

Solar Photovoltaics: Principles, Technologies and Materials
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Lecture - 17
P-N Junction Characteristics

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Recap

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

- P-N Junction (dark)
- n-Si - p-Si

$J_N^{total} = J_N^{Drift} + J_N^{Diff} = 0$
 $J_P^{total} = J_P^{Drift} + J_P^{Diff} = 0$

The diagram illustrates a P-N junction with the following labels:

- bulk regions**: The n and p regions.
- Depletion region**: The region where the Fermi levels align.
- acceptor atoms**: Located in the p-region.
- Donor atoms**: Located in the n-region.
- V_{bi}** : Built-in potential across the depletion region.

So, welcome again to the new lecture of this course Solar Photovoltaics, Principles, Technologies, and Materials. So, we will just recap the last topic that we started. So, in the last lecture, we started discussion on P-N junction. And this is basically P-N junction and dark without any light illumination. P-N junction is essentially a junction between two semiconductors, one N-type and one P-type typically.

And generally, we look at in the context of identical homo junction that is n-silicon on one side, and p-silicon on other side. And as you put them together, so you have n-silicon, and p-silicon making a junction. And as you put them together the Fermi levels of those these two semiconductor are (Refer Time: 01:04), because Fermi level has to be through same throughout the device and equilibrium.

And this condition of acquiring the Fermi levels of two semiconductors leads to band bending at the interface, because on one side you have electrons in excess, on the other side you have holes in excess on the p-side. As a result electrons have a built in

concentration gradient from n to p side, and holes have a built in concentration gradient from p to n side.

However, at equilibrium diffusion current cannot drive. Once the junction is established, there is a there is a opposing, because the band bending at the interface there is a opposing electric field, which gives rise to drift current which opposes the diffusion current. And these two diffusion and drift current balance out each other leading to low current at equilibrium.

However, because you are formed the junction, so the junction shows a region which is called as depletion region at the interface. So, this is for example the P and N. And this region at the interface is depletion region in which there are no excess there are no free electron, there are no holes and electrons. Basically, what you have at the interface, this is the interface.

So, if this is n-side you are going to have donor atoms here, so these are donor atoms positively charged these are donor atoms. And on this side you are going to have the density of this these atoms will depend upon or the under width of this depletion as we will see it will depend upon how dope the semiconductor is on this side, you will have accepted atoms. And holes and electrons would have neutralized each other.

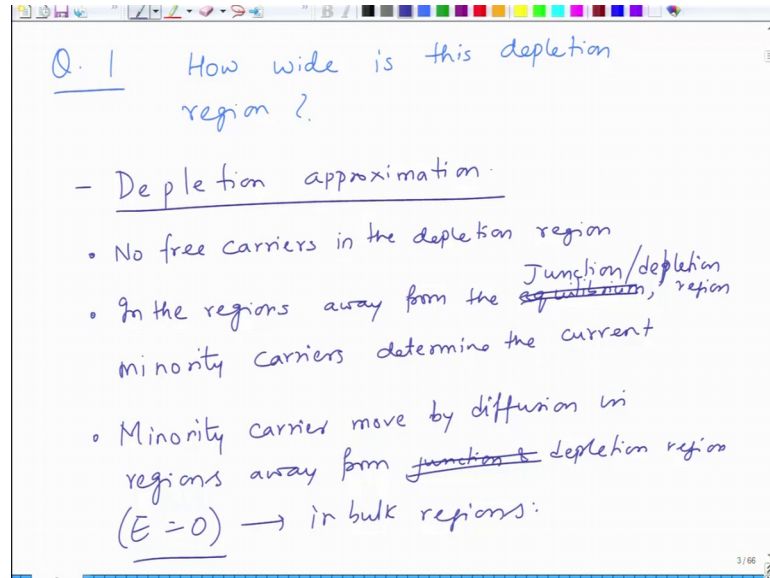
So, as a result this region is called as depletion region depletion region which means, it is depleted of any free charges, whereas in these regions the regions were the bands of flat. So, these regions are called as bulk regions. You still have excess carriers on both sides. So, this side will have lot of electrons, these are these are electrons, and this and here you will have some holes. And on this side you will have lot of holes, and few electrons.

