Properties of Materials: Nature and Properties of Materials III
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Lecture 3
Normal and Shear Stress, Transformation of Axes

So, welcome again to the new lecture of this course properties of materials. So, let me just briefly recap what we discussed in the last class.

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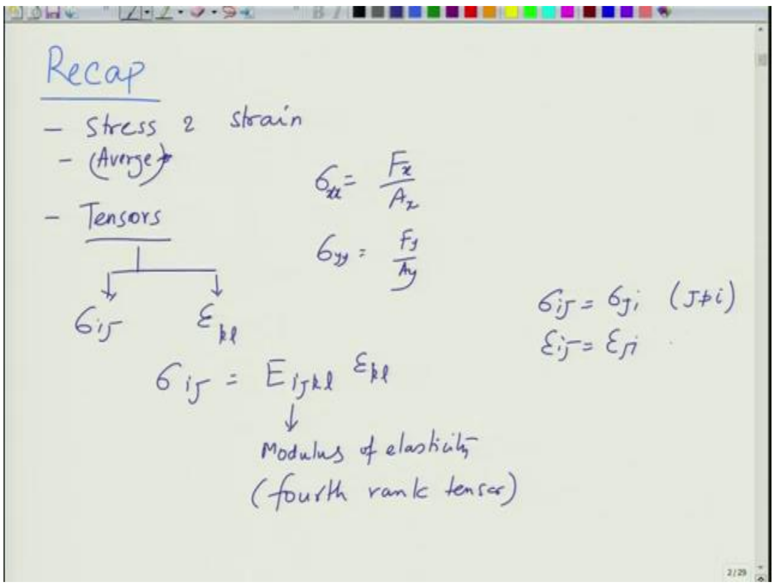

So, in the last class we essentially introduced what stress is, what strain is and basically you can say that, so stress we defined in the beginning as average stress and average strain. So, average values of both of them. But then later we introduced the concepts of tensors and this is because the stress and strain both are vectorial quantities as a result they have different components in different directions.

So, although for example, stresses made of forced and area, force and area both being scalar quantities, the magnitude of forces magnitude of force and area of force along different directions are different as a result as stresses are different in different directions. So, while sigma xx could be Fx by Ax sigma yy is Fy by Ay. The magnitude, so these are all principal stresses sigma xx and sigma yy normal stresses we can say them.

So, these normal stresses are different to each other because the Fx and Fy magnitudes can be different as well as Ax and Ay magnitudes can be different. So, although these two are scalar quantities sigma itself is a is not a scalar quantity.

This is what we introduced in the last thing last lecture and we also said that sigma can be represented as sigma iJ stress strain can be represented by (())(01:56) iJ and these two are related with respect to each other by a quantity called iJkl EiJkl and well this E is nothing but modulus of elasticity which is a fourth rank tensor and by symmetry sigma iJ is equal to sigma Ji when J is not equal to i. Similarly, epsilon iJ is equal to epsilon Ji when J is not equal to i and this is essential to provide to avoid the instabilities in the system otherwise the system will become unstable.

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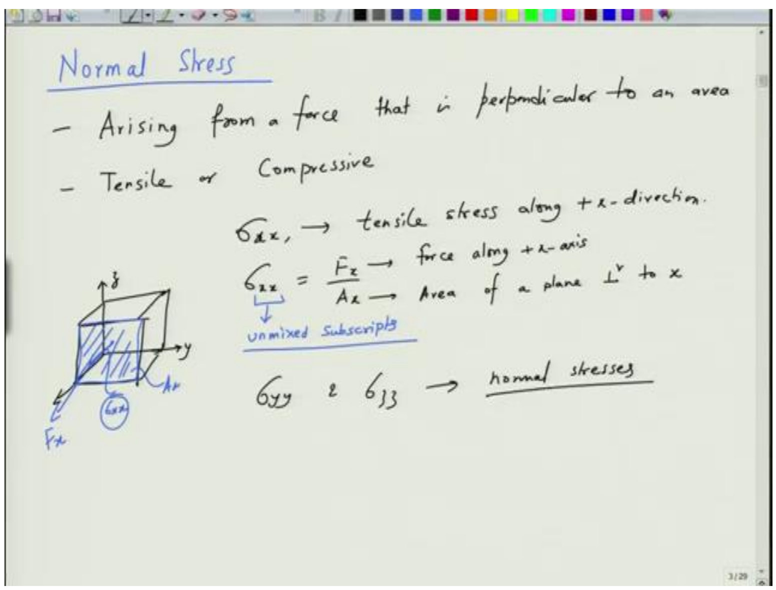

