**Defects in Crystalline Solids (Part-I)**

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**Lecture –29**

**Critical Resolved Shear Stress + Examples Contd…**

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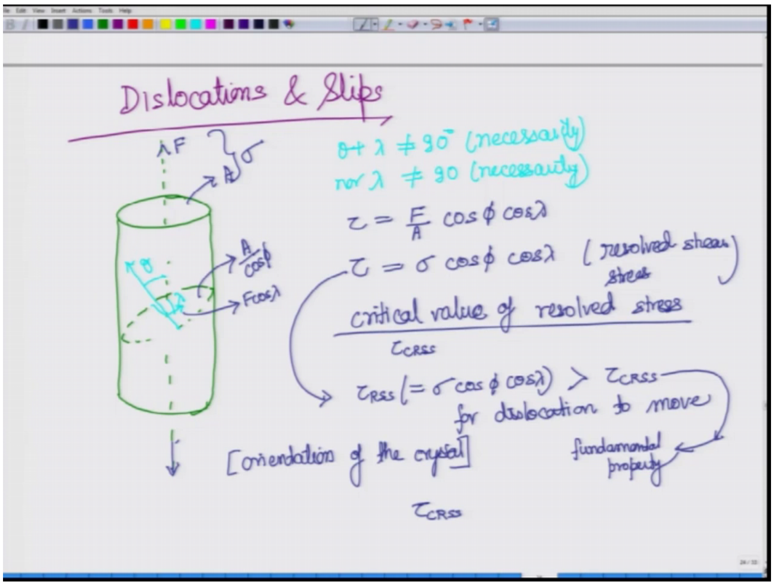


Ok so, coming back to our dislocation and slips, we are talking about resolved shear stress. Over here as we indicated in the previous class this denotes the normal to this plane, whose area and the area here is A therefore, this area will become, this is sorry A by cos phi and if the force in this direction is F and this is the slips direction, slip direction then the resolved force in this direction would become F cos lambda and therefore, you can see that tau equal to which is equal to F by A, which is shear stress will become F by A cos phi cos lambda and therefore, in this F by A, which we have already used the term sigma for it. So, as if you are applying a stress sigma, remember this is a single crystal. So you are applying a stress sigma on to the single crystal. So, tau is equal to sigma cos phi cos lambda. Now this is the tau or the resolved shear stress, which is acting on this dislocation along the slip direction. So, this much we have established yet so far.

Now let us say that, there is some critical value of this shear stress resolved shear stress ok, what do we mean by critical value of resolved stress? Meaning even though, you are applying the stress the force here and the normal stress sigma, which comes down to as tau over here, but there is certain value beyond which dislocation will start to move. So, let us term it as tau CRSS critically resolved shear stress.

So your tau, this tau, which we have obtained here, which can be termed as tau resolved shear stress equal to sigma cos phi cos lambda, this must be greater than tau CRSS for dislocation to move. Ok, so this we must know about this value tau CRSS and this tau CRSS value, it is something like a fundamental property for a given slip system. So, you would know and from there you should be able to find out. So, there will be several place for each of them you will find the resolved shear stress and whichever comes out greater than tau CRSS, they will be over there, you will be able to see or you should expect the dislocation would start to move.

Or in other words we can say that, if you keep increasing stress. So, this direction you are applying force or on a particular area so which is equivalent to applying a stress. So, if you keep increasing the stress, then the one plane on which the resolved shear stress exceeds, the critical resolved shear stress value first will start to slip and which particular one will start to slip or which will get activated first will depend on the orientation of the crystal. So, you will also need to know the orientation of the crystal. So, orientation of the crystal will help you understand given the sigma value, what will be the tau CRSS? And from that you would compare for all the different sets of possible slip planes slip systems and whichever exceeds tau CRSS first, will start to slip or will get activated.

So, this is a important quantity tau CRSS and what is it? It is the critical value of resolved stress, but when you look at it and you look at the term critical value of stress you would be it would sound similar to something, we have already introduced and what is that? It is the tau PN the Peierl Nabarro stress.

