**Solar Photovoltaics: Principles, Technologies and Materials**
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**Lecture - 20**
**P-N Junction Analysis (Dark)**

So, welcome again to the new lecture on Solar Photovoltaics Principles, Technologies, and Materials. So, we will just do a brief recap of the last lecture.

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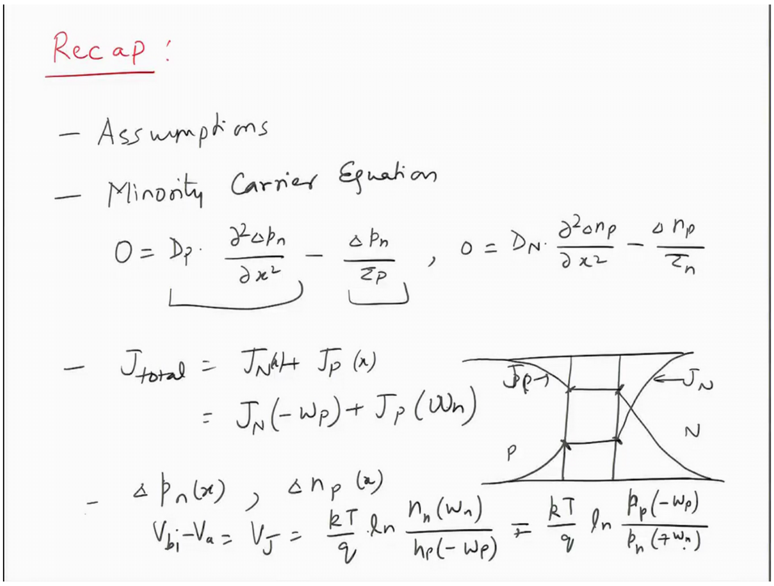


So, in the last lecture we looked at things like so, we wanted to establish a mathematical framework. So, we first looked at assumptions for p-n junction analysis. So, we assume that there is no generation of carriers in the semiconductor, there is a depletion region, there is no generation recombination within the depletion region, the conditions are low level injections. And within the bulk of the semiconductor, the field is flat, the bands are flat as a result electric field is equal to 0 and, within the bulk of the semiconductor, the change in the majority carrier concentration with respect to distance is equal to 0. And then we made the depletion approximation appropriately and for then we wrote from the continuity equation, the minority carrier equations. And these equations work for the holes, it would be D p del 2 delta p n by del x 2 minus delta p n divided by tau p and for electrons it would be D n del 2 delta n p by del x 2 minus delta n p divided by tau n. these were the equations that we looked at in terms of ah.

So, basically what you have here is, because you have, you are only considering the diffusion of carriers and recombination and we also said that the total current within the semiconductor J total was equal to J N plus J P at any given x. And what basically it meant was this is identical to saying at the current you have at minus W p plus J P at W n also makes up this relation, because as we saw that if you have the total current. So, if you have this as a depletion region the total current goes like this sorry, it goes like this and similarly on this side the total current goes like this and then it goes like this.

So, basically we are saying that so this is for, if this is P side, this is N side. This would be hole current J P and this would be the electron current J N. So, basically that if the total current is equal to J N plus J P at any given position, it also means that this is equal to J N and J P at this point and J N and J P at this point or you can say it is equal to J N at minus w p plus J P at plus W n, because of depletion approximation that nothing goes in a, whatever goes in depletion region goes out ok. There is no generation recombination within that depletion region.

So, basically what we have to do is that we have to solve equation for delta p n x and delta n p x in order to calculate the currents and that is what we were doing. Finally, what we did was we calculated that junction potential, the junction potential V J was equal to k T by q which is equal to V bi. So, equilibrium it will be equal to V bi and non equilibrium condition it will be equal to V bi minus V A and this is equal to k T of l n n at the w n which is a majority carrier concentration divided by n p at minus w p. Similarly you can write the expression in terms of holes which will be equal to k T divided by q l n and p p p at minus w p divided by p n at plus w n right from the same, because if you recall in the previous in the, we took the electron term, you could have taken the whole term also. If you take the whole term then, you can do the similar calculation for electric field using the instant relation from the whole equation. We use the electron equation that is why we have everything in electron things.