And so, now let us move on to the next topic of this particular course. So, we first define what we call as normal stress. So, normal stress is basically the stress which arises from a force so arising from a force that is perpendicular to an area and this could be either tensile or compressive nature. So, we represent these normal stresses as sigma xx. So, basically sigma xx would be tensile stress along, let us say plus x direction and it can be written as sigma xx is equal to Fx divided by Ax.

So here, Fx could be the force, x is the direction and this is the area of a plane perpendicular to x. So for example, if you want to do it in a cube So, let us say this is x, this is y, this is z then this would be sigma xx which is basically perpendicular to this phase here. So, this is the area Ax and this is the direction of Fx, basically you can say Fx which gives rise to sigma xx.

So, basically when the subscripts are unmixed, then we look at what we call as is as normal stress. Similarly, you will have notions for sigma yy and sigma zz. See if these are all normal stresses explained in a similar manner.

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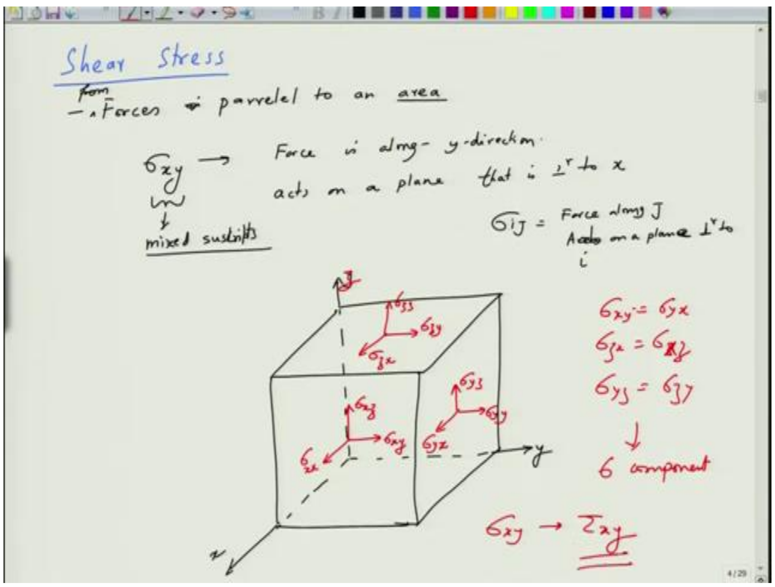

Next thing that you need to know is what we call as shear stress, a shear stress is basically, the resulting from forces, from forces which is parallel to an area. So, shear stresses are generally depicted as let us say sigma xy. So, sigma xy means that force is along y direction and which acts on a plane that is normal to x, normal to x.

So, generally these will be mixed. So, not generally always these will be mixed subscripts. So, this is what it is. So, if it is sigma iJ you say that force along J and the area on a, that acts on a plane which is, sorry acts on a plan that is perpendicular to i. So, which is, so essentially what would it mean is that if you draw now a, and these.

So let us say for the front face we have here sigma xx and sigma xy. So, this is on the front plane acting along the y direction on a plane that is perpendicular to x and then we will have sigma xz.

Similarly on this plane we can have sigma yy and then we can have these shear stress components, we can have sigma yz and sigma yx and for this we will have sigma, this is z not y sigma zz.

So, this should be sigma zx, by symmetries as I said sigma xy will be equal to sigma yx and sigma zx will be equal to sigma xz and sigma yz will be equal to sigma zy. So, this will reduce the total number of components to 6 and this sigma xy is often referred as tau xy. So, this is shear stress tau xy.

