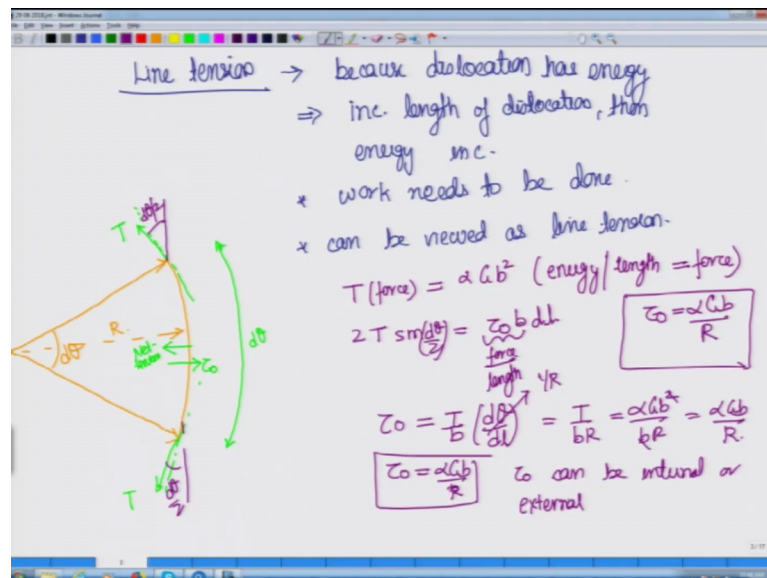


**Defects in Crystalline Solids (Part -I)**  
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**Lecture - 24**  
**Climb Forces on Dislocations + Interaction Between Dislocations**

So hopefully, you have tried to get this relation; if not let us see what do we need to do, it is a very straightforward thing from this point onwards. So, we will have, when you have this relation the only thing the additional information that you need is that when you, place it like this, which is  $T$  by  $b$   $d\theta$  by  $dL$ .

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The only additional information you need is that  $d\theta$  by  $dL$  becomes  $1$  over  $R$  and therefore, this becomes, this becomes  $T$  by  $bR$ , but  $T$  we already said is equal to  $\alpha G b^2$ . So, this becomes  $\tau_0$  becomes  $\alpha G b$  by  $R$ . So, this is  $\tau_0$  equal to  $\alpha G b$  by  $R$ .

Now, what is the meaning of this shear stress  $\tau_0$  and when we write it like this, what it is saying is that if you have a dislocation and you want to maintain a radius of curvature with radius,  $R$  a curvature with radius  $R$ , then you need to apply a minimum shear stress not a minimum, but this is shear stress  $\tau_0$ . So, these shear stress  $\tau_0$  must be applied if you want a curvature with radius  $R$ . In other words, if there is a curvature in

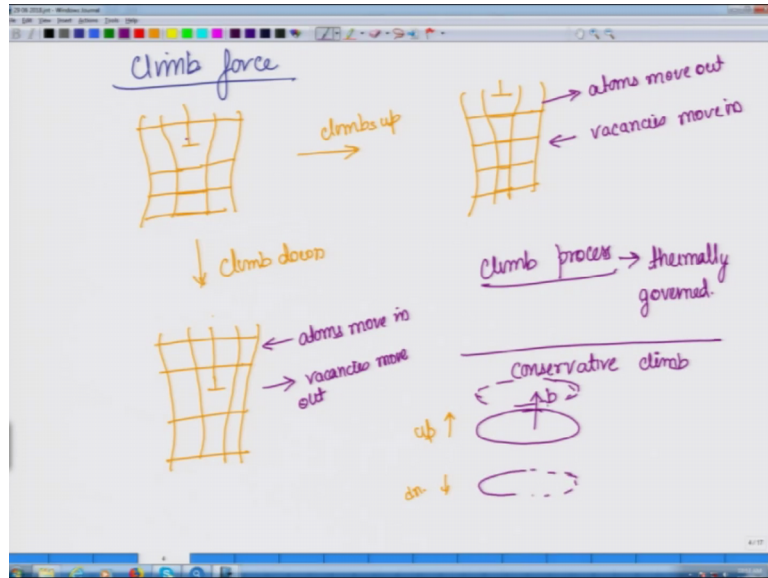
the dislocation then a resolved shear stress equivalent to  $\tau_{naught}$  is such being applied on to this dislocation.