So, so as we said earlier that in this case J_N plus J_P total is equal to 0 ok. J_N total is equal to $J_{P,N}$ drift plus $J_{N,N}$ drift, this is equal to 0. And J_P total is $J_{P,P}$ drift plus $J_{P,N}$ drift this is also equal to 0, so which means drift and diffusion currents equal are equal and opposite to each other for both holes and electrons.

Any more migration of electrons and holes is not possible, because you have a built in electric field, and this is called as V_{bi} on both sides. So, this is called as V_{bi} . And as we saw last time this V_{bi} was determined by acquiring a Fermi levels on both sides. And this V_{bi} turns out to be $kT/q \ln$ of $N_A N_D$ divided by n_i^2 . So, basically

the built in field increases as you in sorry it should be N_a into N_d built in field increases as increase the acceptor and donor doping concentration in the in the semiconductors. This is what we did last time.

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Now, what will do is that we will look at some more aspects of P-N junction, and little bit more quantitative manner. For example, how wide is the depletion region. So, first question is question number-1, how wide is this depletion region that is the first question that we want to answer, and then we also know we also want to answer.

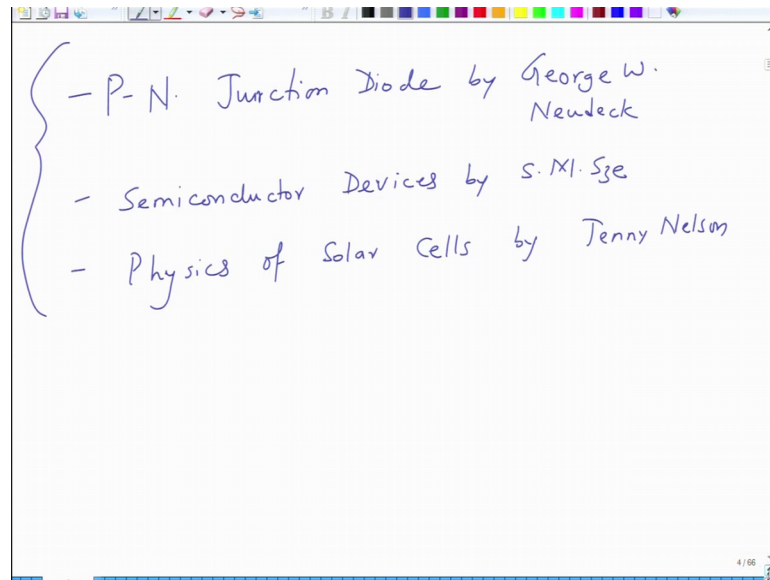
So, for this what we take into consideration is first we make a approximation which is depletion approximation ok. Now, depletion approximation basically means is that you do not have any free carriers in the depletion region. By the way if you want to do analysis of this, if you want to go into details of this, I would recommend you to go through the book, and I will write it here probably, I will write it after this ok.

So, first is that known there are no carriers in the depletion region. Second is in the regions away from the equilibrium minority carrier determines from the junction sorry minority carrier determine the current, because majority carriers are nearly close to equilibrium all the time, because minority carriers get perturb it perturb the most.

And minority carriers move so at junction and depletion region, here also it should be depletion region basically, the region where electric field is equal to 0, where the bands

are flat. So, minority carriers move by the diffusion and regions away from depletion region, and where E is equal to 0. Since, electric field is equal to 0, they cannot move it, they cannot move by diff they can only move by diffusion ok, so which are called as n-semiconductor in bulk regions. So, the regions are called as bulk regions ok.