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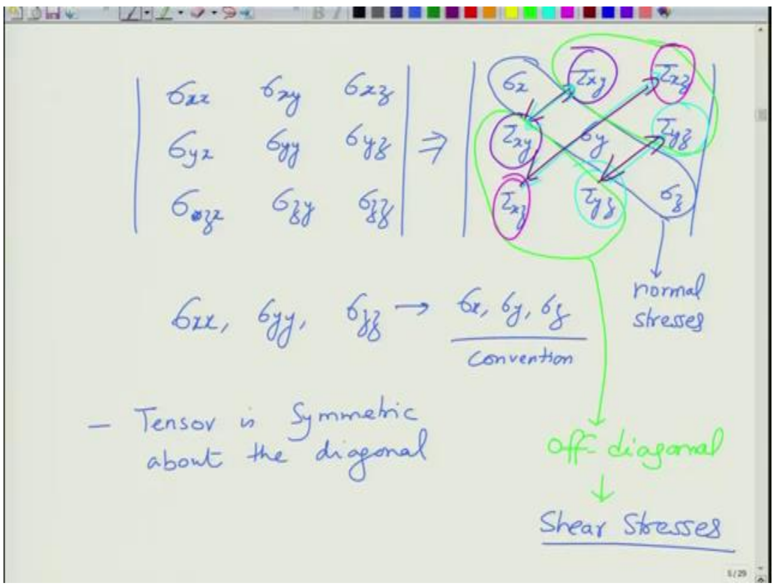

So, when you now want to write the tensor the tensor is sigma xx, sigma xy, sigma xz, sigma yx, yy sigma yz, sigma xz, zx. You will also see instead of xx xy and z people use 1, 2, 3 that is also perfectly legitimate. So, this becomes sigma x. So, often you will see sigma xx sigma yy and sigma zz which are normal stresses are referred as sigma x, sigma y, sigma z.

So, this is a convention. So, this could be sigma x, this could be tau xy, this could be tau xz and then we have tau yx which is nothing but tau xy and then we have sigma y and then we have tau yz then we have tau xz, then we have tau yz and then we have sigma z. So, we can see the diagonal components are these are normal stresses and the off diagonal components which are these and these.

So, these are we can say shear stresses. So, and the tensor is symmetric about the diagonal. So, basically we can say the tensor is symmetric about the diagonal and that is that we can see because this is corresponding to this, this corresponds to this and this corresponds to that. So, these two are related to, connected to each other, similar to each other, since they are similar to each other, these are all off diagonal components which are shear stresses symmetric So, this is just a definition of what is the stress and strain in the victorial notation and how do you write shear and normal stresses.

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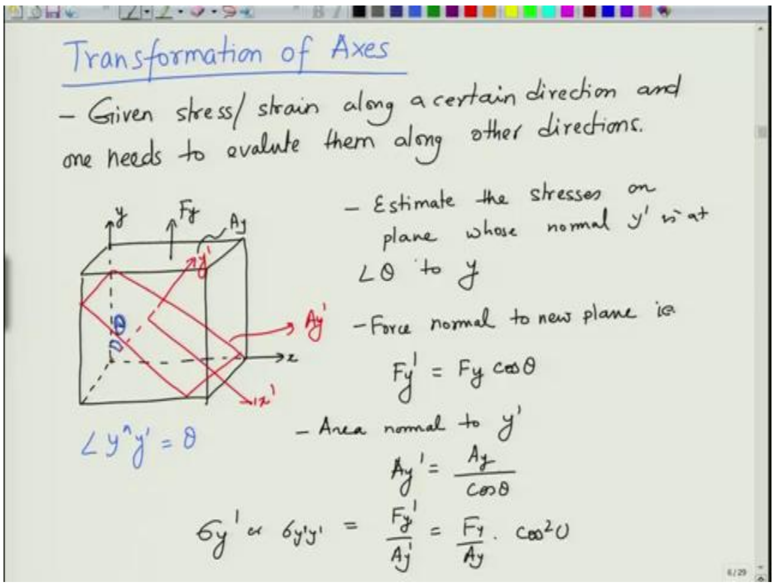

Now, often what you will see is that often in mechanics on mechanical behavior of materials, you might want to do what we call us transformation of axes. So, which means you are given stress or strain along a certain direction and in need to and this is necessary because as when we apply when we define the metals we apply load along certain direction, but the stress components are different in different direction and this is where we need to calculate the stresses in different directions from the from the one which is available.

