Defects in Crystalline Solids (Part-II)

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Lecture – 13
Dislocation Interaction & Strain Hardening

So, in the previous module, we found how the shear strength of the material increases with increase in the dislocation density. But, how does dislocation density increase, now that is a question that we will try to answer our part of it in this particular module.

$$\Delta G = \Delta H - T\Delta S$$

So, the our topic right now is origin and multiplication of dislocation. Now, if we are talking about increment or decrement in the number of dislocation, the first question is, is there any equilibrium number of dislocation or in other words our dislocations also a thermodynamically stable structure like we found that the point defects are. And when the an when a particular entity is thermodynamically stable, then there is a inherently equilibrium density of these entities that is what we saw, when we look at point defects like vacancies. So, the first question we will answer is, is it a thermo dynamical stable quantity, and what will be its equilibrium concentration.

So, let us start with the equation which we started for point defects, which is delta G equal to delta H minus T delta S. So, delta H is the energy of formation, and it can be given as approximately so this value. And I am not again going into the details 5 electron volt by b, and T delta S 2 k T by b, and this is per unit length. So, of course the denominator of length would be so common in both of these.

Now, so the you can see that this quantity is increasing with temperature, and therefore this will become more and more negative as we increase the temperature. But, at what temperature will it be sufficiently high this quantity, so that it becomes negative. So, for delta G to be negative which is when we can say that that at this particular temperature, this is a thermodynamically stable quantity. So, is maybe that whole range of temperature is possible that you can find out by putting T over here. So, for delta G with to be

negative, which would mean that this becomes a thermodynamically stable quantity, we have to find a temperature range.

And when you equate this, you would find that T must be very high, what is what do we mean by very high in almost all cases, it means T should be greater than T melting point. So, the dislocations are thermodynamically stable only at temperature above melting point, where the crystal structure does not exist anyways. So, it means that dislocations are not thermodynamically stable; at least until the crystal structure exists, unlike point defects. We saw that point defects can be stable.

But, then the question arises, then why do we see dislocations I mean if you anneal it, no matter for how much time. And if you try to grow a single crystal, you still we talk about dislocations presence of dislocations something order of the order of 10 to the power 10 per meter square, so why is there dislocation, and that is because no matter how you have created your material, if it is a even a little bit large decent amount of size, there will be enough stresses inside it that dislocations will get generated. So, however real materials are never stress free. And when they are not stress free, it implies dislocations will always be present.

And approximately depending on the stress, you will get the different amount of dislocation density. But, on an average for a material, for a metal or in the lower system, there is an approximate range, and what is that range that in a annealed material approximately 10 to the power 10 per meter square. And why do we write the unit per meter square, if you remember from part one, it is meter per meter cube. And therefore, it becomes per meter square.

While a deformed if you have done some process like rolling, forging, extrusion, then the dislocation density increases. And it does not increase like two times, three times, four times, it is increasing by order of magnitude. So, four orders of magnitude increase can be seen. And these are approximate numbers; it will be here and there depending on which particular system we are talking about.

So, now there are dislocations inside it, and if you talk about a single small amount of stresses acting on to the material very very small, then and inside a single crystal what

would happen is that the dislocations would be moving on only one set of parallel planes, and that is something that we discussed, when we talked about critically it is all shear stress.

So, let us say you have different sets of the slip planes inside a single crystal, and you are applying stress the one which reaches the critical results shear stress first, we will start to have dislocation glide on it initial for in the first and foremost. So, initially, dislocations glide on single set of parallel planes. And when you increase the stress, then that critical resolved shear stress of more than one plane is a achieved and so more than one plane can get activated.

Now, when we have more than one set of plane, then of course somewhere some of some of these planes will intersect. And if the planes are intersecting, then the dislocations moving on these will intersect. And this is what leads to dislocation intersection, and their multiplication. So, you get rapid multiplication and work hardening. So, rapid multiplication because of intersection is one way the dislocations increase in number, and we will come back to this which leads to work hardening.

$$E_{loop} = \frac{Gab^2}{2\Pi} \ln(\frac{a}{b}) \left[\frac{2+\vartheta}{1-\vartheta}\right]$$

But, there is also at least theoretically, another way that dislocations can originate, and that is homogeneous nucleation of dislocations. So, first we will look into this theoretical in the sense that such high stresses are required that you may assume that this happens, it is not the most preferred way buts it gives us a lot of insight about the material behaviour. So, first let us look at how does a nucleation nucleate homogeneously.

So, the next topic we want to look at is ok. So, when the dislocations are created, inside a defects region that is what we mean by homogeneous nucleation. So, when just like nucleation of the particular phase or the like the solidification, so there are solids forming inside the liquid. And when it is forming uniformly, homogeneously all across not because of any defect region, but is throughout in the defect free region that is called homogeneous nucleation, and just like that.

