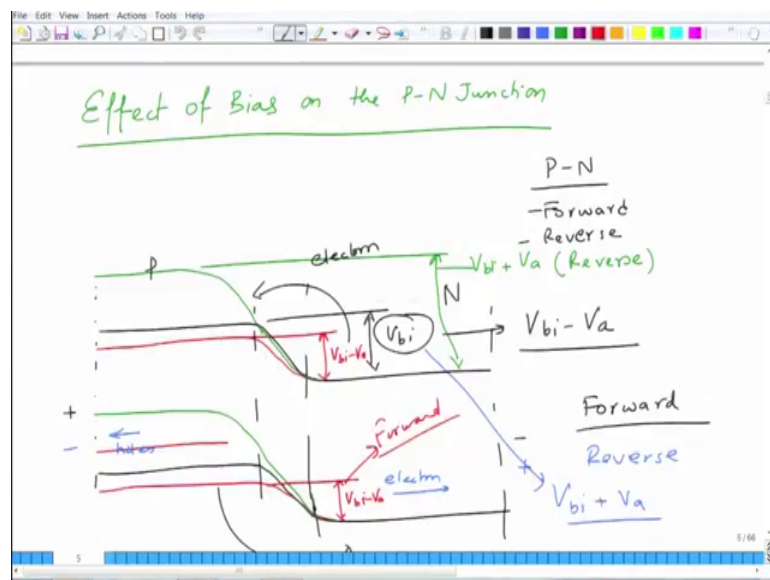


**Solar Photovoltaics: Principles, Technologies and Materials**  
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**Lecture - 19**  
**P-N Junction Analysis (Dark)**

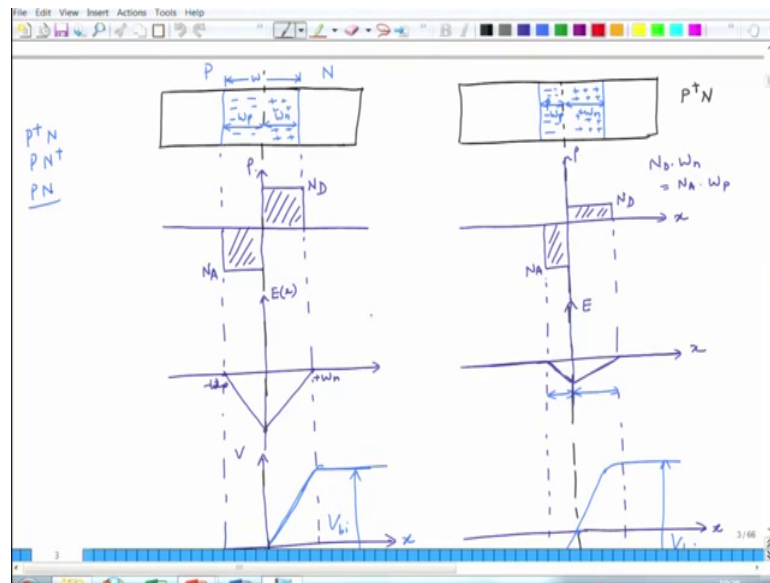
So, welcome again to a new lecture of this course Solar Photovoltaics Principles Technologies and Materials. So, in the, so we will just do a brief recap of last lecture. So, let me just go back to last lecture what we did in that lecture.