Maybe it is being generated somewhere inside, maybe it is being applied from outside, but whatever be the source, this  $\tau_{naught}$  shear stress is being applied on to the dislocation, if it has a curvature of radius  $R$ . The only unknown thing over here is  $\alpha$  and we know that from first order of approximation, we can take  $\alpha$  as equal to 1. So, and it varies at the most between 0.5 to 1.5.

So, you will be, you will have error in that range only, but nevertheless you would have a relation which will describe how much stress has to be applied or the converse. How much stress is being experienced by that shear stresses is being experienced by the dislocation, if we already know that it has some radius of curvature  $R$ . So, the important thing here, to note is that this  $\tau_{naught}$  can be internal or external. External for example, when you are doing tensile test so, there is some load being applied or when you are doing deformation, let us say in rolling extrusion or something.

So, some stress is being applied externally, which gets translated to a result shear stress  $\tau_{not}$  or internally, because you have already done deformation and therefore, there are lots of dislocation. So, because of a bunch of dislocation, a stress field is created and, because of the stress field, some results shear stress  $\tau_{naught}$  is acting on to the given dislocation, which has radius curvature  $R$ . So, this is some information about the glide force.

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Now, let us discuss a little bit about climb force. We will not go as much detail in climb force, because it does not have a straight forward relation, the way we derived for glide force. However, there are few things that you can understand. Let us say you have dislocation somewhere like this. So, there are planes over here, there is this location line and then again there are planes like this.

Now, this dislocation line, sorry this dislocation, which is a line perpendicular to the screen is this can either move up. So, it moves up, climbs up, it will look something like this or it can climb down. So, it will become so now, you see that the when the dislocation which is climbing, it can either climb up or climb down.

Now, what happens when it is climbing up? When it is climbing up, then the one particular layer or, row of atom which was present over here, this must go out. So, in this particular case, atoms move out and it would mean that vacancies must move in, in this particular case there was the atom. Extra atom layer was only up to this, this particular row, but now it has extended to additional row. Therefore, atoms must move in. If atoms move in, it implies that vacancies move out and because of this; there is squeezing action or expanding action, which is causing the dislocation to move up or down.

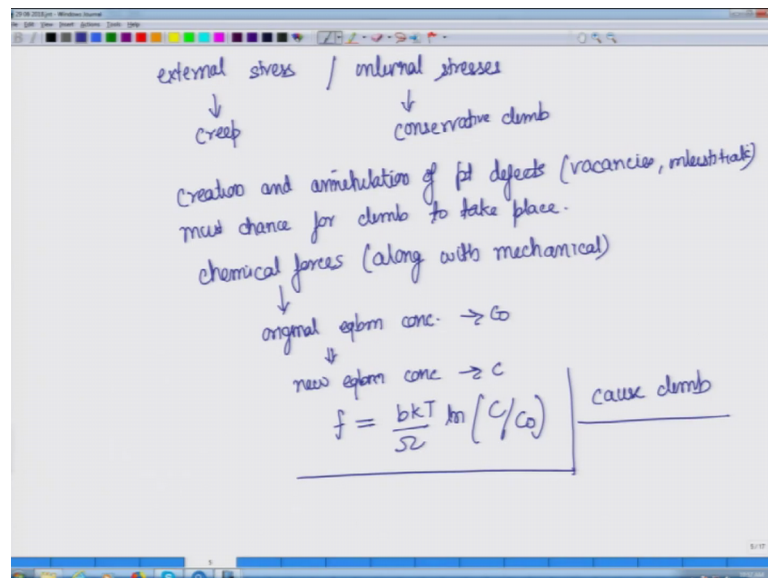
What do you, what you realize is that in this particular case a lot of movement of not only the atoms take place, but also vacancies take place. And therefore, most of the time this climb process is thermally agitated or thermally governed, because when you change the temperature vacancy, its concentration will increase. If you decrease the temperature

vacancy concentration will decrease, and therefore, it will cause dislocation movement in one way or the other.