So, for this let us draw a diagram here. So let us say we have a, so let us say this is x and this is y for the sake of convenience and let us say then there is a there is an area which is like this. Let us say this is an area Ay prime. So, we have a normal load which is. So this is Fy and this Fy acts on Ay and then we transformed on the coordinate system, let us say for which I have a…

So this is y prime which is normal to this plane Ay prime, and this y prime is has another direction which is, so let us say this is x prime and so let us say in this case, these are both orthogonal axis and the y prime is oriented with respect to xy, previous x is in such a manner so that the angle here is theta.

So, this angle here between y, so y and y prime is equal to theta. So, this is the angle between the y and y prime. So, what we want to do is that, so we want to, so what is our objective The objective is to calculate the stress to estimate the stress on a plane whose normal y prime is at theta angle to y.

So, the normal we know that original stresses forces Fy and the area is Ay. So let us say we calculate what is the normal force to this new plane. So, that is let us say, Fy prime, so this Fy prime by geometry you can say it is equal to Fy into cos theta and now we calculate, what is area normal to y prime and this area normal to y prime is written as ay prime which is ay in this case it has now become.

So, this is the area of this phase is A, this is Ay. So, it becomes Ay divided by cos theta by geometry. So, basically you can say that sigma y prime or you can say sigma y prime sigma y prime y prime to be precise is equal to Fy prime divided by Ay prime which is equal to you can see now it becomes Fy divided by Ay into cos square theta.

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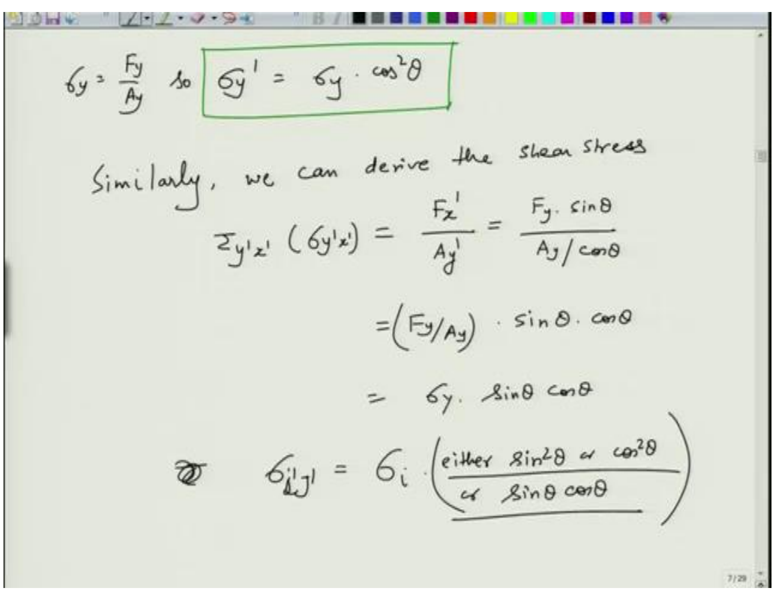

So, essentially this sigma y prime is equal to sigma y, sigma y was equal to Fy divided by Ay. So, this becomes sigma y into cos square theta. So, this is the first relation that we have worked out by transforming the axis. Now, similarly, if you want to calculate the shear stress, the shear stress and the shear stress we can write as tau in this case it becomes y prime x prime.

So, basically the forces acting along x prime on a plane whose normal is perpendicular to y prime. So, this is nothing but sigma y prime x prime this is equal to basically you can say Fx prime divided by Ay prime and this is equal to you can say this is equal to Fx prime we have this will become Fy divided by sin theta and this will become Ay divided by cos theta and in this case it becomes essentially Fy by divided by Ay into, not divided by sin theta into sine theta into cos theta.

So, this is sigma y into sine theta, cos theta. So, we can see that when you transform the axes and so when you transform the axes, what you are going to obtain is. So, let us say you want to write a general expression of sigma i prime J prime. So, this will come to be, let us say, I do not know some sigma i into either sine square theta or cos square theta or you will have a product of these two sine theta into theta, you will have one of these terms coming in the multiplication factor and this can vary.