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So, before I go into little bit more details of this, let me just mention the book. Books could be P-N Junction Diode by George, let me see what the name of the author is George W Neudeck, and by S.M Sze. Then in the context of solar cells you can also look at Physics of by Jenny Nelson. These are three books which can be useful to understand this whole P-N junction electrical characteristics in detail.

So, let me now go back to where we were. So, first approximation is depletion approximation which says that as we solve than the previous slide that, there are no free carriers in the depletion region in the regions away from the depletion region. Minority carriers determine the current. And minority carrier move by diffusion in the regions away from depletion region, where electric field is equal to zero, wherever the wherever you have flat electric field flat energy bands which means electric field is equal to 0, because you have to take the derivative of potential. And since potential is related to energy which is minus v by minus e by q , and from that you will determine electric field.

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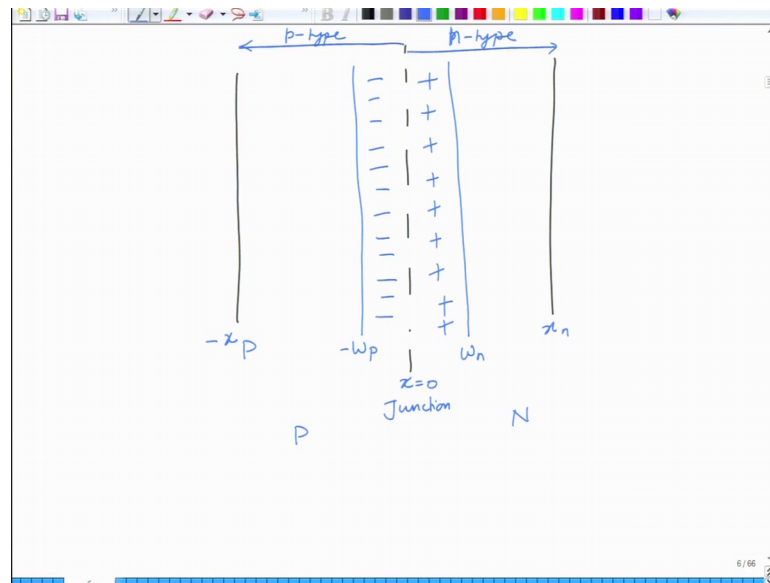
The image shows a whiteboard with handwritten text and equations. The title is 'Super-position Approximation'. Below it, there is a bullet point: '- Recombination rates, R, are linear in the neutral region (minority carriers) (bulk)'. This is followed by two equations: $R_p = \frac{\Delta p}{\tau_p}$ (n-type) and $R_n = \frac{\Delta n}{\tau_n}$ (p-type). A second bullet point states: '- Effects of light & applied bias are independent or decoupled.' The whiteboard has a toolbar at the top and a status bar at the bottom right showing '5/66'.

So, next approximation is called as super-position approximation. This basically means is that the recombination rates R rate rates R and basically R and are p are linear in the neutral regions; basically, neutral regions are the ones which are that the electric field is zero. The neutral field is you can say is the bulk region bulk region. There are multiple name quasi neutral regions, and bulk region and so on and so forth. And this is basically applies to minority carrier ok.

So, basically R of n is equal to R p sorry R of p is equal to Δp by τ_p , this will be in n -type ok. And R n would be equal to Δn by τ_n , this would be in p -type all right this we have seen, so they are basically linear. They go as linear function of Δp , and Δn . And if you have light present, if you have bias present, so you might have presence of light some light may be present, and you might have applied potential, the effects produced by these two are decoupled.

So, effects by effects of light, because light can change the carrier concentration right, as a result it can change the diffusion gradient, whereas bias can lower or lower the barrier at the interface. So, effect of light and applied bias that is external field are independent or decoupled to be more precise ok.

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So, let us see the situation is so we define the solar, we define this P-N junction as you know if you have a P-N junction like this. This is the interface somewhere here we have the boundary of depletion region ok. So, this let us say is x is equal to 0, which is the perfect which is the junction. And we assume that this is a perfect interface ok.

