by square root of 5 into square root of 2 into 3 MPa. So, you can do the mathematics yourself the stress will turn out to be 1.4 to 1.5 mega Pascal. So, this will be the magnitude of shear stress that will add long 10 bar 1 direction when the stress applied normal to the phase of the bar is 3 MPa.

This can be worked out by doing transformation of axes just like we have done in this case. So, you can work out for any system equations like these and then determine what the stresses are going to be.

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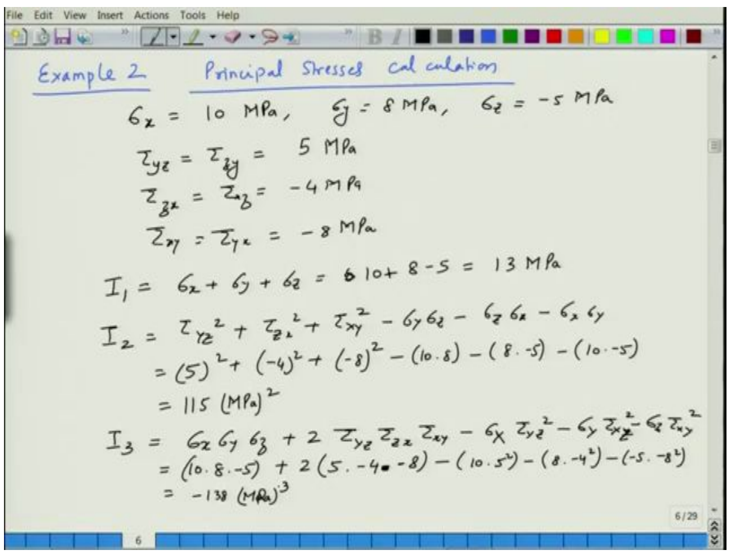

Now let us look at another example of related two principal stresses in, principal stress calculation in a body which has been subjected to certain stress state. So, let us say we define sigma x is equal to 10 MPa sigma y is equal to 8 MPa, and sigma z is equal to minus 5 of MPa. So, we can calculate what is tau y z, and tau z y. Tau y z and tau z y is given as minus of 4 MPa.

Tau z x is equal to tau x z which is given as minus of 4 MPa again, sorry this is, let us see the above is 5 MPa and then tau x 5 which is equal to tau y x which is equal to minus of 8 MPa. So, we can calculate what I1 is I1 is as we are saying I1 is sigma x plus sigma y plus sigma z. If I1 is sigma x plus sigma y plus sigma z then we can calculate this as 5, sorry 10 minus, 10 plus 8 minus 5, this is equal to 13MPa.

Let us do the calculation for I2, I2 will be as we wrote earlier sigma y z square plus sigma z x square plus sigma x y square so we can replace the sigma with essentially tau. Minus of sigma y sigma z minus of sigma z sigma x minus sigma x sigma. So, we will replace the sigma y y with sigma y as the notation is.

So, let us replace the values now here. So this is 5 square plus minus 4 square plus minus of 8 square this will be equal to and this is again sigma y and sigma z. So, this is equal to 5 minus 4 minus 8 and then this going to be 10 into 8 minus of 8 into minus 5 minus of 10 into minus 5, and we do the sum this is equal to 115 MPa square. And I3 as we know is sigma x sigma y sigma z plus we can write tau y z tau z x tau x y minus of sigma x into tau y z square minus of sigma y into tau x y square minus of sigma z into tau sorry y x z square.

So, sigma x is, so we can say this is 10 into 8 into minus 5 plus 2 into 5 into minus 4 into minus 8 minus sigma x is 10 into tau y z is 5 square minus of sigma y which is 8 into tau x z which is minus 4 square minus of minus 5 into tau x y which is minus 8 square and if you do the Math this will turn out to be minus of 138 MPa cube.

