Properties of Materials (Nature and Properties of Materials: III)
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Lecture 37 - Fundamentals of Semiconductors

So, welcome again to the new lecture of the course, Properties of Materials. Let us just briefly first see what we did in the last class.

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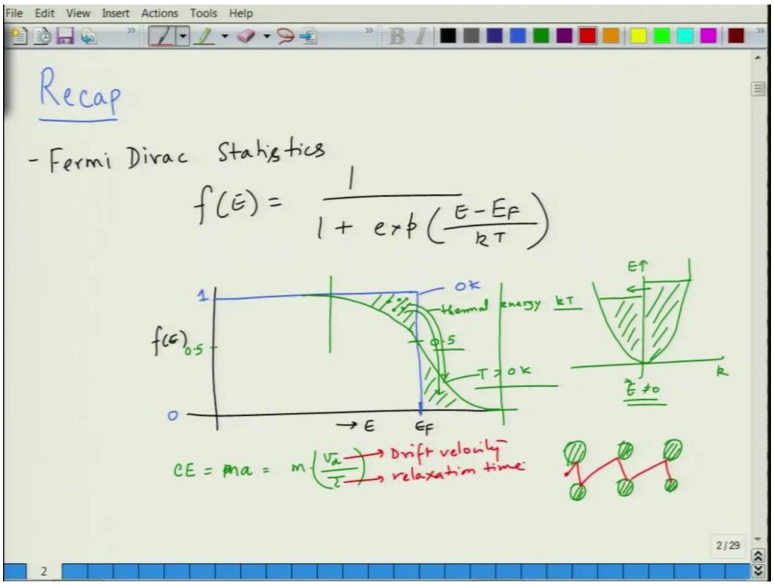

Recap

- Fermi Dirac Statistics

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

$$CE = ma = m\left(\frac{v_d}{\tau}\right)$$

So, in the last class, we were talking about Fermi Diarc Statistics, which is basically probability of finding an electron in the energy space. So, essentially, when you plot f E as a function of E, if you have energy, Fermi energy is energy at which, Fermi energy is the energy above which all the energy levels are empty at 0 Kelvin and below which all the energy levels are filled at 0 Kelvin. So, at 0 Kelvin, the plot shows this kind of nature.

So, this is at 0 Kelvin. And this is the E F. So, everything is so, this is the probability 1 and this is probability 0. So, at 0 Kelvin, everything every energy state up to E F is filled and all the energy states above E F are empty and of course, energy states are quantised. As you increase the temperature, increasing the temperature leads to some electrons crossing from this side. So, these are the electrons which have crossed on this side. They have gone over here.

So, these two areas are equal and this is 0.5. So, basically, at temperature greater than 0 Kelvin, the energy level in the vicinity of E F just below E F, they lose their electrons which get thermal energy. So, these are the electrons which are present here. They get excited by thermal energy. So, basically, thermal energy, it moves them from here to here. So, essentially, this is the sort of band in which then they conduct because some states are empty, some states are free.

So, this is the band in which they conduct. So, basically, now you have room for movement of electrons and this also causes imbalance in the velocity of electrons. If you plot the E versus k plot, so on one side the velocity is lower in the k space, on the other side the velocity is higher. So, this results in net velocity. So, this is k, this is E so, basically, you can say that there is a net velocity of electron which allows them to conduct under the influence of electric field.

So, this is when electric field is not equal to 0. And then we looked at basically, what happens when you apply electric field, electric field gives rise to acceleration m a, but you cannot have infinite acceleration. So, as a result, a is replaced by drift velocity which is average out velocity divided by relaxation time tau, tau is the so when electron travels in a lattice, so, this is the lattice let us say.

And you have electron travelling. So, electron undergoes collision. So, it undergoes collision, this collision tends to decelerate the electrons. As a result, there is average velocity of electron V d which is drift velocity. And tau is the relaxation time which is the average time spend between two successive collisions.

