Solar Photovoltaics: Principles, Technologies and Materials
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Lecture - 22
P-N Junction Analysis (Light)

So, welcome again to this new lecture on Solar Photovoltaic Principles, Technologies and Materials. So, we were discussing P-N junction light, we were not able to finish this analysis in the last lecture. So, we will try to finish this in this lecture. So, what we did in the last class we will just recap first.

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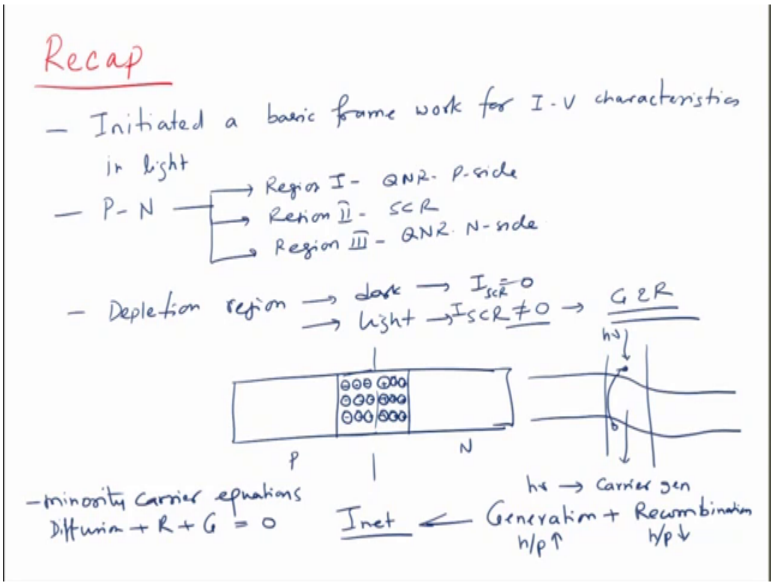

So, in the last class, what we did was we initiated a basic framework for determining I-V characteristics in light ok. We had already done some analysis in dark. Quite a few of those conditions are still valid. So, basically what we did was you still consider this as a typical P-N junction. We divided the device in three regions; one is region 1 that is quasi neutral region on P-side. Then we divided this we had region 2, which is the space charge region that is a depletion region. And then we have region 3, which is the QNR on N-side right.

We assume that the device is in is in steady state, so basically we consider a fixed voltage of fixed intensity. And at a given time, we have taken the depletion approximation as valid, and then we solve for basically carrier concentration and current from all the regions, so from bulk region as well as now from the depletion region.

See the earlier in the in the depletion region in dark no current was produced I was equal to 0, which means whatever goes in goes out. But, in light the space charge region, you can say this is I SCR in I SCR, it is not equal to 0 because of generation and recombination ok. Because, remember this region, so although it is a depletion region what did it mean, it is a depletion region. So, on this side you have, so on this side if this is P, this is N, on this side you are going to have acceptor ions, on this side you have donor ions all right.

Now, it is depletion region because every free hole and free electron is cancelled out here, they have neutralized with each other. As a result there are only ionic cores, but still it is still has a valence band and conduction band right. So, electrons can exceed still so if you look at the energy band diagram, the energy band diagram is something like this alright, so which means in this region also this is so see, however it is changing as a function of distance.

However, when you shine light on it, the electrons from this side can still go to this side and creating a hole. So, this region is still capable of producing. So, when you shine light, it will produce carriers. So, you can have carrier generation, when you shine light. But, when you generate carriers, then you also recombine carriers, because generation and recombination are dynamic processes, so because they are away from equilibrium condition.

So, excess and valence band will always have tendency to return back to the valence band to minimise their energy. As a result, they will they will lead to the combination. The combination can happen because of traps, it can happen because of radiation radiative recombination, whatever the recombination mechanism is. Generation will always be accompanied with recombination right. So, while generation will increase n and p, recombination will decrease n and p. And the combination of these two will give you to net current ok.