So, most of the time this can climb force processes thermally governed, but there are also certain conservative climb motions. For example, you can have a prismatic dislocation loop like the one where we said that the Burger vector is like this. Now, this particular dislocation loop, if it moves up or down as you can see in either case, no vacancy is being involved, because let us say this is the extra half plane.

This extra half plane can be here, it can be in a row below this or it can be in a row above this. But wherever it is, it does not change the number of atoms in the extra half plane, which is  $b$  or if you assume it as a extra plane of vacancies then even in that case there is no change in the vacancy, total number of vacancies. And therefore, this can be a conservative climb; so coming to the approximate relation for, this climb.

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Now, here what you have to first realize is that again here, you can have external stresses as well as internal stresses, which can cause, which can also cause. So, the thermal agitation, you need thermal agitation, but also stresses can cause the dislocation to, or will, which will basically add additional stress for the dislocation to move and this additional stress can be external or internal. How can be it external?

For example, in creep; so, if you are doing it at very high temperature, there are stresses acting, because of the load. And, because of the temperature, there is vacancy generation can take place and in those cases you can have external stresses acting, which will cause the dislocations to climb. Internal is the one for example that we discussed in the conservative climb.

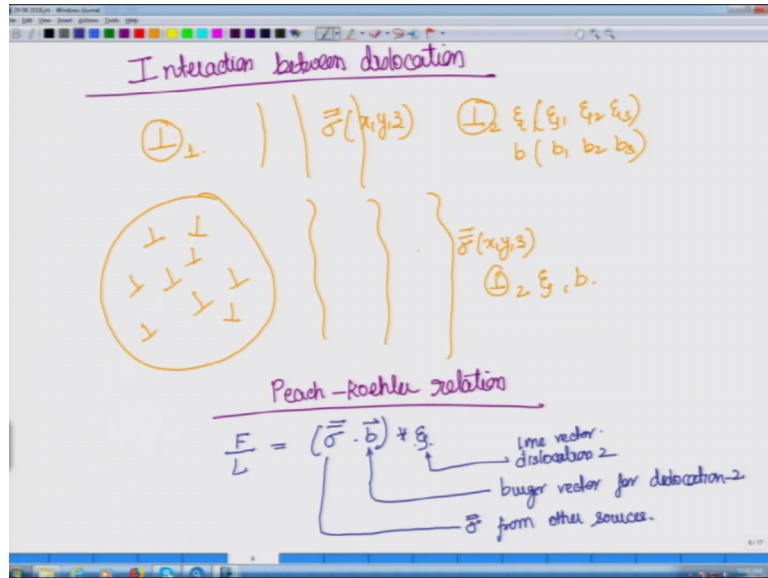
Now, except for the conservative climb, we have seen that creation and annihilation of point defects. In the simplest case, it will only be vacancies, but people who are familiar with steel would know that carbon atom also sets into the dislocation core and therefore, that will also act as that those interstitials would also act as point effect. So, those concentrations would also must also change for the dislocation to climb and therefore, creation and annihilation of point defects vacancies, interstitials must change for climb to take place.

Now, this can this particular, concentration can also change, because of chemical forces. So, you have to take into account chemical forces along with the mechanical forces and whenever there are chemical forces, then the equilibrium concentration of these point defects will change. So, let us say the original, equilibrium concentration was  $C_0$  then it may change to and let us say, it has changed to  $C$ . Then the force, if the, the it can be easily shown that the force, this chemical force can be given as  $f = b k T \ln \frac{C}{C_0}$  by  $C_0$ .

So, the chemical forces that are acting which, because of which the equilibrium concentration changes from  $C_0$  to  $C$  can be given by  $f = b k T \ln \frac{C}{C_0}$ , where  $\omega$  is the atomic volume  $k$  is the Boltzmann constant,  $b$  is the Burger vector  $T$  is the temperature in Kelvin and  $C$  and  $C_0$ , we have defined over here. So, this gives you the force, the chemical forces, which will cause climb. So, this is the fundamental relation for the chemical forces required for the climb motion.