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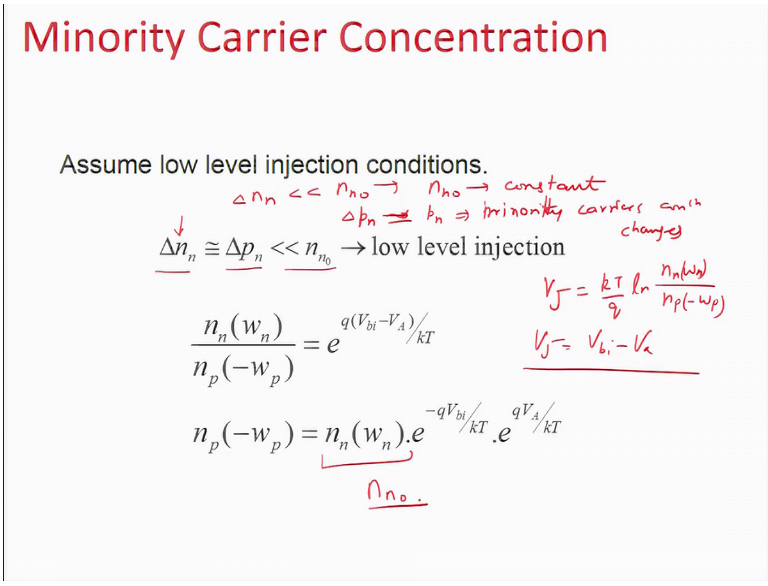


So, now let us go forward on this. So, in the, so if you want to talk about the minority carrier concentration assuming that we have low level injection conditions. If you have low level injection conditions, if you have low level injection condition we are saying that delta n n is equal to delta p n and this delta p n is a much smaller than n n naught ok. So, we are, so the perturbation in both is similar, but this perturbation in majority carrier concentration is very small, whereas, in case of minority carrier concentration. So, this is, so this n this delta n n is very very small as compared to n n naught.

So, which means n n naught is virtually constant right; however, this delta p n is nearly comparable to p n as a result minority carrier concentration changes ok. So, since delta p n is large enough as compared to p n, but it does not affect the majority carrier concentration. These conditions are called as low level injection, because you are not injecting too many majority carriers, you are injecting mostly minority carriers. So, if that is true then from the previous expression where we wrote that V J is equal to k T divided by q ln of n n w n at w n divided by n p at minus w p. I can now flip this equation change this equation. So, if I write in terms of electron concentration it becomes n n at W n divided by n p at minus W p is equal to exponential of q into V b i minus V a divided by k T, where V J is equal to V b i minus V a.

So, now using this equation if I flip it over I can get n p at minus w p is equal to n n w n multiplied by exponential of minus q V by V V b i divided by k T into exponential of q V a divided by k T. What is n n W n? Nothing, but n n not right, it is nothing, but n n naught. Similarly you can write an expression for whole concentration also.

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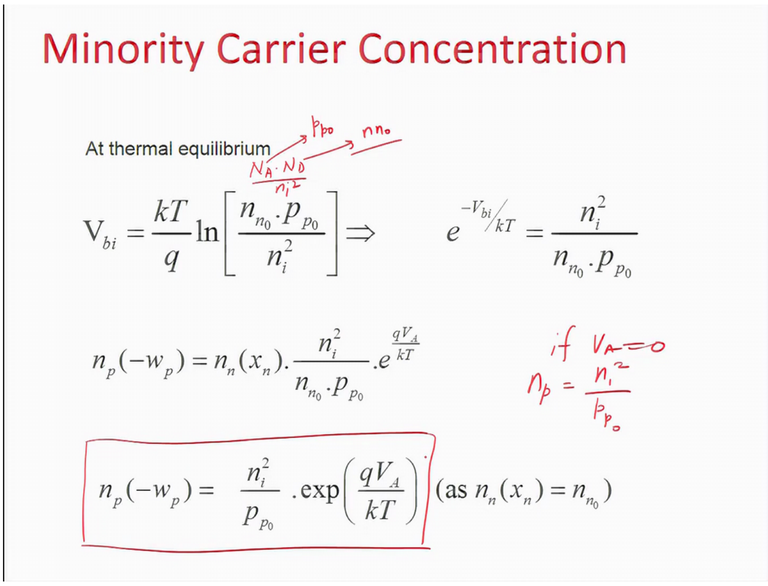


So, now, if I move forward at thermal equilibrium, if I say V bi is equal to this. We derived earlier that V bi was equal to k T divided by q into ln of N A into N D divided by n i square and N A is equal to p p naught and N D is equal to n n naught. So, this was basically if you remember n a into n d divided by n i square n a is equal to p p naught n d is equal to n n naught. So, I can calculate a relation for V bi. Now if you substitute this what you get is a relation. So, you just need to now substitute if you do that you get a relation for n n n p at minus w p n p at minus w p turns out to be a n i square divided by p p naught n i.

So, this is whole concentration on the, this is the electron concentration on the p side at the edge of depletion region, so it becomes a minority carrier right. So, minority carrier concentration on the p side is equal to n i square divided by p p naught exponential of q V A divided by k T. Suppose V A was equal to 0 then this will be equal to 1, then this will be nothing, but if V A was equal to 0 then what you get is n p is equal to n i square divided by p p naught which is the minority carrier concentration in a normal n or p type semiconductor. Similarly if you do it for holes you will get the same answer ok. So, basically I have got the concentration of minority carriers in terms of bias that is applied.