So, now depletion region produces current, and we will have to see what kind of what current does it how much current does it produce, so that is what we were doing. And we also wrote the minority carrier equations, which was basically three terms. First was the diffusion term, then you have recombination term, then you have generation term, all these three are equal to 0. And we now need to integrate these, we now need to analyse these equations. We will not go through the complete derivation of it, but we will just do a few things for this.

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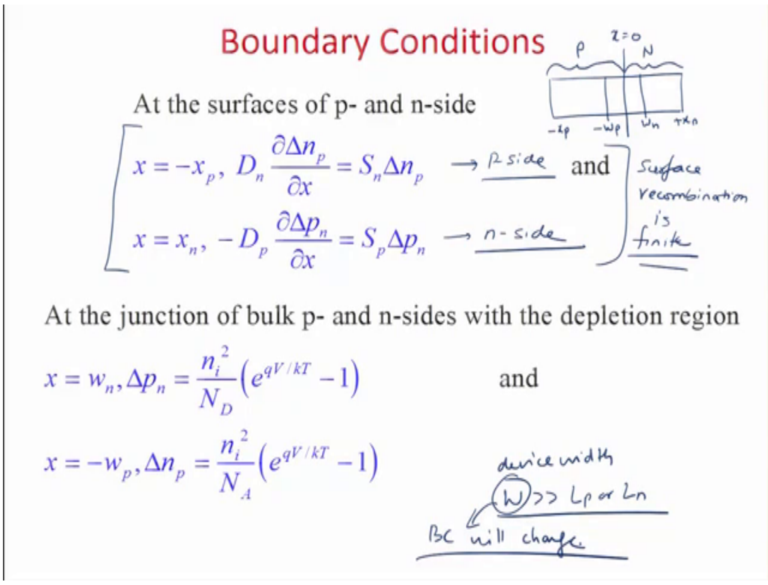

So, first thing is we have to analyse, what are the boundary conditions in order to solve the equations. So, boundary conditions are at the surface of p and n side, so that is at x is equal to minus x p and x is equal to x n, so that is if you take this as a junction this is at x is equal to 0, this is w n, this is minus w p, this is minus x p, and this is plus x n. So, this side is N side, this side is P side ok.

So, at x is equal to minus x p the that is all when you have a surface the carriers will go to the surface because of presence of dangling bonds, because of presence of other surface defects, there is surface recombination. So, at the surfaces of P and N side that diffusion diffusive component D n del of delta n by dx, which is the diffusion current because of perturbation of charges is equal to surface recombination that is S n into delta n p.

This is a generation this is a recombination term that we saw, when we are talking about the recombination surface recombination term. So, so at this on the surface basically, you can say the diffusion current is limited by the surface recombination. So, the carriers which diffuse out, they get combined at the surface.

On the other side at x is equal to x n, you have the other condition that is minus D p del of delta p n b y d x del x is equal to S p into delta p n. So, this is for you can say this is for p side, and this is for n side. So, first boundary condition is the diffusion term is equal to the surface recombination rate on the surface.

And then we have the next term that is at the junction of bulk p and n side with the depletion region that is at this and this point that is at x is equal to w n and minus w p. So, at x is equal to w n delta p n is equal to n i square by N D into exponential of q V by k T minus 1. And delta n p is n i square N A into exponential q V by k T minus 1. V is nothing but same as applied, so V would be as we have seen as we have got it from the other, and previous analysis in P-N junction in dark.

So, these condition is similar to what we saw in dark at the boundary delta P-N is equal to n i square by N D exponential q V by k T minus 1. And at minus w p delta n p is equal to same n i square by N A into exponential q V by k T minus 1. So, these are the boundary conditions at the surfaces of p and n side, and at the junction of p and n sides of the depletion region.