So; now, we have looked at the fundamental derivation of forces that can cause glide, for the glide motion and the forces that can cause the climb motion. We, these are the two primary way that a dislocation makes a movement. Now, when the dislocations are moving, then you can realize that it will also interact. Next step of understanding in terms of motion is, how do they interact.

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So, there may be one dislocation, because of which the other dislocation will experience the stress and that will cause some force acting on to it or there may be a bunch of dislocation, because of which there may be stresses acting on to another dislocation. So, let us say that we have a dislocation standing over here. So, this is the one that is causing the stress field and this is another dislocation whose motion we want to study. Now, this one will create certain stress field. So, let us say these are stress contours.

So, there will be  $\sigma_{x y z}$  and this dislocation (Refer Time: 13:30), this dislocation will have some line vector  $E$ . It will also have some, which can be resolved as  $E_1 E_2 E_3$  way, where 1 2 3 can be taken as  $x y z$  direction and it will also have a Burger vector. So, Burger vector can be resolved as  $b_1 b_2$  and  $b_3$ .

Now, this so far what we have said is that this stresses are acting onto this dislocation 2, because of 1 dislocation. However, what we are taking into account is just the stress field and this stress field can be generated by not just 1 dislocation, it could be a bunch of dislocation. So, let us say there is this bunch of dislocation, because of which there is some stress field, again which will have the form  $\sigma_{x y z}$ . So, this is a matrix quantity  $\sigma_{x y z}$  and, because of that you want to calculate the stresses on to 2 for which we know  $E$  and  $b$ .

Now, this is  $b$ , this is given by what is called as Peach-Koehler relation. So, what is this Peach-Koehler relation? It describes the forces that we want to calculate on 2, because of either the case of 1 or 2, but what we have is basically stresses acting, because of or

arising from different dislocations. So, it will be given like this. So, like we said this is, this E belongs to dislocation 2, the line vector this Burger vector belongs to dislocation 2. So, dislocation 2 is the dislocation onto which we come on to find the forces acting and again you see that we are calculating as force per unit length.

Now, what is this stress? Where is the stress coming from? This stress is arising, because of dislocation 2 or bunch of dislocation or any other source. So, you can call it simply sigma from other sources. We have already seen how to calculate stresses from a single dislocation. So, if it is a single dislocation, we can directly find at a particular distance x y z, what will be the stresses and then we can put it over here, sigma dot b cross E to get the forces acting per unit length, on to the dislocation.

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For edge dislocation  $\xi (0 0 1)$   $b (b_1 b_2 b_3)$

$$E = \begin{bmatrix} \sigma_{yx} b_1 + \sigma_{yz} b_2 & \rightarrow F_x \\ -\sigma_{zx} b_1 - \sigma_{zy} b_2 & \rightarrow F_y \\ 0 & \rightarrow F_z \end{bmatrix}$$

$b_2 = 0 ; b_3 = 0$

$$F_x = \sigma_{yx} b_1 = \frac{Gb^2 x (x^2 - y^2)}{2\pi(1-\nu) (x^2 + y^2)^2} \quad (\text{slide})$$

$$\frac{F_y}{L} = -\sigma_{zx} b_1 = -\frac{Gb^2}{2\pi(1-\nu)} \frac{y (3x^2 + y^2)}{(x^2 + y^2)^2} \quad (\text{climb})$$

$\sigma$  is arising from single edge dislocation

For a simple case that we will discuss here to begin with, let us say we have a for edge dislocation, whose line vector is given as 0 0 1 meaning, it is along the z direction would you be able to show that the force acting on to this, oh sorry yeah, there is another (Refer Time: 17:19) term here.

So, force will have x y z component. So, this is F x this is F y and this is F z and, it goes without saying that when we have written it like this, it means that the Burger vector of the dislocation onto which we want to calculate as Burger vectors b 1 b 2 and b 3. However, since the dislocation is oriented along z the third Burger vector b 3 does not

appear in over here. Now, if we further make the simplification that  $b_2$  is equal to 0, what does it turn out to be  $b_2$  equal to 0.