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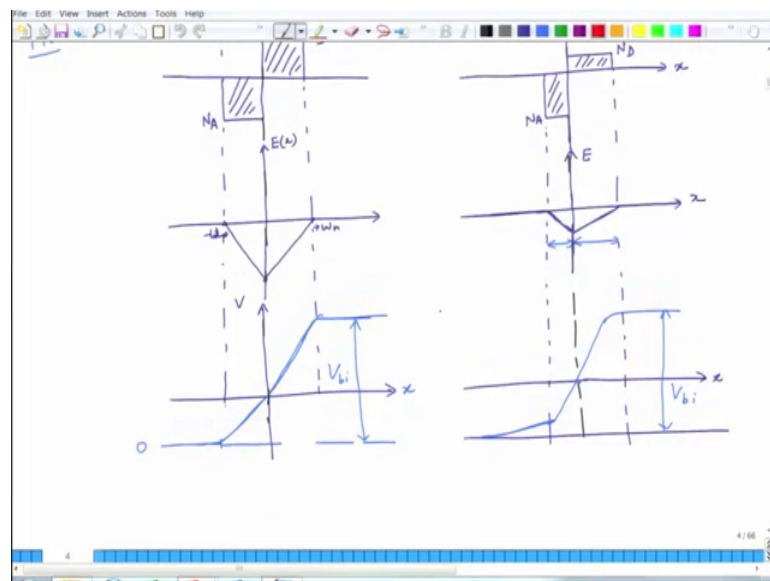


So, in the last lecture we looked at, we looked at effect of bias on P-N junction and we also looked at properties, we also looked at the various diagrams as to what happens what happens with what happens to charge carrier density, electric field and potential as a function when the when the P-N junction is; so, you can have P and N equally doped or your P and N unequally doped.

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So, we took two cases in which P and N were nearly similarly doped, and another case of P side being more heavily doped as compared to N side and this has repercussions on the charge density on the both sides and width of depletion region. So, depletion region is equally wide in N and P side if both are equally doped.

However, depletion region is more narrower in case of heavily doped semiconductor than in case of lightly doped semiconductor. However, you have to maintain the relations of charge neutrality on both sides as a result you have this  $N_D \cdot w_n = N_A \cdot w_p$

w p. And likewise you have more drop in the electric field in the narrow region than in the wider region and this has similar repercussion in terms of field as well potential as well.

So, now, then we looked at effect of bias on the P-N junction, we looked at qualitative way of understanding it. So, effect of bias is essentially you can have two biases forward and reverse bias. Forward bias is essentially it reduces the barrier height. So, your barrier height becomes  $V_{bi}$  minus  $V_a$ , because  $V_a$  is positive as a result more electrons can flow and more holes can flow across the junction. Whereas, in case of reverse bias you increase the barrier height leading to reduced hole and electron flow across the junction as a result the current lower.

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$V_a$  - applied bias  
 +ve - F.B.  
 -ve - R.B.

$V_{bi} \rightarrow V_{bi} - V_a$

For n-side -  $0 \leq x \leq w_n$

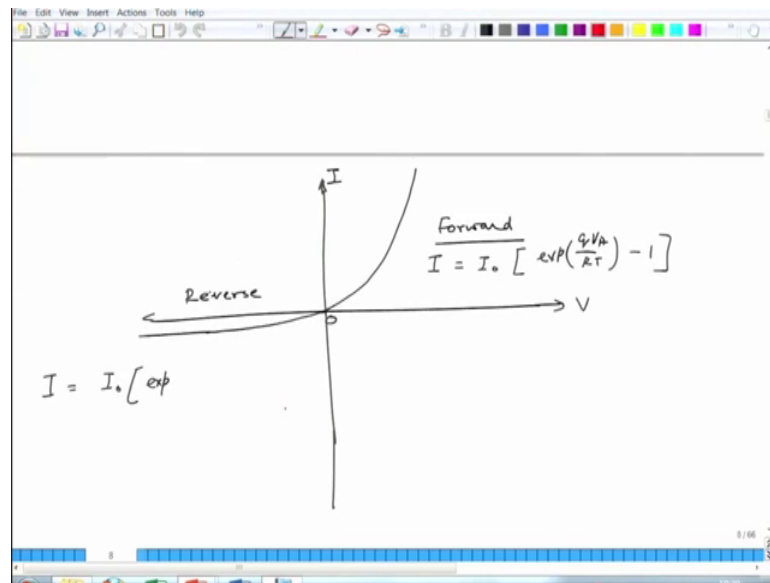
$$w_n = \left[ \frac{2\epsilon_s}{q} (V_{bi} - V_a) \cdot \frac{N_A}{N_D(N_A + N_D)} \right]^{\frac{1}{2}}$$

$$V(x) = (V_{bi} - V_a) - \frac{q N_D}{2\epsilon_s} (w_n - x)^2$$

$$E(x) = - \frac{q N_D}{\epsilon_s} (w_n - x)$$

So, this leads to or this, so you can modify this equation accordingly just replace  $V_{bi}$  minus  $V_a$  and this has implications in terms of both electric field and potential.