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General solution for $n_p(x)$ and $p_n(x)$

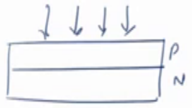
$$n_p(E,x) = A_n \cosh\left(\frac{-x-w_p}{L_n}\right) + B_n \sinh\left(\frac{-x-w_p}{L_n}\right) + \gamma_n e^{-\alpha(x+x_p)}$$

where $\gamma_n = \dfrac{\alpha(1-R)I_s L_n^2}{D_n\left(\alpha^2 L_n^2 - 1\right)}$

and

$$p_n(E,x) = A_p \cosh\left(\frac{x+w_n}{L_p}\right) + B_p \sinh\left(\frac{x+w_n}{L_p}\right) - \gamma_p e^{-\alpha(x+x_p)}$$

where $\gamma_p = \dfrac{\alpha(1-R)I_s L_p^2}{D_p\left(\alpha^2 L_p^2 - 1\right)}$



So, now when you apply these boundary conditions and solve the differential equation, we I mean it is a it is a level tds differential it is a tds solution, but it is not very difficult to do, you can do it yourself. In fact, I would recommend you to go, and do it yourself as well. So, if you do the solution, the general solution for n p X, and p n X works out to be like this.

n p E, x that is minority carriers on p side ok. So, electrons on side their concentration as a function of energy, and distance varies as A n cos of hyperbolic minus x minus w p divided by L n plus B n sign of hyperbolic minus x minus w p by L n plus gamma n, exponential of minus alpha into x plus x p, where this term gamma n is alpha into 1 minus R I n L n square divided by D n into alpha square L n square minus 1 ok.

You can see that we will see this later on later in detail, but let us first just go through the general forms. And then for p and x you get, similarly for P-N as a function of energy and distance you have similar terms. So, A p cos of hyperbolic instead of minus x minus w p by L n, now we have x plus w n. So, this is for electrons and holes thus, and as a result you have this minus w p term appearing, because this is from the p side.

On the n side for a holes, you will have w n term coming. And since you talking about holes, the minority carrier diffusion distance is considered. So, you have a cos term, you have a sin hyperbolic term, and similar term minus of so instead of plus gamma, and you have minus of gamma p exponential of minus alpha into x plus x p. Now, this again so this gamma p is that is alright, I think that is ok, because it is coming from the.

So, you assume that you have a junction like this ok. So, this is what P, this is what N, and the radiation comes like this ok. So, every carrier that has to reach N side has to go through P ok. So, some terms will remain, similar because it has to still cover that distance. So, again you have expression for gamma p you have a expression for gamma p, which is alpha into 1 minus R I s L p square divided by D p into L. So, you can see that the only difference is that you have you replace w p by w n, L n by L p A n by A p, and the electron terms are basically replaced by the whole terms.

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Application of Boundary Conditions to previous equations gives the values of coefficients.

$$A_n = n_{po}\left(e^{qV/kT}-1\right) - \gamma_n e^{-\alpha(x_p-w_p)}$$

and

$$B_n = \left(\frac{\gamma_n\left[e^{-\alpha(x_p-w_p)}\left(\frac{S_nL_n}{D_n}\cosh\left(\frac{x_p-w_p}{L_n}\right)+\sinh\left(\frac{x_p-w_p}{L_n}\right)\right)-\left(\frac{S_nL_n}{D_n}+\alpha L_n\right)\right] - n_{po}\left(e^{qV/kT}-1\right)\left[\frac{S_nL_n}{D_n}\cosh\left(\frac{x_p-w_p}{L_n}\right)+\sinh\left(\frac{x_p-w_p}{L_n}\right)\right]}{\frac{S_nL_n}{D_n}\sinh\left(\frac{x_p-w_p}{L_n}\right)+\cosh\left(\frac{x_p-w_p}{L_n}\right)} \right)$$

So, when you apply boundary conditions to these equations to find out the value of coefficients, we have this value for A n, which is for the electrons an is n p naught into exponential of q V by k T minus 1 minus gamma n exponential of minus alpha x p minus w p. Similarly, you have a term for B n, it is a very complex looking term, but you have here is you have if you if you look at this carefully, you have a term which is dependent upon the bias which is here, but you can see there are other terms where the bias does not come into picture. So, there is only one term which is bias dependent. Similarly, here you have only one term which is bias dependent, and when you plug in the value of these A n and B n.