So, n p at minus W p is equal to n i square divided by p p not into exponential of q V a by k T, and I can see that if I increase the applied bias then my minority carrier concentration also increases exponentially which means my current should also increase exponentially, I am seeing a trend here.

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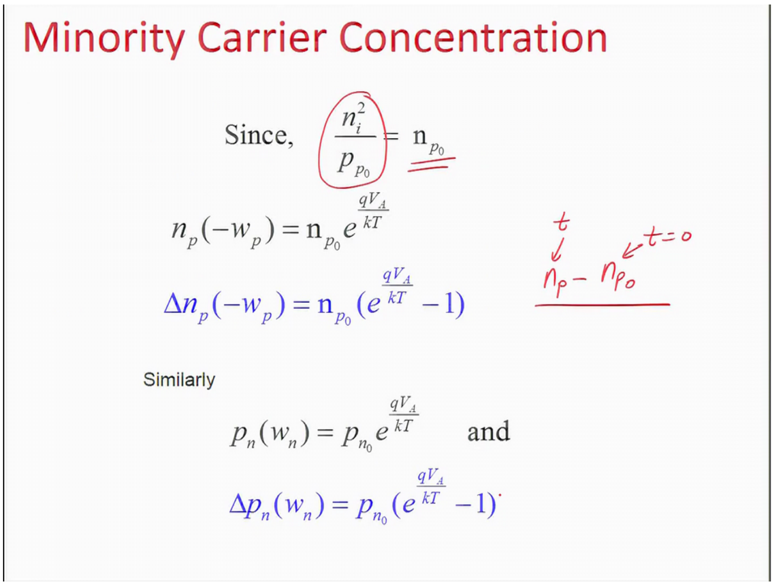


Similarly, you can write for, so now, we have got an in and we. So, we know that since n i square divided by p p naught is equal to n p naught ok, n i square divided by p p naught is equal to n p naught at 0 bias right, this is a equilibrium carrier concentration. So, I can replace this here. So, n p at minus w p is equal to n p naught into exponential q V a by k T. So, if V i is 0 then n p at minus wp is equal to n p naught.

Now, I can write what is delta n p, the change in carrier concentration. So, delta n p at minus w p is equal to delta n p at minus w p is equal to n p at minus w p minus n p naught with respect to the original one. So, this is the, so essentially basically it is n p minus n p naught. This is the at t is equal to 0 this is at some t after the perturbation has been made, so this is straight forward. Similarly if you do the same analysis for holes, you will get delta p n at plus w n is equal to p n not into exponential q V A by k T minus 1 using the similar analysis.

Now moving forward, so you now need to solve these equations to get the currents ok. So, for that first we make what we call as long base assumption condition.

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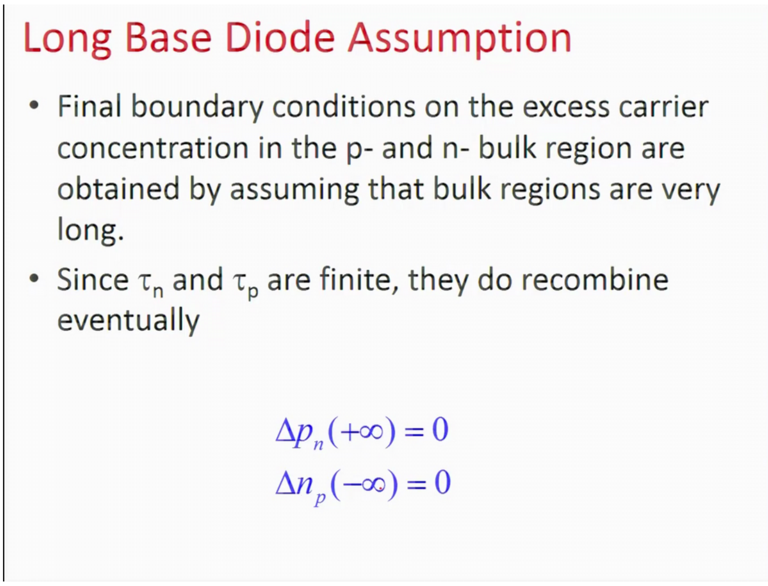


Long base assumption long base diode assumption basically means that we need to get the final boundary conditions to solve. Final boundary conditions on the excess carrier concentration in the, because remember we have got the expression for delta p, but we also have the recombination term in the minority carrier that we wrote earlier the continuity equations. So, we actually need to solve those equations alright. So, for that we first need to assume that bulk regions are very long; bulk regions are very long in comparison to the depletion region.