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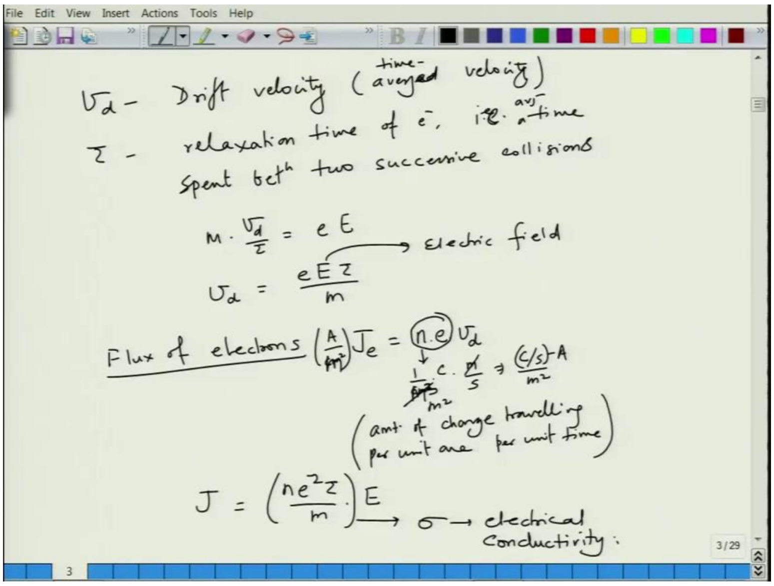

v_d - Drift velocity (time-averaged velocity)

τ - relaxation time of e^-, i.e. avg. time spent bet^n two successive collisions

$$m \cdot \frac{v_d}{\tau} = eE \longrightarrow \text{Electric field}$$

$$v_d = \frac{eE\tau}{m}$$

Flux of electrons $\left(\frac{A}{m^2}\right)$ $J_e = \boxed{n \cdot e}\, v_d$

$$\frac{1}{m^2} \cdot C \cdot \frac{\cancel{m}}{s} \Rightarrow \frac{(C/s) \to A}{m^2}$$

(amt. of charge travelling per unit area per unit time)

$$J = \left(\frac{ne^2\tau}{m}\right) E, \quad \left(\frac{ne^2\tau}{m}\right) \longrightarrow \sigma \rightarrow \text{electrical Conductivity}$$

So, V d we can define as, drift velocity or you can say averaged, time averaged velocity and tau is the relaxation time of electron that is time, so this is we can say average time spent between two successive collisions. So, as a result, we saw that m into V d divided by tau is equal to e E. So, from this we got drift velocity is equal to e E tau divided by m.

So, this E is basically electric field not the energy. And from this, we calculated what is the flux of electrons, flux of electrons is nothing but so J let us say is the flux of electrons, which is essentially nothing but number of electrons of charge e moving at a velocity V d. So, if you look at this number so, if you look at quantity, this is basically in Amps per centimetre square.

Now, let us see, so this m is in per centimetre cube. So, this is 1 over centimetre cube, e is Coulomb, V d is metre per second. This is let us say metre cube per second and this is ampere per metre square. So, this cancels with this. So, you have so this becomes m square from m cube. What we have essentially, Coulomb per second per metre square.

And what is Coulomb per second? Is the ampere. So, basically, it is number of charges, total amount of charge flowing to certain length per unit time. So, that is essentially the so, basically, it is amount of charge travelling per unit time per unit area. So, that is what it is. So J is equal to n e V d, you can say amount of charge travelling per unit area per unit time. That is what the flux says basically.

So, if you now make the substitution, we make we write J as n e square tau divided by m into E and this term is nothing but sigma that is electrical. So, this is where we were yesterday.