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$$A_p = p_{no}\left(e^{qV/kT}-1\right)-\gamma_p e^{-\alpha(x_p+w_n)}$$

and

$$B_p = \left(\frac{\gamma_p e^{-\alpha(x_p+w_n)}\left[\left(\frac{S_pL_p}{D_p}\cosh\left(\frac{x_n-w_n}{L_p}\right)+\sinh\left(\frac{x_n-w_n}{L_p}\right)\right)-\left(\frac{S_pL_p}{D_p}-\alpha L_p\right)e^{-\alpha(x_n-w_n)}\right] - p_{no}\left(e^{qV/kT}-1\right)\left[\frac{S_pL_p}{D_p}\cosh\left(\frac{x_n-w_n}{L_p}\right)+\sinh\left(\frac{x_n-w_n}{L_p}\right)\right]}{\frac{S_pL_p}{D_p}\sinh\left(\frac{x_n-w_n}{L_p}\right)+\cosh\left(\frac{x_n-w_n}{L_p}\right)}\right)$$

So, similarly you will get the values of A p and B p also, when you apply boundary conditions.

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Currents

- Since currents are due to diffusion only, the expression are

$$j_p = -qD_P\frac{\partial p_n}{\partial x}$$

$$j_n = qD_N\frac{\partial n_p}{\partial x}$$

The currents. So, let us leave it there right now right, we know that we can calculate. So, basically what you have done is you have just taken the solution of differential equation, the general solution, boundary conditions calculate the value of coefficients. For any differential equation is just that it is too complex to read out, but you can you can you can look at them from your from your notes as well as from the books that these are fairly complex terms for A n and B n, but they are not that complex when you think of it.

So, we will we will do that analysis in a little while. So, we have A and B and A p, B p, and we know that the currents are due to diffusion only. So, currents j p is equal to minus q D p d p n by d x. And j n is equal to q D N del p del n p by del x not D p n, but del n p by del x, these are partial equations.

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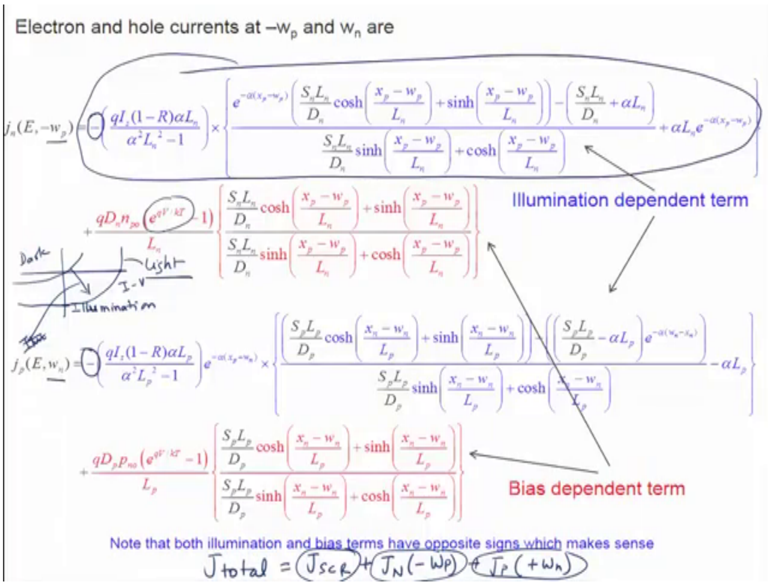

So, when you now replace these expressions and calculate the current, so you do not have to worry about the complexity of the equation, what we have here is you get two terms for so this is electron current at minus w p, and hole current at plus w n ok. So, when we look at total current, hole current is calculated at plus w n. Electron current is calculated at minus w p, and then we will do another current which is the SCR current that we will take at the end.