Secondly, since tau n and tau p are finite numbers, there is a finite lifetime of carriers. The carriers do recombine eventually at some point they do recombine. So, we can say that delta p n at plus infinity is equal to 0 and delta n p at minus infinity is equal to 0, which means whatever carrier concentration, minority carrier concentration you had at the boundary of depletion region with respect to bulk region, it goes down to a equilibrium concentration eventually, all right because the perturbation becomes equal to 0.

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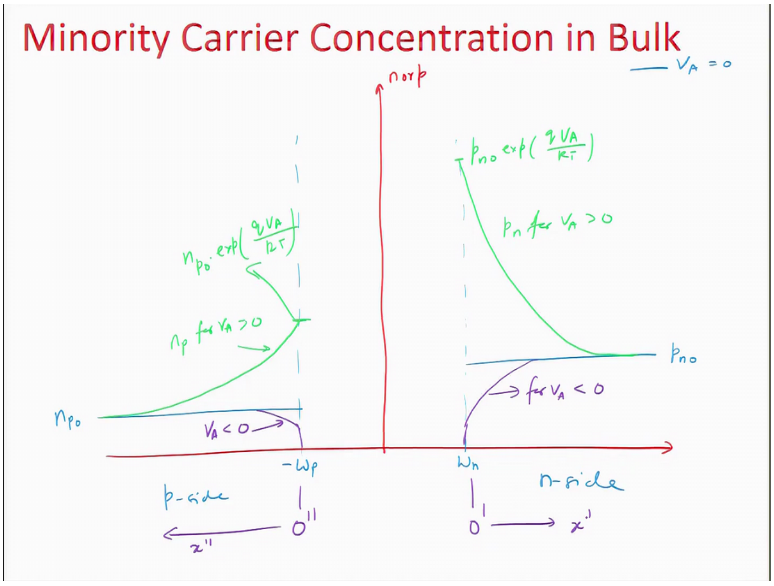


So, if that is the case we now do. So, basically what we are saying this if you plot the minority carrier concentration the bulk it goes like this. So, this is n or p. So, if this is the edge of one depletion region; that is w n, another edge of depletion region, this is minus w p. So, I can say that, so if you have bias that is equal to 0; if your bias is equal to 0 then this is n p not if this is p side ok. If this is n side then my p n naught is somewhere here, they can they will not be equal, so this would be p n naught. So, when bias is equal to 0 right, so it is blue color means V A is equal to 0 bias is positive. When bias is positive then they have exponential dependence. So, this varies like this. And similarly it varies like this right, exponential behavior is there you can see from the equations.

So, this is for case which is n p for V A greater than 0 and this is for p n for V A greater than 0 right and this value would be given by the expression that we wrote earlier. So, this would be equal to p n naught exponential of q V A by k T and this value will be given by similarly n p naught into exponential of q V A by k T alright. This is simple straight forward mathematics, there is nothing too clumsy about it and if you now want to plot the, sorry if I use a different color now here. If I use a purple color for the negative bias this will go as something like that on both sides. This would be the behavior for the negative bias. So, this would be for, it will eventually go to 0 very close to 0. So, this would be for V A that is smaller than 0 this would be for V A smaller than 0 ok.

So, you can see that carrier concentration is there is hardly any carrier concentration in the vicinity of junction in the reverse bias which means no carriers cross on both sides.

So, carry actually carriers are taken away from the junction which means carrier concentration it is very small, so no junction current flows when reverse bias is there, and large junction current flows when you have a forward bias and let us assume for the sake of analysis new positions, this position. Let us say we define as 0 prime, this position we define as 0 double prime. So, on this side we take x double prime, on this side we take x prime.

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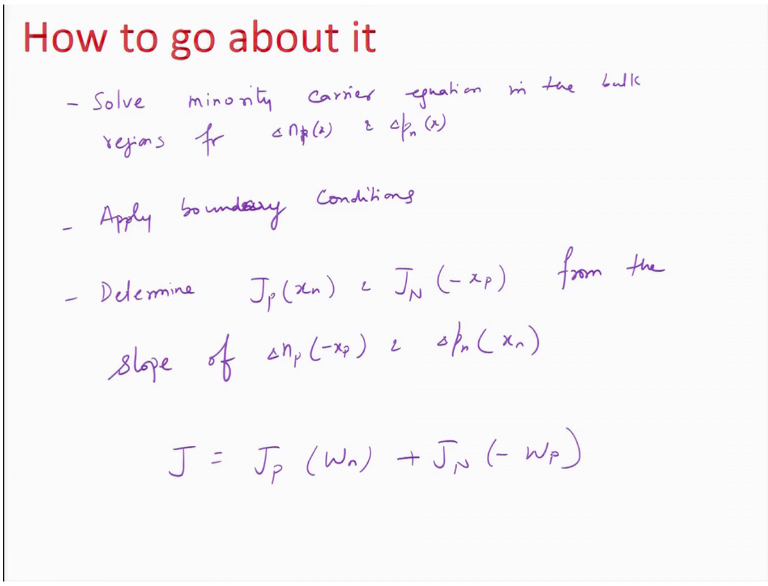