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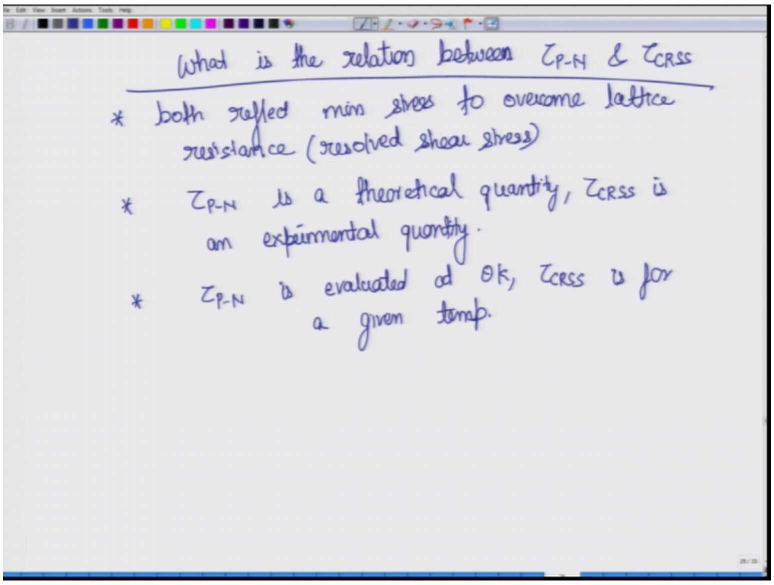


So, what is the relation between tau PN, which is the Peierl Nabarro stress and tau CRSS?

Remember what were these quantities? Both of them, both reflect minimum stress to overcome lattice resistance. So in this sense, it looks like both of them are same and we are talking about resolved shear stress that that we must keep in mind. So, I will put this in braces here, but there is one major difference here tau PN is a theoretical quantity, you saw that we had a relation that was obtained by Peierl and Nabarro. So, tau PN is a theoretical quantity and what about tau CRSS? Tau CRSS as you would imagine is a experimental quantity. So, from experiment you get to know, what is critical resolved shear stress for a particular slip system?

There is one more big difference between tau PN and tau CRSS, tau PN that the one that has been evaluated is usually evaluated at 0 Kelvin. On the other hand, tau CRSS since it is experimental quantity, it is temperature dependent where, whatever temperature you are doing the experiment, it gives you the resolved shear, minimum resolved shear stress for that particular temperature. So, tau CRSS is for a given temperature. So you can see that, we started with tau PN, we found a different slip system and then from there we set that in a single system. If you are taking talking about a single crystal, you apply stress and then there we inherently invoke the term critical resolved shear stress, which will need to be overcome by the resolved shear stress for dislocation to slip and so this was a experimental quantity well this one tau PN was a theoretical quantity, but both of them represent the same concept, which is the lattice resistance. So, in order to overcome lattice resistance there has to be a stress applied, which is represented by tau PN and tau CRSS.

Now, the I showed you that, there will be particular orientation and for that particular orientation, you will get to have several slip systems and of these several systems, one of them, which will first achieve the critical resolved shear stress then, it will get activated.

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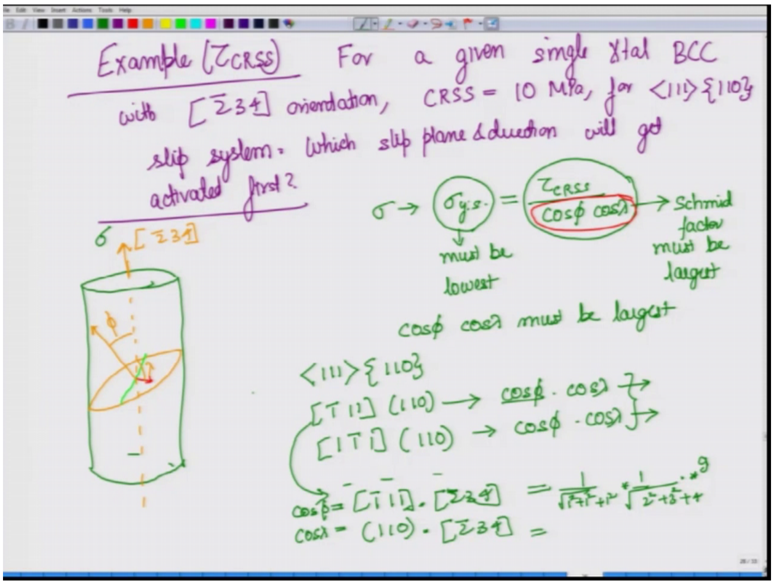


So, let us solve the example based on that. So, let us move on to an example for this tau CRSS. So, what is the question, it is given that for a given single crystal remember, we are talking about when we are talking about critical resolved stress, the derivation we had they we are using a single crystal. For a given single crystal BCC and it is given what type of system it is? Why do you need that?