So, this term which is the illumination dependent term, the first term you can see you have terms like q electronic charge, reflection coefficient, absorption coefficient, minority carrier diffusion length. All these are constants, this is the only term which is varying that is I s the intensity of radiation. Everything else so x is the n is the distance in the device, w p is also distance in the device for holes. So, you can see all the terms and S n is the surface recombination velocity. So, we can see that the only variable here is illumination. So, this term big term is the illumination dependent term.

The second term which is the red one has bias. Again all other things are known, it is bias which varies. Similarly, so we for holes you get the similar expressions. So, what you have here is you have two terms in the current. One term is the illumination dependent term, second term is the bias dependent term. And these are two independent terms, they are not dependent.

So, the good thing is that if they were dependent, you would have lot of difficult time. But, since they are independent term, you can analyse both of them separately. So, suppose, what will happen in case of illumination is equal to 0? If illumination is equal to 0, then these two will become similar to what you calculated in dark right, they will not be very different. So, if you make I s equal to 0, if you just have this term, and this term, it should give you the answer which are similar to what must have happened in dark condition. And so essentially you can do that very easily.

And similarly, when you put the bias equal to 0, then what should happen? And then so another thing that you need to notice that, the illumination term has this minus sign, whereas the bias term has a positive sign to it. So, this tells that at zero bias at zero bias solar cell is going to produce the current, the current is going to be negative alright. So, when you plot the I-V characteristics for a P-N junction. The I-V characteristics will not be like this, you should start at somewhere here, this is the illumination current.

And then as increase the bias, this current reduces and goes to 0 at certain point at certain bias. And we will see what these terms are and this is what basically the I-V curve, you should obtain for a solar cell, when you plot this for a given illumination ok. So, when you when you make when you fix I when you fix I, then you should obtain I-V curve like what you so this is this would be dark this would be light.

And whatever is extra coming is because of illumination, so this shift from this side to that side is because of illumination effect alright. And it is it is a we are lucky to have these two terms independent of each other, they are not so that so that they are easy to analyse. And the meaning of this you see, I just wanted to do a little bit upon this boundary condition, this boundary condition essential it is a realistic boundary condition, because we are assuming that surface recombination is finite ok and surfaces basically within a few diffusion length of junction.

So, surface is not very far from the junction ok. If it happens that surface is very far from the junction, then basically you can say minority carrier concentration must be so essentially, you will have to change this boundary condition when the surface is very far.

But, generally the solar cells are like this that the surface recombination is finite in the solar cells. So, this is a realistic boundary condition that you want to apply for solar cells. But, you will you may have other boundary conditions, if your solar cell junction is if solar cell device is very thick as compared to the depletion region.

So, when your device is very thick, then obviously or you can say that the width is much larger than L p or L n, then you will have to change the width of the device, then boundary condition will change, and hence your solution will also change. So, this is for more realistic condition of having a device where so as I said the good thing about this expression is that it gives you two terms. One is the elimination dependent term, second is the bias dependent term.

The elimination term is negative in nature, bias dependent term is positive in nature. So, at zero bias whatever current you get is because of illumination ok. And as you increase the bias the current reduces, and it becomes zero at certain bias, when both of these terms cancel each other for both electron and hole current. So, this is what we obtain from this expression. Now, we look at the so what we have done is we said earlier that J total was equal to J SCR plus J N at minus w p plus J p at plus w n ok. So, instead of because these two currents can be calculated from the diffusion equations quite easily.