So question now is how do we evolve a general framework? Because for each of these, when it gets to three dimensions, it gets even more complicated. So the question is, do we want to do in this manual way or do we want to evolve a general framework to calculate them easily and we will see how do we do that. Now, let us look at some of the things related to sign convention.

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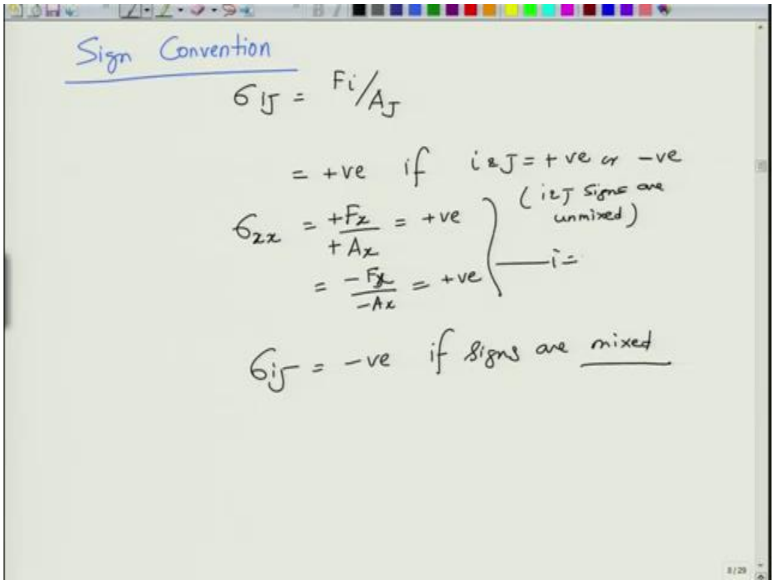

So, Sign convention we follow sigma iJ is equal to Fi divided by AJ. So, sigma iJ will be positive if i and J are both positive or both negative. So, essentially what it means is that, let us say we calculate sigma xx. So, it forces acting along plus x direction and the area is also along plus x. So, these both will come plus and it will remain positive vice versa if it was minus Fy minus Fx, this was a longer to get to direction as well then again it remains positive.

So, this is what it be. So, as long as i and J signs are unmixed this remains positive and sigma iJ will become negative if signs are mixed. So, for principal stresses i will equal to i will be equal to J so, it will be nothing but basically in this case what i was equal to J. So, you are only worried about the sign.

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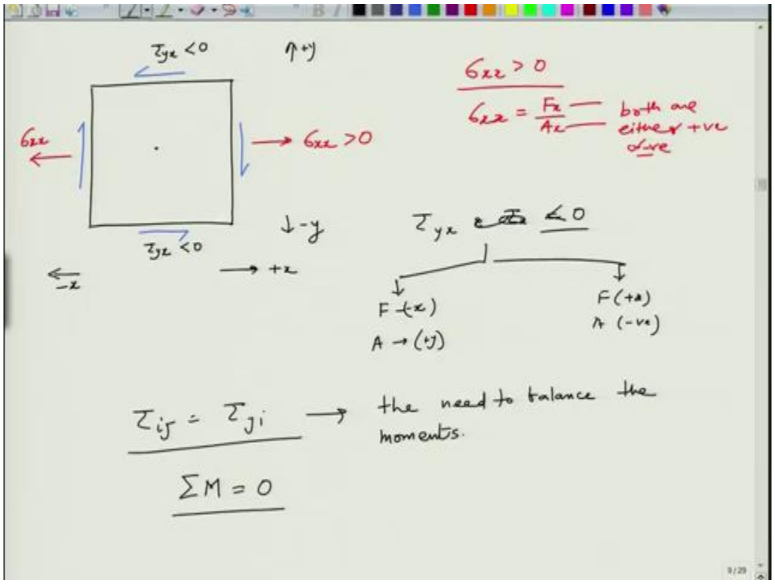

So let's say we draw a picture here. So, in this case, let us say, so this will be positive x, this will be negative x, this will be positive y, this will be negative y. So, let us now draw the stresses. So, if the principle stress or the normal stress acts in this direction which is sigma xx. So, sigma xx whether you take this direction or this direction.