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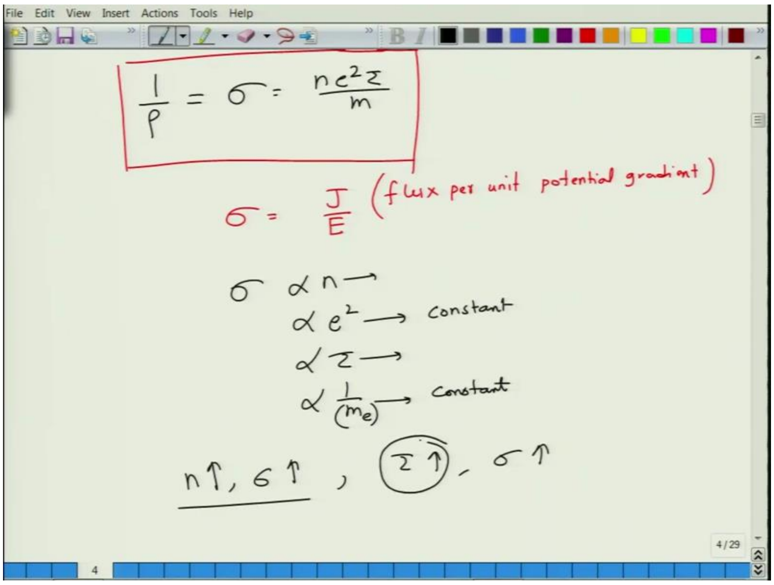


$$\boxed{\frac{1}{\rho} = \sigma = \frac{ne^2\tau}{m}}$$

$$\sigma = \frac{J}{E}$$ (flux per unit potential gradient)

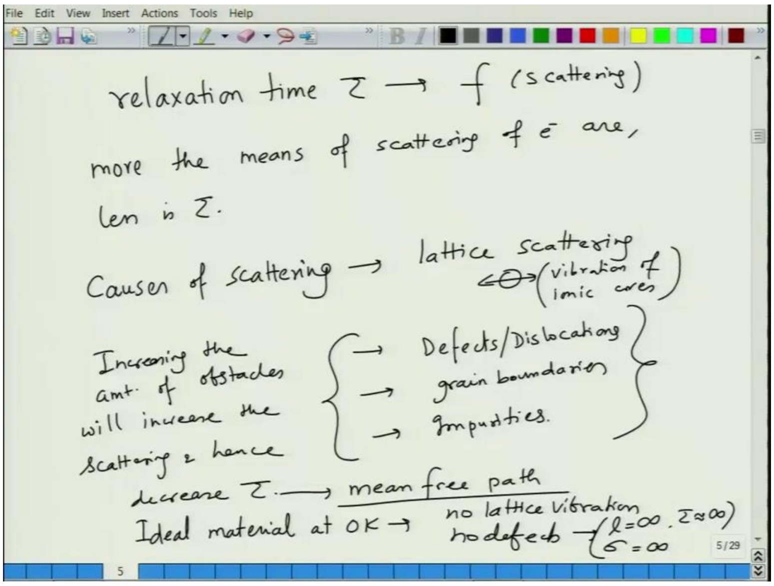
$\sigma \propto n \rightarrow$

$\propto e^2 \rightarrow$ constant

$\propto \tau \rightarrow$

$\propto \frac{1}{(m_e)} \rightarrow$ Constant

$\underline{n\uparrow, \sigma\uparrow}$, $\tau\uparrow$, $\sigma\uparrow$



relaxation time $\tau \rightarrow f$ (scattering)

more the means of scattering of e^- are, less is τ.

Causes of scattering → lattice scattering $\Leftrightarrow$ (vibration of ionic cores)

→ Defects/Dislocations
→ grain boundaries
→ Impurities.

Increasing the amt. of obstacles will increase the scattering & hence decrease τ. → mean free path

Ideal material at 0K → no lattice vibration, no defects → $\{ l = \infty,\ \tau \approx \infty),\ \sigma = \infty$

So, we define this sigma is equal to n e square tau by m and this is equal to 1 over rho. And this is a fundamental relation for metals or conductors. So, basically, you can say conductivity, so you can write basically sigma is equal to J divided by E. So, essentially, it is flux per unit potential gradient. This is what it is. Now, you can see that, sigma is proportional to, proportional to, n e square tau divided by m.

So, sigma is proportional to n, it is proportional to e square, it is proportional to tau, it is inversely proportional to 1 over m. In this, this is a constant, this is a constant because it is mass of electron let us say m e, this is e which is a constant. What varies is n and tau. So, if you can increase so basically, if n increases, sigma increases. Tau increases, sigma increases. Now, n is generally something which is fixed for a given material. So, and does not change too much.