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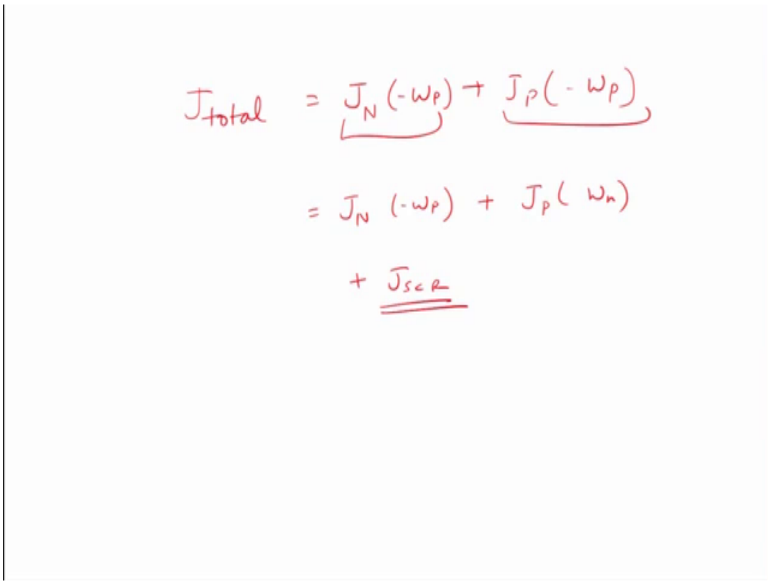

And this SCR current has to be so now you will appreciate that why did we separate, so when I was explaining this particular point, anyway so let us let me just, so when I was saying that J total, so I said earlier that J total was equal to J N at minus w p plus J p at minus w p, it is easy to calculate this, but it is not easy to calculate this, because carriers will immediately diffuse away. So, the so instead of doing this you do J N at minus w p which can be calculated easily using diffusion using the solutions. J p at w n, this is also calculate able. And then whatever is generated from SCR region that is what we are going to look at now current, which is which is going to.

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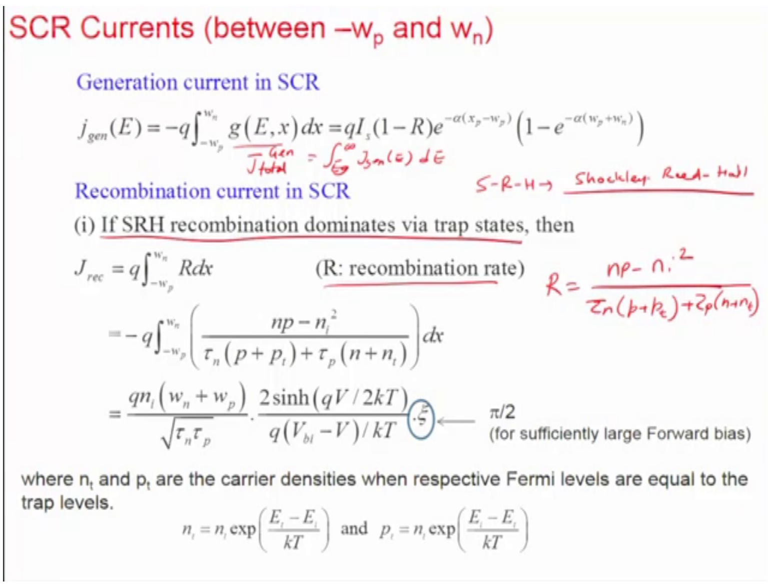

So, let us look at the space charge region current within between minus w p and plus w n. So, the generation current in space charge region j gen E be defined as is equal to minus of q minus w p plus so first you look at the generation current, because the space charge you will have generation as well as recombination.

So, generation current is basically integration of g E, x dx over minus w n to plus w p, and this turns out to be like this. So, we know the expression for g E, x. So, you plug in there you get this term q I s 1 minus R exponential of minus alpha x p minus w p into 1 minus e to power minus alpha p into w p plus w n.

Now, you look at the recombination current. But, when if you look at the recombination, the recombination current is so we assume that it is Shockley Read Hall trap assisted recombination dominates wire trap states. So, this is SRH means, Shockley Read Hall ok. This is basically for trap assisted recombination. So, we are not assuming radiative recombination, we are saying it is a non-radiative trap assisted recombination.

Then, J recombination is q minus w p to w n R of dx right. So, of course this current is negative in nature, and this current is positive in nature, because generation current is produced you create electrons. And recombination current means you consume electrons. So, both have opposite signs.