So, how do we go about it? Well, so how do we go about it, is first we need to solve the solve minority carrier equation in the bulk regions for delta n p x, and for delta p n x both of these and then apply boundary conditions, because these are differential equations. So, solution requires application of boundary conditions. And then determine J P at x n and J N at minus x p, which means at the far ends of the semiconductor device away from a depletion region from the slope of delta n p at minus x p and delta p n at x n.

That is what we are going to do and we know that total current J is equal to J Pat **w** n plus J N at minus w p this is. So, let us first do the analysis.

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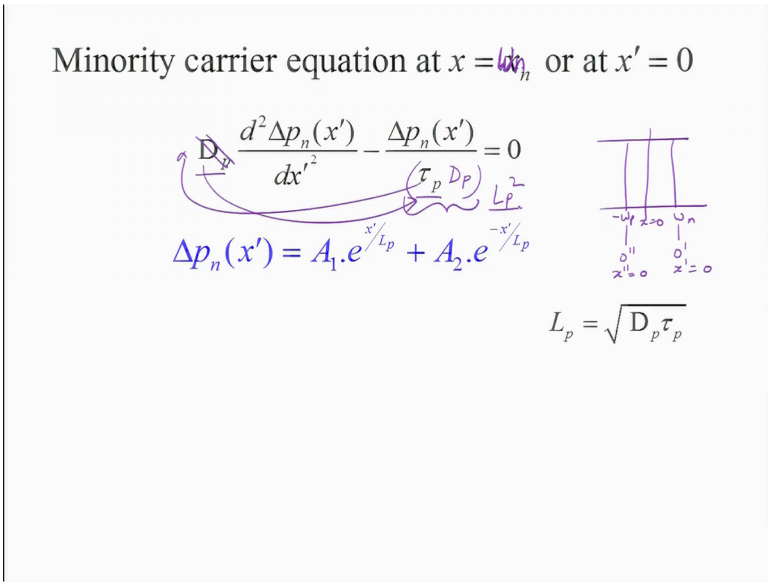


So, now let us look at the minority carrier concentration equation at x is equal to x n; that is at x is equal to, sorry x not x n should be w n ok. So, x is equal to w n that is so if this is the depletion region. We said that this is x is equal to 0 originally this is w n, this is minus w p. We redefined our coordinates, so this becomes 0 prime double prime, this becomes 0 prime ok. So, or x prime is equal to 0, this is x double prime is equal to 0. So, basically x is equal to w n, means x prime is equal to 0. So, when we do that, then we redefine our minority carrier equation which is D p into del d 2 delta p n x prime dx prime square minus delta p at x prime divided by tau p that is equal to 0.

So, from this equation is a second order differential equation, the solution of this is delta p n x prime is equal to A 1 exponential of x prime divided by L p plus A 2 into exponential of minus x prime divided by L p, this is a standard solution for a second order differential equation. Since we know that diffusion length is equal to square root of diffusivity into minority carrier lifetime, we have replaced in this. So, you can see that you have a term D; you have a term tau right. You can take this on this side or you can take this on this side. So, essentially if you solve the differential equation you can take this becomes D p tau p and D p tau p is nothing, but L p square ok.

So, when you do this, when you do the solution you get something like that. So, basically we need to determine, now A 1 and A 2. H ow do you determine A 1 and A 2? By applying boundary conditions.

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Boundary Conditions

BC-1 : $\Delta p_n(+\infty) = 0$

BC-2 : $\Delta p_n(x = w_n) = \Delta p(0') = p_{n_o}\left[\exp\left(\frac{qV_A}{kT}\right) - 1\right]$

$$A_1 = 0, A_2 = \Delta p_n(w_n)$$

$$\Delta p_n(x') = p_{n_o}\left(e^{\frac{qV_A}{kT}} - 1\right)e^{-x'/L_p}$$

$$p_n(x') = p_{n_o} + p_{n_o}\left(e^{\frac{qV_A}{kT}} - 1\right)e^{-x'/L_p}$$

Similarly, you do the same analysis for now boundary; what are the boundary conditions? The boundary conditions are, the first boundary condition is delta p n at infinity is equal to 0, second boundary condition is delta p n at x is equal to w n that is at 0 prime is equal to delta p n naught exponential qV A by k T minus 1, this we have established earlier. Similarly so from this we can get A 1 is equal to 0 and with A 2 is equal to minus delta p n at w n. Sorry I just wanted to make here when we are looking at, we are looking at actually w n; w n at minus w p similarly here minus w p n w n.