And we derive this as w_n , so this is P-side, this is N-side. So, this is depletion region let us say here we have positive donor ions right, on this side we have negative acceptor ions, but there are no free carriers all right. So, this is minus w_p , this is plus x_n , this is minus x_n . So, this side is N-type, and this side is P-type ok.

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Essential Equations

$$E = -\frac{d\phi}{dx}$$

Poisson's Equation \rightarrow for a region with charge distribution.

D - dielectric displacement

$$\text{div. } D = \rho \Rightarrow \nabla \cdot D = \rho$$
$$D = \epsilon_s \cdot E \rightarrow \text{electric field.}$$

\uparrow
dielectric constant

$$\nabla \cdot E = \frac{\rho}{\epsilon_s} \text{ or } \nabla^2 \phi = -\frac{\rho}{\epsilon_s}$$

So, the basic let us first look at the essential equations. Like essential equations is that we know that electric field is equal to minus of d phi by d x all right. Now, we have a equation which is called as Poisson's equation, which basically tells us it applies in a region where you have a charge distribution. So, for a region with charge distribution all right.

So, this is so if you have let us say D is the dielectric displacement, which is related to displacement D is the displacement, then divergence of D is equal to rho. Rho is the charge density so or you can say del dot D is equal to rho. And from basic physics that we know that D is equal to epsilon s into E, where epsilon s is the dielectric constant of the semiconductor, and E is the electric field all right.

So, from this we can say that del dot E is equal to rho divided by epsilon s or you can write del 2 phi is equal to rho divided by E s I am sorry epsilon s minus of, so that is how you can you can write it as so now so this so basically Poisson's equation says that that del of E or divergence of E is equal to rho divided by E s rho is the charge density, this is divided by so divergence of electric field is a basically proportional to the charge density in the material this comes from the basic physics right.

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P-N Junction

(i) $-w_p \leq x \leq 0 \quad \frac{\partial E}{\partial x} = -\frac{q N_a}{\epsilon_s} \rightarrow \text{p-side}$

(ii) $0 \leq x \leq w_n \quad \frac{\partial E}{\partial x} = \frac{q N_d}{\epsilon_s} \rightarrow \text{n-side}$

Electric Field (P-side) $-w_p \leq x \leq 0$

$E(-x_p) = 0$

$$\int_0^{E(x)} dE = -\int_{-w_p}^x \frac{q N_a}{\epsilon_s} dx$$

$$E = -\frac{q N_a}{\epsilon_s} (x + w_p) \quad -w_p \leq x \leq 0$$

So, now in the context of P-N junction first we write for minus x being between minus w p and 0, so within this region we have the acceptor ions right. So, del E by del x in this region is equal to minus of q N a divided by epsilon s, because we have negative charges in that region. So, rho will be minus of q into N a. N a is the expected ion density whatever accepted ions are there within that region into q the electronic charge, and since they are minus we have since the negative charges we have added a minus here divided by epsilon s. For the region on n side x between 0 and w n, it would be del E upon del x being equal to q N d divided by epsilon s by the same analogy right. On one side you have positive charges, on the other side you have negative charges.

Now, if you look at the electric field on p-side, so first look at the electric field on p-side, so for p-side we know which means that it is a we are talking of x minus of w p, so this is p-side, this is on n-side. So, for electric field we know that from the we need to apply first condition that is the boundary condition E at minus x p is equal to 0, which means far ends of the semiconductor electric field is equal to is equal to 0, because the bands are flat right.

So, E at minus x p is equal to 0, so we then write d E to be equal to minus of q N a divided by epsilon s into d x. Now we need to integrate this, so we integrate this from 0 to E x, and this is integrated from minus of w p to x. So, this is this is within the depletion region right.