So, what you can, however, effect for a given material is tau and this tau is relaxation time which is basically is a function of basically scattering. So, more the means of scattering of electron are, lesser is tau. So, essentially, what we say is that, if you are if you have things in the material which make electron collide more or scatter more, so it will have less average time spent between the two collisions.

So, essentially, what are the things which lead to, so causes of so, one is we have seen is lattice scattering because ionic cores are vibrating so, this is because of vibration of ionic cores. And as a result, you have lattices scattering. That will be present in any material, other means could be defects you can say dislocation, for example.

You can have grain boundaries, and you can have impurities. All of these things will cause scattering and so if you increase, increasing the amount of obstacles will increase the scattering and hence decrease tau. So, this is what basically tau will be. So, and this is this tau is essentially related to what we say, is the mean free path.

Mean free path is the average distance travelled between two collisions. So, of course, when you do not have for ideal crystal, for ideal material at 0 k, there are no lattice vibrations, no defects. So, which means the L should be equal to infinite or tau should be equal to infinite. So, basically, conductivity should also be infinite for ideal material. But that is not the case because materials realistly have defects, some defects always present and they also have impurities. As a result, you will always have some finite.

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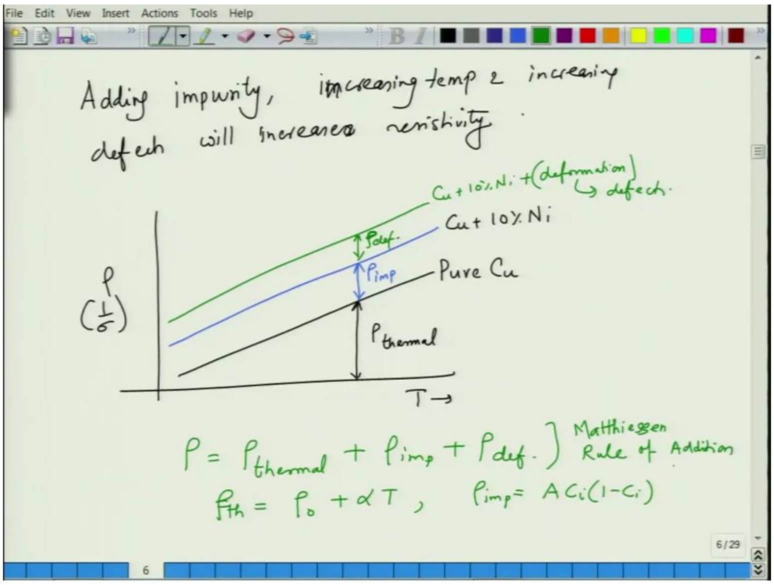
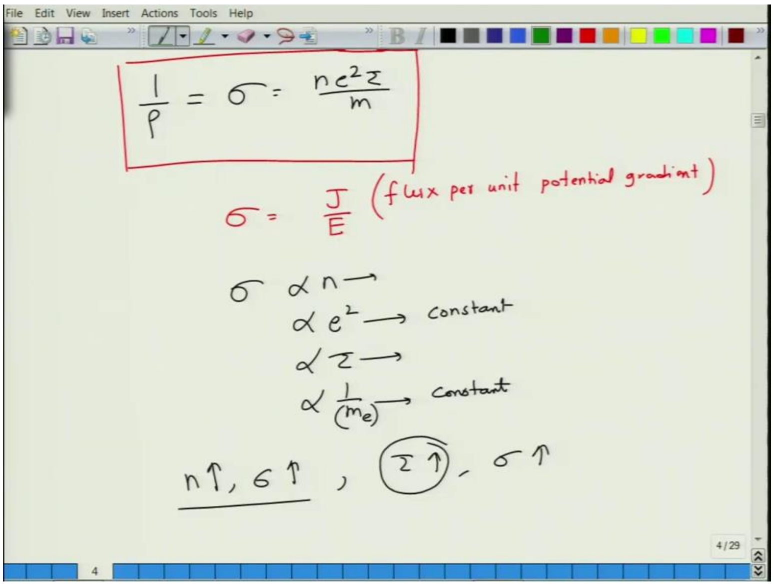