Because, we need to know what's what different slip systems can get activated, with orientation bar 2 3 4? So, the orientation of the single crystal is given and what is given? Is the tau CRSS, which is equal to 10 MPa and since BCC, if you remember we had more than one type of slip system. So, we are defining which particular slip system this CRSS value is it is given, that it is for when you have 1 1 1 direction and 1 1 0 plane for 1 1 0 1 1 1 1 1 1 1 0 slip system.

Now, you have to find which slip plane and direction will get activated first? Ok. So, let us understand the geometry first and then we will explain what this problem is? What this is asking? So, as in the derivation we have a single crystal will assume, it is a cylindrical bar something like this and over here, there is let us there will be several planes will talk about it more when we start solving it.

Let us, let us draw one particular plane. So, this is one particular plane, which has the normal like this and this is the central axis. So, with this central axis this is making angle phi as we said earlier and there is dislocation line somewhere over here and for this dislocation line, let us say this is the slip direction. So, for this slip direction, this is there is angle lambda. So phi and lambda we are denoting it by phi and lambda, but this is the bar 2 3 4 orientation, which means that this particular orientation is bar 2 3 4.

So, this is the orientation bar 2 3 4 and let us assume that, there is a some stress being applied, let us denote it by sigma. Let us say, this is the slips plane on which gets activated first then, it would mean at certain value of sigma. So, it would mean that this is the sigma required for yielding of the material. So, we can also called, it sigma yield. So, it has being increased and at a particular value this is starts to get slipping or it starts dislocation start to move.

So, at that particular stress that be sigma becomes sigma yield stress and the value at which it starts to move, it has to be related like we showed earlier, it is related to the critical resolved shear stress, cos phi by cos lambda. So, this is the one plane that gets activated first, it means that for this the stress requirement was lowest, it is you keep increasing and at this particular value. This achieved the critical resolved shear stress on the plane first meaning this had this requirement for this was the lowest.

And therefore, this must be a very, this must be smallest quantity and for that this is a smallest quantity which means this which is also called a schmid factor, must be largest but compared to what will look at in a moment. So, for this to be lowest, for this to be lowest, this also is lowest, this particular quantity, which I will now circle in red, this must be largest. So, what we are saying is that cos phi cos lambda must be largest, but what are we comparing? Will come that come to that in a moment. So, ours slip system is 1 1 1 1 1 0. Now this is a general class of slip system given to you, it is not given which particular plane it can be 1 1 1 1 0, it can be 1 bar 1 0, it can be bar 1 1 0. So, that is what you need to find out? To get the different values of cos phi and lambda.

For example, it could be as well bar 1 1 1 1 0, this could be one particular slip system and remember for it to be a slip system; this particular direction must lie on this plane. So, the dot product between these two also has to be 0 and at the same time. So, it cannot be all the combination of 1 1 1 into all the combination of 1 1 0 only those combination of 1 1 1 and 1 1 0, where the dot product will be 0, why is that so?

Because, this is a slip system and the slip direction must lie on the slip plane. So, this particular direction must lie on this plane. So, that dot product must be 0 ok. So, this is one example of this there can be several more that, we can find 1 bar 1 again 1 1 0 and so on. So, for each of these you will have to find cos phi cos lambda cos phi cos lambda and the one which gives you the largest will be the one that will get activated first.

So, how do we find cos phi? So, this phi is you already know the 1 bar 1 1 1, which is the orientation or sorry the slip direction. So, this is the bar 1 1, you have to find the angle between this and bar 2 3 4. So, you just take a dot product. So, you what you will do is? Let us say, we are talking about this particular.

So, cos phi will be equal to dot product with bar 2 3 4 cos lambda will be equal to 1 1 0 dot product with bar 2 3 4. So, this will be some you will have to take the square. So, this is the denominator term and in the numerator, what you will have is you will multiply this with this which is minus 1 into minus 2, which is 2 then add it with the second term to multiplication of second 2 terms, which is 1 into 3 plus 3 plus 1 into 4. So, it will become 4 plus 3 plus 2.