So, from this we got A 1 and A 2 and then we can get what is delta p n at x prime. This is so, it is just the substitution now right A 1 and A 2. So, if you substitute A 1 A 2 you get this. So, basically p n x prime is now equal to p n naught plus p n naught into exponential of q V A by k T minus 1 exponential of minus x prime divided by L p.

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$$J_p(x') = -qD_p \frac{\partial \Delta p_n(x')}{\partial x'}$$

$$J_p(x') = -qD_p . p_{n_o}\left(e^{\frac{qV_A}{kT}} - 1\right)\left(-\frac{1}{L_p}\right)e^{-x'/L_p}$$

$$J_p(x') = \frac{qD_p}{L_p} . p_{n_o}\left(e^{\frac{qV_A}{kT}} - 1\right)e^{-x'/L_p}$$

$x' = 0$ at $x = w_n$

$$J_p(x) = \frac{qD_p}{L_p} . p_{n_o}\left(e^{\frac{qV_A}{kT}} - 1\right).e^{-(w_n + x)/L_p}$$

Now, so, from this we can get what is diffusion current. Diffusion current is nothing, but J p x prime is equal to minus q D p into del of delta p n x prime divided by d x prime right. So, this is a simple diffusion current which is derivative of delta p n. Now you need to substitute the value of delta p n here. I f you substitute the value of delta p n here, you get J p x prime as some function alright, it is very simple differential equation, there is nothing complicated about it. I f you just keep doing it what we get is, J p x prime is equal to q D p divided by L p into p n naught into exponential of q V A by kT minus 1 into exponential of minus x prime divided by L p. Substitute x prime by x, x prime is equal to 0 is equal to x is equal to w n. So, we substitute x prime by x and what we get is x prime is nothing, but x plus w n all right.

So, because x prime is 0 where x is equal to w n as a result x prime is x plus w n. So, we have made the substitution we get the equation for J p . If you do the similar analysis for electrons you will get J N; if you do that so, by doing similar like.

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At $x = w_n$ or $x' = 0$

$$J_{p(Depl)} = J_p(w_n) = J_p(0')$$

$$J_P(w_n) = \frac{qD_p}{L_p} . p_{n_o} \left( e^{\frac{qV_A}{kT}} - 1 \right)$$

So, by now at x is equal to w n or x prime is equal to 0. We know that when J p at the depletion edge is equal to J p at w n that is equal to J p at w 0 prime. So, J p at w n is equal to q D p by L p into p n, naught into exponential q V A by kT minus 1. So, essentially what we have done is, if you look at the yeah. So, basically we have taken this x prime is equal to 0 all right. So, when you take x prime is equal to 0, this should be hang on this should be 1, mine mistake here it should be minus w n, because you are starting from x.

So, we are saying that x prime is equal to 0 at x is equal to w n. So, basically x prime will be w x minus w n. So, when you take x is x as w n then that exponential factor becomes equal to 0 at the edge of the depletion region. So, we get J p at w n; that is equal to q D p by L p into p n naught into exponential of q V A by kT minus 1. If you do the same analysis you can get the electron current as well, and so, I am not going to go through that, but if you do the same thing you get the you get an expression for the hole current. So, this is what the whole analysis is.

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## for P-region$(x \le -w_p \text{ or } x'' \ge 0'')$

$$\Delta n_p(x \le -w_p) = n_{p_0}\left(e^{qV_A/KT} - 1\right)e^{-(x+w_p)/L_N}$$

*or*

$$\Delta n_p(x'') = n_{p_0}\left(e^{qV_A/KT} - 1\right)e^{-x''/L_N}$$

*or*

$$n_p(x'') = n_{p_0} + n_{p_0}\left(e^{qV_A/KT} - 1\right)e^{-x''/L_N}$$

where $L_N = \sqrt{D_N \tau_N}$ (minority carrier diffsion length for electrons)

You can look at these slides later on, because they will be available here, but basically you are doing for the region x prime x double prime greater than 0 double prime; that is for the region x on that in the bulk region of P side which means x smaller than equal to minus w p.

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$$J_N(0'') = J_N(-w_p)$$

$$J_N(x'') = \frac{qD_N}{L_N} n_{p_0}\left(e^{qV_A/KT} - 1\right)e^{-x''/L_N}$$

at $x = -w_p$ i.e., $x'' = x + w_p = 0$

$$J_N(-w_p) = \frac{qD_N}{L_N} n_{p_0}\left(e^{qV_A/KT} - 1\right)$$

So, if you do that you can calculate using similar logic J N at minus w p is equal to q D N by L N into n p naught into exponential of q V A by kT minus 1.