Adding impurity, increasing temp & increasing defects will increases resistivity.

$$\rho = \rho_{thermal} + \rho_{imp} + \rho_{def}. \Big]$$ Matthiessen Rule of Addition

$$\rho_{th} = \rho_0 + \alpha T, \quad \rho_{imp} = A C_i(1 - C_i)$$



$$\boxed{\frac{1}{\rho} = \sigma = \frac{ne^2\tau}{m}}$$

$$\sigma = \frac{J}{E}$$ (flux per unit potential gradient)

$$\sigma \propto n \rightarrow$$
$$\propto e^2 \rightarrow \text{constant}$$
$$\propto \tau \rightarrow$$
$$\propto \frac{1}{(m_e)} \rightarrow \text{constant}$$

$$\underline{n\uparrow, \sigma\uparrow}, \quad \textcircled{\tau\uparrow}, \sigma\uparrow$$

So, basically what we are saying is that, adding impurity, increasing temperature and increasing defects will increase resistivity. So, essentially, if you look at that case, for instance copper, let us say we make a case for copper, this is temperature axis, this is resistivity. So, let us say this is the case for pure copper. The resistivity increases and basically, whatever contribution you have at this point is basically you can say rho thermal.

And rho is nothing but 1 over sigma. Now, let us say, we add little bit of nickel to it. When we add nickel, little bit of nickel to it, so this is pure copper. When you add little bit of nickel to it, so let us say copper plus 10 percent nickel so, when you add 10 percent nickel, it increases the resistivity. So, as a result, the extra contribution is rho impurity.

Now let us say, you add some amount of deformation to it. So, when you add some amount of deformation to it, then this is the extra contribution that you create as a result of rho defects. Because deformation causes dislocations and defects to be created. As a result, resistivity for this you can say is copper plus 10 percent nickel plus deformation.

So deformation causes defects and defects so, that is how the resistivity will vary. So, essentially, resistivity increases with increasing impurity content and increasing temperature as well as increasing defect content. So, basically, we can write this rho as rho thermal which is the basic component plus rho impurity plus rho defect and where, rho thermal is approximated as rho not plus alpha T and rho impurity is given as A some constant into C i, which is a impurity concentration into 1 minus C i.

And basically, this is, this you can say is additive kind of rule. So, this is basically you can say is Matthiessen Rule of Addition. So, I would suggest you to go through a few examples so, what you do is that you take different values of n, different values of e, different values of tau and try to calculate, so you can take different value of e but only different value of n and tau. To see what are the different conductivity values you can calculate from just, and this will be also part of assignments that will be given to you.

So, now what we have said is, we have first talked about the conductivity of metals. So, conductivity of metals say, sigma is equal to n e square tau over m which is a reciprocal of resistivity. And conductivity decreases or resistivity increases with increasing temperature, increasing impurity concentration and increasing the defect concentration. So, this is what it is about the metals. Now, what we do is that, we so let us just work out certain numbers for, just look at the table for certain materials.

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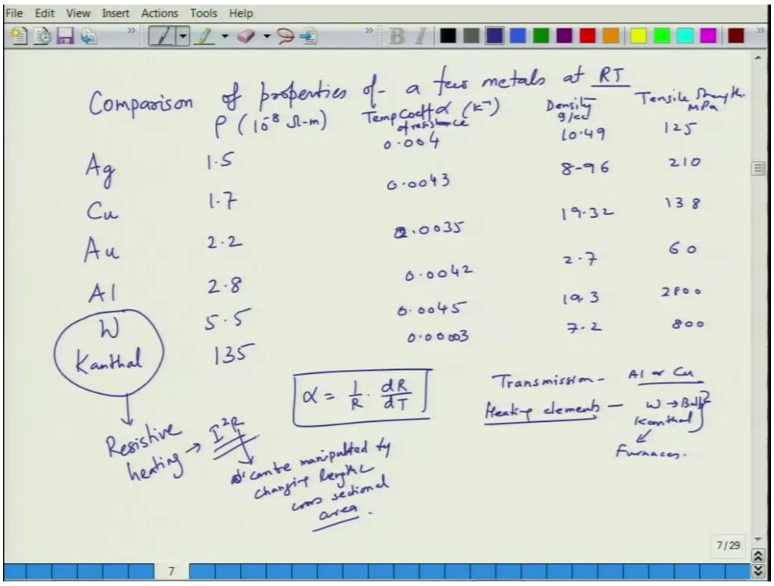