It means that we have the Burger vector only in the, perpendicular direction to the line vector. So, it is edge dislocation. So, we can say that Burger  $b_2$  is also equal to 0  $b_3$  does not matter, but we will say that it is also equal to 0 and therefore,  $F_x$  by  $L$  would come out to be throw. I am missing something, make note of this. I had earlier, mistakenly written at  $\sigma_y y$ , it should be  $\sigma_y x$ . So, it is  $\sigma_y x b_1$  and now, we can simply put  $\sigma_y x$ , which we know what the relation for, which we know from our earlier derivation.

So, the  $\sigma_y x$  is the stress, because of a particular dislocation at our given distance  $x$  and  $y$ . Therefore, it turns out to be  $x^2$  square minus  $y^2$  square and you can also find; so, this is the  $F_x$  per unit length that is the glide force so on. Dislocation 2, this is the glide force that we have calculated and this is the climb force that we have calculated and since, we have expanded  $\sigma_y x$  like, this it means that we are assuming that this stress  $\sigma$  is arising from a single dislocation, from single edge dislocation.

Now, at this point, it would look like how did we arrive to this relation, but this,  $F_y L$  equal to this and I would very much like that you do it on your own with I will just, what I will try to do. Here, is show you the full mathematical multiplication, which gets us to the relation  $\sigma \cdot b \times E$ , not to derive the relation, but how to expand the relation  $\sigma \cdot b \times E$ . So, our next step, once we have looked at one example; where we were looking at the forces acting on edge dislocation is the next step, is to look at the full expansion of the relation, Peach-Koehler relation  $\sigma \cdot b \times e$ .

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Peach-Koehler relation

$$\frac{F}{L} = \underbrace{[\vec{\sigma} \cdot \vec{b}]}_{\text{vector}} \times \vec{e}$$

$$[\vec{\sigma} \cdot \vec{b}] = \left\{ \sum_i \sum_j \sigma_{ij} \right\} \left\{ \sum_k b_k \right\}$$

$$= \begin{bmatrix} \sigma_{11} b_1 + \sigma_{12} b_2 + \sigma_{13} b_3 \\ \sigma_{21} b_1 + \sigma_{22} b_2 + \sigma_{23} b_3 \\ \sigma_{31} b_1 + \sigma_{32} b_2 + \sigma_{33} b_3 \end{bmatrix}$$

$$\underbrace{[\vec{\sigma} \cdot \vec{b}]}_{\text{vector}} \times \vec{e} = \begin{bmatrix} (\sigma_{21} b_1 + \sigma_{22} b_2 + \sigma_{23} b_3) \hat{e}_3 - (\sigma_{31} b_1 + \sigma_{32} b_2 + \sigma_{33} b_3) \hat{e}_2 \\ (\sigma_{31} b_1 + \sigma_{32} b_2 + \sigma_{33} b_3) \hat{e}_1 - (\sigma_{11} b_1 + \sigma_{12} b_2 + \sigma_{13} b_3) \hat{e}_3 \\ (\sigma_{11} b_1 + \sigma_{12} b_2 + \sigma_{13} b_3) \hat{e}_2 - (\sigma_{21} b_1 + \sigma_{22} b_2 + \sigma_{23} b_3) \hat{e}_1 \end{bmatrix}$$

So, coming back to our Peach-Koehler relation, which was ok. So, now, if you look at it there are two components to it; first what we need to do is do this dot product where sigma is a tensor and b is a Burger vector. So, you can realize that when you do this, do this multiplication what you will get is a vector. So, this vector will be cross multiplied with this, vector and there again, we will get a vector of force, where we will have F x F y and F z. So, first let us look at the expansion for sigma dot b.

This can be in a summarized way, it can be written like this, which will expand to like this. So, you can see that this is sigma i j into sigma b k. So, we are looking at in those forms. So, what you would see is that what we have now obtained is a vector, which gives us sigma dot b. So, this made, this matrix sigma has now, been, taken a dot product with b, which is given by this condensed equation, when we expand it, this looks like this.