So, we got two expressions; one for electron, one for hole. Now you need to just sum them up. So, when you sum them up what you get is this, J total is equal to J N plus J P and J total is equal to q into D N by L N into n p naught plus D p by L p into p n naught.

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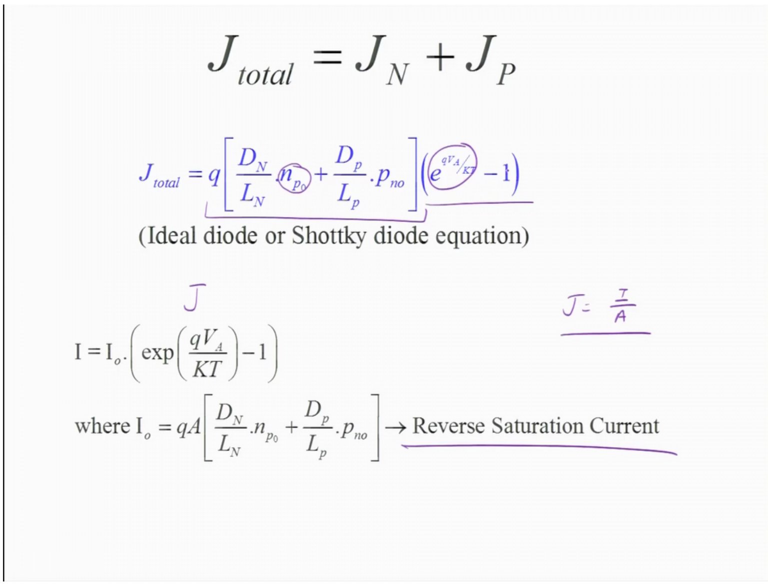


So, you can see that this is equilibrium concentration. This is D N by L N the diffusivity divided by diffusion length, this is D P by L P. So, this is for hole specify electrons into a factor exponential q V A by kT minus 1. This current you can see that there is no bias dependence here, so you can write this as J total. So, I think J have already written it here, so I can write this as. So, if you convert the current into current density into current you can write this as I is equal to I naught exponential of q V A by KT minus 1, and where I naught can be written as q into area, because if you convert J to I then so, because J is equal to I divided by A. So, q into A D N divided by L n p naught plus D P divided by L P into p n naught. This is the reverse saturation current that you get at 0 bias or negative bias or at negative bias, not 0 bias at a negative biases. So it will be minus. So, basically when the voltage is very negative this will be 0, so this will be minus 1. So, what you get on the other side is minus I naught ok. So, this is what basically we get.

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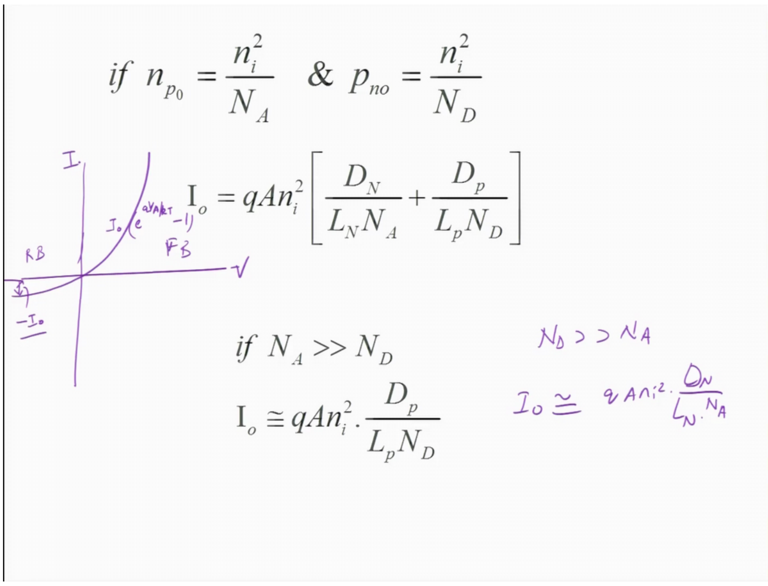


So, we know that what is n p naught and p n naught and if you replace them we get I naught as equal to q A into n i square D N divided by L N N A plus D P divided by L P and D. So, it is just so, you can further tweak these equations, for example, if N A was a lot larger than N D which means P side was more heavily doped than the N side, then this is almost equal to q A n i square into D P divided by L P into N D. So, reverse saturation current is governed by the whole diffusion length. Whereas, if it was the other way around, so if you had, it other way around N D much greater than N A then you can write that I naught it will be equal to q A and I square into L N D N divided by L N into N A.