The dislocations when it is happening across the material throughout uniformly, and not at a particular preferred site or at energetically high energy site or at in preferred site, then it is called homogeneous nucleation, so when nucleate when dislocation is created in a defect free region. And now like I have already warned you that the stress requirement will be very high, which is what we will see very large stress is required. And hence it is not preferred phenomena by which dislocations now density increases ok.

So, now let us say that a dislocation loop has been created, what is the shape of this dislocation loop. So, we will say that let us say a square dislocation loop has been created. And why we are talking about dislocation loop and not a straight line, because if a dislocation straight line dislocation is created, then it must end at one of the grain boundaries or at the surface and both of which are another defect, so it would mean that it is a preferred site. But, we do not want to talk about those preferred site, we want to talk about homogeneous nucleation. So, only we can think about dislocation loop.

So, it the budget vector is in the plane, and let us say the lengths of the sides are a. So, what will be the energy of total dislocation length? Now, you can see if this is b, then this is screw, and this is also screw, and this is edge, and this is edge. Now, in part one we have seen, how to write the equation for a screw dislocation, and how to write the equation for a for a edge dislocation. And here we know the total length usually the energy is written in per unit length, but now we can multiply it with that length to get total energy not per unit length.

So, the energy of the loop can be written as 2 into a into E edge per unit length plus 2 a into E, so this is energy of the screw dislocation per unit length. And these are the quantities which we have derived for a screw dislocation, and for the edge dislocation we directly introduced it in part one of this course. Therefore, E loop can now be written as if you put it is not a very complicated task, you will see that it will come out to like this. So, nu is the poison ratio. And we could have simplified our task if we take nu equal to zero, and the overall discussion that we do can still be continued. But, we will let it be here, because it does not interfere with our equation derivation. So, this is the energy of the loop meaning this much energy must have been done.

$$a_c = \frac{Gb}{4\Pi\tau}\left[\frac{2+\vartheta}{1-\vartheta}\right]\left[1+\ln(\frac{a}{b})\right]$$

And when a dislocation loop has been created, there is also some amount of work done. And this work done is basically, because the dislocation loop you can assume that it is going from a point towards loop, so some work was done. So, this work must have been done by the loop. So, the work done can be given as area into the force acting on to it, so it can be written as a square tau b. Now, this is the energy higher and increase in the energy, this is the decrease in energy. So, the change in energy can be written as minus a square tau b, so this is the delta E or the extra amount of energy that must be presented.

Now, this can be looked at in some few in one another way. So, if we are plotting delta E versus a. So, this plot would have a variation like this. So, if the this delta E a variation delta E variation, you see over here. If the size of this loop is beyond certain critical size, then on further increase of the loop because of this force acting on to it, then on further increase on the of the size of the loop delta E would decrease.

On the other hand, if the size of the loop is below this critical size that any increment in the size of the dislocation loop would cause or would require or cost extra energy. So, this side is not stable. On the other hand if the dislocation loop is formed with size more than critical, then it will be stable, because its energy can decrease with increasing size of the dislocation loop, so that is what we mean by delta that is what we mean by the critical size of the loop. And how do we find that critical size of the loop, it is not very difficult. We just need to you have to differentiate delta E with respect to a, and that will give you the critical size of the loop.

So, let us do this to find out what will the critical size be, so this is delta E. And you have a over here, a over here, so we differentiate it over here. And therefore, I will write down what we get, so G a b square, so you will have to do it in parts, because a is over here, a is over here. So, you can take this as common. And one times once you will have to differentiate this keeping this constant. Once you will have to differentiate, this keeping this constant.

So, first we will keep this constant and differentiate this, so this becomes G a b square by p, and on differentiating this, we become it becomes 1 by b into ln 1 by a by b becomes ln b by a plus now this time we will keep this constant, and will differentiate this. So, this

becomes G b square by pi, and this remains constant ln a by b. And the whole thing is can be multiplied by 2 plus nu by 1 minus nu, and differentiating this it is 2 a tau b.

So, from here we can get the critical size. So, let us take this common, so p this b gets cancelled, and what we get is that a critical now this is a implicit equation. So, the a c is on the left hand side, and also a c will be on the inside we are not in a position to convert it to a explicit equation. So, this will have to be solved in an implicit in the way implicit equations are solved to be able to get the value of a c. Other things would be known in such a equation. And now that we are calculating a c, let us also exercise something which is calculating the delta E at this critical size, which would be this value or the maximum amount of energy that would be required by let us say some thermal fluctuations to cause the generation of such dislocation loops.

$$\Delta E_c = \frac{G a_c b^2}{2\pi}\left[\frac{2+\vartheta}{1-\vartheta}\right]\left\{ln\left(\frac{a_c}{b}\right)-1\right\}$$

So, here we will put this a critical value that we have obtained over here into this equation that we have over here, but in a slightly you can say we will apply some trick over here to make this equation simpler, what is that trick we will see in a moment. So, this is the equation we had, if you put it like this. Now, here the trick that we will apply is that we will put this a tau a square tau b as a c into a c tau b, why we have done this.