So, if you take this direction this is positive because the forces along positive x direction as well as the area is along also the positive x direction in this case the forces along minus x direction and the area is also minus x direction. So, as a result sigma xx its being a positive. So, sigma xx is greater than 0 because both Fx and Ax are, so this is nothing but sigma xx will be equal to Fx divided by Ax. So both are either positive or negative.

So on the right, they are positive, on the left they are negative. If you look at the shear stress on the other hand, so shear stresses tau y x, this ends up being equal less than 0. Because if you look at the top one the stress x along the minus x axis. But the area is along the plus y axis as a result there is unevenness. So, tau yx and tau, tau yx is basically negative 0.

Because two cases in the first case, the force is along minus x and area is along plus y, in the other case the force is along plus x and the area is along negative one as a result is a mismatch and we say that tau x ,tau iJ is equal to tau Ji and this is essential basically because of the need to balance the moments. Because moment sigma M has to be equal to 0, otherwise, the body will become unstable and it will go in sort of a perpetual rotation or acceleration.

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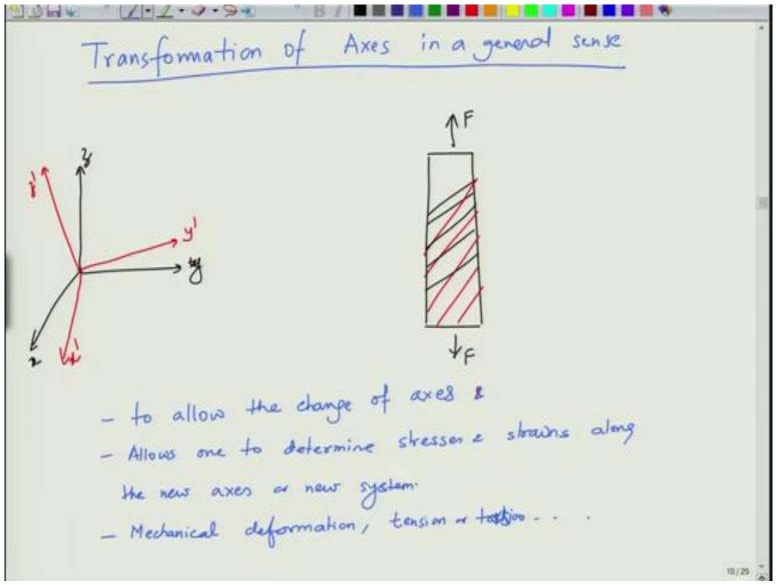

Now, let us look at another topic. The next which is the important one is the transformation of axes in a general sense. So, now this is required let us say you have stress axes which is normal stress axes is x y z. So, suppose you have a bar like this you apply the forces f. However, the deformation does not always happen does not happen perpendicular direction of force. Deformation happens on entities called as, as we will explain later on different planes.

So, we have Miller indices. So, we want to calculate what are the stresses which are principal stresses with respect to these planes. So, you could have one set of planes you could have another set of planes depending upon with a sample as single crystal, a poly crystal, you may require to calculate the stresses, principal stresses for different planes and directions.

Similarly, when it comes to tensile and torsion stress, you may have different tensile and shear stresses and you need to evaluate them for different directions. So, basically you need to do transformation on axes to allow the change of axes. So, from let us say, in this case, we go from, so let us maintain the orthogonal system y prime and then z prime.

So, essentially what you have done is you have taken the x y, x y z system and rotated it by certain angle with respect to x y or z and then you have a different, different system different axes system altogether at a certain angle. So, basically it allows you to change the axis and hence allows one to determine stresses and strains along the new axes or new system.

So, this is what we are going to do and this is very important for mechanical deformation because deformation is an isotopic phenomena. Because crystallinity of material. It could be also related to things like tension or torsion calculating different stresses and so on and so forth. So, we will get back to this in detail in the next lecture.

So, what basically we have done in this, this lecture is we have defined what the normal and shear stresses are, how do we transform the stresses in the different axes and what is the sign conventions. In the next lecture, we will produce a general framework for transforming the axes and then calculating the stresses and strain along those new transformed axes, thank you.