So, when we do that we have E as minus of so this is this condition we know that dielectric field at minus x p is equal to 0, because at the end of the semiconductor the bands are flat. As a result there is no electric field there, but we need we are interested in calculating the electric field in this so in that since, it is not a boundary condition in that sense. So, it is minus of q N a divided by epsilon s integrate. So, if you if you if you do integrate, what you will get is x plus w p. So, this is E is equal to minus q N a divided by epsilon x into x plus w p for x being between 0 and minus w p ok. This is for the electric field on p-side.

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Electric field (N-side) $0 \leq x \leq w_n$

$$E(w_n) = 0$$

$$E(x) = \int_{E(x)}^0 dE = \int_x^{w_n} \frac{qN_d}{\epsilon_s} \cdot dz$$

$$E(x) = - \frac{qN_d}{\epsilon_s} (w_n - x) \quad \underline{0 \leq x \leq w_n}$$

Now, the same thing if you want to calculate for electric field on N-side. So, when we look at the electric field on N-side between x being from 0 to w n, then first condition is that E electric field at w n is equal to 0, because it is 0 at the boundaries of depletion region in the quasi neutral region. So, E x is now calculated by integrating the electric field from E x, because electric field is continuous in the depletion region, and then it becomes 0. So, this is d E, and this is integrated from x to w n, and this becomes q N d divided by epsilon s into d x. And if you integrated, you get E x as minus of q N d divided by epsilon s into w n minus x. So, remember it is integrated from E x to 0 that is why, this minus sign comes. And this is valid for x being between w n to 0. So, this is how we calculate the electric field.

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Potential for both p- and n-side

$$\frac{d\phi}{dx} = -E(x)$$

$$\phi(x) = \frac{qN_a}{2\epsilon_s} (x+w_p)^2 \quad -w_p \leq x \leq 0$$

$$\phi(x) = -\frac{qN_d}{2\epsilon_s} (w_n-x)^2 + V_{bi} \quad 0 \leq x \leq w_n$$

For p-side $\frac{d\phi}{dx} = \frac{qN_a}{\epsilon_s} (x+w_p)$ For n-side $\int \phi(x) = V_{bi} - \phi(x)$

And next we look at the potential across the depletion region from the edge of depletion region on p-side to the edge of depletion region in the n-side. So, we know that since $d\phi$ is equal to $d\phi$ by dx is equal to minus of $E(x)$, again you can do this by integrating. So, I will not do that, but you will obtain for p-side $\phi(x)$ as qN_a divided by $2\epsilon_s$ into x plus w_p square. So, this is less than equal to 0 minus w_p .

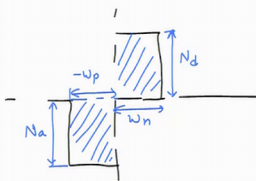
And for the n-side, you will obtain $\phi(x)$ as minus of qN_d divided by $2\epsilon_s$ into w_n minus x square plus V_{bi} ok, because let me just write the basic expressions, because for p-side you will have $d\phi$ by dx as qN_a divided by ϵ_s into x plus w_p .

Whereas, for n-side this will be you will have to integrate it from. So, if you look at $\phi(x)$, it will be integrated from. So, basically the integration will take place from $\phi(x)$ to V_{bi} , because you are going from so because you are starting like this right, this is V_{bi} . The maximum value on the other side it takes is V_{bi} , so this is for so if you are starting from zero on this side, it will achieve some value at the interface, and then maximum value it can take is V_{bi} , so that is the maximum value. So, the integration will take place from $\phi(x)$ to V_{bi} and V_{bi} we already know. So, basically $\phi(x)$ would be this, so this would be V_{bi} minus $\phi(x)$. So, whatever $\phi(x)$ you will have, it will be V_{bi} minus of something all right.