Now, if you go over back over here, you will see that this is a c tau can be brought here. And if you multiply it by b, then there is a b square term over here, so it becomes G b square by 2 pi times this term, which is very similar to this term and that is where it makes our task easier. So, this term becomes a c into G b square, so we have this minus this. And how do we handle that we can take out some of the common terms G a c b square, and this we will multiply by 2 on the numerator and denominator, so that we have 2 pi as the common, and this 2 plus nu is still the common factor by 1 minus nu.

And therefore, this is 2 ln a c that we have here or delta E critical the amount of critical energy that is required for by any mechanism, so that a dislocation loop can be created, which is stable. If you create smaller loop, then you will require lesser energy. So, this is the critical energy required to form a loop of criticals of size a c critical.

So, as long as you can provide this smart this type of energy as long as this amount of energy is available maybe by thermal fluctuations, then dislocation loops can generate stable dislocation let me add stable dislocation loops can be generated. And remember that a c over here is your critical loop size, which has been defined earlier.

$$a_c = \frac{Gb}{4\Pi\tau}\left[\frac{2+\vartheta}{1-\vartheta}\right]\left[1+\ln(\frac{a}{b})\right]$$

And let me just add one point here, when I say a c, it is the critical loop size and loops bigger than this size will be term will be stable meaning, they will not contract. Because, if they want to contract, you will need more energy, so that is energetically not a favourable thing, they will you will have to supply additional energy. So, it will like to increase the size, because that way the energy can be reduced. So, any amount of work that is done on this, it will lead to increase where it can increase in the size, then it will reduce in that total energy.

$$\tau = \frac{G}{10}$$

Now, that we have this equation there is something very interesting that we can talk about or we can obtain from this relation. Now, let us say that there is no thermal fluctuation. In absence of any thermal fluctuations, what will happen? How much is the delta E that can be provided, the delta E that can be provided is z a 0 actually. Now if delta E is E 0, then from this what do we get? We get that ln a critical by b is equal to 1, because these terms cannot be 0, so this must be 0. So, ln a c by b is equal to 1, so we get that in the terms of critical loop size, this is the relation that we obtain.

Now, let us put this equation back into the a c, so we have this a c equation. And let us see what it predicts for us, because there is a tau term which talks about the amount of stress that is being applied. So, let us say so this implies a critical so a critical is equal to 2.71 b, so on the left hand side we have this. So, this is coming from this equation, you just keep in mind this is the equation that we are using which is a critical.

So, on the left hand side a c we are using 2.71 b as we just obtained, when we have no thermal fluctuation meaning delta E is equal to 0. So, 2.71 b is equal to G b by 2 pi tau 2 plus nu, and over here we have ln a c by b which is nothing but 1. So, this is ln a c by b

we have seen is 1. So, this is 1 plus 1, so this is equal to 2, and this 2 gets cancelled over here, and this badger vector gets cancelled over here.

So, what we have, we have this tau this shear stress that is applied on to the dislocation. So, this is the tau that is applied not on to the dislocation, but on to the system. So, this is the shear stress that is being applied can be given in terms of when there is no energy external energy, then this can be given by G by 2.71 pi into this sum factor which is close to one. Now, what is this approximately equal to this is equal to G by 10. You remember seeing this equation somewhere, this is the same value that we obtained as the amount of stress required to deform a material, if there are no dislocations inside the material.

So, again we come back to the same relation, which describes that the shear strength of defect free material is of the order of G by 10. So, we have obtained similar the same thing, but in a different way where it shows that a defect free material will have very very high strength.

So, we started with talking about how the dislocations multiply, and we found that first thing that we found is that thermodynamically, the dislocations are not stable at temperatures below melting point. Then we said that dislocations generate on in one plane or dislocations glide on individual plane as you keep increasing the applied stress. When the applied stress exceeds more than the critical resolved shear stress of more than one plane, more than one plane starts to glide or the dislocations on more than one plane starts to glide. And they can interact, and they can multiply. We will look about that multiplication again in the next module which later on.

However, here we talked first about, if there is a possibility of homogeneous nucleation of dislocation loop, and we show that yes it is possible, and that there is a very large amount of energy required for that there is a critical size of the loop below above which it will be stable. And we took that analysis further ahead, and showed that when there is no thermal fluctuation which can provide the delta a c, then the stress required to deform the material is very high of the order of G by 10 something we had seen earlier from a different kind of analysis, so that is what we have discussed today. And next time, we will talk more about dislocation multiplication.

Thank you, and see you in the next module.