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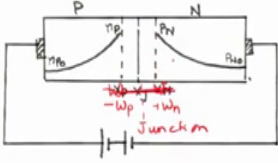
And in case of current and voltage characteristics what you have is, you have a exponential increase in the current in case of forward bias and in case of reverse bias you have you have very low current in the in the reverse bias. So, this is the saturation current is nothing, but  $I$  minus  $I$  naught which is essentially the reverse current that you get which is nearly very small. So, diode basically will acts like a switch, so in the reverse bias it has 0 current in the forward bias it has very high current.

Now, what will do is that in this lecture we will take forward this discussion to calculate the P-N junction current. So, we will go we will go a little quickly about it because we do not have too much time in these this lecture, but we will try to establish a framework of how to derive the current.

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### P-N junction in dark: I-V Characteristics

- 1) No generation of carriers
- 2) Valid depletion approx
- 3) No generation/recombination in Depletion region.
- 4) Low level injection in the bulk region.  
i.e.,  $n_p \ll p_p$  &  $p_n \ll n_n$   $\Delta n_p \approx \Delta p_n$
- 5) In bulk  $\frac{d\phi}{dx}$  or  $E = 0$
- 6) In bulk  $\frac{dp_p}{dx} = 0, \frac{dn_n}{dx} = 0$



So, essentially P-N junction in dark when you want to drive the I V characteristics there are certain assumptions that we make. So, first is there is no generation of charge carriers as a result because you have no light based generation or thermal generation. So, we assume that carrier concentration whatever it was to begin with that stays constant. And the depletion region approximation is valid which means you have a depletion region in which there is no generation recombination that takes place and its free of free charges there are no free there are no electrons and holes there. It is completely depleted of charges basically.

And there are no recombination generation in depletion region and we also assume that a bulk of the semiconductor has this low level injection condition, which means that this  $n_p$ , the minority carrier concentration in the P side that is electron concentration is much smaller than the hole carrier hole concentration in the P side and the minority carrier concentration N side that is hole concentration N side is much smaller than the electron constitution N side. But it also means is that the perturbations, so you can say that the  $\Delta n_p$  you can say  $\Delta n_p$  and  $\Delta p_n$  are such that they do not disturb the majority carrier concentration they only affect the minority carrier concentration. So, injection is lower in that sense.

In the bulk of the semiconductor the electric field is equal to 0, so because of flat bend condition. And in the bulk we also have the carrier concentration as a function of

distance in both P and N side for the majority carrier they are equal to 0. So, because the majority carrier majority carrier concentration remains unperturbed as a result you do not have a variation in these.

However, if you look at the minority carrier concentration minority carrier concentration is something like this. So, this should be minus  $w_p$ , it should be, so let me just say this is minus  $w_p$  this is plus  $w_n$  and this is the junction, ok. So, minority carrier concentration at the junction at the at that boundary of the depletion region and the bulk the concentration is  $n_p$  and deep in the bulk you have  $n_p$  naught. Similarly, on the N side minority carrier concentration that is hole concentration is  $p_n$  at the boundary between depletion region and bulk that is at  $x$  is equal to  $w_n$  and it goes to equilibrium value of  $p_n$  naught in the bulk of the semiconductor. So, these are certain conditions that we will follow to begin with.