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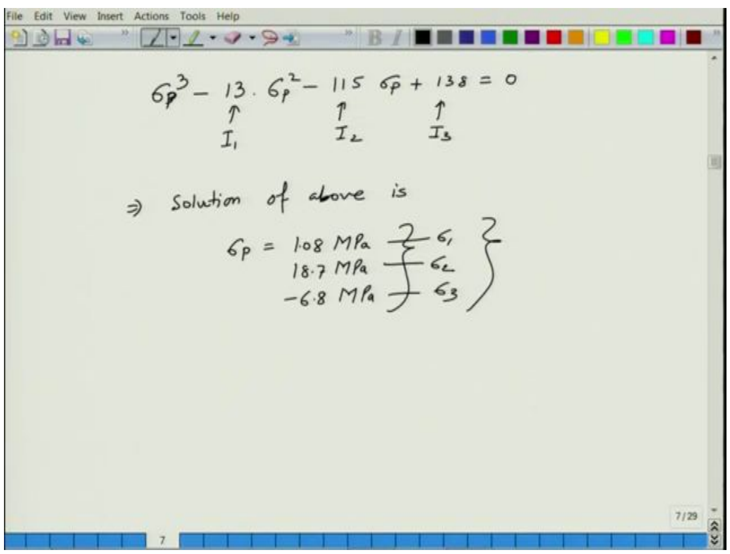

So, now let us do the substitution so sigma p cube minus of I1 which is 13 into sigma p square minus 115 sigma p plus 138 is equal to 0. So, this is I1 this is I2 and this is I3. So, basically this was the equation in which we have repeated values. And if you solve this, above is sigma p value is turn out to be 1.08 MPa, 18.7 MPa and minus 6.8 MPa. So, this is the solution you are going to obtain for the three principal stresses that is sigma 1, sigma 2, and sigma 3. So, I hope it is clear from the formula that how does 1 determine the stresses sigma.

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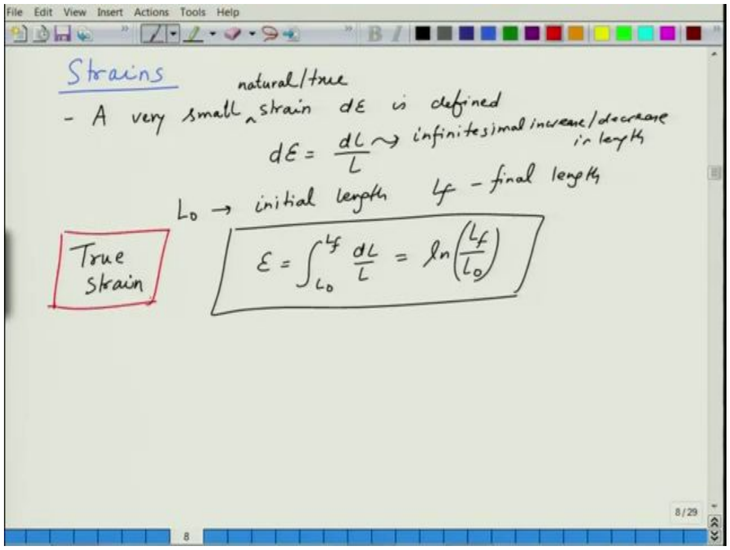