Comparison of properties of a few metals at RT

	ρ (10^{-8} Ω-m)	Temp Coeff of resistance α (K^{-1})	Density g/cc	Tensile Strength MPa
Ag	1.5	0.004	10.49	125
Cu	1.7	0.0043	8.96	210
Au	2.2	0.0035	19.32	138
Al	2.8	0.0042	2.7	60
W	5.5	0.0045	19.3	2800
Kanthal	135	0.00003	7.2	800

$$\boxed{\alpha = \frac{1}{R} \cdot \frac{dR}{dT}}$$

W, Kanthal → Resistive heating → I^2R → R can be manipulated by changing length & cross sectional area.

Transmission – Al or Cu

Heating elements – W → Bulb; Kanthal → Furnaces.

So, let us say, comparison of a few metals at room temperature. So, let us say, let us make a list. Silver, copper, gold, aluminium, we can also add tungsten because it is important thing. So, let us see and we can have kanthal. Kanthal is basically, molybdenum di-silicide which is used for heating filament. So, resistivity of, resistivity in 10 to power minus 8 of Ohm metre. So, if you write for silver, it is about 1.5; for copper, it is about 1.7; for gold, it is about 2.2.

And then aluminium, it is about 2.8; for tungsten, it is about 5.5 and for kanthal, it is about 135. So, these are certain resistivity values. And you can also write things like what is the temperature coefficient alpha, which is nothing but Kelvin inverse, then we can write density and we can also write tensile strength which is MPa. So, for silver, it is 0.004 for 10.49 density which is gram per CC and then it is 125 MPa.

And if you look at copper, it is 0.0043; this is 8.96 and this is 210. Gold is 2, 0.0035, this is 19.32. Gold is very heavy and 138 is the tensile strength. For aluminium, it is 0.0042, this value is 2.7 and this is 60 MPa, very weak. Tungsten on the other hand, which is used in bulbs, it has a coefficient thermal expansion of 0.0045 and then 19.3 is the density, but its tensile strength is extremely high. That is why we do not use gold. We use tungsten for making the….

So, this is, sorry, temperature coefficient of resistance, not coefficient thermal expansion by the way. So, you can define this as alpha is equal to 1 over R into d R over d T. This is alpha. And the value for kanthal is 0.00003 and this is 7.2 and this is 800. That is why kanthal is generally used in furnaces, where you do not want very high coefficient, where you want high strength as well as high resistivity for heating.

And basically, the resistance of this tungsten and kanthal is essentially useful for resistive heating because of I square R term. So, higher the resistances because of I square R losses and that is why if you have copper and aluminium, they will not lead to too many losses. But if you have things like tungsten, kanthal that will lead to and you can manipulate the resistance also by changing the, can be manipulated by changing length and cross-sectional area.

So, essentially, for transmission line for example, for power transmission we use either aluminium or copper because of their fabricability and mechanical strength and low I square R losses. And for heating elements, which should not melt easily, we use tungsten as well as kanthal. So, these are used in bulbs, for example, these are used in furnaces. So, essentially, these are the and of course, other things are for heating elements, we used to use good oxidation resistance and high temperature strength and so on and so forth.

So, let us, this is about metals, basically basic properties of metals, that metals have the important thing that you consider about metals is resistivity and conductivity, and so which is conductivity is given as n e square tau by m. So, by you can manipulate the conductivity of materials by changing the changing n and tau and it is tau basically makes a lot of difference to conductivity of these materials. Now, what we are going to do is that, we are now going to discuss what we say, is the properties of semiconductors.