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### Minority Carrier Equations

<p>n-type</p> $0 = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p}$ $J_p \cong -qD_p \frac{\partial \Delta p_n}{\partial x}$ $p_n = p_{n_0} + \Delta p_n(x)$	<p>p-type</p> $0 = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n}$ $J_n \cong qD_n \frac{\partial \Delta n_p}{\partial x}$ $n_p = n_{p_0} + \Delta n_p(x)$
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$J_N^{\text{drift}} \approx J_p^{\text{drift}} = 0$   
 $G_M \neq 0$

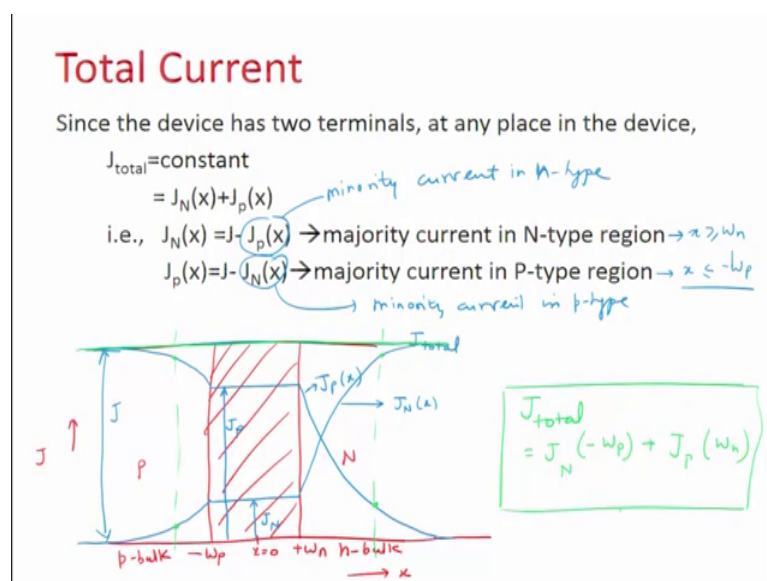
So, minority carrier equations can be written as. So, if you remember we talked about the continuity equations, right. So,  $\frac{\partial n}{\partial t}$  or  $\frac{\partial p}{\partial t}$  now if we take a steady state conditions if we take a steady state condition then for n-type semiconductor one can write minority equation as. So, only the, so the only because you have electric field that is equal to 0 as a result you not going to have drift current you are not going to have generation you are going to have some trap assisted recombination in the bulk region.

So, the only current that is available for, because we are ignoring the depletion region we are looking at the first bulk region.

Within the bulk region the minority carrier equation can be written as the diffusion component that is  $D_p \frac{d^2 \delta p_n}{dx^2} - \frac{\delta p_n}{\tau_p} = 0$ . And we know that the hole current in this case is  $-q D_p \frac{d \delta p_n}{dx}$  and  $p_n$  which is the minority carrier concentration at a distance  $x$  is equal to  $p_{n0} + \delta p_n$  that is the perturbation the change in the minority carrier concentration as a result of sweeping of carriers from one side to another side when you apply bias.

Now, for P type you can write similar equations as in the study state this become  $0 = 0$  is equal to  $D_N \frac{d^2 \delta n_p}{dx^2} - \frac{\delta n_p}{\tau_n} = 0$ .  $J_n$  in this case sorry this should be  $J_n$ ,  $J_n$  in this case is  $q D_n \frac{d \delta n_p}{dx}$  which is the diffusion current for electrons and  $n_p$  is similarly  $n_{p0} + \delta n_p$ . So, of course, you can see here you have  $J_N$  or  $J_P$  drift that is equal to 0 and then generation is equal to 0. So, you do not have any generation of carriers because it is in dark. So, there are no carriers that are been generated, ok. But you will have recombination in the bulk because carriers will once you once the carriers go from P to N side or N to P side they will be recombine with the holes and electrons on the other side.

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So, the total current that we have in the device is since the device has two terminals and at any, so at any place in the device we can say  $J_{total}$  is constant which means its equal to  $J_{N x}$  plus  $J_{P x}$ . So,  $J_{N x}$  would be  $J$  minus  $J_{P x}$  that is a majority current in N-type region and  $J_{P x}$  would be  $J$  minus  $J_{N x}$  that would be a majority current in the P-type region. So, if you look at it in the in the a schematic term I can make a diagram like this let us say this is the depletion region,. So, this is minus  $w_p$  this is plus  $w_n$  and at somewhere here we have  $x$  is equal to 0, ok.