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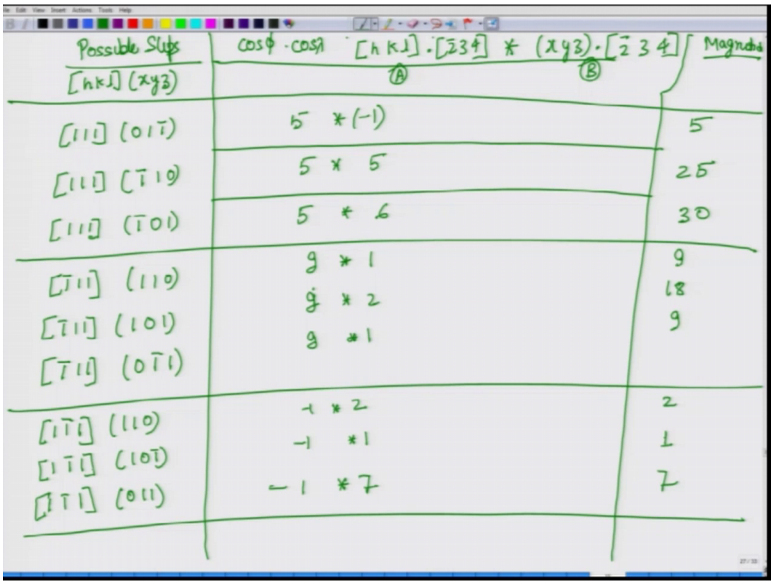


So in this particular case, it will become 9 by this quantity. So, this is the cos phi term similarly, you will find the cost lambda term and you have to do it for all the possible planes. So, what are the possible planes?

Now, since we are doing it as an example. So, let us do it do all the, calculate it for all the possible planes. So, was first will look at what are the possible systems, possible slips? Then we have to find cos phi into cos lambda? Which is nothing, but so if we say that possible slip is h k l direction x y z plane, then cos phi is h k l dot bar 2 3 4 as we said earlier and we have to multiply it with cos lambda, which is nothing, but you have to take again x y z whatever is the x y z dot product with again bar 2 bar 2 3 4 and then we have to just find the magnitude, we are not interested in the sin because, sin can be changed by taking the opposite direction. So, we are only interested in the magnitude of this product cos phi and cos lambda.

And remember to make it easy, what will do here? Is that there is there was a denominator term here. So, we can call it A, there was a denominator term here B, since both of them will remain constant, we need not look at the A and B, we we will only compare these 2 products h k l dot bar 2 3 4 and x x y z into bar 2 3 4, will not normalize it which will that is will not divided by the term, which will which was come in the denominator. So now, let us look at, what are the possible system? First let us look at all the possible 1 1 1 direction, for all the 1 1 1 direction, what are the planes on to which it can be contained? 1 is 0 bar 1 then we have bar 1 1 0 then, we have bar 1 0 1.

So, these are all the possible planes on which this 1 1 1 direction can be 1 1 1 slip direction can be contained. So, this takes care of 3 systems this is not, all will go to other systems, but first let us find out the cos phi and the cos lambda term. So, you can see here, you have to multiply it 1 with minus 2 1 with 3 1 with 4 and add this. So, 4 plus 3 minus 2, this becomes 5 and you do the same thing over here 0 bar 1. So, you replace x y z by 0 1 bar 1 and do the dot product. So, you have to just this one gets nullified 0, this is 1 3 into 1 3 4 into minus 1 minus 4. So, the net is minus 1 and therefore, this is equal to 5, but we are interested only in magnitude. So, it comes out 5.

So, you have to do the same thing, now I will just show you the different planes and give you the final values, you can cross check it on your own. So, this for the second one, it will come out to 5 into 5 equal to 25 for this third one, it will come out to 5 into 6 equal to 30. So, this is all the 1 1 1. Now let us look at, another possible one, which is bar 1 1 1. Now what are the planes that can contain it? 1 1 0 1 0 1 and you can check for yourself, these are the only possibilities we do not have any other possibilities here and how do I find whether this is contained this the particular direction is contain on a plane? I again do a dot product here it is dot product should be 0.

So, that is how I find out once, I take bar 1 1 1, I take all the possible combinations of 1 1 0, which will give a dot product of 0. So, that is the way to find out this part and once you have this also remember that the negatives are possible so, but we are only interested in magnitude because, we want the largest magnitude.

So, we need not worry about the negatives and the positives. So, these are the 3 possibilities over here and what you will see? It will come out to 9 into 1 9 into 2 and again 9 into 1. So, this comes out to 9, 18 and 9. So, this takes care of these 2 different. So this another, set of bar 1 1 direction. Now will go to third type of 1 1 1 direction, which is 1 bar 1 1 and the possible planes are listed here. So again, I will just note down the.