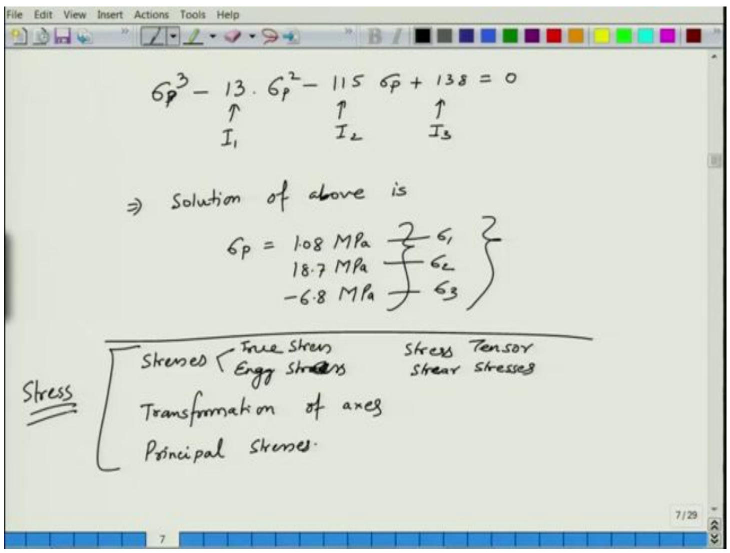

So, now what we do is that having looked at the stresses in little detail there is something which is called as more stresses, if time permits we will come to that later on, but now let us go back to the concept of strains. So, what we have worked out in stresses until now is basically what stresses are true stress, engineering stress, what is stress tensor, the transformation of axes, and so true stress, engineering stress, stress tensor we also looked at shear stresses and we looked at the transformation of axes and then we looked at what is, how do you work out the principal stresses.

So, these are the few topics that we looked at under stress. Now let us move on to the next topic that is strain in the materials. So, if you consider a solid is being formed. So, let us say a very small strain that is infinite decimally strain normal strain d epsilon is defined as a very small let us say natural strain slash true strain is defined as d epsilon is equal to the L by L. So, this is the infinitesimal increase slash decrease in length.

So, let us say if L naught is the initial length and Lf is the final length then this is from L naught to Lf d L by L. So, integrated from L naught to Lf and we get a strain which is ln of Lf divided by L naught. This is called as true strain or natural strain. It is other strain which is, so let us say this is true strain.

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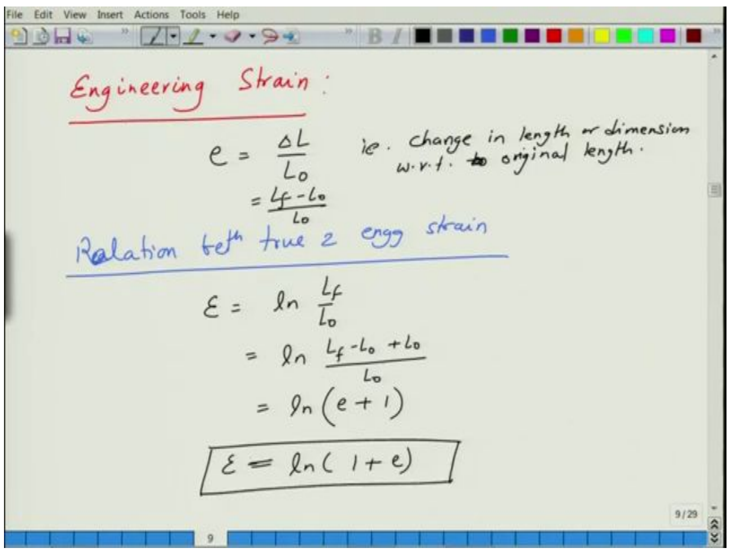

There is other strain which is called as engineering strain, which is given as e is equal to delta L divided by L naught. Which is that is change in length or dimension with respect to original length. The problem is engineering strain is not a very accurate measure of real strain in the material and there are few differences.

So, at very small strains these two values correspond with respect to each other but when the value is increased to large strains then there is loss of correspondence between the two value and the reason is true strain is the spontaneous true strain natural strain which gets added which gets which is equal in both in tension and operation whereas engineering strains has problems with respect to addition as well as when you calculate them with respect to tension and compression as we will see later.

Now, let us see what is the relation between the two things so relation between. So, we can write this as Lf minus L naught divided by L naught. So, we write this epsilon as ln of Lf divided by L naught I can write this as Lf minus L naught plus L naught divided by L naught and this becomes e plus 1. So, basically epsilon is equal to ln of 1 plus e as it is always, as it is expressed. So, this is the relationship between true strain and engineering strain.