So, since I am and if I plot on this axis  $J$  and on this axis I have this  $x$  the distance this is the p-bulk region this is n-bulk, ok. So, this is P this is N and this is somewhere that depletion region. So, if I just change the colour, ok. So, I am saying that this is  $J_{total}$ . So, this is  $J_{total}$ , all right.  $J_{total}$  is sum of  $J_N$ ,  $J_P$  it is a constant current. So, what it means is that in case of J P N it goes a something like that and then in the depletion region it stays constant then within. So, if I extend it to other side, all right.

And now if I draw the how, so this is what current this is going to be? This is going to be. So, here you can see that this is this the this current is high in the P side its constant in the depletion region because in depletion region we assume that there is no recombination in generation as a result nothing happens to carriers, but the moment they come on this side the current drops. So, which current is this going to be? This is going to be hole current this is going to be  $J_{P x}$  because it holes become a minority carriers on the N side. So, when they come to this side they are large in number, but as they go travelling in the N region the number reduces because of recombination because recombination takes place with the electrons available there.

Similarly, if you plot for electrons the electrons will show variation like this. So, this is a systematic variation this can be and this goes as something like this. So, this would be  $J_{N x}$  and this would be  $J_{P x}$ . So, basically you can say within this region this is  $J_N$  and this would be  $J_P$ . So, if you want to find out the majority current in the N-type region the majority current in the N-type region would be  $J_{total}$  minus  $J_{P x}$ . So, right because that becomes the majority current in that.

So, similarly on this side if you want to find out the majority current  $J_{P x}$  that would be total  $J$  minus  $J_{N x}$ , similarly you can find a minority current. So, this would be for example, the minority current in current in N-type, if you take  $J_{N x}$  as the majority



current and the N-side. Similarly, this will become majority minority current in the in P-type. So, basically we are looking at this would be for the region which is  $x$  greater than  $w_n$  and this would be for the reason which is  $x$  greater than equal to minus  $w_p$ , right

So, now, so this a for some carrier concentration we have we are going to have this kind of profile. So, this is the total magnitude  $J$ , all right and at any at any given point of time the total current is going to be this current if I use different colour for that let us say I make it green. So, this is  $J$  total. So, this green one is sum of this and that. So, at any given point of time is going to be sum of this and that, ok. Similarly here at any given point time its going to be sum of this and that.

Now, so  $J$  total I can write now since in these regions they are going to vary, but within this region they are going to stay constant. So, I can also write  $J$  total is nothing, but  $J_N$  at minus  $w_p$  plus  $J_P$  at  $w_n$ , all right. Is that a fair assumption? So, this is what is the conclusion from this slide the  $J$  total is equal to  $J_N$  at minus  $w_p$  and  $J_P$  at  $w_n$ .

Now, instead of calculating this currents at any arbitrary place I am now going to calculate these currents at this fixed places which are the boundaries of depletion region on both bulk sides N and P sides.

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### Methodology

- First solve for  $\Delta p_n(x)$  in the n-bulk region to compute  $J_p(x)$
- Then solve for  $\Delta n_p(x)$  in the p-bulk region to compute  $J_n(x)$

So, what is the method that you that we are going to follow? The method that we have to follow is first we solve for  $\Delta p_n(x)$  in the n-bulk region to compute  $J_P(x)$ , ok. So,

basically solve for minority carrier concentration in the N side. So, that we can calculate the minority carrier current, similarly we solve for  $\delta P$  in the p region to solve the electron current that is the majority current that is the minority current in the P side.