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Depletion region width

$$\phi(0^-) = \phi(0^+)$$

$$\left(\frac{q N_A}{2 \epsilon_s}\right) w_p^2 = \left(-\frac{q N_D}{2 \epsilon_s}\right) w_n^2 + V_{bi}$$


$$w_p N_A = w_n N_D$$

$$w_p = \frac{w_n N_D}{N_A}$$

$$w_n = \frac{1}{N_D} \sqrt{\frac{2 \epsilon_s V_{bi}}{q \left(\frac{1}{N_A} + \frac{1}{N_D}\right)}}$$

So, now let us see, what is the depletion region width. Now, since we are saying that the interface the potential is continuous, what it means is that phi at 0 minus is equal to phi at 0 plus all right. So, as a result if you if you if you now since you have these two expressions, phi x for this is for this would be w n in this side.

So, since we are saying that potential is continuous at the interface, so which means phi 0 minus is equal to phi 0 plus. And then you apply this to previous equations you get q N a divided by 2 epsilon s into w p square, which is equal to minus of q N a divided by 2 epsilon s, and w n square plus V b i. So, essentially you substitute x is equal to 0 in both the equations, so you get this equality.

And we know that we said that charge density distribution is like this, this is a interface. And on one side at the depletion region, on one side you have acceptor ions, on the other side you have donor ions right. So, this is up to a width w n, and this is N d right. This is N a, and this is minus of w p all right ok.

Since the solid is charged neutral, which means that on both sides are depletion region w p, because of charge and mass conservation w p N a should be equal to w n N d that area of this rectangle is equal to area of this rectangle it maybe a symmetric. So, one may be high, one may be low. But, so you can see that if N a goes up, w p will go down. If N d goes up, w n will go down. However, the charge density the positive charge density on one side of depletion region is same as is matched by the negative charge density on the

other side of the region. So, otherwise you will you will not have electronic equality or junction will not be at the equilibrium.

So, you can say that from this I can write down w_p is equal to $w_n N_d$ divided by N_a . So, if you now make the substitutions of this in the previous equation, you can calculate what is w_n . So, if you do that, you will find that w_n is equal to 1 over N_d into square root of $2 \epsilon_s V_{bi}$ divided by q into 1 over N_a plus 1 over N_d .

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$$w_p = \frac{1}{N_A} \sqrt{\frac{2 \epsilon_s V_{bi}}{q \left(\frac{1}{N_A} + \frac{1}{N_D} \right)}}$$

Depletion region width

$$W = w_n - (-w_p)$$

$$= w_n + w_p$$

$$= \sqrt{\frac{2 \epsilon_s V_{bi} (N_A + N_D)}{q N_A N_D}}$$

$$W = \sqrt{\frac{2 \epsilon_s V_{bi}}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right)}$$

And if you do the similar thing for w_p , you will obtain w_p has p as 1 over N_a into square root of $2 \epsilon_s V_{bi}$ into q into 1 over N_A plus 1 over N_D . So, this is sorry this is w_n , and this is so basically you got this equation ok, you got this equation, you got this equation, you use these two equations to find out what is w_n , and what is w_p ok. In terms of donor concentration, acceptor concentration V_{bi} and electric constant ok.

So, we have got two expressions for w_n and w_p . So, we know that capital W which is the depletion region width. So, depletion region width W is equal to what is it w_n minus of minus w_p all right. So, basically it is w_n plus so if we sum them together, what you will obtain is square root of $2 \epsilon_s V_{bi}$ into N_A plus N_D divided by $q N_A N_D$ or alternatively you can write this as $2 \epsilon_s V_{bi}$ divided by q into 1 over N_A plus 1 over N_D , this is the relation of charge density or you can say acceptor and donor density, and the built in electric field with respect to the depletion region width.

So, what it tells you is that when the donor and acceptor density increase, w goes down. So, this is what we have done in this lecture. So, basically we have looked at some electrical parameters, the continuity of potential electric field, the basic equations for electric field and potential.

Remember that they are continuous in nature at the interfaces, and from this we can determine what is the what is the so you can plot using those equations what is the electric field, what is the potential, and what is the charge density, and we can estimate what is the built in what is the depletion region width ok. So, we will stop here in this lecture, and we will continue this in the next lecture to work out the electrical characteristics of P-N junction ok.

Thank you.