So, now you put in the R of recombination rate. For Shockley for Shockley Read Hall mechanism, the recombination rate is given by this expression that we wrote earlier also. R is equal to n p minus n i square divided by tau n into p plus p t plus tau p n plus n t. This is the expression, where n p is the carried electron hole concentration, n i is the intrinsic carrier concentration, tau n is the lifetime of electron, p t is the hole trap density, and n t is the electron trap density.

So, when you do this analysis, so this integration turns out to be in this fashion. So, you have q n i w n plus w p divided by square root of tau n tau p into 2 of sin of hyperbolic q V by 2 k T divided by q V b i minus V divided by k T into some term eta ok. This is approximately pi by 2, if you have forward bias, which is sufficiently large in nature.

Whereas, n here n t and p t are carrier densities, when respective Fermi levels are equal to trap levels ok. So, they are basically carrier density corresponding to when Fermi levels are equal to equal to trap levels. So, they are not actually trap density they are carrier densities. So, n t is again it is a similar expression and n t is equal to n i exponential E t minus E i divided by k T. And this is E i minus E t by K t. Similar they are similar to electron and hole concentration in a n type semiconductor, you calculate the n electron hole concentration with respect to trap.

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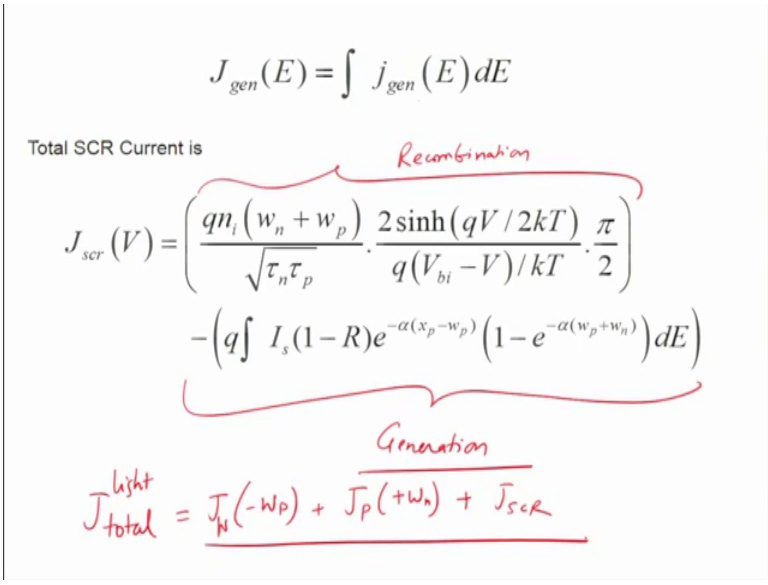

So, now this generation current that the total SCR current now is total SCR current is you have to now integrate. So, total gen total of course we have taken the spectral response for each energy, you have integrate these. So, this generation current is j gen. So, if you have to now look at j total gen, you have to integrate right E g to infinity j gen E d E right.

So, this has to be integrated, so that is why I have written in the next slide that j gen E is integration of this j gen small j gen E overall energies. Then, the total current that you get is essentially, this recombination current this is the recombination, and this is the generation ok. And you can see that it has certain bias dependent here. And you have a again these two terms are decoupled, you have a term which is bias dependent, you have a term which is illumination dependent, but these are finite terms they are not zero unlike, it was not dark ok.

So, so now this J total of a P-N junction device in light that we have calculated is minus J N at minus w p plus J p at plus w n plus J SCR. This is the total current that you will get out of a solar cell device P-N junction in light condition alright. So, we will now we have done a P-N junction analysis for a dark as well as light conditions which gives you a fair idea of how to get these expressions. And this current as we will see, it will be called as photo current or a short circuit current for a solar cell device; we will see later on. So, we will do now solar cell solar cells in detail in the next class onwards ok.

Thank you.