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## Depletion Region

- what goes in, goes out
- no generation & recombination

If the minority carrier diffusion current at the edges of the depletion region is known, then it is known throughout the depletion region

$$J_{p|depl} = J_p(w_n) = -qD_p \left. \frac{dp_n}{dx} \right|_{x=w_n}$$

$$J_{n|depl} = J_n(-w_p) = qD_n \left. \frac{dn_p}{dx} \right|_{x=-w_p}$$

Total current

$$J = J_p(w_n) + J_n(-w_p)$$

So, this is what we do. So, basically the assumptions that we make in depletion region we assume that what goes in goes out, and as a result there is no generation and recombination. And if the minority carrier, so we saw from the previous diagram that if the minority carrier diffusion current at the edges of the depletion region is known then we know the total current throughout the depletion region as well as total current throughout the device, ok.

So, basically we can write  $J_p$  depletion edge is equal to  $J_p$  in the depletion region is equal to  $J_p$  at  $w_n$  which is equal to  $-qD_p \frac{dp_n}{dx}$  at  $x = w_n$ , right. Similarly, I can write  $J_n$  at depletion region edge that is  $J_n$  at  $-w_p$  this is  $qD_n \frac{dn_p}{dx}$  at  $x = -w_p$ , right. These are very straight forward equations just taken at the boundary of the depletion region because it stays constant throughout the depletion region. So, for hole current I have taken the edge of the depletion region on the N side and for the electron current I have taken the edge of the depletion region near the P side. So, that and we know that both are minority currents on

each side. So, total current is this as we saw earlier  $J$  is equal to  $J_p$  at  $w_n$  plus  $J_n$  at  $-w_p$ .

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**Boundary Conditions at the edges of Depletion Region**

→ low level injection (additional current due to applied voltage are also very small)  
 →  $E$  &  $n$  are constant

In depletion region  $E \neq 0$ , hence

$$J_N = J_N|_{drift} + J_N|_{diff}$$

$$= q\mu_n n E + qD_N \frac{dn}{dx} \cong 0$$

$J = qE = ne\mu_n E$

$V_{bi} = \int_{-w_p}^{w_n} E dx$

$$E = -\frac{qD_N \frac{dn}{dx}}{q\mu_n n} = -\frac{D_N}{\mu_n} \cdot \frac{dn/dx}{n} = -\frac{kT}{q} \cdot \frac{dn/dx}{n}$$

(integration of this will give  $V_{bi}$ ;  $V_{bi} = -\int_{-x}^{\infty} E dx$ )

Einstein Relation

$\frac{D_N}{\mu_n} = \frac{kT}{q}$

$\frac{D_p}{\mu_p} = \frac{kT}{q}$

→  $V_{bi}$  → Junction Potential

Now, to solve these equations we need the boundary level, boundary conditions. So, boundary conditions at the edge of the depletion region first we assume that there is low level injection condition, right. And also the additional current that we apply because of applied voltage is very small at least in the depletion region, and we assume that electric field and the carrier concentrations are the majority carrier concentrations are constant.

So, in the depletion given region however, in the depletion region we have a condition where electric field is not equal to 0, within the depletion region we know that electric field is not equal to 0. As a result we can write  $J_N$  is equal to  $J_N$  drift plus  $J_N$  drift what it means is that  $J$ . So, this is going to be equal to drift current is going to be equal to  $q n$ ,  $q n \mu_n$  into  $n e$  right, this is  $e$ , the electric field I have noted at this because there is a confusion between  $e$  and electric field and energy. So, that is why so we can say that this is electric field, ok.

So, basically now, we know from the ohms law that this is  $J$  is equal to  $\sigma E$  which is equal to  $n e \mu_n E$ ,  $e$  right. So, this is what it is it is  $n$  electronic charge into  $\mu_n$  into  $\epsilon E$  plus the diffusion current that is  $q D_N \frac{dn}{dx}$  and that is equal to 0 in the equilibrium. Of course, so first we are taking at the equilibrium.