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And since epsilon is equal to ln or 1 plus e if we do if you open the v if you do the series expansion then we can write this as e minus e square divided by factorial 2 plus e cube by factorial 3 minus e 4 by factorial 4 and so on and so forth. So, we can see that as e tends to 0 epsilon tends to e. But this happens only at which means at very, at very small strains. So, let us do a little calculation.

So, we have a value of e let us say we go from 0.001 to 0.0005 to 0.001 to 0.005, 01 to 0.05 to 0.1 to 0.5 to 1.0. These are the values of small e which are changing. So, if you now calculate epsilon which is ln 1 plus e, at 0.001 of strain engineering strain the value is 0.0000999 which is equal to 0.0001 equivalent. And if you calculate the ratio this turn out to be 1.001. If you calculate for 0.00005 this turns out to be 0.000499 and this is equivalent to 0.0005 and the ration turns out to be 1.002.

And if you do for this this is 0.000999 and this turns out to be 0.001 ratio is nearly 1.002 and 0.005 will show value of 0.00498 which is again sort of a approximated to 0.005 and the ratio will increase to 1.004, and then you keep on doing this when you come to let us say 0.01 then the value that you get is 0.0098 and you can again approximated to 0.001, 0.005, 0.05 will give you a value of 0.0487, 0.1 will give you a value of 0.095, 0.5 will give you a value of 0.405 and 1 will give you a value of 0.693.

And you can see this ratio for example if you calculate here it will be1.443 for this it will be 1.233 for this it will be 1.049 for this it will be 1.025 and for this it will be something like 1.01. So, you can see that these two values start diverging somewhere around 0.01. So, this is where you have sort of increased differences. So, it is only at small strains there is a good correspondence but at large strains there are large differences.

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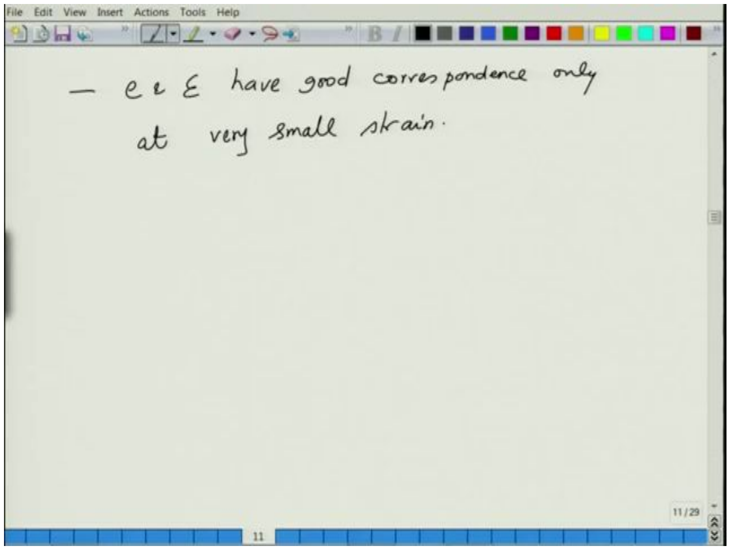

So, this is one major difference that e and E have good correspondence only at very small strains. That is why true strain is much better measure of strain then the engineering strain. What we are going to do in the next class now is we will take up more difference we will understand more difference in between the true strain and engineering strain. Especially in terms of how they add up and how are they related to tension and compression as well as how are they related to volume change?

So, what we have done in this class we have looked at basically the principal stresses how do you work out the principal stresses and then how do you calculate the principal stresses in a, we just had a simple example of tensile test in which forces applied to normal to the faces and then we calculate the stress on plain on which slip occurs we will come to the phenomena of slip later on. Then we also worked out what the magnitude of principle stresses.

So, in the previous case we worked out a shear stress and then we worked out the value of principal stresses once we know the value of principle and other normal and shear stresses, and then we looked at the conserve of strain the difference between true and engineering strain, and we will develop on it further in the next lecture. Thank you.