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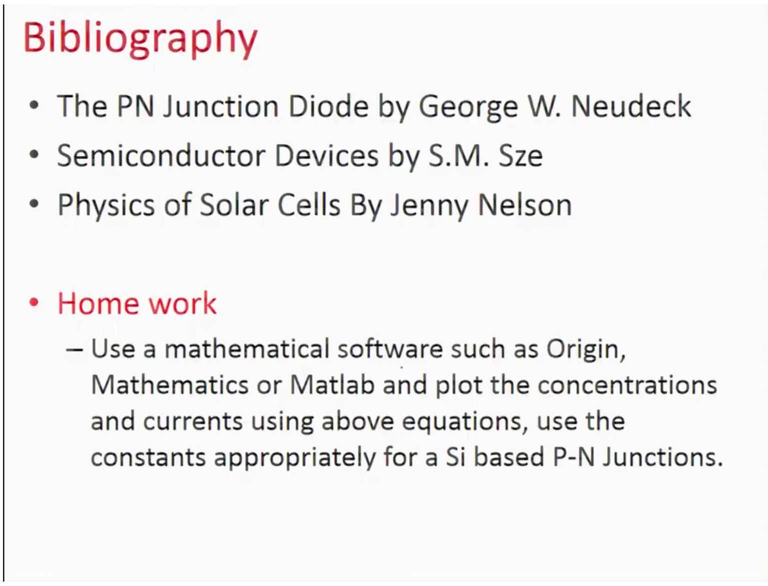


So, basically this is what the expression for current that you get is. So, when you plot it now, so this is the forward current which has a e to power q V A by kT minus 1 dependence and this is basically the minus I naught you get in the reverse bias. So, this is reverse bias, this is forward bias; if this is voltage, this is current.

So, this is the quick analytical solution of reverse and forward current for a P N junction in forward and reverse bias ok. So, you can go through these slides again. So, basically if I just recap this just for this of clarity what we did was, we first we first worked out the electron concentration in the as a function of applied bias right and this is nothing from this is very simple from the junction potential expression. Junction potential can be obtained from the balance between current and diffusion current and drift current and apply Einstein relation you get a expression for minority carrier concentration in terms of applied bias and this you can do for both electrons and holes. So, if you do for both electrons and holes, we get n p at minus w p for electrons, and similarly you can get for holes also. And from n p you can get delta n p. similarly for p n you can get delta p n. These delta p n and delta n p are going to be useful to solve the diffusion minority carrier equations. So, then we solve the minority carrier equations.

So, basically that is what we did. We had this minority carrier equation, we just change the references from x is equal to w n to x is x prime. There is no need to do that, but it just makes it as a little simpler. So, you just solve the differential equation from this, then you apply the boundary conditions for both the regions. From the boundary conditions, you can work out the minority carrier concentration at a given position and then you calculate it at the edge of the depletion region, because there is no point in calculating at any x, because you all do always know that J total is equal to J N at minus w p plus J P at plus w n. So, you just want to simplify that.

So, you replace x prime as 0. When you replace x prime as 0 this is what you get. So, J P x is equal to q p d p by l p into p n not exponential q V A by kT minus 1 into exponential minus of minus w n plus x divided by kT. So, x at x prime x is equal to w n, this factor becomes 1. So, at x is equal to w n we get J P w n is equal to q D p by L p into p n naught divided into exponential q V A by kT minus 1.

Similar expression you get for the electrons. Again you need to solve for, you need to first get the diffusion, you need to plug in the value of delta p and in terms of exponential q V A in the differential diffusion equation in the continuity equation and then you apply the boundary condition, just like we applied in case of electrons for the holes and when you apply the boundary condition after applying the boundary condition you get this x. So, you get this expression delta n p x double prime is equal to n p naught exponential q V A by kT minus 1 into exponential of minus x prime x double prime divided by L N.

So, they are similar equations, it is just that you replace n by p and p by n that is it and the positions are different. So, you just, then again you do the simple thing you, since we want to calculate J N at minus w p you replace x double prime by so, at minus w p x double prime is equal to 0. So, hence this J N at minus w p is nothing, but q D n divided by L N into n p naught into exponential q V A by kT minus 1 and then you sum them up and what we get is the famous diode equation that you obtain and that is what gives you the exponential relationship of a diode current.

So, this is the analysis that we have done in dark. In the next class what we will do is that we will take this same analysis forward by applying a light term to it and we will see what we get in terms of a P N junction which operates in light condition that will set up the framework for photo voltaic junctions ok. So, from the next class onwards we will be moving into Solar Photovoltaics in real sense, because we will be looking at PN junctions and light all right so.

Thank you. So, we will meet in the next lecture.