So, we here we can calculate what is the electric field. The electric field is then going to be equal to  $q D N$  by  $dx$  divided by  $q \mu n$  right, from this equation we can find out this is the electric field and this is nothing, but minus  $D N$  by  $\mu n$  into  $dn$  by  $dx$  divided by  $n$ . And  $D N$  by  $\mu n$ , there is a something called as Einstein relation which we have not derived, but it is simple to derive at equilibrium. At equilibrium  $D N$  by  $\mu n$  is equal to  $k T$  by  $q$ , similarly  $D P$  by  $\mu p$  is equal to  $k T$  by  $q$ . So, what it does is that basically Einstein relation is equation that relates the diffusivity with the mobility. You can see that this is  $D$  is diffusivity.  $\mu$  is mobility, this is carrier mobility, this is carrier diffusivity. So, it relates the microscopic parameter mobility to diffusivity.

So, hence this if I replace this  $D N$  by  $\mu n$  is as ask by  $k T$  by  $q$  due to virtue of Einstein relation I get this relation, and if you integrate this electric field from minus infinity to plus infinity you will get built in field basically, right. And if you if you integrate this over this; so, junction.

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### Junction Potential

$$V_j = V_{bi} - V_A = \int_{-w_p}^{w_n} E dx$$

$$V_j = V_{bi} - V_A = \int_{-w_p}^{w_n} -\frac{kT}{q} \cdot \frac{dn/dx}{n} dx$$

$$V_j = \frac{kT}{q} \ln \frac{n_n(w_n)}{n_p(-w_p)}$$

So, this will be the built in field that you get, if you integrate it. The electric field total electric net electric field within the depletion region since you got that electric field is 0 outside the depletion region, whether you infinity from minus infinity to plus infinity, whether you infinity from minus  $w_p$  to plus  $w_n$  its going to remain the same because on the sides its. So, if you integrate it from minus infinity plus infinity you have you are going to get built in field.

Now, what you do is that when you apply bias. So, this built in field is nothing, but junction potential right at equilibrium right, ok. This is the junction potential at equilibrium. When you apply bias this junction potential is going to get modified. So, this junction potential will become  $V_{bi}$  minus  $V_A$  or  $V_{bi}$  plus  $V_A$ . So, let us say we do not worry about the sign right now we just write it as  $V_{bi}$  minus  $V_A$ . So, if I now apply the bias and measure the junction potential calculate the junction potential the junction potential now becomes equal to  $V_{bi}$  minus  $V_A$  which is  $\int_{-w_p}^{w_n} (-E) dx$ .

In the previous case you can integrate it from minus infinity to plus infinity or  $V_{bi}$  can also be  $\int_{-w_n}^{w_p} (-E) dx$ , because since the bands are flat beyond  $-w_n$  and  $w_p$  and  $w_n$  it does not really matter whether you integrate from minus infinity to plus infinity or  $-w_p$  to  $w_n$ .

So, if I now do this, so what I have here is, if I now do this I get this junction potential this junction potential is equal to  $V_{bi}$  minus  $V_A$  and it this is equal to  $\int_{-w_p}^{w_n} (-E) dx$ , ok. And if you do the substitution for  $E$  you get a relation for junction potential that is  $kT$  divided by  $q$  into  $\ln$  of  $n_p/n_n$ ,  $n_p$  minus  $w_p$ ; on the. So, far electrons it would be since we are taking in terms of electrons we get the relation for electron concentration. So, when you look at the elect  $n$ . What is  $n$ ? So,  $n$  is nothing but majority carrier concentration on the N side at  $w_n$ ,  $n_p$  is the minority carrier concentration at  $-w_p$ . So, the ratio of log of ratio of majority carrier concentration on the N side to the minority carrier concentration on the P side multiplied by  $kT$  by  $q$  is the junction potential. So, so.

So, this is this is what we have established till now. We will take this further in the next class. So, next class what we will do is that we will look at the minority carrier concentration calculation. So, from this now you have got this relation and junction potential, you can calculate what is the minority carrier concentration on either side, ok. You can write a similar equation for holes, if you write a similar equation for a holes and then calculate the carrier concentration in terms of junction potential we should be moving forward with the work analysis of carrier concentration calculation and current estimation, ok. So, we will do that in the next lecture. We are running out of time, we will finish here today.

Thank you.