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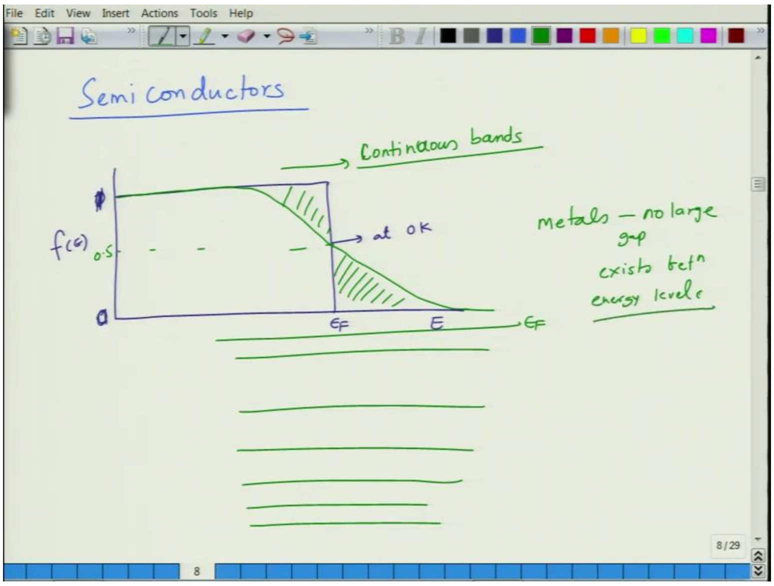

Semi conductors

metals — no large gap exists betn energy levels

Now, earlier we said that Fermi Dirac statistics says that, at 0 Kelvin so, f E versus E at 0 Kelvin you have this kind of step function so, 1 and 0 not, other way around 1 and 0. So, at 0 Kelvin, all the energy levels are filled and all the energy levels are empty above E f. As you increase the temperature, the Fermi level, the curve becomes asymptotic around 0, around E f, around 0.5 value.

And then the electrons leave the lower part to upper part. What we are looking at metals is basically, you can say metals have continuous bands. Semiconductors on the other hand as different. So, if you look at these quantised state of energy, so in quantised states of energy, you have bands like these. At some point you will have E f. So, you will have E f, so of course, you have quantised bands but the bands are, there is no large gap.

These gaps are smaller. As a result, electron can get energy and then you can move around in those energy levels. In semiconductors, what happens is that, so you can say in metals, no gap no large gap exists between energy levels.

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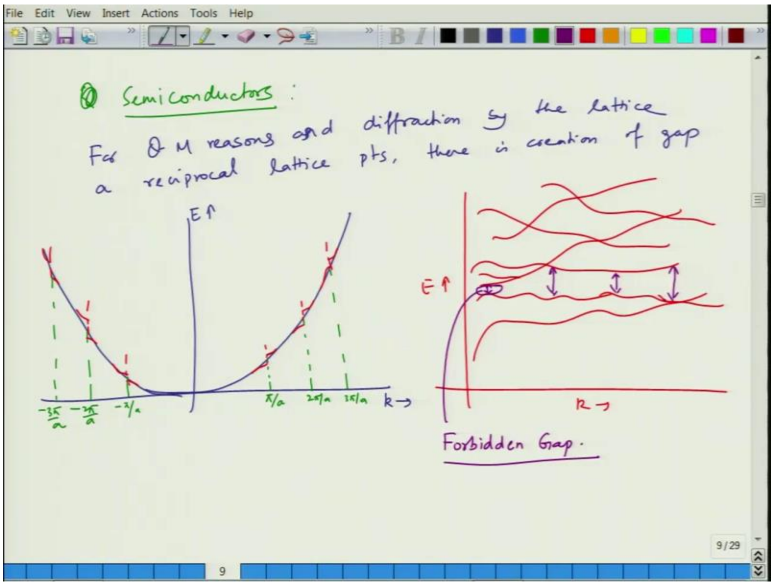

Semiconductors :

For Q M reasons and diffraction by the lattice a reciprocal lattice pts, there is creation of gap

Now, in semiconductors for quantum mechanical reasons and diffraction by the lattice at reciprocal lattice points, there is a creation of gap. So, essentially, when you plot this E k diagram, so E k diagram is something like this, this is E, this is k. Now, this is k space, so k space basically will have now reciprocal lattice points will be plus pi by a, 2 pi by a, 3 pi by a, minus pi by a, minus 2 pi by a, and minus 3 pi by a.