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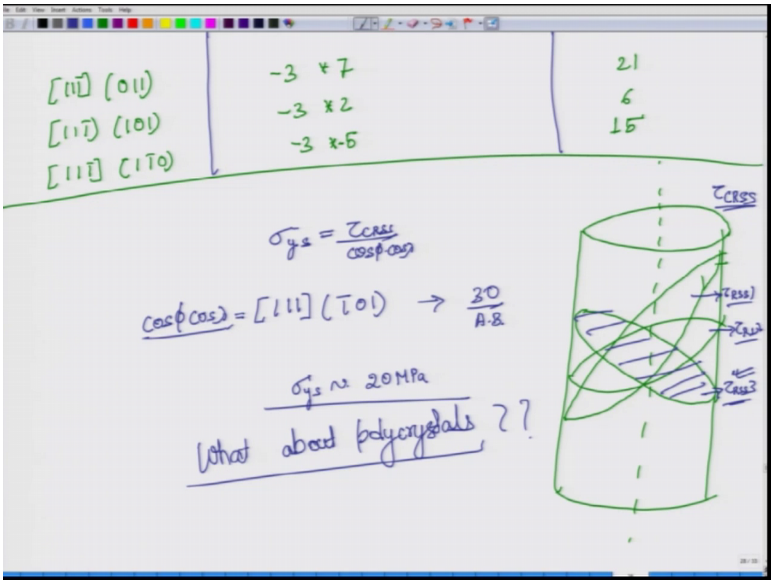


So now, at this point you have to ask yourself, how many possible 1 1 1 directions are? And the answer would be 4 different types of 1 1 1 are possible and we have listed 3 here. So, there is one more possible. 1 1 1 system and that is 1 1 bar 1 and the planes which can contain it are 0 1 1 1 1 bar 1 1 0 1 1 1 bar 1 and 1 bar 1 0.

So, now that takes care of the last set of planes possible and the product for this would be minus 3 into 7, I am not showing you I am not going through the same steps. I have explained the steps for the first 3 and you can keep applying it over here. So, this is the all the possible planes that, you have looked at. Now look at the value, now we will start to compare which one has the highest value.

So here, the highest one is 21 and if you go over here, what you see is this particular system gives the maximum value 30, which means that when if you were to calculate, sigma yield strength by tau CRSS by cos phi cos lambda, this particular plane which is if you go back the plane, which gives you the highest one is 1 1 1 bar 1 0 1. So, for this it came out to the 30 and there was in denominator here A and B what are those denominator A and B, which we had showed earlier? This is nothing, but these 2 terms, this will become A, this will become B.

But we are not interested in finding out the sigma yield strength value, but you can see that once you know, what is the tau CRSS and we have already calculated which one which particular plane to select and it can A and B. So, you will be able to find cos phi cos lambda for this. So, it will be 30 by A B and you can substitute with over here to find, what will be the yield strength? In fact, for this particular system, you would be able to show that the yield strength would come out to approximately 20 mega pascal, but our problem was only to find which particular slip system will get activated?

Which means our answer is just we have to just calculate or we have to just show this one this part to this part. So, we have as long as we have showed that, this is the largest value we are done we do not need anything more. I am just showing you that there are can be additional steps that you can do and also calculate the yield strength value or the 6 stress value at which, this slipping will occur on this particular plane.

So, there will be several number of planes. So, let if I could, so let me try to now visualize, what I have just explain. So, there were several of these 1 1 1 direction 1 0 1 planes. So, there are several of these 1 1 1 planes, for each of them there will be a different the tau CRSS is constant for all of these.

Tau CRSS is same for all of these, but tau RSS 1 for this tau RSS 2 for this tau RSS 3. So, all of them will have different resolved shear stress and the one which is the smallest will reach the tau CRSS, first because you are a keep we keep increasing the stress value or even if you are not increasing, but what it is showing is that this particular one will be able to get activated on application of stress the earliest and therefore, that particular slip system, let us say it is tau CRSS, tau RSS 3 then this particular slip system gets activated.

So, that is the purpose of using or going through this exercise to be able to identify in a single crystal system, which one will start to deform? So, here the dislocations will start to move and it leads to rotation of the crystal, which we will look at more when we get to the plasticity part. So, it will lead to the rotation of the crystal and slowly, when the rotation become such that some other plane gets activated. So, there are different stages of deformation in a single crystal, first there is a one single particular one particular slip system gets activated then multiple slip systems get activated and then entanglement of these dislocations lead to further hardening of the material.

We will talk about these later on and the later stages, when we talk about the plasticity part. So for now, what we need to do or we at this stage what we want is? To understand, this is for singles crystal what about polycrystals? So, this is the question, I will leave you with. So, I have showed you here for what happens in a single crystal? Think what will happen what will be the scenario, when you have a polycrystal, which is what most of the materials or metals and alloys are. So, I will leave you with that thought and will come back to this in the next lecture.