Now, what we need to do is we want the next step; we wanted is now, that we already have this, which is a vector. So, we multiply this vector cross, take a cross product of this vector with E and this is again in, overall the column vector would look complicated, but the multiplication is straightforward, if you know the matrix multiplication and it will come out to be, ok.

So, this is the full expanded form of sigma dot b cross e once, if you are in a position to write your sigma matrix in terms of sigma 1 1 1 1 2 1 3 2 1 2 2 2 3 3 1 3 2 3 3. And similarly, your Burger vector b 1 b 2 b 3 and line vector E 1 E 2 E 3, then this thing just

boils down to this particular line, vector. And, again it must seem like again very-very imposing and intimidating, which has 6 terms in each of the rows, but again if when we make certain assumptions about our dislocation, it quickly balls down to a very simple number.

For example, we had shown earlier that our line vector was  $0\ 0\ 1$ , which means  $\epsilon_1$  and  $\epsilon_2$  would become 0 and when you do that  $\epsilon_1\ 0\ \epsilon_2\ 0\ \epsilon_2\ 0$  here and therefore, you would, what you would get? So,  $\gamma$  again 2 into this will also turn out to be 0, this 0, this 0 what we will, what you will be left with is just this quantity and this quantity and this will also boil down to 0. So, as you can see just by making one simple assumption that your dislocation line is along  $0\ 0\ 1$ , which does not take away any information from your system.

You are able to simplify this to just two terms right,  $Z$  it has, if you look at it, 2 if you take this as one big term then it has 6 terms. So, from 6 terms, it boils down to 2 terms and even in those 2 terms, if you look at  $\sigma_{21}$  to  $2\ b\ 2$  and all these expansions. We, what we have is dislocations, if you assume that what is the, what we have is an edge or a screw dislocation then again you can say ok,  $\sigma_{21}$  or  $2\ 2$  would for example, in the case of our screw dislocation all your  $2\ 1\ 1\ 2\ 2$  and  $3\ 3$  will go away and therefore, you will again reduce. This smaller number of parameters and you will be able to easily obtain, the final relation.

So, coming back to our first problem, not the problem, but the first example that we looked at where we said that we have  $a$ , we have to find the force on a dislocation and we assume that the starting force, the stress is being generated, because of edge dislocation. We showed a relation now, you can obtain that directly from here, like I said you take  $\epsilon$  along  $0\ 0\ 1$  and therefore, your the, what you are left with only these to and over here too what we will do is.

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$\xi = (001) \quad \& \quad \vec{b} = (100) \quad \text{edge dislocation}$   
 $\frac{F}{L} = \begin{bmatrix} \sigma_{21} b_1 \\ -\sigma_{11} b_1 \\ 0 \end{bmatrix} \begin{matrix} \rightarrow F_x \\ \rightarrow F_y \\ \rightarrow F_z \end{matrix}$   
 $\frac{F_x}{L} = \frac{Gb^2x(x^2+y^2)}{2\pi(1-\nu)(x^2+y^2)^2}$   
 $\frac{F_y}{L} = -\frac{Gb^2y}{2\pi(1-\nu)} \frac{1}{(x^2+y^2)^2}$

So, what we have is and we will say that Burger vector is equal to therefore, you would see that  $F$  by  $L$  is equal to  $\sigma_{21} b_1$   $\sigma_{11} b_1$  and the  $z$  quantity is 0. So, this is  $F_x$   $F_y$   $F_z$ . So, what we are talking about here is a edge dislocation. So, we are still very general, we have just said that the line dislocation is along  $001$ . And, since it is a edge dislocation, we can talk about  $100$  as the Burger vector and when we have this Burger vector our force per unit length boils down to this, just from that intimidating matrix.

We have come down to just small, two term matrix, not even a matrix, but a column vector. And therefore, it is much more easier to calculate over here and now, we can like I said  $21$  you can take as  $yx$  for a starting dislocation. And therefore,  $F_x$  by  $L$  can be written as  $G b^2 x$  and  $F_y$  by  $L$  is equal to minus. So, we will leave it at this point.

We will come back to the discussion on forces in the next lecture.