What happens in E k diagram is, because at diffraction by the lattice, there is a gap created at this stage. So, there is a gap. So, this curve tends to become like this around this point. So, basically, what you have to, lot of these gaps are created. But you have, for a 3D solid you have lot of energy states. So, you may have situation like for a 3D solid when you make the band structure for a given 3D solid, it may be slightly different.

For a 3 D solid, it may, let me show you the picture. So, what so you may have now energy levels going like this. So, you will have to calculate the band structure. But what you can see here is, if you have energy state linkages which means, this is E, this is k space. You can see that in this energy space, there is a gap here, but then electron can move to different k space. It can always find energy state.

So, there is no continuous gap across the k space for electrons to travel. So, as a result, electrons can flip flop their way around the k space to higher energy state. Now supposing, instead of this what happened was, that there was a clear gap. So, you can see that there is a gap like this. So, there are gaps here, there are gaps here, there are gaps here. So, whatever there is a minimum gap is, so this is the minimum gap.

So, maxima and minima of upper and lower part. So, maxima and minima of lower and upper part, this is basically what we say is the forbidden gap and this happens because of diffraction of electrons at the reciprocal lattice points. So, if this happens, then this leads to creation of energy gap and this energy gap is basically forbidden energy gap. So, you may have situation like this in semiconductors, the situation is something like this.

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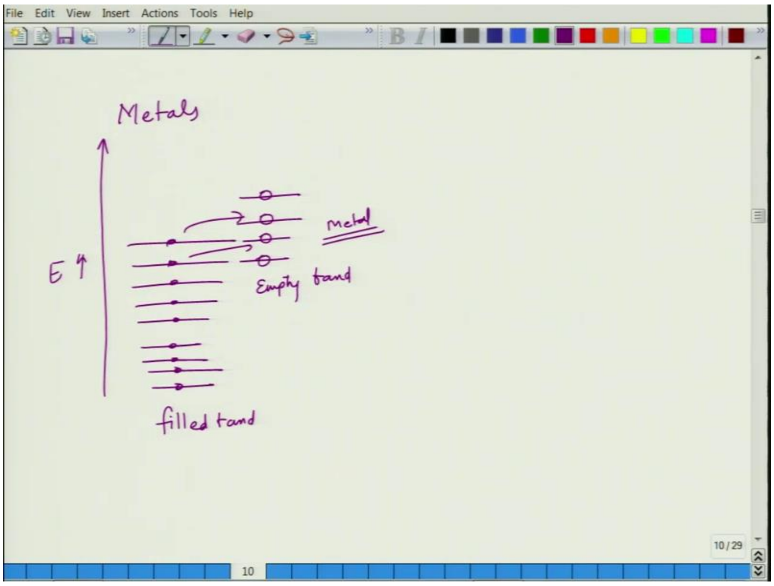

In case of metals, so when you plot metals, metals will have let us say you are plotting energy states so, you will have these energy states like these. The electrons are sitting here and then you have another, let may be little gaps somewhere here. And so this is but at the same time in k space somewhere in parallel, you will have these empty states. So, these are all empty bands.

So, these are all filled bands you can say and this is empty band and this is energy. So, at the same energy state, there are some empty state so, there is a room for electron to move. So, as a result, electron can flip flop here and there and this is what metal is. Metal has no empty energy state. In the semiconductor, the situation is different. We are running out of time now. So, what we are going to do is that, we will discuss that in the next class.

So, in this class basically what we have discussed is, we have discussed about the little bit details of how the conductivity of metals can be varied by changing impurity concentration, by making causing deformation or by increasing the temperature and then we move to semiconductors. We just started talking about creation of energy gap, but we will discuss this in more detail in the next lecture. Thank you.